ICASSP 2021 - 2021 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP) | 978-1-7281-7605-5/20/$31.00 ©2021 IEEE | DOI: 10.1109/ICASSP39728.2021.9413529

A MULTIPLE ACCESS CHANNEL GAME USING LATENCY METRIC

*Andrey Garnaev** *Athina Petropulu*† *Wade Trappe**

*WINLAB, Rutgers University, †ECE, Rutgers University

ABSTRACT

The paper considers a multi-access channel scenario, where several users communicate with a base station, and investigates power allocation is a game-theoretic framework. The communication metric is the inverse signal-to-interference-plus-noise ratio (SINR) at the base station, which, for low SINR reflects communication delay. Each user faces a trade-off between the latency of the signal received by the base station, and the price that the user pays for using a specific amount of power that causes interference in the system. The equilibrium is derived in closed form and its uniqueness is proven. It is shown that the resulting strategy allows each user to maintain uninterrupted communication. For comparison purposes, we construct a specific three user network scenario, and study the SINR and throughput metrics. In that setting, we show that, unlike the latency metric, SINR and throughput may give rise to multiple equilibria, which may cause destabilization of communication.

1. INTRODUCTION

In recent years there has been increased interest in wireless networks in which mobile devices (nodes, users) act in a selfish manner, i.e., they independently select their resource allocation strategies to optimize their individual performance objectives. As such resource allocation and power control problems are multi-objective, they have been studied under a game theoretic framework [1]. In [2–4], game theory has been used to study a fading multi-access channel (MAC) scenario, and in [5–8] to study an orthogonal frequency division multiplexing (OFDM) scenario. In all those works [2–8], the user communication utility (UCU) is throughput.

In this paper, different from prior works, we study the multi-user non-cooperative power control transmission problem with communication latency as UCU. Latency is an important metric in many applications, including video streaming, and high speed communications. Latency as UCU was considered in [9–11] to address an anti-jamming power control scenario. However, the mathematical treatment of the anti-jamming power control problem is significantly different than that of the multi user non-cooperative power control problem considered here. In the aforementioned works the latency was taken to be equal to the inverse SINR [10, 11], or to an age-of-information metric, modeled by an affine function of the inverse SINR [9].

According to Little's Law of queueing theory, in a stable system, the average number of packets in a queue equals the product of the packet arrival rate and the average time packets spent in the queue (latency). Thus, in steady state, for a fixed queue length, the latency is proportional to the inverse arrival rate, or to the inverse throughput. At low SINR, the throughput can be approximated by the SINR, therefore, in that case, in steady state and for a fixed queue length, the latency is proportional to 1/SINR [10].

In this paper, based on the above justification, we model latency by the inverse SINR. We consider a flat-fading MAC scenario in a single cell network, and propose closed-form, optimal power control strategies of users in a game-theoretical framework using the latency metric. Each user faces a trade-off between the latency of the signal received by the base station, and the power that the user uses (user cost), which causes interference in the system. The equilibrium is derived in closed form via a fixed point algorithm, whose convergence is proven. Further, it is shown that the latency metric yields a unique equilibrium for any set of values for network parameters, and provides each user with uninterrupted communication. As a comparison, for a three user network, it is shown that the SINR and throughput metrics may give rise to multiple equilibria, which may lead to destabilization of the communication. An intuitive explanation of this important difference is as follows. In each case, finding the *Nash equilibrium* (NE) is reduced to a system of equation/inequalities defined based on whether the equilibrium is achieved with inner/boundary strategies of a set of feasible strategies. In general, such system may have multiple solutions. For SINR and throughput metrics, these equation/inequalities are linear while in latency they are non-linear, and its this non-linearity that makes the solution unique.

2. COMMUNICATION MODEL

We consider a time-slotted flat-fading MAC in a single cell network, in which n mobile terminals (users) are simultaneously sending data to a base station. Let the strategy of user i be the user's transmit power level P_i, with $P_i \in [0, \overline{P}_i]$ and let (P_i, P_{-i}) be a strategy profile, a set consisting of one strategy for each user, where P_{-i} denotes the strategies of all the users except user i. The SINR of user i at the base station is $\mathrm{SINR}_i(P_i, P_{-i}) = \lambda_i h_i P_i/(N + \sum_{j\neq i} h_j P_j)$, where h_i is the path gain of user i to the base station; λ_i is the spreading gain, and can be defined as W/r_i, with W denoting the available bandwidth, and r_i the transmission rate [12]; and N is the variance of the additive white Gaussian noise (AWGN), which causes degradation of the received signal at the base station. Note that, in contrast to [4], we introduce an upper bound on the user power, i.e., $\{\overline{P}_i\}$, to make scenario more realistic. Of course, the unlimited power resource case is the limit of the case considered here.

Latency is here modeled by the inverse SINR [10]. The user faces a trade-off between a reduction in latency of the signal received by the base station and the price that user pays for using a specific amount of power that causes interference in the system [4]. The latter price is a linear function of the power level of the user, i.e., $C_i P_i$ for user i, with $C_i > 0$ being the price per unit power level. Then, the payoff to the user i is defined as

$$V_i(P_i, P_{-i}) = -1/\mathrm{SINR}_i(P_i, P_{-i}) - C_i P_i. \qquad (1)$$

Let us introduce the following notation to avoid bulkiness in the formulas: $p_i \triangleq h_i P_i$, $\overline{p}_i \triangleq h_i \overline{P}_i$ and $c_i \triangleq C_i/h_i$. Then, in this notations, payoff (1) can be presented as follows:

$$V_i(p_i, p_{-i}) = -(N + \sum_{j\neq i} p_j)/(\lambda_i p_i) - c_i p_i. \qquad (2)$$

We assume that each of the users has complete information about all the network parameters $\{c_i, \lambda_i, \overline{p}_i\}$ and N. We look for a NE. Recall that $\boldsymbol{p} = (p_1, \dots, p_n)$ is an NE if and only if the following inequalities hold:

$$V_i(\tilde{p}_i, p_{-i}) \le V_i(p_i, p_{-i}),\ i = 1, \dots, n, \qquad (3)$$

for any $\tilde{p}_j \in [0, \overline{p}_j]$ and $j = 1, \dots, n$.
Each of the users wants to maximize its payoff. Denote this non-zero sum game by Γ_L.

Finally, note that, by (3), $\boldsymbol{p}$ is a NE if and only if each of these strategies is the best response to the others, i.e.,

$$p_i = \mathrm{argmax}\{V_i(p_i, p_{-i}) : p_i \in [0, \overline{p}_i]\},\ i = 1, \dots, n. \qquad (4)$$

Proposition 1 *In the game Γ_L there exists at least one NE.*

PROOF: By (2), we have that $\frac{\partial^2 V_i(p_i,p_{-i})}{\partial p_i^2} = -2(N + \sum_{j\neq i} p_j)/(\lambda_i p_i^3) < 0$. So, $V_i(p_i, p_{-i})$ is concave in p_i. Thus, by [1], there exists at least one NE. ■

3. NASH EQUILIBRIUM

In this section we prove uniqueness of the equilibrium and find it in closed form. First be provide a lemma on the root of an auxiliary equation. In Theorem 1, that root enters the total power expression of each user.

Lemma 1 **(a)** *The following fixed point equation has the unique positive root*

$$x_* = F(x_*), \qquad (5)$$

where

$$F(x) \triangleq \sum_{i=1}^{n} \min\{f_i(x), \overline{p}_i\} \qquad (6)$$

and

$$f_i(x) \triangleq (\sqrt{1 + 4c_i\lambda_i(N + x)} - 1)/(2c_i\lambda_i). \qquad (7)$$

(b) *The root of Eqn. (5) can be found via the fixed point algorithm. Namely,*

$$x_{i+1} = F(x_i) \text{ for } i = 0, 1, \dots \text{ and } x_0 = 0.$$

This sequence $\{x_i\}$ converges to the unique fixed point x_*.

(c) *Eqn. (5) is equivalent to*

$$\Phi(x_*) = 1, \qquad (8)$$

where $\Phi(x) \triangleq F(x)/x$. Since $\Phi(x)$ is non-increasing on x this root also can be found via the bisection method.

PROOF: Note that

$$\frac{df_i(x)}{dx} = 1/(\sqrt{1 + 4c_i\lambda_i(N + x)}) > 0 \qquad (9)$$

and

$$\frac{d^2 f_i(x)}{dx^2} = -2c_i\lambda_i/\sqrt{(1 + 4c_i\lambda_i(N + x))^3} < 0. \qquad (10)$$

Thus, $f_i(x)$ is strictly increasing and concave. Also,

$$\min\{f_i(x), \overline{p}_i\} = \begin{cases} f_i(x), & N + x < \overline{p}_i + \lambda_i c_i \overline{p}_i^2, \\ \overline{p}_i, & N + x \ge \overline{p}_i + \lambda_i c_i \overline{p}_i^2. \end{cases} \qquad (11)$$

Then, $F(x)$ given by (6) has the following properties: (p-i) $F(x)$ is concave as the sum of concave functions and (p-ii) $F(x)$ is positive and strictly increasing in $[0,\overline{x}]$ where $\overline{x} \triangleq \max\{\overline{p}_i + \overline{p}_i^2 \max_i(\lambda_i c_i) - N, 0\}$, and $F(x)$ is constant for $x > \overline{x}$. Since the left side of Eqn.(5) is equal to zero for x and linearly increasing for $x > 0$, by (p-i) and (p-ii) we have that Eqn.(5) has the unique root, and (a) follows.

(b) By (5), (p-i) and (p-ii), we have that $F(x) > x$ for $x \in [0, x_*)$. This jointly with (p-i) and (p-ii) imply that $0 = x_0 < F(x_0) = x_1 < F(x_1) = x_2 < F(x_2) < ....$ Thus, x_i converges as increasing and upper bounded sequence. Moreover, it converges to the fixed point x_* since Eqn. (5) has the unique root.

(c) First note that for $a > 0$ and $b > 0$ we have that

$$(1+(b+x/2)a)^2 - 1 - a(b+x) = a(b+a(b+x/2)^2) > 0.$$

Then,

$$\frac{d\phi(x)}{dx} = -\frac{1+(b+x/2)a - \sqrt{1+a(b+x)}}{x^2\sqrt{1+a(b+x)}} < 0. \quad (12)$$

with $\phi(x) \triangleq (\sqrt{1+a(b+x)} - 1)/x$.

Thus, $\phi(x)$ is decreasing. Assigning $a = 4c_i\lambda_i$ and $b = N$, this jointly with (6) and (7) imply the result. ■

In the following theorem, via solving the best response equations we prove the uniqueness of equilibrium as well as design it in closed form.

Theorem 1 *In game Γ_L, the NE $\boldsymbol{p}$ is unique. Also,*

$$p_i = \min\{f_i(x), \overline{p}_i\}, \quad (13)$$

where x is uniquely given by (5) or, equivalently, by (8).

PROOF: We find an NE as solution of best response equations. By (2), we have that $\frac{\partial V_i(p_i,p_{-i})}{\partial p_i} = (N + \sum_{j\neq i} p_j)/(\lambda_i p_i^2) - c_i$. Thus, for a fixed p_{-i}, the best response p_i is

$$p_i = \begin{cases} \sqrt{\dfrac{N+\sum_{j\neq i} p_j}{\lambda_i c_i}}, & \sqrt{\dfrac{N+\sum_{j\neq i} p_j}{\lambda_i c_i}} \le \overline{p}_i, \\ \overline{p}, & \sqrt{\dfrac{N+\sum_{j\neq i} p_j}{\lambda_i c_i}} > \overline{p}_i. \end{cases} \quad (14)$$

Thus, if $N + x < p_i + \lambda_i c_i \overline{p}_i^2$, where $x = \sum_{j=1}^n p_j$, it holds that $\lambda_i c_i p_i^2 + p_i = N + x$. Then it holds that $p_i = f_i(x)$, with $f_i(x)$ given by (7). If $N + x \ge p_i + \lambda_i c_i \overline{p}_i^2$, then $p_i = \overline{p}_i$. This and (11) imply that p_i is given by (13). Summarizing (13) implies (5) with $F(x)$ given by (6), and the result follows from Lemma 1. ■

The complexity of designing the NE of the game Γ_L, by Lemma 1(c) and Theorem 1, is $\kappa = \log_2(\overline{p}n/\epsilon)$, where ϵ is the tolerance of the bisection algorithm.

The following proposition establishes the monotonous property of the users' strategies and the corresponding latency.

Proposition 2 *In game Γ_L, the equilibrium strategies as well as the corresponding latency are non-decreasing as the background noise variance, N, increases.*

PROOF: Note that $F(x)$, given by (7), is non-decreasing on N. Thus, by (p-i), (p-ii) and (5), x_* is increasing. Therefore, by (7) and (13), the equilibrium strategies are also increasing. By (14), $p_i c_i = (N + \sum_{j\neq i} p_j)/(\lambda_i p_i)$ while $p_i < \overline{p}_i$. Thus, the latency is proportional to user's equilibrium strategy, and the result follows . ■

Similarly to Proposition 2 we can prove that, as the number of users increases, the applied power by the users increases and so does the latency of their communication.

4. A COMPARISON SCENARIO USING SINR AND THROUGHPUT METRICS

In the previous section we proved that the latency metric, independent of the network parameters, always gives rise to a unique equilibrium. In this section we present an example showing that in contrast to the latency metric, the SINR and throughput metrics may give rise to multiple equilibria.

For SINR as UCU, the payoff to the user i equals

$$W_i^S(p_i, p_{-i}) = \lambda_i p_i/(N + \sum_{j\neq i} p_j) - c_i p_i, \quad (15)$$

and for throughput as UCU the payoff to the user i equals

$$W_i^T(p_i, p_{-i}) = \ln(1 + \lambda_i p_i/(N + \sum_{j\neq i} p_j)) - c_i p_i. \quad (16)$$

Let us consider a network consisting of three users, and for simplicity, set each user's power level price to coincide with the spreading gain, i.e., $c_i = \lambda_i$. Let us also assume that the background noise is not high, i.e., $N < 1$, which will motivate at least some of the users to be active. Also, let the power resources be unlimited, i.e., $\overline{p}_i = \infty$.

Let us denoted by Γ_S and Γ_T the games with payoffs as given in (15) and (16), respectively. In the following two propositions, the NE of Γ_S and Γ_T are derived under the considered scenario.

Proposition 3 *In game Γ_S there are the following seven equilibria: (i) $(\infty,0,0)$, $(0,\infty,0)$, $(0,0,\infty)$, (ii) $(\overline{N},\overline{N},0)$, $(\overline{N},0,\overline{N})$, $(\overline{N},\overline{N},0)$ and (iii) $(\overline{N}/2,\overline{N}/2,\overline{N}/2)$ where $\overline{N} \triangleq 1-N$.*

PROOF: Since $W_i^S(p_i,p_{-i})$ is linear in p_i, there is at least one NE. Moreover, since $c_i = \lambda_i$ and $\overline{p}_i = \infty$, the best response of user i to a fixed p_{-i} is given as follows:

$$p_i = \begin{cases} 0, & 1 < N + \sum_{j \neq i} p_j, \\ \text{any}, & 1 = N + \sum_{j \neq i} p_j, \\ \infty, & 1 > N + \sum_{j \neq i} p_j. \end{cases} \quad (17)$$

Since $N < 1$, by (17), $\boldsymbol{p} = (0, 0, 0)$ cannot be a NE. Also, if $p_i = \infty$ then, by (17), $p_j = 0$ for $j \neq i$, and (i) follows. The rest two cases where either two components of $\boldsymbol{p}$ are positive, or three components of $\boldsymbol{p}$ are positive can be considered similarly. ■

Proposition 4 *In the game Γ_S there are the following equilibria: (i) if $\lambda_i \leq 1$ then $\boldsymbol{p}$ with $p_i = \overline{N}/\lambda_i$ and $p_j = 0$ for $j \neq i$, (ii) if $\lambda_i < 1$ and $\lambda_j < 1$ then $p_i = \overline{N}\,\overline{\lambda}_j/(1 - \lambda_i\lambda_j)$, $p_j = \overline{N}\,\overline{\lambda}_i/(1 - \lambda_i\lambda_j)$, and $p_k = 0$ where $k \notin \{i, j\}$ and $\overline{\lambda}_m = 1 - \lambda_m$, (iii) if $\lambda_i < 1$ for all i or $\lambda_i > 1$ for all i then $p_k = \overline{N}\,\overline{\lambda}_i\overline{\lambda}_j/D$ where $\{i, j, k\} = \{1, 2, 3\}$ and $i \neq j \neq k$ with $D \triangleq \lambda_1\lambda_2\lambda_3 - \lambda_1 - \lambda_2 - \lambda_3 + 2$.*

PROOF: Since $W_i^T(p_i, p_{-i})$ is concave in p_i, there is at least one NE. Moreover, since $c_i = \lambda_i$ and $\overline{p}_i = \infty$, the best response p_i of the user i to a fixed p_{-i} is given as follows:

$$p_i = \begin{cases} 0, & 1 < N + \lambda_i p_i + \sum_{j \neq i} p_j, \\ \text{any}, & 1 = N + \lambda_i p_i + \sum_{j \neq i} p_j, \\ \infty, & 1 > N + \lambda_i p_i + \sum_{j \neq i} p_j. \end{cases} \quad (18)$$

By (18), $p_i \neq \infty$ for any i. Let $p_i > 0$ and $p_j = 0$ for $j \neq i$. Then, by (18), $1 = N + \lambda_i p_i$. So, $p_i = (1 - N)/\lambda_i$. Substituting $\boldsymbol{p}$ with such p_i and $p_j = 0$ for $j \neq i$ into (18) implies that such $\boldsymbol{p}$ is a NE if $1 \leq N + (1 - N)/\lambda_i$. Since $N < 1$, this implies (i). The remaining cases where either two components of $\boldsymbol{p}$ are positive or three components of $\boldsymbol{p}$ are positive can be considered similarly. ■

5. RESULTS

In this section we illustrate the difference in the users' equilibrium strategies and associated UCU depending on the type of metric used.

We consider a network of three users, with spreading gains $\lambda = (3, 4, 5)$, equal power level prices $C = (1, 1, 1)$ and equal power budget $\overline{\boldsymbol{p}} = (1, 1, 1)$ (Fig. 1). Then, in game Γ_T (UCU is throughput), the NE is given as the fixed point of $(p_1, p_2, p_3) = (\lfloor 1 - (N + p_2 + p_3)/\lambda_1 \rfloor_+, \lfloor 1 - (N + p_1 + p_3)/\lambda_2 \rfloor_+, \lfloor 1 - (N + p_1 + p_2)/\lambda_3 \rfloor_+)$. In game Γ_S (UCU is SINR), the NE is: $(1, 1, 1)$ for $N \leq 1$, $(0, 1, 1)$ for $1 < N \leq 3$, $(0, 0, 1)$ for $3 < N \leq 5$ and $(0, 0, 0)$ for $5 < N$. Thus, an increase in background noise leads to a decrease in the users' equilibrium strategies in games Γ_S and Γ_T. In game Γ_L, the users' strategies increase as the background noise increases. In games Γ_S and Γ_T, all the users are non-active for $N > 5$, which results into infinite latency. In Γ_L, all the users are active although latency increases with an increase in N. We should note that although game Γ_S gives the simplest strategies for the users (either transmit or not), the corresponding SINRs are in general discontinuous functions in the network parameters. This makes equilibrium strategies of Γ_S less fair for the users as compared to those given by Γ_T and Γ_L, where throughput and latency vary continuously.

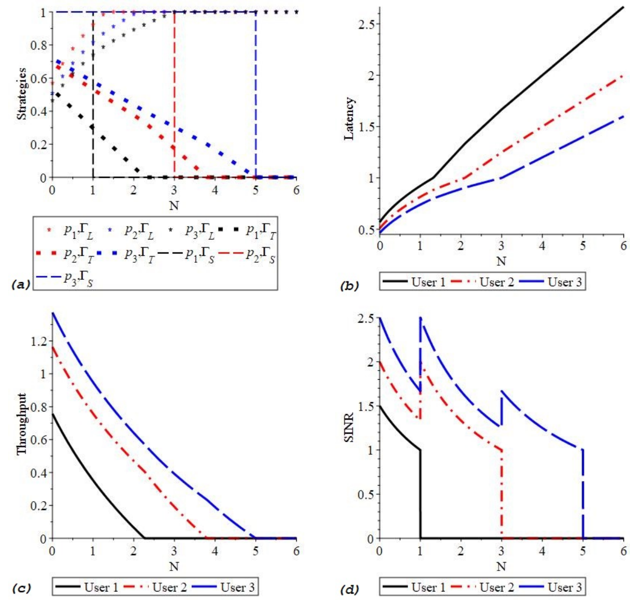

Fig. 1. (a) Users' strategies, (b) latency in game Γ_L, (c) throughout in game Γ_T and (d) SINR in Γ_S.

6. CONCLUSIONS

A fading MAC problem where several users communicate with an receiver has been investigated in a game-theoretical formulation. Equilibrium strategies have been found in closed form for the inverse SINR communication metric to reflect communication delay. It has been established that the latency metric returns a unique equilibrium and this equilibrium allows each user uninterrupted communication for any network parameters. Thus, a decision made on such metric allows for stability in communication. In contrast, SINR and throughput communication metrics, reflecting communication reliability, depending on network parameters may give rise to multiple equilibria, potentially causing destabilization of communication.

7. REFERENCES

[1] Z. Han, D. Niyato, W. Saad, T. Basar, and A. Hjrungnes, *Game Theory in Wireless and Communication Networks: Theory, Models, and Applications*. Cambridge University Press, 2012.

[2] G. He, M. Debbah, and E. Altman, "k-player Bayesian waterfilling game for fading multiple access channels," in *Proc. 3rd IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, pp. 17–20, 2009.

[3] F. Meshkati, M. Chiang, S.C. Schwartz, H.V. Poor, and N.B. Mandayam, "A non-cooperative power control game for multi-carrier CDMA systems," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, 2005.

[4] T. Alpcan, T. Basar, R. Srikant, and E. Altman, "CDMA uplink power control as a noncooperative game," *Wireless Networks*, vol. 8, pp. 659–670, 2002.

[5] W. Yu, G. Ginis, and J.M. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE Journal on Selected Areas in Communications*, vol. 20, pp. 1105–1115, 2002.

[6] E. Altman, K. Avrachenkov, and A. Garnaev, "Closed form solutions for symmetric water filling games," in *Proc. 27th IEEE International Conference on Computer Communications (INFOCOM)*, pp. 1346–1354, 2008.

[7] E. Altman, K. Avrachenkov, and A. Garnaev, "Closed form solutions for water-filling problem in optimization and game frameworks," *Telecommunication Systems Journal*, vol. 47, pp. 153–164, 2011.

[8] S. Fu, Z. Su, Y. Jia, H. Zhou, Y. Jin, J. Ren, B. Wu, and K.M.S. Huq, "Interference cooperation via distributed game in 5g networks," *IEEE Internet of Things Journal*, vol. 6, pp. 311–320, 2019.

[9] G. Nguyen, S. Kompella, C. Kam, J. Wieselthier, and A. Ephremides, "Impact of hostile interference on information freshness: A game approach," in *Proc. 15th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, 2017.

[10] A. Garnaev, A. Petropulu, W. Trappe, and H.V. Poor, "A switching transmission game with latency as the user's communication utility," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 2912–2916, 2020.

[11] A. Garnaev, A. Petropulu, W. Trappe, and H.V. Poor, "A multi-jammer game with latency as the user's communication utility," *IEEE Communications Letters*, vol. 24, pp. 1899–1903, 2020.

[12] M.R. Musku, A.T. Chronopoulos, D.C. Popescu, and A. Stefanescu, "A game-theoretic approach to joint rate and power control for uplink CDMA communications," *IEEE Transactions on Communications*, vol. 58, pp. 923–932, 2010.