On the Statistical Analysis of Space-Time Wave Physics in Complex Enclosures

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***Abstract*—The paper proposes a physics-oriented, mathematically tractable statistical wave model, named as the space-time stochastic Green's function, for analyzing the wave physics of high frequency reverberation within complex confined electromagnetic environments. The model characterizes both spatial and temporal variations and correlations of wave fields without the need for a detailed knowledge of the complex environment. Experimental results are supplied to validate the proposed work.**

***Index Terms*—Chaos, electromagnetic coupling, Green function, intentional electromagnetic interference, statistical analysis.**

I. Introduction

The study of electronics in strongly confined electromagnetic (EM) environments has been a longstanding topic in applied electromagnetics and electronic engineering [1]–[6]. One well-known example is the mode-stirred reverberation chamber, which has been used as a standard laboratory facility for EM compatibility testing [7]. Another important application is the EM interference (EMI) to electronics hosted inside protective metallic enclosures (e.g. computer chassis, aircraft carbit). The EMI may take the form of intentional coupling from external radio-frequency (RF) sources or unwanted interaction among electronic components within an enclosure. Due to increasingly complex electronic systems and continually evolving RF sources, it is expensive and impractical to perform experimental tests for all possible EMI effects. Therefore, it identifies a timely and critical need for physics-oriented computational models, which characterize the fundamental wave physics of confined EM environments.

It is known that wave propagation inside electrically large enclosures may undergo multiple reflection/scattering from boundaries and internal structures, thus leads to randomized phase, polarization, and direction of wave fields. In the short wave length limit, the wave scattering process may exhibit chaotic ray dynamics, albeit the underline wave equation is linear [8], [9]. From the eigenmode perspective, the complex boundary of the enclosure can lead to high modal density and high modal overlap. Under the high frequency reverberation, the wave fields inside enclosures are very sensitive to the exact geometry of interior structures. Minor differences in the system configuration can result in significantly different EM field distributions inside the enclosure.

Given the complexity of such environment, it is crucial to develop stochastic models to account for the probabilistic nature of wave fields. Recently, a stochastic Green's function (SGF) approach [10] is introduced to model EM wave physics inside target enclosure with some approximately know information of cavity interior. At its heart, the SGF is based on a statistical description of the eigenmodes of an enclosed EM environment based on random matrix theory (RMT) [11]. Compared to related works [3]–[5], [12]–[14], it rigorously separates the coherent and incoherent influences currents in one element have on fields of another element. Moreover, the statistics of the SGF are determined by generic, macroscopic properties of the cavity environment, including the operating frequency, cavity volume, loading and wall losses.

Remark that the SGF approach in the previous work was derived in the frequency domain with time-harmonic RF signals. It becomes less efficient for analyzing broadband, short-pulse EMI sources. Moreover, electronics are routinely equipped with nonlinear components, e.g. diodes, transistor amplifiers, multipliers, mixers. The temporal and space-time statistical model becomes an appealing approach to interface with circuit simulators in order to analyze the nonlinear effects.

The proposed work advances the theory of SGF from the spatial domain (narrowband) to the spatio-temporal domain. The resulting space-time SGF characterizes both spatial and temporal variations and correlations of EM fields in the high frequency reverberation within confined EM environments. The key results and their applications are presented in Sec. II. The theoretical study is validated by representative experiments in Sec. III.

II. Methodology

Consider the scalar wave equation in a cavity with reflecting boundary conditions, the time-dependent Green's function, $G(\mathbf{r},t;\mathbf{r}',t')$, for an impulse at location $\mathbf{r}'$ and time t' satisfies:

$$\left(\nabla^2-\mu\epsilon\frac{\partial^2}{\partial t^2}\right)G(\mathbf{r},t;\mathbf{r}',t')=-\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t') \quad (1)$$

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After taking Fourier transform of (1), the corresponding time-harmonic Green's function can be obtained by the eigenfunction expansion:

$$G\left(\mathbf{r},\omega;\mathbf{r}',t'\right)=\frac{1}{\mu\epsilon}\sum_{i}\frac{\psi_i\left(\mathbf{r}'\right)\psi_i\left(\mathbf{r}\right)}{\left(\omega-j\frac{1}{2\tau_d}\right)^2-\omega_i^2}e^{-j\omega t'} \quad (2)$$

where ω_i and ψ_i are the eigenfrequency and eigenfunction of the cavity. $\tau_d = Q(\omega)/\omega$ is the decay factor related to the cavity quality factor Q and angular frequency ω.

The time-dependent Green's function is then be obtained by inverse Fourier transform of (2). Note that the Green's function in (2) has poles at $\omega = \omega_i + j1/2\tau_d$. The Fourier integral is replaced with a contour integral closed in the lower half frequency plane. By applying the complex residue theorem and Cauchy's integral formula, we have for $t \geq t'$:

$$G\left(\mathbf{r},t,\mathbf{r}',t'\right)=\frac{1}{\mu\epsilon}\sum_{i}\psi_i\left(\mathbf{r}'\right)\psi_i\left(\mathbf{r}\right)e^{-\frac{t-t'}{2\tau_d}}\frac{\sin\omega_i(t-t')}{\omega_i} \quad (3)$$

To construct the space-time stochastic Green's function (ST-SGF), $\tilde{G}\left(\mathbf{r},t,\mathbf{r}',t'\right)$, we substitute a statistical description of eigenfunctions and eigenfrequencies into (3). In this work, the eigenfunction statistics are derived using a time-domain version of Berry's random wave model [15] and Karhunen-Loeve expansion. The eigenspectrum statistics are generated with Wigner's RMT [11] and Weyl expansion [9].

Built upon this theoretical framework, we have analyzed key statistical properties of ST-SGF, as elaborated below.

(**1**) The ensemble average of ST-SGF results in a retarded Green's function in homogeneous media, briefly derived as:

$$\left\langle \tilde{G}\left(\mathbf{r},t;\mathbf{r}',t'\right)\right\rangle$$

$$=\sum_{i}\frac{c^3e^{-\frac{t-t'}{2\tau_d}}}{V\omega_i^2}\frac{\sin(\omega_i\frac{|\mathbf{r}-\mathbf{r}'|}{c})\sin(\omega_i(t-t'))}{|\mathbf{r}-\mathbf{r}'|} \quad (4)$$

$$\approx\frac{e^{-\frac{t-t'}{2\tau_d}}}{4\pi|\mathbf{r}-\mathbf{r}'|}\delta\left(t-t'-\frac{|\mathbf{r}-\mathbf{r}'|}{c}\right),\ t\geq t' \quad (5)$$

where c is the wave velocity and V is the enclosure volume. In the above derivation, we first utilize the spatial correlation property of eigenfunction, then invoke Weyl's law for the mean density of eigenfrequencies, $\Delta\omega = 2\pi^2c^3/(\omega^2V)$.

Equation 4 shows an encouraging result as it implies the spatio-temporal causality of ST-SGF in analyzing transient wave dynamics. Based on the ST-SGF, we obtain a time-domain stochastic integral equation formulation. It can be placed at arbitrary-shaped surfaces on electronic/antenna components inside the enclosure, with applications to time-domain EMI/EMC testing in overmoded reverberation chambers.

(**2**) At a small temporal scale, we have derived the spatial-temporal cross-correlation function of a chaotic wave field using the ST-SGF. For the purpose of elucidation, we consider the case of a sinc pulse excitation with bandwidth B_T centered at frequency f_c. Choosing the reference time $t' = 0$ for convenience, the temporal autocorrelation of ST-SGF $C_G(\tau) =< \tilde{G}(\mathbf{r},t;\mathbf{r}',0),\tilde{G}(\mathbf{r},t+\tau;\mathbf{r}',0)>$ in the diffusive regime ($|\mathbf{r}-\mathbf{r}'| \gg \lambda$) is calculated by:

$$C_G\left(\tau\right)=\frac{c\tau_d}{4\pi^2V}e^{-\frac{\tau}{2\tau_d}}\cos\left(2\pi f_c\tau\right)\text{sinc}\left(B_T\pi\tau\right) \quad (6)$$

The normalized form of 6 agrees with the autocorrelation of impulse response described in [16]. Now consider a time snapshot t_0, the spatial autocorrelation of nearby spatial samples, $C_G(d) =< \tilde{G}(\mathbf{r},t_0;\mathbf{r}',0),\tilde{G}(\mathbf{r}+d,t_0;\mathbf{r}',0)>$, is derived as:

$$C_G(d)=\frac{c^2e^{-\frac{t_0}{\tau_d}}}{4\pi^2V}\frac{\text{Si}\left[\frac{(\omega_c+\pi B_T)d}{c}\right]-\text{Si}\left[\frac{(\omega_c-\pi B_T)d}{c}\right]}{2\pi B_T d} \quad (7)$$

We remark that a through understanding of spatial-temporal cross-correlation plays a vital role in the time-reversal imaging and fidelity decay study in chaotic scattering environments.

(**3**) On a large temporal scale, one can easily derive the time-dependent average power profile from the ST-SGF. The semi-analytical results are expressed in the forms of power delay profile (PDP) and root mean square delay spread. The input only requires macroscopic features of the enclosure, including decay factor τ_d, cavity volume V, excitation bandwidth B_T, rather than the explicit geometry and internal configuration. The outcomes expect to provide physical insights to statistical analysis of time-domain EMI effects on advanced electronics under broadband signals.

III. Experimental Validation

The experimental validation is performed with a 3D metallic cavity with two X-band waveguide antennas mounted on opposite sides of cavity walls as transmitter and receiver. A Fourier transform of the frequency domain measurement data, 8.2-12.4 GHz with 0.042 MHz interval, is used to obtain the time-domain channel impulse response. The 3D cavity is equipped with paddle-wheel stirring structures, thus a configuration ensemble of measured data was collected by rotating the mode stirrer 200 positions over 360 degrees.

In the predictive framework, the ST-SGF is constructed with the cavity volume 0.42m^3, estimated decay factor 2.62e-7, frequency bandwidth, and the RMT for eigenfrequency statistics. We then integrate the ST-SGF, as a statistical model for the cavity environment, with the antenna component-specific response. The comparison between measurement and simulation results is given in Fig. 1. We observed very good agreement in the statistical impulse responses.

The next study is the dynamic range of fluctuation in the channel impulse response, $h(t)$. We have chosen three different time periods, $[2-2.1]\mu$s, $[4-4.1]\mu$s, $[6-6.1]\mu$s, and plot the probability density functions (PDFs) of the measured data and ST-SGF predicted results. A very good agreement is seen in Fig. 2. Then, we proceed to evaluate the time-domain autocorrelation of measured impulse response at small temporal scale. The calculated result is compared to the theoretical prediction using the ST-SGF. As shown in Fig. 3, the results of these two match very well.

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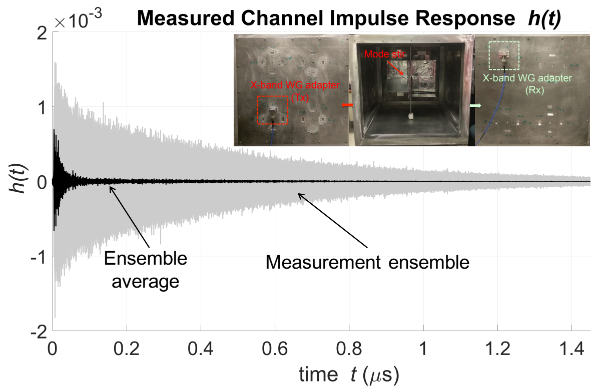
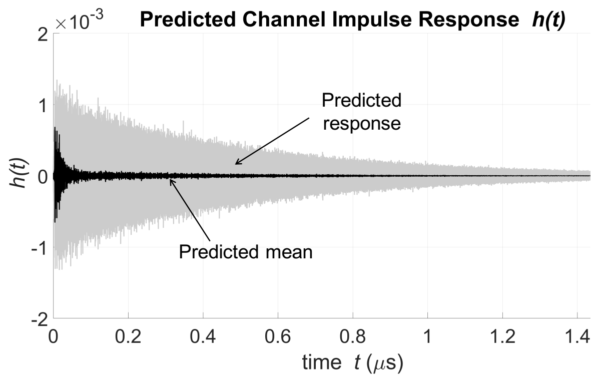


(a) Measurement data



(b) Predictive results

Fig. 1: Statistical impulse responses inside a 3D enclosure.

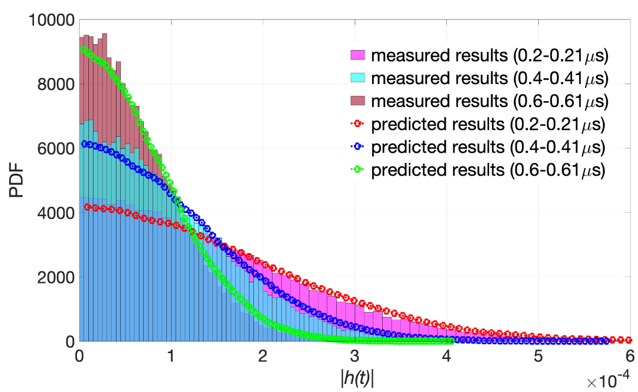


Fig. 2: Probability density functions of $|h(t)|$

IV. Conclusion

The objective of this work is to investigate physics-oriented mathematical and statistical models, which characterize the fundamental wave physics of confined EM environments. The results can then be used as an EMC/EMI virtual testbed for electronic components of interest. The unique contribution is a novel space-time stochastic Green's function method, which enables a systematic study of temporal, spectral, and spatial statistics of wave fields. To our best knowledge, this is the first time available in the literature. The key statistical properties are examined and validated experimentally.

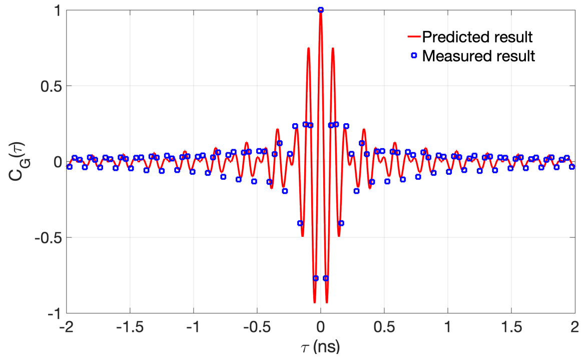


Fig. 3: Temporal autocorrelation of impulse response

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