Pricing Conditional Value at Risk-Sensitive Economic Dispatch

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***Abstract*—There are growing concerns over the ability of current electricity market designs to adequately model and optimize against the stochastic nature of renewable resources such as wind and solar. In this paper, we consider an economic dispatch problem that explicitly accounts for said uncertainty and enforces network and generation limits using conditional value at risk. Our key contribution is the definition and analysis of risk-sensitive locational marginal prices (risk-LMPs) derived from such a market clearing problem. Risk-LMPs extend conventional LMPs to the uncertain setting. Settlements defined via risk-LMPs compensate resources for both energy and reserve schedules. We study these prices via sample average approximation (SAA) on example power networks to demonstrate their viability for electricity pricing with large-scale integration of renewables.**

I. Introduction

The need for innovative market design with uncertain renewable sources such as wind and solar have been widely recognized [1]–[8]. Ad hoc measures with point forecast and fixed reserve margins cannot cope with said uncertainty. We take a principled approach to process uncertainty in forward energy procurement through a coherent risk measure–the conditional value-at-risk (CVaR) measure [9]. Our key contribution is the definition and analysis of locational marginal prices from a CVaR-sensitive ED problem.

Why CVaR? While this risk measure has seen widespread adoption in finance and gained traction in engineering applications, it has received limited attention in power systems. Examples include [10]–[12]. CVaR benefits from being a coherent risk measure, which ensures that the risk-sensitive problem retains the convexity of the deterministic variant, irrespective of the distribution of the underlying uncertainty. Convexity is a useful property for two reasons: it permits the design of efficient algorithms via sampling, and the mature duality theory of convex programming allows the derivation and analysis of meaningful prices for electricity.

In this paper, we begin by presenting the CVaR-based ED problem which imposes risk-sensitive network and generation limits in Section II. We adopt the perspective of a system operator (SO) solving the ED problem to manage operational and power delivery risks across the network. CVaR is equipped with a tunable parameter, enabling the SO to tradeoff economic efficiency and network security with varying levels of conservativeness. In Section III, we apply duality theory to define risk-sensitive locational marginal prices (risk-LMPs). These prices endogenize stochasticity of renewable generation and compensate dispatchable generators for nominal generation and regulation commitments in response to forecast errors. Our prices reduce to conventional LMPs as the forecast error goes to zero. Section IV presents a sample average approximation (SAA) approach to solving the CVaR-sensitive dispatch and prices. We demonstrate key properties of the prices through a numerical example on a five-bus power network. Finally, Section V addresses revenue adequacy of our settlement process. All proofs are left to the appendix.

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II. The CVaR-Sensitive ED Problem

We consider a power network with n buses connected by ℓ lines. Each bus is equipped with some dispatchable generation, non-dispatchable (renewable) infeeds, and inflexible demand, the nodal values of which are collected in the vectors, $\boldsymbol{g}$, $\boldsymbol{\xi}$, and $\boldsymbol{d}$, respectively. Dispatchable generators are those whose output can be altered on command, such as conventional thermal and hydro-electric plants. The nodal renewable power injection, $\boldsymbol{\xi}$, is random and takes values over a compact set Ξ. These samples, $\boldsymbol{\xi}$, can be obtained, either from historical measurements (e.g., the NREL wind database) or from generative models (e.g., generative adversarial networks in [13]).

Accommodating uncertainty in renewable availablility, we formulate an ED problem that schedules both nominal dispatch and a power output adjustment (reserve policies) for dispatchable generators, allowing generators to respond to forecast errors in renewable supply by making $\boldsymbol{g}$ depend on $\boldsymbol{\xi}$. We restrict attention to affine recourse policies, i.e.,

$$\boldsymbol{g}(\boldsymbol{\xi}) = \boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi}, \tag{1}$$

where $\boldsymbol{g}_0$ is the nominal generation, $\boldsymbol{\xi}_0$ the nominal or forecasted renewable infeeds and $\boldsymbol{G}$ encodes the adjustments to forecast errors $\Delta\boldsymbol{\xi} := (\boldsymbol{\xi} - \boldsymbol{\xi}_0)$. Specifically, with n wind resources in the system, $\boldsymbol{G} \in \mathbb{R}^{n\times n}$ and G_{ij} denotes the portion of deviation in renewable generation $\Delta\xi_j$ at node j picked up by generator at node i. Here, $\mathbb{R}$ is the set of real numbers. Assume that forecast errors are zero-mean, i.e.,

$$\mathbb{E}[\Delta\boldsymbol{\xi}] = 0. \tag{2}$$

We enforce three types of constraints: network-wide power balance, generation capacity limits, and transmission line flow limits. We adopt a linear power flow model via the widely used DC approximations that assume small voltage angle differences between nodes, lossless lines and (per unit) voltage magnitudes close to unity. Under these assumptions, reactive power is neglected and line flows become linear maps of the power injections across the network.

Power balance across the network for all $\boldsymbol{\xi}$ requires $\mathbf{1}^\intercal \left(\boldsymbol{g}(\boldsymbol{\xi}) + \boldsymbol{\xi} - \boldsymbol{d}\right) = 0$, where $\mathbf{1}$ is the vector of all ones of appropriate size. Under an affine recourse policy and assuming zero-mean forecast errors, this is equivalent to

$$\mathbf{1}^\intercal (\boldsymbol{g}_0 + \boldsymbol{\xi}_0 - \boldsymbol{d}) = 0, \quad \boldsymbol{G}^\intercal \mathbf{1} = \mathbf{1}. \tag{3}$$

Dispatchable generators can only produce power within their capacities, i.e., within $[0, \overline{\boldsymbol{g}}]$. Instead of requiring these capacity limits be met for each $\boldsymbol{\xi}$, we enforce them in a risk-sensitive fashion using CVaR. To describe the modeling approach, consider a scalar random variable, Z, with a continuous cumulative distribution, F. Then, $\text{CVaR}_\delta[Z]$ is the expectation over the $(1-\delta)$-tail of the distribution,

$$\text{CVaR}_\delta[Z] := \mathbb{E}[Z \mid Z \geq F^{-1}(\delta)].$$

Here, $\mathbb{E}$ stands for the expectation operator and δ is a tunable parameter that takes values in $[0, 1)$. As $\delta \downarrow 0$, $\text{CVaR}_\delta[Z]$ becomes the average value of Z. Taking $\delta \uparrow 1$, it approaches the highest value that Z can take. CVaR over arbitrary distributions is defined via the following variational characterization by Rockafellar and Uryasev in [14].

$$\text{CVaR}_\delta[Z] := \underset{u \in \mathbb{R}}{\text{minimum}} \left\{ u + \frac{1}{1-\delta} \mathbb{E}[(Z-u)^+] \right\}, \tag{4}$$

where $[A]^+$ is the positive part of A. Equipped with this definition, we impose risk-sensitive local generation constraints,

$$\text{CVaR}_\gamma [\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi}] \leq \overline{\boldsymbol{g}}, \quad \text{CVaR}_\gamma [-\boldsymbol{g}_0 + \boldsymbol{G}\Delta\boldsymbol{\xi}] \leq \mathbf{0} \tag{5}$$

for a parameter $\gamma \in [0, 1)$. The CVaR-based constraints in the above relation are imposed element-wise. As $\gamma \uparrow 1$, constraints become tighter, requiring generation limits to be imposed for almost every sample. However, when $\gamma \downarrow 0$, constraints are only enforced on average, allowing for the potential inability to respond to uncertain wind across multiple scenarios.

To enforce risk-sensitive line flow constraints, let $\boldsymbol{H} \in \mathbb{R}^{2\ell \times n}$ denote the (directed) injection shift factor matrix. Under the linear power flow model, the directed flows across the network are $\boldsymbol{H}(\boldsymbol{g}(\boldsymbol{\xi}) + \boldsymbol{\xi} - \boldsymbol{d})$. Denoting the (directed) line flow capacity limits by $\boldsymbol{f} \in \mathbb{R}^{2\ell}$, we impose

$$\text{CVaR}_\beta [\boldsymbol{H}(\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi} + \boldsymbol{\xi} - \boldsymbol{d})] \leq \boldsymbol{f} \tag{6}$$

for risk parameter $\beta \in [0, 1)$. Altogether, the CVaR-sensitive ED problem can be formulated as

$$\underset{\boldsymbol{g}_0, \boldsymbol{G}}{\text{minimum}} \ \ \boldsymbol{c}^\intercal \boldsymbol{g}_0, \quad \text{subject to } (3), (5), (6). \tag{7}$$

Here, we seek to minimize a linear procurement cost, where $\boldsymbol{c}$ is the vector of offer prices submitted by all generators. We assume that renewable power suppliers are price-takers, offering energy at zero marginal cost. It can be seen that

$$\boldsymbol{c}^\intercal \boldsymbol{g}_0 = \mathbb{E}[\boldsymbol{c}^\intercal (\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi})] = \mathbb{E}[\boldsymbol{c}^\intercal \boldsymbol{g}(\boldsymbol{\xi})],$$

owing to the zero-mean forecast error assumption in (2). Thus, (7) seeks to minimize expected generation costs, while imposing risk-sensitive constraints. By virtue of coherence, the CVaR-sensitive ED problem in (7) is a convex optimization problem, regardless of the distribution of $\boldsymbol{\xi}$.

Remark 1. *CVaR-sensitive constraints are intimately related to chance-constraints. Specifically, CVaR-sensitive constraints are inner approximations of chance constraints, i.e.,*

$$\textit{CVaR}_\beta[Z] \leq 0 \Rightarrow \mathbb{P}\{Z \leq 0\} \geq \beta$$

for a scalar random variable Z. Chance-constrained formulations typically require assumptions on the distribution to admit convex reformulations, while those that are CVaR-sensitive do not. By nature, chance constraints seek to control the frequency of constraint violations, while CVaR-sensitive constraints control both the frequency and severity of violations.

III. Risk-Sensitive Locational Marginal Prices (risk-LMPs) and the Settlement Process

We now proceed to define prices for electricity from the CVaR-sensitive ED problem in (7). To that end, we first reformulate (7) using the characterization of CVaR in (4).

$$J^\star(\boldsymbol{d}) := \underset{\boldsymbol{g}_0, \boldsymbol{G}, \boldsymbol{u}, \underline{\boldsymbol{v}}, \overline{\boldsymbol{v}}}{\text{minimum}} \ \ \boldsymbol{c}^\intercal \boldsymbol{g}_0, \tag{8a}$$

subject to

$$\mathbf{1}^\intercal (\boldsymbol{g}_0 + \boldsymbol{\xi}_0 - \boldsymbol{d}) = 0, \tag{8b}$$

$$\boldsymbol{G}^\intercal \mathbf{1} = \mathbf{1}, \tag{8c}$$

$$\boldsymbol{u} + \frac{1}{1-\beta} \mathbb{E}\left[(\boldsymbol{H}(\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi} + \boldsymbol{\xi} - \boldsymbol{d}) - \boldsymbol{u})^+ \right] \leq \boldsymbol{f}, \tag{8d}$$

$$\underline{\boldsymbol{v}} + \frac{1}{1-\gamma} \mathbb{E}\left[(-\boldsymbol{g}_0 + \boldsymbol{G}\Delta\boldsymbol{\xi} - \underline{\boldsymbol{v}})^+ \right] \leq 0, \tag{8e}$$

$$\overline{\boldsymbol{v}} + \frac{1}{1-\gamma} \mathbb{E}\left[(\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi} - \overline{\boldsymbol{g}} - \overline{\boldsymbol{v}})^+ \right] \leq 0. \tag{8f}$$

Associate Lagrange multipliers $\lambda, \boldsymbol{\nu}$ and $\boldsymbol{\mu}$ with constraints (8b),(8c) and (8d), respectively. Let $z^\star$ denote the optimal value of any (primal or dual) variable z in (8). We assume that the set of primal-dual optimizers is nonempty and compact.

Definition III.1. *The vector of risk-sensitive locational marginal prices (risk-LMPs) is defined as*

$$\boldsymbol{\pi} := \lambda^\star \mathbf{1} - \boldsymbol{H}^\intercal \boldsymbol{\mu}^\star. \tag{9}$$

These prices are nodally uniform, i.e., the prices can vary across buses, but the same price π_i is observed by all participants at bus i. Our definition of risk-LMPs mimics that of LMPs derived from a deterministic ED problem (e.g., see [15]). Risk-LMPs comprise of two terms–$\lambda^\star \mathbf{1}$ and $\boldsymbol{H}^\intercal \boldsymbol{\mu}^\star$. The first defines a common base price across the network that emanates from the network-wide power balance constraint for the nominal scenario. The second term arises due to congestion and introduces locational dependency. Unlike the deterministic case, the congestion component in the CVaR-sensitive problem does not only depend on the nominal dispatch. That is, adverse scenarios can result in a non-zero congestion component despite an uncongested nominal dispatch.

Our first result relates risk-LMPs to the sensitivity of the optimal cost of (8) to nodal demands–a result that holds for LMPs derived from a deterministic ED problem.

Proposition 1. *$J^\star$ is convex in $\boldsymbol{d}$. Suppose $\boldsymbol{X}^\star := (\boldsymbol{H} - \boldsymbol{H}\boldsymbol{G}^\star)\,\Delta\boldsymbol{\xi}$ has a smooth cumulative distribution function and the optimal set of Lagrange multipliers of (8) is bounded. Then, $\boldsymbol{\pi} \in \partial_{\boldsymbol{d}} J^\star(\boldsymbol{d})$, where $\partial_{\boldsymbol{d}}$ computes the subdifferential set of $J^\star$.*

The pricing mechanism is incomplete without defining a settlement process, i.e., how every market participant is compensated under risk-LMPs. We adopt a similar model to existing literature for consumers, wherein they are charged $\pi_i d_i$ for consuming quantity d_i at bus i. Under the risk-sensitive model, generators incur an additional cost of maintaining reserve capacity in the form of $\boldsymbol{G}\Delta\boldsymbol{\xi}$ and must be compensated accordingly. We propose the dual multiplier $\boldsymbol{\nu}^\star$ to reflect the price of maintaining this reserve capacity. Thus, each generator i is paid $\pi_i [g_0^\star]_i$ for the nominal cost of generation and $\sum_{j=1}^n G_{ij}^\star \nu_i^\star$ for maintaining reserve capacity. On the other hand, renewable supplier i is paid $\pi_i [\xi_0]_i - \nu_i^\star$, corresponding to a payment for nominal supply and a penalty levied for their induced uncertainty. The penalty reflects the principle of *cost allocation based on cost causation*.

IV. Solving the CVaR-Sensitive ED Problem using Sample Average Approximation

We compute the dispatch and prices from (8) using N independent and identically distributed (iid) samples of renewable generation $\boldsymbol{\xi}^1, \ldots, \boldsymbol{\xi}^N$ as follows. We replace the expectations with empirical means in each constraint, e.g., the empirical mean variant of (8d) can be reformulated as

$$\boldsymbol{u} + \frac{1}{N(1-\beta)} \sum_{j=1}^{N} \left[\left(\boldsymbol{H}(\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi}^j + \boldsymbol{\xi}^j - \boldsymbol{d}) - \boldsymbol{u} \right)^+ \right] \le \boldsymbol{f}$$

$$\equiv \begin{cases} \boldsymbol{u} + \frac{1}{N(1-\beta)} \sum_{j=1}^{N} \boldsymbol{t}^j \le \boldsymbol{0}, \\ \boldsymbol{t}^j \ge \boldsymbol{H}(\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi}^j + \boldsymbol{\xi}^j - \boldsymbol{d}) - \boldsymbol{f} - \boldsymbol{u}, \quad \boldsymbol{t}^j \ge 0. \end{cases}$$

Proceeding similarly with (8e)–(8f), we arrive at the following SAA-based CVaR-sensitive ED problem.

$$\widehat{J}_N^\star(\boldsymbol{d}) := \text{minimize} \quad \boldsymbol{c}^\intercal \boldsymbol{g}_0, \tag{10a}$$

subject to

$$\boldsymbol{1}^\intercal(\boldsymbol{g}_0 + \boldsymbol{\xi}_0 - \boldsymbol{d}) = 0, \tag{10b}$$

$$\boldsymbol{G}^\intercal \boldsymbol{1} = \boldsymbol{1}, \tag{10c}$$

$$\boldsymbol{u} + \frac{1}{(1-\beta)N} \sum_{j=1}^{N} \boldsymbol{t}^j \le \boldsymbol{0}, \tag{10d}$$

$$\boldsymbol{t}^j \ge \boldsymbol{H}(\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi}^j + \boldsymbol{\xi}^j - \boldsymbol{d}) - \boldsymbol{f} - \boldsymbol{u}, \tag{10e}$$

$$\underline{\boldsymbol{v}} + \frac{1}{(1-\gamma)N} \sum_{j=1}^{N} \underline{\boldsymbol{s}}^j \le \boldsymbol{0}, \tag{10f}$$

$$\underline{\boldsymbol{s}}^j \ge -\boldsymbol{g}_0 + \boldsymbol{G}\Delta\boldsymbol{\xi}^j - \underline{\boldsymbol{v}}, \tag{10g}$$

$$\overline{\boldsymbol{v}} + \frac{1}{(1-\gamma)N} \sum_{j=1}^{N} \overline{\boldsymbol{s}}^j \le \boldsymbol{0}, \tag{10h}$$

$$\overline{\boldsymbol{s}}^j \ge \boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi}^j - \overline{\boldsymbol{g}} - \overline{\boldsymbol{v}}, \tag{10i}$$

$$\boldsymbol{t}^j, \underline{\boldsymbol{s}}^j, \overline{\boldsymbol{s}}^j \ge \boldsymbol{0}, \quad j = 1, \ldots, N$$

over $\boldsymbol{g}_0, \boldsymbol{G}, \boldsymbol{u}, \boldsymbol{t}^j, \underline{\boldsymbol{v}}, \overline{\boldsymbol{v}}, \underline{\boldsymbol{s}}^j, \overline{\boldsymbol{s}}^j$. In effect, (10) is an approximation to (8) for computing an optimal dispatch. This SAA-based ED problem can be solved as a linear program. Linear programming duality allows us to define electricity prices from this approximate problem. Specifically, let $\widehat{\lambda}_N, \widehat{\boldsymbol{\mu}}_N$ denote the dual multipliers for constraints (10b) and (10d), respectively.

Definition IV.1. *The vector of SAA-based risk-sensitive LMPs is defined as $\widehat{\boldsymbol{\pi}}_N = \widehat{\lambda}_N^\star \boldsymbol{1} - \boldsymbol{H}^\intercal \widehat{\boldsymbol{\mu}}_N^\star$.*

One can show that $\widehat{\boldsymbol{\pi}}_N \in \partial_{\boldsymbol{d}} \widehat{J}_N^\star(\boldsymbol{d})$, i.e., the SAA prices measure the sensitivity of the optimal cost of the SAA-based ED problem to the vector of nodal demands. The proof is similar to that of Proposition 1 and is omitted for brevity. Notice that since the N samples $\boldsymbol{\xi}^1, \ldots, \boldsymbol{\xi}^N$ are drawn randomly, the output of the SAA problem is itself random. Hence, the prices calculated from this sampled problem are random as well. Next, we study properties of these prices empirically.

A. Numerical Experiments On A Five-Bus Network Example

We consider the heavily-loaded PJM 5-bus network from the IEEE PES Power Grid Library v17.08 [16]. The network, depicted in Figure 1a, is augmented with renewable generation at buses 1, 2, and 4, with 2.3, 1.5, and 0.9 per unit capacity, respectively. Wind outputs from three wind power plants from the New York area were used from NREL's synthetic dataset [17], treating the tuple of wind power outputs every 5 minutes over 2008-2011 as a single sample, $\boldsymbol{\xi}$.

The SAA approach is an effective mechanism for solving CVaR-sensitive ED, as Figure 1b reveals. The variance of the prices at each bus drops below 6×10^{-5} with just $N = 100$ samples, and below 3×10^{-6} with $N = 1000$ samples. With that in mind, all other simulations use 1000 samples.

The CVaR-sensitive dispatch presents a natural extension to its commonly studied deterministic variant. As Figure 1d shows, scaling the forecast error by η and taking $\eta \downarrow 0$ recovers the deterministic ED solution and the risk-sensitive LMPs reduce to conventional LMPs. Notice that nominal dispatch costs increase as the SO becomes more risk-averse in enforcing line limits (see Figure 1c). Regulation prices exhibit similar behavior with increased risk aversion, per Figure 1e. Hence, greater risk aversion to line limit violations results in dispatchable generators collecting more payment for reserve provision. Renewable suppliers, on the other hand, incur higher penalties for their impending uncertainty.

V. Revenue Adequacy

The SO should ideally never run cash negative after settling all payments with market participants, i.e., the dispatch and pricing mechanism should be revenue adequate. In this section, we analyze when our risk-sensitive dispatch and pricing mechanism is revenue adequate. To that end, define the merchandising surplus (MS) as the aggregate payments received from consumers less the aggregate payments made to dispatchable and renewable suppliers:

$$\text{MS} = \boldsymbol{\pi}^\intercal \left(\boldsymbol{d} - \boldsymbol{g}_0^\star - \boldsymbol{\xi}_0 \right) - \boldsymbol{1}^\intercal \boldsymbol{G}^\star \boldsymbol{\nu}^\star + \boldsymbol{1}^\intercal \boldsymbol{\nu}^\star = -\boldsymbol{\pi}^\intercal \boldsymbol{p}_0^\star,$$

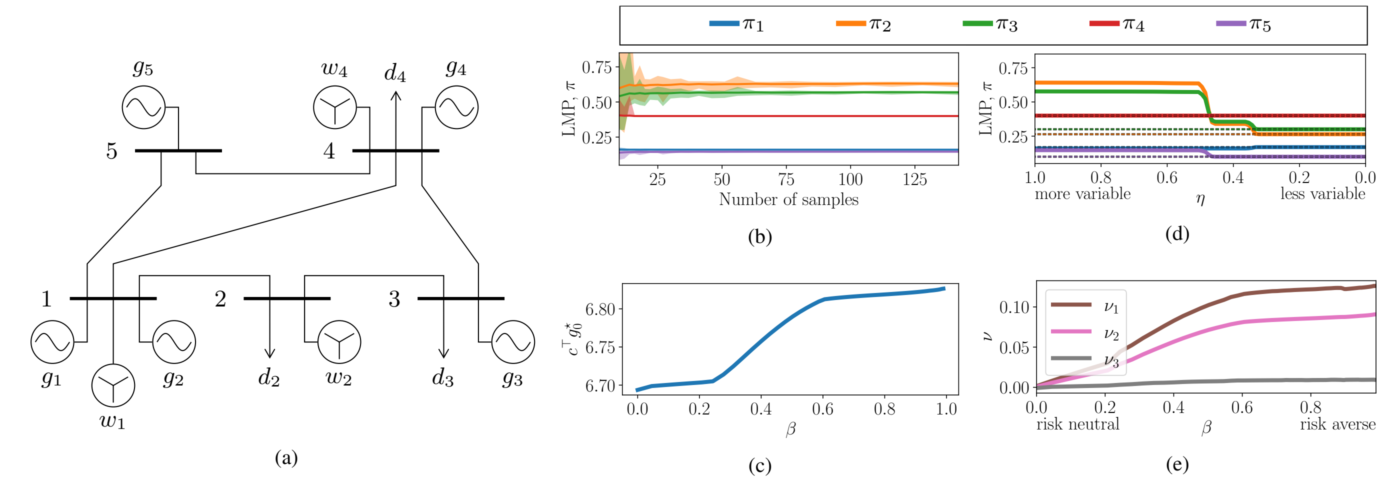

Fig. 1. Evaluation of the prices for the 5-bus network in (a), depicting the (b) mean and range as a function of sample complexity with $\gamma = \beta = 0.9$, (c) nominal cost of generation as a function of β with $\gamma = 0.6$, (d) effect of scaling forecast error by η for $\gamma = \beta = 0.6$, where the dashed line shows the LMP of the nominal ED, and (e) reserve price, ν, as a function of β with $\gamma = 0.6$.

where $\boldsymbol{p}_0^\star = \boldsymbol{g}_0^\star + \boldsymbol{\xi}_0 - \boldsymbol{d}$. We analyze conditions under which MS ≥ 0 from the CVaR-based ED problem.

Proposition 2. *Suppose $\boldsymbol{X}^\star := (\boldsymbol{H} - \boldsymbol{H}\boldsymbol{G}^\star)\Delta\boldsymbol{\xi}$ has a smooth cumulative distribution function. Then, the merchandising surplus is given by*

$$MS = \boldsymbol{\mu}^{\star,\intercal} \boldsymbol{H} \boldsymbol{p}_0^\star = \boldsymbol{\mu}^{\star,\intercal} \left(\boldsymbol{f} - CVaR_\beta \left[\boldsymbol{X}^\star\right]\right). \tag{11}$$

$MS \geq 0$, if $f_i \geq CVaR_\beta[X_i^\star]$ for each $i = 1, \ldots, 2\ell$ or $\boldsymbol{\mu}^\star = 0$.

Proposition 2 reveals that MS from our CVaR-sensitive ED problem arises due to congestion that results in price separation across different buses in the network. Without congestion in any scenario, $\boldsymbol{\mu}^\star = 0$, that implies MS $= 0$. Such a property is shared by MS obtained with LMPs from a deterministic ED problem. In fact, for a deterministic ED problem, MS equals the congestion rent $\boldsymbol{\mu}^{\star,\intercal}\boldsymbol{f}$. We recover that result by driving $\Delta\boldsymbol{\xi}$ to zero, that in turn makes $\boldsymbol{X}^\star$ identically zero making MS equal to $\boldsymbol{\mu}^{\star,\intercal}\boldsymbol{f}$ as is the case with conventional LMPs.

MS from our CVaR-sensitive ED problem can be viewed as the congestion rent with a modified line flow, $\boldsymbol{f} - \text{CVaR}_\beta\left[\boldsymbol{X}^\star\right]$. To gain more insights into the modifier, notice that $X_i^\star$ is composed of two terms: $\boldsymbol{H}_i\Delta\boldsymbol{\xi}$ and $\boldsymbol{H}_i(\boldsymbol{G}^\star\Delta\boldsymbol{\xi})$, where $\boldsymbol{H}_i$ denotes the i-th row of $\boldsymbol{H}$. The first term equals the induced flows on the i-th line due to nodal forecast errors $\Delta\boldsymbol{\xi}$. The second term equals the same from the dispatchable generator responses $\boldsymbol{G}^\star\Delta\boldsymbol{\xi}$ to said forecast errors. That is, $X_i^\star$ captures the net effect of forecast errors on the i-th line flow. The modifier satisfies

$$0 = \mathbb{E}[X_i^\star] \leq \text{CVaR}_\beta\left[X_i^\star\right] \leq \max X_i.$$

Thus, $\text{CVaR}_\beta\left[X_i^\star\right]$ increases from zero to $\max X_i$ as β ranges from zero to unity. While we cannot guarantee nonnegativity of MS for all problem instances, this range provides valuable insights into when we might expect it. Specifically, when β is small (the SO is close to being risk neutral) or $\max X_i$ is smaller than f_i (when forecast errors are small or do not induce large enough net flows on the i-th line), we expect MS ≥ 0. It is encouraging that we obtained MS ≥ 1.14 for all our experiments on the five-bus network.

VI. Conclusions

In this work, we derived and analyzed prices from a CVaR-sensitive ED problem where transmission line flow constraints and generation limits are imposed in a risk-sensitive fashion. We showed that these prices have properties similar to LMPs derived from a deterministic ED problem. Approximations to these prices were derived using sample average approximation (SAA) that were empirically shown to asymptotically converge. Forward settlements and revenue adequacy issues were also analyzed.

There are interesting future research directions that we aim to pursue. First, we want to incorporate unit commitment decisions and analyze CVaR-sensitive prices for day-ahead market operations. Second, we hope to analytically establish the asymptotic convergence of the SAA-based approximate prices. Third, we want to empirically compare our prices to current market practice.

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VII. Appendix (Proofs)

We make use of the following result (stated without proof).

Lemma VII.1. $\frac{d}{dy}\mathbb{E}_z\left[(z+y)^+\right] = \Pr\{z+y \geq 0\}$ *for z with a smooth cumulative distribution function and $y \in \mathbb{R}$.*

Proof of Proposition 1. Convexity of $J^\star$ follows from [18, Proposition 1]. Recall that $\boldsymbol{p}_0 = \boldsymbol{g}_0 + \boldsymbol{\xi}_0 - \boldsymbol{d}$ and $\boldsymbol{X} = (\boldsymbol{H} - \boldsymbol{H}\boldsymbol{G})\Delta\boldsymbol{\xi}$. Then, we have

$$\boldsymbol{H}(\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi} + \boldsymbol{\xi} - \boldsymbol{d}) - \boldsymbol{f} = \boldsymbol{H}\boldsymbol{p}_0 + \boldsymbol{X} - \boldsymbol{f}.$$

In the sequel, we use the notation $\boldsymbol{A}_i$ to denote the i-th row of a matrix or vector $\boldsymbol{A}$. Assign Lagrange multipliers $\underline{\boldsymbol{\phi}}$ and $\overline{\boldsymbol{\phi}}$ to (8e) and (8f), respectively. The Lagrangian of (8) is then

$$\begin{aligned}\mathcal{L}\left(\boldsymbol{g}_0, \boldsymbol{G}, \lambda, \boldsymbol{\nu}, \boldsymbol{\mu}, \underline{\boldsymbol{\phi}}, \overline{\boldsymbol{\phi}}\right) &:= \boldsymbol{c}^\intercal \boldsymbol{g}_0 + \lambda\left(\boldsymbol{1}^\intercal \boldsymbol{p}_0\right) - \boldsymbol{\nu}^\intercal\left(\boldsymbol{G}^\intercal\boldsymbol{1} - \boldsymbol{1}\right)\\
&+ \sum_{i=1}^{2\ell} \mu_i \left(u_i + \frac{1}{1-\beta}\mathbb{E}\left[\left(\boldsymbol{H}_i\boldsymbol{p}_0 + X_i - f_i - u_i\right)^+\right]\right)\\
&+ \sum_{i=1}^{n} \underline{\phi}_i \left(\underline{v}_i + \frac{1}{1-\gamma}\mathbb{E}\left[\left(-g_{0i} + (\boldsymbol{G}\Delta\boldsymbol{\xi})_i - \underline{v}_i\right)^+\right]\right)\\
&+ \sum_{i=1}^{n} \overline{\phi}_i \left(\overline{v}_i + \frac{1}{1-\gamma}\mathbb{E}\left[\left(g_{0i} - (\boldsymbol{G}\Delta\boldsymbol{\xi})_i - \overline{g}_i - \overline{v}_i\right)^+\right]\right).\end{aligned}$$

By hypothesis, $X_i = (\boldsymbol{H} - \boldsymbol{H}\boldsymbol{G})_i\Delta\boldsymbol{\xi}$ has a smooth cumulative distribution. Then, $\mathcal{L}$ at optimality of all primal and dual variables except u_i is a smooth function of u_i. Setting its derivative at $u_i^\star$ to zero using Karush-Kuhn-Tucker (KKT) optimality conditions yields

$$\mu_i^\star\left(1 - \frac{1}{1-\beta}\Pr\{\boldsymbol{H}_i\boldsymbol{p}_0^\star + X_i^\star - f_i - u_i^\star \geq 0\}\right) = 0, \quad (12)$$

from Lemma VII.1. Denote by $\mathcal{L}^\star$, the value of $\mathcal{L}$ at optimality. Lemma VII.1 implies continuous differentiability of the constraint functions, while nonemptiness and compactness of the primal-dual optimal set implies satisfaction of Mangasarian-Fromovitz regularity [19]. Thus, we can apply [20, Theorem 5.3] to derive the following.

$$\begin{aligned}\partial_{d_j} J^\star(\boldsymbol{d}) \ni \frac{\partial \mathcal{L}^\star}{\partial d_j}&\\
&\overset{(a)}{=} \lambda^\star - \sum_{i=1}^{2\ell}\frac{\mu_i^\star H_{ij}}{1-\beta}\Pr\{\boldsymbol{H}_i\boldsymbol{p}_0^\star + X_i^\star - f_i - u_i^\star \geq 0\}\\
&\overset{(b)}{=} \lambda^\star - \sum_{i=1}^{2\ell}\mu_i^\star H_{ij}\\
&= \pi_j,\end{aligned}$$

where (a) follows from Lemma VII.1 and (b) from (12). This completes the proof of Proposition 1. □

Proof of Proposition 2. Since $\boldsymbol{1}^\intercal\boldsymbol{p}_0^\star = 0$, we have

$$\mathrm{MS} = -\boldsymbol{\pi}^\intercal\boldsymbol{p}_0^\star = -\left(\lambda^\star\boldsymbol{1} - \boldsymbol{H}^\intercal\boldsymbol{\mu}^\star\right)^\intercal\boldsymbol{p}_0^\star = \boldsymbol{\mu}^{\star,\intercal}\boldsymbol{H}\boldsymbol{p}_0^\star.$$

To simplify $\boldsymbol{\mu}^{\star,\intercal}\boldsymbol{H}\boldsymbol{p}_0^\star$, consider i for which $\mu_i^\star > 0$. Then,

$$\begin{aligned}(12) &\implies \Pr\{X_i^\star \geq f_i + u_i^\star - \boldsymbol{H}_i\boldsymbol{p}_0^\star\} = 1-\beta\\
&\implies \mathrm{CVaR}_\beta[X_i^\star] = \mathbb{E}[X_i^\star | X_i^\star \geq f_i + u_i^\star - \boldsymbol{H}_i\boldsymbol{p}_0^\star].\end{aligned} \quad (13)$$

The second line utilizes the smoothness of the distribution of $X_i^\star$. Complementary slackness condition on the i-th line flow constraint further gives

$$\mu_i^\star u_i^\star + \frac{\mu_i^\star}{1-\beta}\mathbb{E}\left[\left(\boldsymbol{H}_i\boldsymbol{p}_0^\star + X_i^\star - f_i - u_i^\star\right)^+\right] = 0. \quad (14)$$

Notice that $\mathbb{E}\left[Z^+\right] = \mathbb{E}\left[Z | Z \geq 0\right]\Pr\{Z \geq 0\}$ for any random variable Z. Using this together with (13), we simplify the second term in the relation to

$$\begin{aligned}&\frac{\mu_i^\star}{1-\beta}\mathbb{E}\left[\left(\boldsymbol{H}_i\boldsymbol{p}_0^\star + X_i^\star - f_i - u_i^\star\right)^+\right]\\
&= \mu_i^\star\mathbb{E}\left[\boldsymbol{H}_i\boldsymbol{p}_0^\star + X_i^\star - f_i - u_i^\star \;|X_i^\star \geq f_i + u_i^\star - \boldsymbol{H}_i\boldsymbol{p}_0^\star\right]\\
&\overset{(a)}{=} \mu_i^\star\boldsymbol{H}_i\boldsymbol{p}_0^\star - \mu_i^\star f_i - \mu_i^\star u_i^\star + \mathbb{E}\left[X_i^\star \;|X_i^\star \geq f_i + u_i^\star - \boldsymbol{H}_i\boldsymbol{p}_0^\star\right]\\
&\overset{(b)}{=} \mu_i^\star\boldsymbol{H}_i\boldsymbol{p}_0^\star - \mu_i^\star f_i - \mu_i^\star u_i^\star + \mathrm{CVaR}_\beta\left[X_i^\star\right].\end{aligned}$$

Here, (a) follows from linearity of expectation and (b) is a consequence of (13). Using the above relation in (14) yields

$$\mu_i^\star\boldsymbol{H}_i\boldsymbol{p}_0^\star = \mu_i^\star f_i - \mathrm{CVaR}_\beta[X_i^\star].$$

The result follows from summing the above over $i = 1, \ldots, 2\ell$. □