A Multi-Jammer Power Control Game

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***Abstract*— We consider a single carrier communication system subjected to multi-jammer interference, and study optimal power control in the framework of game theory. The Nash and Stackelberg equilibria are derived in closed form. Conditions involving the background noise variance and the transmission cost are established for determining whether the equilibria are unique. It is proven that in the case of multiple equilibria, the user has the same payoff at each equilibrium, which reflects the stability of the communication. Also, consistent with the literature on single-jammer scenarios, it is shown that when the jammers are in a Stackelberg game, although they act as followers, they gain in payoffs as compared to the Nash game.**

***Index Terms*— Multi-jammer, equilibrium strategies.**

I. INTRODUCTION

AS WIRELESS networks are vulnerable to jamming attacks, anti-jamming strategies are of considerable interest. Jamming problems are multi-objective problems, dealing with different agents (users and jammers), with each agent having its own objective [1]. Non-cooperative game theory (GT) is a natural framework to study such problems and capture different agents' behaviors [1]. According to the Nash game (NG) framework, the agents choose their strategies simultaneously to maximize their payoffs, while according to the Stackelberg game (SG), the agents choose their strategies sequentially.

A classical power control (PC) anti-jamming game with the signal to interference plus noise ratio (SINR) as user communication utility (UCU) - referred to here as a PC-SINR game - was studied in [2], [3], where a jammer acting as an agent in an NG was referred to as a *regular-type jammer*, while a jammer acting as a follower in an SG was referred to as *smart-type*. The latter term reflects the jammer's ability to quickly learn the user's transmission strategy and accordingly adjust its own. Several works have addressed the PC-SINR game with different types of uncertainty, such as uncertainty on the type of jammer strategy (NG or SG) [4], the channel fading gain and agent transmission cost [5], or the presence of observation noise [6]. The aforementioned works [3]–[6] involve only two agents, i.e., one user and one jammer. The multi-agent scenario has also been considered. A multi-user relay-assisted PC problem with one jammer was studied in [7]. A multi-user PC problem with one jammer was investigated in [8]. In [9], single user communication supported by relay nodes facing a single jammer was studied. A PC game with one jammer acting as the leader and several users acting as followers was studied in [10]. The special cases of a single user and two jammers NG and a multi-jammer NG with latency as UCU were studied in [11] and [12], respectively.

In this letter we generalize the single jammer PC-SINR problem [2], [3] to the multi-jammer case under both the NG and SG frameworks. In addition to the novelty of our analysis of that scenario, we prove that under SG, the jammers gain in payoffs as compared to NG. This warrants the characterization of SG jammers as *smart-type*, a term that was initially coined in the single-user single-jammer scenario [2], [3]. *The main contributions of this letter are as follows:* (1) We derive the Nash equilibrium (NE) and Stackelberg equilibrium (SE) in closed form in a multi-jammer PC-SINR game; (2) We establish the condition on the network parameters for the equilibrium to be unique; (3) We prove that when multiple equilibria arise, the user has the same payoff for each of them which reflects the stability of the communication; (4) We prove that, although in the SG the jammers act as followers they gain in payoffs as compared to the NG. This supports the use of the term "smart-type jammer" in the multi-jammer scenario.

II. COMMUNICATION MODEL

We consider a network with a user that needs to communicate with a receiver in the presence of K jammers. Each jammer intends to degrade the user's communication by generating interference. The strategy of the user is to adjust its transmission power P, with $P \in [0, \overline{P}]$, and for the k-th jammer, to adjust its jamming power J_k, with $J_k \in [0, \overline{J}_k]$, for $k \in D_K \triangleq \{1, \ldots, K\}$. Here, $\overline{P}$ and $\overline{J}_k$ are the total power budgets of the user and the k-th jammer, respectively. Also, let $\boldsymbol{J} = (J_1, \ldots, J_K)$ [1] be the vector of jammers' strategies, arranged by the jammer's index. The SINR at the receiver equals

$$\mathrm{SINR}(P, \boldsymbol{J}) = hP/(N + \sum_{k \in D_K} g_k J_k) = hP/\mathcal{N}(\boldsymbol{J}), \quad (1)$$

where h is the source-destination fading channel gain, g_k is the jammer-destination channel gain (or interference channel gain)

[1] We use bold fonts to denote vectors.

Manuscript received April 15, 2021; revised May 29, 2021; accepted June 14, 2021. Date of publication June 28, 2021; date of current version September 10, 2021. This work was supported in part by the U.S. National Science Foundation under Grant CCF-1908308 and Grant CNS-1909186. The associate editor coordinating the review of this letter and approving it for publication was J.-B. Seo. *(Corresponding author: Andrey Garnaev.)*

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Digital Object Identifier 10.1109/LCOMM.2021.3093235

associated with the k-th jammer, N is the background noise variance, and $\mathcal{N}(\boldsymbol{J})$ is the total noise power, which equals

$$\mathcal{N}(\boldsymbol{J}) \triangleq N + \sum_{k \in D_K} g_k J_k. \tag{2}$$

As commonly assumed in jamming games literature [2]–[4], [8], [9], [11], [12], the network parameters (channel gains, power costs, noise variance) are assumed to be known. Such knowledge can be available in a stationary network environment and an agents' allocation, which allows the estimation of such parameters based on past observations.

We should note that even with that knowledge, each agent still has uncertainty about the SINR, since it depends on the choice of power used by the other agents.

The user payoff is taken to be the difference between the user's SINR and cost of transmission power, i.e.,

$$v_U(P, \boldsymbol{J}) = \text{SINR}(P, \boldsymbol{J}) - C_U P, \tag{3}$$

where C_U is the transmission cost per unit of power applied by the user. The user and k-th jammer ($k \in D_K$) as its adversary are antagonists with respect to SINR which is reflected via opposite signs of SINR into the payoff to the k-th jammer as follows:

$$v_{J,k}(P, (J_k, \boldsymbol{J}_{-k})) = -\text{SINR}(P, \boldsymbol{J}) - C_{J,k} J_k, \tag{4}$$

where $C_{J,k}$ is the jamming cost per unit of jamming power, and $\boldsymbol{J}_{-k} = (J_1, \dots, J_{k-1}, J_{k+1}, \dots, J_K)$.

A. Auxiliary Assumptions and Notations

Let us denote by $\boldsymbol{J}^k = (J_1^k, \dots, J_K^k)$ with $k \in D_{K+1}$ the vector of the jammers' strategies corresponding to boundary feasible values, i.e.,

$$J_j^k \triangleq \begin{cases} \overline{J}_j, & j \le k-1, \\ 0, & k \le j \end{cases} \quad \text{for } j \in D_K. \tag{5}$$

Thus, $\boldsymbol{J}^1 = \boldsymbol{0} \triangleq (0, \dots, 0)$ and $\boldsymbol{J}^{K+1} = \overline{\boldsymbol{J}} \triangleq (\overline{J}_1, \dots, \overline{J}_K)$.

Let N_k be the sum of background noise and the interference generated by the jammers' strategies $\boldsymbol{J}^{k+1}$, i.e.,

$$N_k \triangleq \mathcal{N}(\boldsymbol{J}^{k+1}) = N + \sum_{i=1}^{k} \overline{J}_i g_i. \tag{6}$$

It can be seen that $N = N_0 < \ldots < N_K < N_{K+1} \triangleq \infty$. Finally, let us, without loss of generality, index the jammers in increasing order based on the ratio

$$\overline{C}_k \triangleq C_{J,k}/(h g_k), \tag{7}$$

i.e.,

$$\overline{C}_0 \triangleq 0 < \overline{C}_1 < \ldots < \overline{C}_K < \overline{C}_{K+1} \triangleq \infty. \tag{8}$$

III. Nash Equilibrium

Each of the agents wants to maximize its payoff. For a non-zero sum game, $(P, \boldsymbol{J})$ is an NE [1] if and only if for all $(\tilde{P}, \tilde{\boldsymbol{J}})$ it holds that

$$v_U(\tilde{P}, \boldsymbol{J}) \le v_U(P, \boldsymbol{J}), \tag{9}$$

$$v_{J,k}(P, (\tilde{J}_k, \boldsymbol{J}_{-k})) \le v_{J,k}(P, (J_k, \boldsymbol{J}_{-k})), \; k \in D_K. \tag{10}$$

Let us denote this non-zero sum game with known network parameters by Γ_N.

Proposition 1: In game Γ_N there is at least one NE.

Proof: By (3), $v_U(P, \boldsymbol{J})$ is linear in P, while, by (4), $v_{J,k}(P, \boldsymbol{J})$ is concave in J_k, and the result follows [1], since each set of feasibly strategies is compact. ■

By (9) and (10), $(P, \boldsymbol{J})$ is NE if and only if each strategy is the best response to the others, i.e., they are solution of the *best response equations*, i.e.,

$$P = \text{argmax}\{v_U(P, \boldsymbol{J}) : P \in [0, \overline{P}]\}, \tag{11}$$

$$J_k = \text{argmax}\{v_{J,k}(P, (J_k, \boldsymbol{J}_{-k})) : J_k \in [0, \overline{J}_k]\}, k \in D_K. \tag{12}$$

Note that for a fixed $P \ge 0$, (10) can be viewed as defining an NE in a sub-game involving only jammers as agents, where each jammer wants to maximize its payoff. Let us denote this sub-game by $\Gamma_J(P)$.

A. Solution of the Sub-Game $\Gamma_J(P)$

In this section we find an NE of sub-game $\Gamma_J(P)$ and the total noise power associated with the NE.

Proposition 2: For a fixed $P \ge 0$, sub-game $\Gamma_J(P)$ has an unique NE, and it is equal to $\boldsymbol{J}(P) = (J_1(P), \dots, J_K(P))$, where

(a) if

$$N_{k-1}^2 \, \overline{C}_{k-1} \le P < N_{k-1}^2 \, \overline{C}_k, \; k \in D_{K+1}, \tag{13}$$

then

$$\boldsymbol{J}(P) \triangleq \boldsymbol{J}^k, \tag{14}$$

(b) if

$$N_{k-1}^2 \overline{C}_k \le P < N_k^2 \overline{C}_k, \; k \in D_K, \tag{15}$$

then

$$J_j(P) \triangleq \begin{cases} \overline{J}_j, & j \le k-1, \\ \dfrac{1}{g_k}\left(\sqrt{\dfrac{P}{\overline{C}_k}} - N_{k-1}\right), & j = k, \\ 0 & j \ge k+1, \end{cases} \quad j \in D_K, \tag{16}$$

where in both (a) and (b), k, to be denoted by $\kappa(P)$, is uniquely defined based on (13) and (15).

Remark 1: We note that formulas (13)-(16) identify the jammers that implement boundary feasible strategies.

Proof of Proposition 2: By (4), we have that

$$\frac{\partial v_{J,k}(P, (J_k, \boldsymbol{J}_{-k}))}{\partial J_k} = \frac{h g_k P}{\mathcal{N}(\boldsymbol{J})^2} - C_{j,k}. \tag{17}$$

By (7) and (17), the best response J_k to $\boldsymbol{J}_{-k}$ is given by

$$J_k = \begin{cases} \overline{J}_k, & \mathcal{N}(\boldsymbol{J})|_{J_k=\overline{J}_k} \le \sqrt{\dfrac{P}{\overline{C}_k}}, \\ \sqrt{\dfrac{P}{\overline{C}_k}} = \mathcal{N}(\boldsymbol{J}), & \mathcal{N}(\boldsymbol{J})|_{J_k=0} < \sqrt{\dfrac{P}{\overline{C}_k}} < \mathcal{N}(\boldsymbol{J})|_{J_k=\overline{J}_k}, \\ 0, & \sqrt{\dfrac{P}{\overline{C}_k}} \le \mathcal{N}(\boldsymbol{J})|_{J_k=0}, \end{cases} \tag{18}$$

where $\mathcal{N}(\boldsymbol{J})|_{J_k=x} = N + \sum_{j \ne k} g_j J_j + g_k x$.

Let $\boldsymbol{J}$ be an NE. By (8) and (18), there is a k such that

$$\boldsymbol{J} = \boldsymbol{J}^k \text{ or } J_j = \begin{cases} \overline{J}_j, & j \le k-1, \\ \in [0, \overline{J}_k), & j = k, \\ 0 & j \ge k+1 \end{cases} \quad \text{for } j \in D_K. \tag{19}$$

Substituting the first equation of (19) into (18) implies that such $\boldsymbol{J}^k$ is an NE if and only if (13) holds, and (a) follows. Substituting the second equation of (19) into (18) implies that (19) is an NE if and only if (15) holds, and $\boldsymbol{J} = \boldsymbol{J}(P)$ with $\boldsymbol{J}(P)$ given by (16). This implies (b). ■

In the following corollary we calculate the total noise power caused by the jammers implementing the NE strategies $\boldsymbol{J}(P)$ and establish its properties.

Corollary 1: (a) The total noise power in the network generated by the jammers using the NE strategies, $\boldsymbol{J}(P)$, is

$$\mathcal{N}(\boldsymbol{J}(P)) = \begin{cases} N_{\kappa(P)-1}, & P \in I_{\kappa(P)}, \\ \sqrt{P/\overline{C}_{\kappa(P)}}, & P \in \overline{I}_{\kappa(P)}, \end{cases} \tag{20}$$

where

$$I_k \triangleq [N_{k-1}^2 \, \overline{C}_{k-1}, N_{k-1}^2 \, \overline{C}_k), \tag{21}$$

$$\overline{I}_k \triangleq [N_{k-1}^2 \overline{C}_k, N_k^2 \overline{C}_k), \; k \in D_K; \tag{22}$$

(b) The intervals I_k and $\overline{I}_k$ have the following properties:

$$I_k \cup \overline{I}_k = [N_{k-1}^2 \, \overline{C}_{k-1}, N_k^2 \, \overline{C}_k), \; k \in D_K, \tag{23}$$

$$I_{K+1} = [N_K^2 \, \overline{C}_K, \infty) \text{ and } \overline{I}_{K+1} = \emptyset, \tag{24}$$

$$\cup_{i=1}^{k} (I_i \cup \overline{I}_i) = \mathcal{I}_k \triangleq [0, N_k^2 \overline{C}_k); \tag{25}$$

(c) $\mathcal{N}(\boldsymbol{J}(P))$ is continuous piece-wise increasing function from N_0 for $P \in [0, \overline{C}_1 N_0^2)$ to N_K for $P \ge N_K^2 \, \overline{C}_K$.

Proof: Note that I_k coincides with the set of user's power levels given by (13), while $\overline{I}_k$ with the set of power levels (15). Then, Proposition 2 implies (a). By (21) and (22), (b) follows. Finally, (c) follows from (a), (b) and Proposition 2. ■

B. Equilibrium Strategies in Closed Form and Uniqueness Conditions

Theorem 1: In game Γ_N, NE is given by $(P, \boldsymbol{J}(P))$, where P is unique except in case (b), where a continuum of user's equilibrium strategies arises. Moreover, $P = \min\{\tilde{P}, \overline{P}\}$ and $\tilde{P}$ is given as follows:

(a) if $h/C_U < N_0$ then $\tilde{P} = 0$,

(b) if $h/C_U = N_{k-1}$ then $\tilde{P}$ is any power level from $[N_{k-1}^2 \overline{C}_{k-1}, N_{k-1}^2 \overline{C}_k]$,

(c) if $N_{k-1} < h/C_U < N_k$ then $\tilde{P} = h^2 \overline{C}_k / C_U^2$.

Note that (b) and (c) define k uniquely. Also, in case (b) of multiple equilibria (with $\overline{P} > N_{k-1}^2 \overline{C}_{k-1}$) the user's payoff, v_U, is equal to zero for any equilibria (see Eqn. (3)).

Proof: By (3), $v_U(P, \boldsymbol{J})$ is linear in P. Thus, $P = \mathrm{BR}_U(\boldsymbol{J})$ is the best response to $\boldsymbol{J}$ if and only if

$$P = \mathrm{BR}_U(\boldsymbol{J}) = \begin{cases} 0, & h/C_U < \mathcal{N}(\boldsymbol{J}), \\ \in [0, \overline{P}], & h/C_U = \mathcal{N}(\boldsymbol{J}), \\ \overline{P}, & h/C_U > \mathcal{N}(\boldsymbol{J}). \end{cases} \tag{26}$$

By (11) and (12), substituting $\boldsymbol{J} = \boldsymbol{J}(P)$ into (26), we obtain the fixed point equation $P = \mathrm{BR}_U(\boldsymbol{J}(P))$ to find user's equilibrium strategy. By Proposition 2, Corollary 1 and (26), all solutions of this fixed point equation are given by (a)-(c), and the result follows. ■

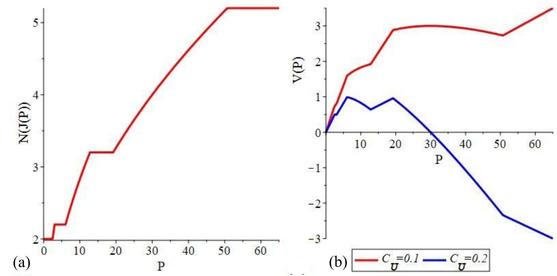

Fig. 1. (a) $\mathcal{N}(\boldsymbol{J}(P))$ and (b) $V(P)$ as functions of P.

To illustrate the continuous, piece-wise increasing structure of function $\mathcal{N}(\boldsymbol{J}(P))$ let us consider a three-jammer network with $g = (0.2, 1, 2)$, $h = 0.8$, $N = 2$, $C_J = (0.1, 1, 3)$ and $\overline{\boldsymbol{J}} = (1, 1, 1)$. This specific network will be used in the remaining of this letter to illustrate the obtained results. To cover all feasible $\overline{P}$, we here additionally assume that the user's power resource is unlimited. Then, $\mathcal{N}(\boldsymbol{J}(P))$ is constant within intervals $D_1 = [0, 2.5]$, $D_2 = [3.02, 6.05]$, $D_3 = [12.8, 19.2]$ and $D_4 = [50.7, \infty)$. If $C_U = 0.25$ then $C_U = h/\mathcal{N}(\boldsymbol{J}(P))$ for $P \in D_2$. Each $P \in D_2$ represents the user's equilibrium strategy, with all of them having the same payoff, equal to zero (see (3) and (26)). If besides its basic goal to maximize its payoff the user has a secondary goal to maximize its SINR, the user has to employ the maximal feasible equilibrium strategy in D_2, i.e., $P = 6.05$. Similarly, we can consider the remaining intervals, where $\mathcal{N}(\boldsymbol{J}(P))$ is constant.

IV. Stackelberg Equilibrium

Suppose that between the user and the jammers there is a hierarchical relation, i.e., the user is the leader and the jammers are the followers. The SE of such scenario are found as the solution of a two-level optimization problem with the user at the top level and the jammers at the low level. The problem can be solved by backward induction.

In the first step of the two-level game, in response to user's strategy P, the jammers implement their equilibrium strategies

$\boldsymbol{J}(P)$ in the sub-game $\Gamma_J(P)$. *In the second step* of the two-level game, the user, taking into account such jammers' behaviour, selects the optimal P to maximize the user's payoff as follows:

$$\max\{V(P) : P \in [0, \overline{P}]\}, \tag{27}$$

where

$$V(P) \triangleq v_U(P, \boldsymbol{J}(P)) = hP/\mathcal{N}(\boldsymbol{J}(P)) - C_U P. \tag{28}$$

Then, $(P, \boldsymbol{J}(P))$ is SE. Let us denote this SG by Γ_S.

Proposition 3: In game Γ_S there is at least one SE.

Proof: The result follows from Corollary 1. ∎

A. Auxiliary Notations and Results

By Proposition 2 and Corollary 1, the user's payoff $V(P)$ can be written in closed form as follows:

$$V(P) = \begin{cases} W_{\kappa(P)}(P), & P \in I_{\kappa(P)}, \\ \overline{W}_{\kappa(P)}(P), & P \in \overline{I}_{\kappa(P)}, \end{cases} \tag{29}$$

where

$$W_k(P) \triangleq (h/N_{k-1} - C_U)P, \tag{30}$$

$$\overline{W}_k(P) \triangleq h\sqrt{\overline{C}_k P} - C_U P \text{ for } k \in D_K. \tag{31}$$

Thus, $V(P)$ is linear in $I_{\kappa(P)}$ and concave in $\overline{I}_{\kappa(P)}$.

In the following proposition we establish monotonuous properties for $W_k(P)$ and $\overline{W}_k(P)$.

Proposition 4: Function $W_k(P)$ and $\overline{W}_k(P)$ have the following monotonuous properties:

(a) If $N_{k-1} < h/C_U$ then $W_k(P)$ increases for $P \geq 0$, if $N_{k-1} = h/C_U$ then $W_k(P)$ is constant for $P \geq 0$, and if $N_{k-1} > h/C_U$ then $W_k(P)$ decreases for $P \geq 0$;

(b) $\overline{W}_k(P)$ is concave and achieves the maximum at $P = P_k$, where

$$P_k \triangleq h^2\overline{C}_k/(4C_U^2); \tag{32}$$

(c) if $h/(2C_U) \leq N_{k-1}$ then $\overline{W}_k(P)$ decreases in $\overline{I}_k$, if $N_{k-1} < h/(2C_U) \leq N_k$ then $\overline{W}_k(P)$ gets maximum in $\overline{I}_k$ at $P = P_k$, and if $N_k < h/(2C_U)$ then $\overline{W}_k(P)$ increases in $\overline{I}_k$.

Proof: By (30), $W_k(P)$ is linear in P and (a) follows. By (31), $\frac{d\overline{W}_k(P)}{dP} = h\sqrt{\overline{C}_k}/(2\sqrt{P}) - C_U$, and (b) follows. By (22), $P_k \in \overline{I}_k$ if and only if $N_{k-1} < h/(2C_U) < N_k$. This jointly with (b) imply (c). ∎

For $N_0 \leq h/C_U$, let us denote by $k_\star$ such index that $\overline{W}_k(P)$ gets its maximum either at inner point of $\overline{I}_k$ or at its left boundary. By (6) and Proposition 4(c), $k_\star$ is such that

$$N_{k_\star - 1} < h/(2C_U) \leq N_{k_\star}. \tag{33}$$

For $N_0 \leq h/C_U$ let us denote by $k_\bullet$ such index that $W_k(P)$ is non-decreasing in $[0, \infty)$ for $k < k_\bullet$ and $W_k(P)$ is decreasing in $[0, \infty)$ for $k \geq k_\bullet$. By (6) and Proposition 4(b), such $k_\bullet$ is defined as follows:

$$N_{k_\bullet - 1} \leq h/C_U < N_{k_\bullet}. \tag{34}$$

By (6), (33) and (34), we have that

$$k_\star \leq k_\bullet. \tag{35}$$

In the following proposition we find $P \in [0, \infty)$, at which $V(P)$ is maximum.

Proposition 5: (a) If $k_\star = k_\bullet$ then $V(P)$ attains its maximum at $P = P_{k_\star}$.

(b) If $k_\star < k_\bullet$ then $V(P)$ attains its maximum within finite set of power levels S, where

$$S \triangleq \{N_{k-1}^2 \overline{C}_k : k = k_* + 1, \ldots, k_\bullet\}. \tag{36}$$

Proof: Proposition 4, with $\mathcal{I}_k$ given by (25), implies that

$$V(P) \text{ is increasing on } \mathcal{I}_k \cup I_{k+1} \text{ if } N_k \leq h/(2C_U). \tag{37}$$

This, jointly with Proposition 4, (29) and Eqns.(33)-(35) imply (a). To get (b), we also has to take into account that $N_{k-1}^2\overline{C}_k$ is the upper bound of interval I_k, and, thus, by Proposition 4 and (34), $V(P)$ attains a local maximum within I_k at $P = N_{k-1}^2\overline{C}_k$, and (b) follows. ∎

Let us illustrate the piece-wise linear structure of payoff $V(P)$ for the specific three-jammer network considered in Section III-A with $C_U \in \{0.1, 0.2\}$. Fig. 1(b) shows that $V(P)$ can have several local maxima (Proposition 4 and Proposition 5). The global maximum depends on C_U and $\overline{P}$. For example, let $\overline{P} \in \{40, 60\}$. For $C_U = 0.2$, in both cases the maximum is achieved at $P = 6.05$. Meanwhile, for $C_U = 0.1$ the maximum is achieved at $P = 30$ and $P = 60$ for $\overline{P} = 40$ and $\overline{P} = 60$, respectively.

B. Stackelberg Equilibrium in Closed Form

In this section we derive SE in closed form and establish a condition for uniqueness.

Theorem 2: In game Γ_S, SE is given by $(P, \boldsymbol{J}(P))$, where P is unique except in two particular cases (b) and (d) given below. In particular,

(a) if $h/C_U < N_0$ then $P = 0$,

(b) if $h/C_U = N_0$ then P is any power level from $[0, \min\{N_0^2\overline{C}_1, \overline{P}\}]$,

(c) if $h/C_U > N_0$ and $k_\star = k_\bullet$ then $P = \min\{\overline{P}, P_{k_\star}\}$,

(d) if $h/C_U > N_0$ and $k_\star < k_\bullet$ then $P = \operatorname{argmax}\{V(P) : P \in \mathcal{P}\}$, where $\mathcal{P} \triangleq ([0, \overline{P}] \cap S) \cup \{\overline{P}\}$ is the finite set of power levels.

PROOF: Since $\mathcal{N}(\boldsymbol{J}(P)) \geq N_0$, by (28), $V(P) = hP/\mathcal{N}(\boldsymbol{J}(P)) - C_U P \leq (h/N_0 - C_U)P$. Thus, if $h/N_0 < C_U$, then, $V(P) < 0 = V(0)$ for $P > 0$, and (a) follows. If $h/N_0 = C_U$, then, by Corollary 1, $V(P) = (h/N_0 - C_U)P = 0$ for $P \leq \overline{C}_1 N_0^2$ and $V(P) < 0$ otherwise, and (b) follows. (c) and (d) follow from Proposition 5 and (37).

C. Jammers as Followers Benefit in Payoffs

By the definition of NE and SE we have that the user acting as the leader in SG benefits in payoff as compared to the user in NG. In the following proposition, we prove that jammers acting as followers in SG also benefit in payoffs as compared with jammers in NG.

Proposition 6: Let $(P_N, \boldsymbol{J}_N)$ and $(P_S, \boldsymbol{J}_S)$ be equilibrium in Γ_N and Γ_S, respectively. Then (a) $P_S \leq P_N$, (b) $\boldsymbol{J}_S \leq \boldsymbol{J}_N$, (c) $v_{J,k}(P_N, \boldsymbol{J}_N) \leq v_{J,k}(P_S, \boldsymbol{J}_S)$ for $k \in D_K$.

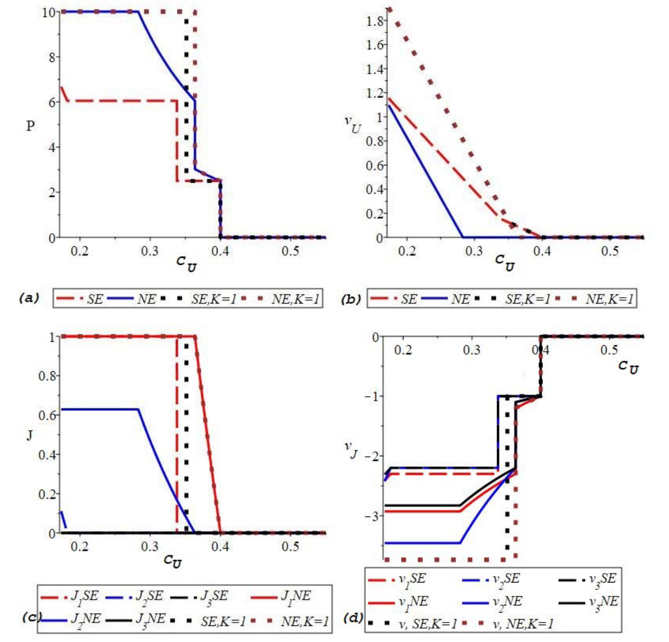

Fig. 2. (a) User's equilibrium strategies, (b) user's equilibrium payoffs, (c) jammers' equilibrium strategies and (d) jammers' equilibrium payoffs as functions of C_U.

Proof: By Theorem 1 and Theorem 2, we have that: (i) $P_N = P_S = 0$ for $h/C_U \le N_0$, (ii) for $h/C_U > N_0$ and $k_\star = k_\bullet$, since $\overline{C}_k/(4C_U^2) < \overline{C}_k/C_U^2$, then $P_S \le P_N$ and (iii) for $h/C_U > N_0$ and $k_\star < k_\bullet$, since the P_N belongs to be inner of $I_{k_\bullet}$ while P_S belongs to left boundary of $I_{k_\bullet}$ then $P_S \le P_N$. This implies (a). By Proposition 2, $\boldsymbol{J}(P)$ is non-decreasing in P, and, so, (a) implies (b). By (17), $g_k hP/\mathcal{N}(\boldsymbol{J}(P)) = \mathcal{N}(\boldsymbol{J}(P))C_{J,k}$ for $J_k \in (0, \overline{J}_k)$. Since the right side $\mathcal{N}(\boldsymbol{J}(P))$ is increasing in P, then its left side $g_k hP/\mathcal{N}(\boldsymbol{J}(P))$ is also increasing in P. Thus, the k-th jammer payoff $v_{J,k}(P, \boldsymbol{J}(P)) = -hP/\mathcal{N}(\boldsymbol{J}(P)) - C_{J,k}J_k(P)$, $k \in D_K$, is non-increasing in P as the sum of two non-increasing functions, and (c) follows from (a). ∎

V. Discussion of the Results

Let us illustrate our results for the three-jammer network of Section III-B and the corresponding single-jammer game, i.e., $K = 1$, with $\overline{P} = 10$. Via Theorem 1 and Theorem 2, the user is inactive in both games, Γ_N and Γ_S, for large transmission cost, $C_U \ge 1 (= h/N_0)$ (see Fig. 2). At the threshold $C_U = h/N_0$, multiple user's equilibria arise for both games, which are reflected by a jump at user's strategy. Namely, each $(P, \mathbf{0})$ with $P \in [0, \overline{C}_1 N_0^2]$ is an equilibrium corresponding to a unique user's payoff equal to zero and the k-th jammer payoff equal to $-PC_U$. In that case, the user's payoff is insensitive to the user's equilibrium strategy, while the jammers' payoffs are sensitive. A decrease in C_U leads to an increase in user's as well as jammers' strategies. It leads to an increase in the user's payoff and in a decrease in the jammers payoffs. In SE, each agent applies smaller power as compared to NE, while the payoff of each agent is greater or equal to that in NE. This result supports the term smart-type jammer in this multi-jammer scenario; the term was originally coined for the jammer acting as follower in single jammer PC-SINR game [2]. Finally, note that an increase in the number of jammers leads to a decrease in the user's payoff and an increase in jammers' payoffs, since each jammer might apply smaller jamming power to achieve the tradeoff between a decrease in the SINR at the receiver and involved jamming cost.

VI. Conclusion

We have considered a scenario in which a user communicates with a receiver in the presence of any number of jammers as a power control game. We have derived all Nash (regular-type jammers) and Stackelberg (smart-type jammers) equilibria in closed form. We have shown that in the case of multiple equilibria, the user has the same payoff for each equilibrium, which indicates the stability of the suggested solution. We have also shown that, smart-type jammers, although they act as followers, gain in payoffs as compared to regular-type jammers.

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