Impact of Sharing Driving Attitude Information: A Quantitative Study on Lane Changing

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Abstract—**Autonomous vehicles (AVs) are expected to be an integral part of the next generation of transportation systems, where they will share the transportation network with human-driven vehicles during the transition period. In this work, we model the interactions between vehicles (two AVs or an AV and a human-driven vehicle) in a lane changing process by leveraging the Stackelberg game. We explicitly model driving attitudes for both vehicles involved in lane changing. We design five cases, in which the two vehicles have different levels of knowledge, and make different assumptions, about the driving attitude of the rival. We conduct theoretical analysis and simulations for different cases in two lane changing scenarios, namely changing lanes from a higher-speed lane to a lower-speed lane, and from a lower-speed lane to a higher-speed lane. We use four metrics (fuel consumption, discomfort, minimum distance gap and lane change success rate) to investigate how the performance of a single vehicle and that of the system will be influenced by the level of information sharing, and whether a vehicle trajectory optimized based on selfish criteria can provide system-level benefits.**

I. INTRODUCTION

The transportation system of tomorrow is envisioned to be connected and automated. While a vast array of safety, mobility, and environmental benefits are anticipated from deploying these technologies [1], [2], [3], [4], [5], [6], training automated vehicles to navigate the transportation network remains an underdeveloped area of research. The difficulty is two-fold: first, to enhance the adoption rate of autonomous vehicles (AVs) and foster a seamless transition from human-driven vehicles to autonomous ones, their driving patterns should mimic those of human drivers, so as to induce a familiar sense of travel in passengers that can facilitate acceptance of AVs as a safe mode of transportation. Second, during the transition period to a fully-automated transportation system, a heterogeneous traffic stream of both human-driven and autonomous vehicles need to share the transportation network. For the interactions between human-driven and autonomous vehicles to be safe, they need to replicate some aspects of human driving.

With the development of machine learning techniques, driving behavior can be sensed and modeled to high levels of accuracy. This provides an opportunity to build human-like AVs and enhance interactions between human drivers and the control entity in AVs [7], [8], [9]. However, such advancements may put forward concerns on whether an AV may gain advantage over human drivers by using its onboard sensors and computational resources to collect and analyze data from human drivers in real-time. On the other hand, there are concerns on human drivers exploiting their knowledge of AVs, e.g., taking advantage of the fact that AVs tend to maintain larger safety gaps. As such, there is interest in *quantifying* the value of knowing other vehicles' driving attitude for a vehicle looking to maximize its own utility. A further interesting problem is to analyze how knowing different extent of knowledge by AVs or human drivers on other vehicles may impact system-level performance metrics.

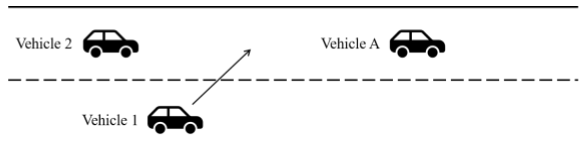

Fig. 1. Lane Changing Case

In this context, this paper models the lane-changing process of an AV, where the AV (vehicle 1 in Fig. 1) intends to change lanes and insert itself downstream of a (human-driven or an autonomous) vehicle (vehicle 2 in Fig. 1) in the target lane. We consider this process to be a continuous one, involving continuous assessment of two vehicles of their relative positions, and revising their trajectories. We follow a game-theoretic approach in order to capture the interactions between vehicles, and investigate the degree to which one entity's perception of the driving attitude of the other affects the success rate of lane changing, the fuel consumption of the two vehicles, and safety and comfort levels of their passengers.

There are a number of studies in the literature that focus on developing models for vehicles while considering the interactions between them. [10] utilizes hierarchical reasoning to model interactions between vehicles and reinforcement learning to update level-k policy in an environment of all level-(k-1) vehicles. They simulate macroscopic traffic with drivers of different levels of reasoning. [11] and [12] also develop vehicle models by the method of hierarchical reasoning, but in different application scenarios. [13] performs an interaction-aware motion

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prediction model for all surrounding vehicles. In their work, the probability of different maneuvers can be calculated based on observed trajectory.

A number of studies focus on the interaction between human drivers and robot drivers. [14] assumes that humans presume robots to behave rationally. As such, robots can predict human behavior and take advantage of it in their motion planning. [15] uses microscopic traffic simulations to show that the average travel time decreases by a factor of 4 if altruistic AVs are introduced to traffic streams. [16] and [17] leverage Kalman filter and machine learning to detect and recover sensor attacks or sensor faults in AVs from surrounding environment. [18] models surrounding vehicles based on level-k game theory. Optimal decisions for an AV at roundabouts are computed after estimating the driver type of the opponent vehicle. [19] and [20] model the interaction between AVs and human drivers using dynamic game theory. [21] utilizes inverse reinforcement learning to model human drivers, assuming they are perfectly rational. AVs can thus purposefully elicit desired changes in the human state. [22] claims that by interacting with humans, robots can learn the humans' internal states and thus optimize their operations. However, this information might lead to the robot taking advantage of the humans.

The difficulties in modeling the interactions between human drivers and robot drivers are mainly from (1) the uncertainty and possible irrationality of human behavior [23], and the fact that not all human drivers have the same model [24], and (2) the difficulty of predicting AV intentions by human drivers [25]. The challenge in ensuring safety without overly impacting performance is acknowledged widely [26].

In this paper, we focus on the interaction of two vehicles involved in a lane changing scenario: the vehicle who intends to change lanes, and its upstream vehicle in the target lane. For each vehicle, we quantify multiple important metrics (including fuel cost, comfort level, safety level, and lane changing success rate) under different levels of knowledge of the other vehicle's driving attitude (measured by its aggressiveness level). Additionally, we investigate how these metrics change at the system level under different levels of information sharing. Insights from this research could help inform regulations on the degree of information sharing. Additionally, this work could shed light on whether introducing incentives to encourage information sharing by drivers could produce system-level benefits.

The rest of this paper is organized as follows. In Section II.A, we introduce our Stackelberg game-based lane changing model. In Section II.B, we present a simulation environment and different case settings for different degrees of information sharing, and list four metrics to measure the individual vehicle and system performance. We then present a number of theoretical observations under simplified models in Section III.A, and analyze the simulation results obtained in Matlab for the full model in Section III.B. We summarize the takeaways in Section IV.

II. METHOD

A. Stackelberg Game-Based Lane Changing Model

We model the lane changing process using a Stackelberg game [27]. In this game, the subject vehicle who intends to change lanes is the leader (vehicle 1 in Fig. 1), and the vehicle upstream the subject vehicle in the target lane is the follower (vehicle 2 in Fig. 1). The game starts by vehicle 1 deciding to either "Stay in the current lane" or "Change lanes", where each of these decisions is accompanied with a decision on longitudinal acceleration. Following vehicle 1's decision, vehicle 2 decides its longitudinal acceleration accordingly. Assuming both vehicles are rational players, each vehicle solves a bi-level optimization problem to maximize its payoff, denoted by U^i, $i \in \{1,2\}$, while assuming that the other vehicle aims to maximize its own payoff.

Following the work by [24], we assume the payoffs of the two vehicles involved in the lane-changing process are affected by their perception of a suitable safety gap with other vehicles, their level of protectiveness of the space ahead of them, and their level of comfort, where all these factors are a function of the drivers' driving attitudes. The driving attitude of a vehicle is modeled by a parameter q, which can be viewed as the aggressiveness factor of the driver and bounded within the range $[q_{min}, q_{max}]$.

Note that while we leverage the work from [24] in our modeling, there are several important modifications we made to make the model more representative of the lane changing behaviour in practice: (i) We utilize a more realistic trajectory model in our work – while [24] utilizes a piece-wise constant acceleration function to describe a vehicle's longitudinal motion, we adopt a time-based quintic function as our trajectory function to guarantee a smooth overall trajectory [28]. (ii) Rather than assuming that the trajectories are computed by a central controller, we assume each vehicle will compute its own trajectory based on its knowledge of the rival. (iii) We explicitly consider the computation/perception-reaction delay and the control period for both vehicles. (iv) We capture a more realistic driving scenario by incorporating a number of additional parameters that are related to aggressiveness, e.g., ideal headway and lateral average speed. We introduce "functions of aggressiveness" to incorporate these factors in our model. (v) We assume that a vehicle's safety payoff is a function of the smallest gap it maintains with other vehicles. For example, in Fig. 1, after vehicle 1 changes lanes, it will create a time gap with vehicle A and one with vehicle 2. The smaller of these two time gaps will be considered as the final safety payoff. In the remaining of this section, we will first layout the vehicle trajectory, and then mathematically present the total payoff of vehicles as a function of their trajectories.

A vehicle's trajectory, including its longitudinal and lateral movements, is modeled by Eq. (1).

$$\begin{cases} x(t) = a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0 \\ y(t) = b_5t^5 + b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0 \end{cases} \quad (1)$$

where the coefficients a_i and b_i can be determined by the boundary conditions in Eqs. (2) and (3).

$$\begin{cases} x(t_0) = x_0, \dot{x}(t_0) = v_{x,0}, \ddot{x}(t_0) = a_{x,0} \\ y(t_0) = y_0, \dot{y}(t_0) = v_{y,0}, \ddot{y}(t_0) = a_{y,0} \end{cases} \quad (2)$$

$$\begin{cases} x(t_{lon}^f) = x_{t_{lon}^f},\ \dot{x}(t_{lon}^f) = v_{x,t_{lon}^f},\ \ddot{x}(t_{lon}^f) = a_{x,t_{lon}^f} \\ y(t_{lat}^f) = y_{t_{lat}^f},\ \dot{y}(t_{lat}^f) = v_{y,t_{lat}^f},\ \ddot{y}(t_{lat}^f) = a_{y,t_{lat}^f} \end{cases} \quad (3)$$

In Eq. (2), t_0 is the starting time of the planned trajectory, and its value is context-dependent. For example, if vehicle 1 is planning its own trajectory, $t_0 = 0$, since this vehicle is aware of its real-time motion data. However, if vehicle 1 is interested in predicting vehicle 2's trajectory, there exists a time delay of t_d^1 in obtaining vehicle 2's motion data, hence $t_0 = -t_d^1$. Let x_0, $v_{x,0}$, and $a_{x,0}$ denote the longitudinal position, velocity and acceleration at time t_0, respectively. Similarly, let y_0, $v_{y,0}$, and $a_{y,0}$ be the lateral position, velocity, and acceleration at time t_0. Let us define t_{lat}^f and t_{lon}^f as the ending time of lateral and longitudinal motions, respectively. Note that t_{lat}^f and t_{lon}^f may have different values, which indicates that the lateral and longitudinal motions do not necessarily take similar time to complete. In Eq. (3), $x_{t_{lon}^f}$, $y_{t_{lat}^f}$, v_{x,t_{lon}^f}, v_{y,t_{lat}^f}, a_{x,t_{lon}^f}, and a_{y,t_{lat}^f} denote longitudinal and lateral positions, velocities and acceleration values at the ending time of the lane changing process. We make the same assumptions as in [28] to model the status of the vehicle at the end of the lane changing process, i.e., the vehicle will aim to achieve the same speed as its leading vehicle, with no longitudinal acceleration, no lateral velocity, and zero acceleration.

We assume that the lateral average velocity during the lane changing process, v_l, is a function of the aggressiveness factor of vehicle 1 – the more aggressive the driver, the larger the value of v_l, as formulated in Eq. (4).

$$v_l = v_l^{\min} + \left(v_l^{\max} - v_l^{\min}\right) \cdot \frac{q_1 - q_{\min}}{q_{\max} - q_{\min}} \quad (4)$$

where $v_l^{\max}$ and $v_l^{\min}$ are the maximum and minimum lateral velocity of the most aggressive ($q_1 = q_{\max}$) and the most cautious ($q_1 = q_{\min}$) drivers, respectively. The ending time of the lateral motion, t_{lat}^f, can thus be computed as in Eq. (5). Hence, $x_{t_{lon}^f}$ and $y_{t_{lat}^f}$ remain the only free decision variables.

$$t_{lat}^f = t_0 + \frac{\left| y_{t_{lat}^f} - y_0 \right|}{v_l} \quad (5)$$

In the scenario shown in Fig. 1, vehicle 1 has both longitudinal and lateral motions, while vehicles 2 and A only have longitudinal motions.

A set of constraints need to be incorporated in motion planning to ensure that the planned trajectory is safe, and to account for the vehicle's mechanical capacity. These constraints impose limitations on vehicular speed, acceleration, and jerk, and guarantee collision avoidance. Details of these constraints can be found in [29].

We adopt the same payoff functions as in [24]. The overall payoff U_{all} is a function of the space payoff, U_{space}, and the safety payoff, U_{safety}, as shown in Eq. (6). Safety payoff reflects the willingness to maintain a large-enough gap with other vehicles to avoid collisions, and the space payoff reflects the competing objective of attempting to keep one's relative position in the traffic stream (i.e., avoiding other vehicles from injecting themselves downstream the vehicle).

$$\begin{aligned} U_{\text{all}} = \quad & f_w(J) \cdot ((1 - \beta(q)) \cdot U_{\text{safety}}\left(x^1_{t_{lon}^f}, y^1_{t_{lat}^f}, x^2_{t_{lon}^f}, t_b\right) \\ & + \beta(q) \cdot U_{\text{space}}\left(x^1_{t_{lon}^f}, y^1_{t_{lat}^f}, x^2_{t_{lon}^f}\right) + 1) - 1 \end{aligned} \quad (6)$$

where $f_w(J)$, as defined in Eq. (6), is the penalty on jerk (the smaller the jerk, the more comfortable the ride, and thus the larger the payoff). $\beta(q)$, as defined in Eq. (8), is the weight to balance space and safety payoffs (a higher weight indicates a more aggressive driver). t_b, as define in Eq. (9), is the ideal safe headway (the driver achieves its maximum safety payoff when its time gap with any other vehicle is larger than t_b).

$$f_w(J) = \exp\left[-\frac{\int_{t_0}^{t_{lon}^f} J^2(t)\mathrm{d}t}{w \cdot \left(t_{lon}^f - t_0\right)}\right] \quad (7)$$

$$\beta(q) = \frac{1}{1 + e^{-q}} \quad (8)$$

$$t_b = t_{\max} - (t_{\max} - t_{\min}) \cdot \frac{q_1 - q_{\min}}{q_{\max} - q_{\min}} \quad (9)$$

where $t_{\max}$ and $t_{\min}$ are the maximum and minimum ideal time gaps of the most cautious ($q_1 = q_{\min}$) and the most aggressive ($q_1 = q_{\max}$) drivers, respectively. For equations to compute U_{safety} and U_{space}, refer to [24].

As shown in [24], if driver aggressiveness factors are publicly known to everyone, then the final solution of the Stackelberg game will be:

$$\left(x^{1*}_{t^f_{lon}}, y^{1*}_{t^f_{lat}}\right) = \arg\max_{\left(x^1_{t^f_{lon}}, y^1_{t^f_{lat}}\right)\in\Gamma^1} \left(\min_{x^2_{t^f_{lon}} \in \gamma^2} U^1_{\text{all}}\left(x^1_{t^f_{lon}}, y^1_{t^f_{lat}}, x^2_{t^f_{lon}}, q_1\right)\right) \quad (10)$$

$$\gamma^2\left(x^1_{t^f_{lon}}, y^1_{t^f_{lat}}\right) \triangleq \{\xi \in \Gamma^2 : U^2_{\text{all}}\left(x^1_{t^f_{lon}}, y^1_{t^f_{lat}}, \xi, q_2\right) \geq U^2_{\text{all}}\left(x^1_{t^f_{lon}}, y^1_{t^f_{lat}}, x^2_{t^f_{lon}}, q_2\right), \forall x^2_{t^f_{lon}} \in \Gamma^2\} \quad (11)$$

$$x^{2*}_{t^f_{lon}} = \arg\min_{x^2_{t^f_{lon}} \in \gamma^2} U^1_{\text{all}}\left(x^{1*}_{t^f_{lon}}, y^{1*}_{t^f_{lat}}, x^2_{t^f_{lon}}, q_1\right) \quad (12)$$

where for $i = 1,2$, U^i_{all} denotes the overall payoff of vehicle i, q_i denotes the aggressiveness factor of vehicle i, $x^i_{t^f_{lon}}$ denotes the longitudinal position of vehicle i at time t^f_{lon}, Γ^i denotes the strategies (action candidates) of vehicle i. $y^1_{t^f_{lat}}$ denotes the lateral position of vehicle 1 at time t^f_{lat}. $\left(x^{1*}_{t^f_{lon}}, y^{1*}_{t^f_{lat}}, x^{2*}_{t^f_{lon}}\right)$ represent the optimal solutions for vehicles 1 and 2.

However, each of the vehicles 1 and 2 may not have perfect knowledge of the other's aggressiveness factor. Moreover, they may have different communication and computation performances. Accordingly, the interaction between the two vehicles during the lane changing process can be captured as in Fig. 2. As the figure shows, vehicles 1 and 2 independently compute the equilibrium state based on their knowledge/perception of the other. Taking vehicle 1 as an example, it obtains vehicle 2's outdated motion data (with delay 1), and computes the solution of the Stackelberg game based on its knowledge of the aggressiveness factors of both vehicles: q_1 and q_{12}, where q_1 is the true aggressiveness factor of vehicle 1, and q_{12} is vehicle 1's perception of vehicle 2's aggressiveness factor. At this iteration, the planned trajectory within the first control period will be executed. Next, the computation process will repeat in a receding-horizon until vehicle 1 changes lane successfully or the trial time reaches the pre-defined maximum value $T_{\max}$.

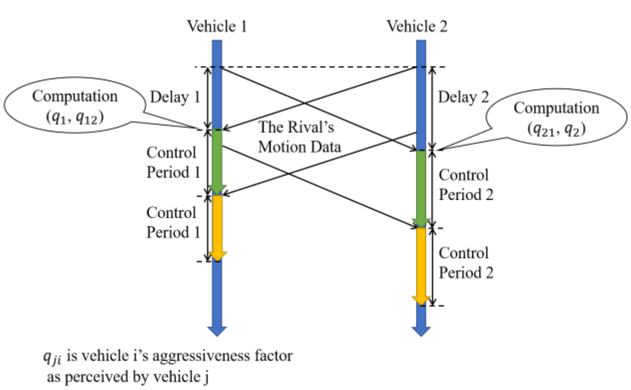

Fig. 2. Two Vehicles Interaction Diagram

B. Simulation Setting

We consider multiple lane-changing scenarios that differ in the extent each vehicle knows about the other's aggressiveness factor. More specifically, we consider the following five cases: (i) Vehicles 1 and 2 have complete knowledge of the aggressiveness factor of each other, i.e., $q_{21} = q_1$ and $q_{12} = q_2$. (ii) Vehicle 2 has complete knowledge of vehicle 1's aggressiveness factor, while vehicle 1 assumes vehicle 2 to be the most aggressive, i.e., $q_{21} = q_1$ and $q_{12} = q_{\max}$. (iii) Vehicle 2 has complete knowledge of vehicle 1's aggressiveness factor, while vehicle 1 assumes vehicle 2 to be the most cautious, i.e., $q_{21} = q_1$ and $q_{12} = q_{\min}$. (iv) Vehicle 1 has complete knowledge of vehicle 2's aggressiveness factor, while vehicle 2 assumes vehicle 1 to be the most aggressive, i.e., $q_{12} = q_2$ and $q_{21} = q_{\max}$. (v) Vehicle 1 has complete knowledge of vehicle 2's aggressiveness factor, while vehicle 2 assumes vehicle 1 to be the most cautious, i.e., $q_{12} = q_2$ and $q_{21} = q_{\min}$.

For each case, we evaluate four metrics: (i) the total fuel cost of vehicles 1 and 2, (ii) the total accumulated discomfort along the trajectories of vehicles 1 and 2, (iii) the minimum distance gap with other vehicles along the trajectories of vehicles 1 and 2, and (iv) the success rate of lane changing. Fuel cost and discomfort level are measured by the integral of the square of acceleration and jerk, respectively.

In our study, we first find the equilibrium solutions analytically, without considering safety and vehicle mechanics constraints. Next, we use simulations that incorporate these constraints to obtain the equilibrium solutions in more realistic settings. We conduct simulations by generating a total of 49 instances in which $q_1 \times q_2 \in \{-3,-2,-1,0,1,2,3\}^2$, where q_{min} is set to -3 and q_{max} to 3. Similarly as in [28], for each instance we simulate two scenarios: in scenario 1, vehicle 1 changes from a high-speed lane (60 km/h) to a low-speed lane (40 km/h); in scenario 2, vehicle 1 changes from a low-speed lane (60 km/h) to a high-speed lane (80 km/h). Table I summaries parameters, along with their definitions and assumed values in simulations.

III. RESULT ANALYSIS

A. Theoretical Intuition

We assume that during the lane changing process, avoiding collisions and respecting a vehicle's mechanical constraints have the highest priority, balancing the space and safety payoffs has the second highest priority, and comfort has the least priority. To enforce the first priority, minimum gaps and the vehicles' mechanical constraints are incorporated as constraints. To address the second priority, we set w in Eq. (7) to a large value, which results in $f_w(J)$ approaching 1 and U_{all} approximating to a linear combination of U_{safety} and U_{space}.

TABLE I

PARAMETER DEFINITIONS AND VALUES

Parameter	Value	Definition
t_d^i	0.7 secs	Computation/perception-reaction delay of vehicle i
t_c^i	0.7 secs	Control period of vehicle i
$v_{x,0}^1$	60 km/h	Initial velocity of vehicle 1
$v_{x,0}^2$	40 km/h or 80 km/h	Initial velocity of vehicle 2
q_i	$\in \{-3,-2,-1,0,1,2,3\}$	Aggressiveness factor of vehicle i
q_{ji}	$\in \{-3,-2,-1,0,1,2,3\}$	Aggressiveness factor of vehicle i as perceived by vehicle j
w	100	Weight in Eq. (7) to compute payoff's penalty on jerk
t_b	$\in [t_{b,\min}, t_{b,\max}] = [1,3]$ secs	Ideal time gap between vehicles
v_l	$\in [v_{l,min}, v_{l,max}] = [0.5,1.75]$ m/s	Average lateral velocity in the lane changing process
t_{lon}^f	10 secs	Length of the prediction horizon (duration of the longitudinal motion during lane changing)
x_0^1	$v_{x,0}^2$ meters	Initial position of vehicle 1
x_0^2	0 meter	Initial position of vehicle 2
x_0^A	$3v_{x,0}^2$ meters	Initial position of vehicle A
Variable	**Domain**	**Definition**
$x_{t_{\mathrm{lon}}^f}^1$	$\in [x_0^1, x_{t_{\mathrm{lon}}^f}^A]$ meters	Longitudinal position of vehicle 1 at t_{lon}^f
$y_{t_{\mathrm{lat}}^f}^1$	$\in \{0, 3.5\}$ meters	Lateral position of vehicle 1 at t_{lat}^f
$x_{t_{\mathrm{lon}}^f}^2$	$\in [x_0^2, x_{t_{\mathrm{lon}}^f}^A]$ meters	Longitudinal position of vehicle 2 at t_{lon}^f

TABLE II

OPTIMAL STRATEGY FOR VEHICLE 2 WITHOUT ACCOUNTING FOR SAFETY AND VEHICLE MECHANICS CONSTRAINTS

Vehicle 1 Action	Driving Attitude of Vehicle 2	
	Aggressive	Cautious
Vehicle 1 changes lane	$\frac{\beta(q_2)}{6} \geq \frac{1-\beta(q_2)}{t_b^2}$ Stay as close to vehicle 1 as possible	$\frac{\beta(q_2)}{6} < \frac{1-\beta(q_2)}{t_b^2}$ Maintain a time headway that is no less than t_b^2
Vehicle 1 stays in the current lane	$\frac{\beta(q_2)}{3} \geq \frac{1-\beta(q_2)}{t_b^2}$ Maintain a larger longitudinal position than vehicle 1	$\frac{\beta(q_2)}{3} < \frac{1-\beta(q_2)}{t_b^2}$ Maintain a time headway that is no less than t_b^2

Next we will first conduct theoretical analysis without considering the safety and vehicle mechanics constraints. Then later in Section III.B, we will use simulations to conduct extensive analyses that take into account of these constraint sets.

Table II shows the optimal actions for vehicle 2 in response to an action by vehicle 1. For example, if vehicle 1 chooses to change to vehicle 2's lane, vehicle 2 will attempt to be as close to vehicle 1 as possible if it is aggressive enough (i.e., $\frac{\beta(q_2)}{6} \geq \frac{1-\beta(q_2)}{t_b^2}$); otherwise it will maintain a time headway that is no less than t_b^2. Following this policy, vehicle 2 can maximize its payoff U_{all}^2.

As discussed in Section II.A, one of the contributions of this work is to consider a vehicle's ideal safety gap to be a function of the driver's driving attitude (aggressiveness). As the results presented in Table II demonstrate, if vehicle 2's ideal safety gap t_b^2 is *not* a function of its aggressiveness factor, the optimal action of vehicle 2 will not depend on the driving attitude of vehicle 1. This indicates that even if vehicle 1 has a wrong estimation for vehicle 2's aggressiveness factor (i.e., $q_{12} \neq q_2$), as long as inequalities $\frac{\beta(q_{12})}{6} < \frac{1-\beta(q_{12})}{t_b^2}$ and $\frac{\beta(q_2)}{6} < \frac{1-\beta(q_2)}{t_b^2}$ hold, its prediction of vehicle 2's behavior remains unchanged. However, in real driving scenarios, people with different driving attitudes will typically require different safety gaps. This also motivates our modification of the model in [24] to assumes that the gap is dependent of the driver's aggressiveness factor.

Table III shows the optimal $x_{t_{\mathrm{lon}}^f}^1$ and its corresponding payoff U_{all}^1 for different actions of vehicle 1 and different aggressiveness factor levels of vehicle 2, without considering the safety gap and vehicle mechanical constraints. The optimal action $y^1_{t^f_{lat}}$ is determined by comparing the payoffs in Table III. If vehicle 2 is aggressive, vehicle 1 will choose to change lanes if Eq. (13) holds, and stay in the original lane otherwise.

$$\begin{aligned}&(1-\beta(q_1))\cdot\left(\frac{2t_{\min}}{t^1_b}-1\right)+\beta(q_1)\cdot\frac{t_{\min}}{3}\\&>(1-\beta(q_1))+\beta(q_1)\cdot\left(\frac{2t_{\min}}{3}-1\right)\end{aligned}\tag{13}$$

where $t_{\min}=(l_{car}+l^{gap}_{\min})/\left(v^F_{t^f_{lon}}\right)$ is the minimum time headway ($v^F_{t^f_{lon}}$ is the following vehicle's velocity at time t^f_{lon}).

If vehicle 2 is cautious, vehicle 1 will change lanes if Eq. ([condition lc_2]) holds, and stay in the original lane otherwise.

$$\begin{aligned}&(1-\beta(q_1))\cdot\min\left(\frac{2t^2_b}{t^1_b}-1,1\right)+\beta(q_1)\cdot\frac{t^2_b}{3}\\&>(1-\beta(q_1))+\beta(q_1)\cdot\left(\frac{2t^2_b}{3}-1\right)\end{aligned}\tag{14}$$

If $t^2_b>t^1_b$, this inequality Eq. (14) will always hold, which indicates vehicle 1 will always change lanes if it is more aggressive than vehicle 2.

From Eqs. (13) and (14), we can see that when vehicle 2 is aggressive, vehicle 1's optimal decision does not depend on the exact value of q_2 (as q_2 does not appear in Eq. (13)). While in the case that vehicle 2 is cautious, vehicle 2's aggressiveness extent will influence vehicle 1's optimal action.

When considering constraints on safety gap and vehicle mechanics (e.g., constraints that enforce speed, acceleration, jerk and collision avoidance), optimal solutions may deviate from those presented in Tables II and III. In our simulation model, $x^1_{t^f_{lon}}$ and $y^1_{t^f_{lat}}$ are vehicle 1's decision variables that indicate the longitudinal and lateral positions of this vehicle at the end of the planning horizon. The decision variable $y^1_{t^f_{lat}}$ determines the lane in which vehicle 1 is positioned at the end of the planning horizon, indicating whether the lane changing process has been completed successfully. The decision variable $x^1_{t^f_{lon}}$ indicates the detailed trajectory of the vehicle.

B. Experimental Results

In this section, we compare fuel cost, discomfort, distance gap and lane changing success rate for different cases and different scenarios. Fig. 3 and Fig. 4 are box-plots for lane changing scenarios from 60 km/h to 40 km/h and from 60 km/h to 80 km/h, respectively. The values showed in these figures are sum of the values for both vehicle trajectories.

Fig. 3 demonstrates that when the subject vehicle (vehicle 1) is moving from a higher-speed lane into a lower-speed lane, case (ii) provides the best general outcome (the smallest fuel cost, highest safety gap, and least discomfort), where vehicle 1 assumes that vehicle 2 is the most aggressive and vehicle 2 has complete knowledge of vehicle 1's aggressiveness level. This can be attributed to the low rate of lane-changing success under this case – since vehicle 1 assumes that vehicle 2 is aggressive, for most cases an intent for lane changing results in an equilibrium solution that does not involve lane changing. As such, vehicles do not experience the higher fuel consumption, discomfort, and safety risks associated with frequent changes in speed and acceleration, which are inherent parts of the lane-changing process. Fig. 3 also indicates that all other cases achieve success rates in lane changing that are similar to each other, and higher than that of case (ii). In general, case (ii) provides fuel consumption, distance gap, discomfort level, and large-changing success rate that are different from the same metrics for other cases in a statistically significant manner (at the 5% significance level), while there is no statistically significant difference between any of the metrics among other cases according to paired Student's t-tests.

TABLE III

OPTIMAL $x^1_{t^f_{lon}}$ AND ITS CORRESPONDING PAYOFF FOR VEHICLE 1 WITHOUT ACCOUNTING FOR SAFETY AND VEHICLE MECHANICS

Driving Attitude of Vehicle 2	Lateral Position of Vehicle 1 at the End of the Horizon, $y^1_{t^f_{lat}}$	
	Stay in the current lane	Change lanes
Aggressive	Maintain a larger longitudinal position than vehicle 2 $(1-\beta(q_1))\cdot\left(\frac{2t_{\min}}{t^1_b}-1\right)+\beta(q_1)\cdot\frac{t_{\min}}{3}$	Catch up with vehicle A (longitudinally) $(1-\beta(q_1))+\beta(q_1)\cdot\left(\frac{2t_{\min}}{3}-1\right)$
Cautious	Maintain a time headway that is no less than $\min(t^1_b,t^2_b)$ $(1-\beta(q_1))\cdot\min\left(\frac{2t^2_b}{t^1_b}-1,1\right)+\beta(q_1)\cdot\frac{t^2_b}{3}$	Maintain a larger longitudinal position than vehicle 2 $(1-\beta(q_1))+\beta(q_1)\cdot\left(\frac{2t^2_b}{3}-1\right)$

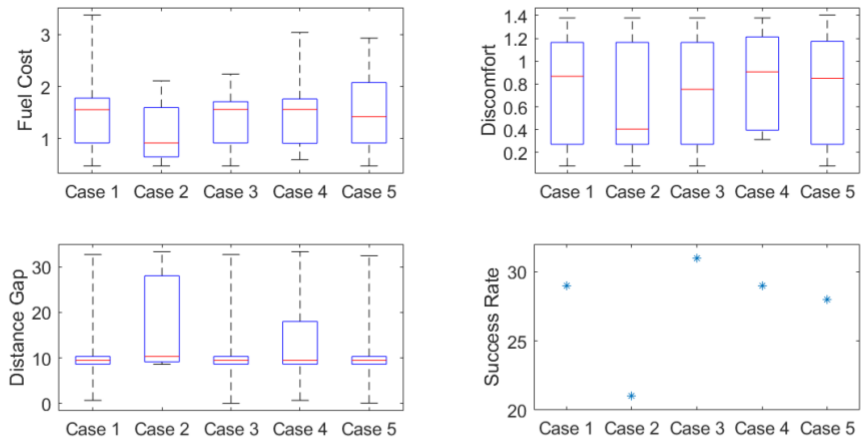

Fig. 3. Four Metrics under Different Cases for the Scenario where Vehicle 1 Changes Lane from 60 km/h to 40 km/h

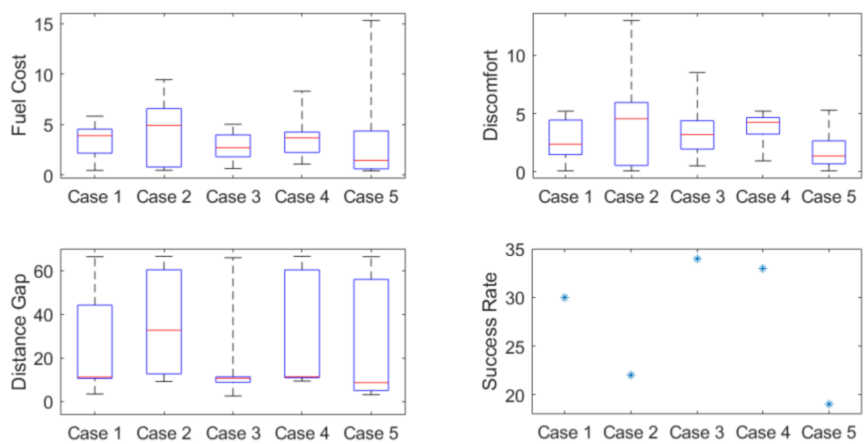

Fig. 4. Four Metrics under Different Cases for the Scenario where Vehicle 1 Changes Lane from 60 km/h to 80 km/h

Fig. 4 shows the fuel cost, distance gap, discomfort level, and lane-changing success rate for the scenario in which the subject vehicle (vehicle 1) intends to change from a lower-speed lane to a higher-speed lane. In this scenario, case (v), in which vehicle 1 has complete information about the aggressiveness factor of vehicle 2 but vehicle 2 assumes vehicle 1 to be the most cautious, provides the best results in fuel consumption and comfort. Similar to the previous scenario, these results can be attributed to vehicle 1's low lane-changing success rate, which is due to its cautious attitude. This figure indicates that the highest distance gaps are maintained under case (ii), in which vehicle 1 perceives vehicle 2 to be extremely aggressive. Due to this perception, lane changing can rarely be completed (note the low success rate), and only under circumstances where a comfortable safety gap can be maintained. However, the failed attempts at lane changing could lead to higher fuel costs and discomfort levels, as demonstrated in Fig. 4.

Fig. 5 shows a successful lane changing example in case (iii). At about $t = 3$ seconds, vehicle 1 starts to change lanes, assuming that vehicle 2 is the most cautious. However, it later cancels this lane changing plan as its computed trajectory does not allow for a safe lane change. At about $t = 5$, vehicle 1 starts another attempt to change lanes, which is successfully executed. Fig. 6 shows another example based on case (iii) in which vehicle 1 fails to change lanes.

Tables IV and V display the best and worst cases for each metric, for vehicle 1, vehicle 2, and at the system level (computed as sum of the relevant metrics for both vehicles). Interestingly, these tables demonstrate that under no scenario are the best metric values obtained under vehicles having full knowledge of each other's aggressiveness factor (i.e., case i). For example in Table V, the best cases for vehicle 1 in terms of the fuel cost and safety metrics are not obtained under having the full knowledge of vehicle 2's aggressiveness factor, but are under making assumptions on vehicle 2. It is only for the comfort metric that vehicle 1 would benefit from having full knowledge of vehicle 2's aggressiveness factor. These tables also suggest that the worst and best cases of information sharing depend on the relative speeds of the original and target lanes. For example, when vehicle 1 attempts to move from a higher-speed lane to a lower-speed lane, case (ii) provides the best results according to all three metrics. However, when moving from a lower-speed lane to a higher-speed lane, vehicle 1 may benefit differently under various information levels depending on the metric of interest. Finally, it is interesting to observe that in Table IV, when vehicle 1 is moving from the higher-speed to the lower-speed lane, the system level benefits align with vehicle 1's benefit. Conversely, Table V suggests that when vehicle 1 attempts to move from a lower-speed to a higher-speed lane, system level benefits align with vehicle 2's benefits. These observations suggest that the system level benefits are in general more aligned with the benefits of the vehicle who is traveling on the higher-speed lane.

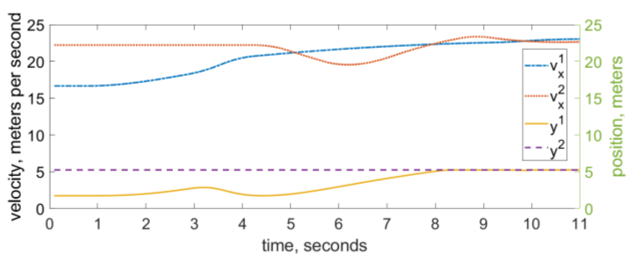

Fig. 5. Vehicle 1 Changes Lane Successfully

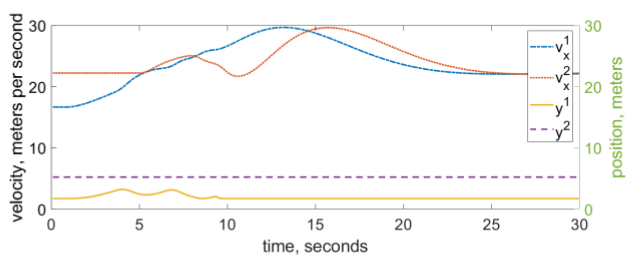

Fig. 6. Vehicle 1 Fails to Change Lane

TABLE IV

VEHICLE 1 CHANGES LANES FROM 60 KM/H LANE TO 40 KM/H LANE

Metrics	Vehicle 1		Vehicle 2		Both Vehicles	
	Best	Worse	Best	Worst	Best	Worst
Fuel cost	*(ii)*	*(i)*	*(iv)*	*(v)*	*(ii)*	*(i)*
Discomfort	*(ii)*	*(i)*	*(v)*	*(iv)*	*(ii)*	*(iv)*
Distance gap	*(ii)*	*(iii)*	*(iv)*	*(v)*	*(ii)*	*(iii)*
Success rate	—	—	—	—	*(iii)*	*(ii)*

TABLE V

VEHICLE 1 CHANGES LANES FROM 60 KM/H LANE TO 80 KM/H LANE

Metrics	Vehicle 1		Vehicle 2		Both Vehicles	
	Best	Worse	Best	Worst	Best	Worst
Fuel cost	*(iii)*	*(ii)*	*(v)*	*(iv)*	*(v)*	*(ii)*
Discomfort	*(iv)*	*(ii)*	*(v)*	*(iv)*	*(v)*	*(ii)*
Distance gap	*(ii)*	*(iii)*	*(ii)*	*(iii)*	*(ii)*	*(iii)*
Success rate	—	—	—	—	*(iii)*	*(v)*

IV. CONCLUSION

In this paper, we developed a realistic game-theoretic lane changing model, in which driving attitude are explicitly modeled. We assume the two vehicles most closely involved in the lane-changing process continuously monitor each other and compute new trajectories accordingly. We account for computation/perception-reaction delay as well as the control period for both vehicles during this process. We relaxed the constraints on safety and mechanical constraints on the vehicles, which enabled us to obtain closed-form equilibrium solutions. Analyzing these closed-form solutions provides insights into how a vehicle's trajectory is impacted by its level of access to information on the driving attitude (aggressiveness level) of the rival. Additionally, we used simulations to quantify the change in four metrics (fuel consumption, discomfort feeling, minimum distance gap and lane changing success rate) under more realistic scenarios where vehicle safety and mechanical constraints are present. Our simulation results suggest that, interestingly, the complete information case, in which both vehicles have full information on the driving attitude of the rival, does not provide the best system-level performance under any of the metrics. Simulation results suggest that the optimal level of information sharing depends on multiple factors, including (1) the entity for which we are optimizing (e.g., the level of information that provides the best trajectory for the vehicle changing lane may be different from the level of information that provides the best trajectory for the vehicle who is traveling on the target lane), and (2) the metric of interest. However, despite the dependency of the equilibrium solution on these factors, some general interesting insights can be drawn from the simulation results; for example, the best system-level solutions, under all metrics, are in line with the the solutions that optimize the trajectory for the vehicle traveling in the higher-speed lane, regardless of whether this vehicle intends to change lanes, or is in the target lane.

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