A Cooperative Game Theory-based Approach to Compute Participation Factors of Distributed Slack Buses

Mukesh Gautam, *Student Member, IEEE*, Narayan Bhusal, *Student Member, IEEE*,
Jitendra Thapa, *Student Member, IEEE*, and Mohammed Benidris, *Senior Member, IEEE*,
Department of Electrical & Biomedical Engineering, University of Nevada, Reno, Reno, NV 89557
(emails: {mukesh.gautam, bhusalnarayan62, jitendrathapa}@nevada.unr.edu, and mbenidris@unr.edu)

***Abstract*—The conventional power flow analysis utilizes a single slack, reference, or swing bus. The use of similar formulation when performing power flow analysis may not necessarily result in minimum cost of generation and minimum power loss. Distributed slack buses, which are used to distribute the slack power (power mismatch) among different voltage controlled buses, can solve such a problem and reduce the cost of generation and power loss. This paper proposes a co-operative game theory-based approach to calculate active power participation factors to distribute slack active power among different participating generators. In the first stage, the worth (or value) of individual participating generators and their coalitions are computed. In the second stage, the Shapley value, one of the solution concepts of cooperative game theory, is used to calculate the participation factors of individual participating generators. Case studies on IEEE 14- and 30-bus systems show that the cost of generation and power loss are reduced and voltage profile is improved in case of systems with distributed slack buses compared to that with a single slack bus.**

***Index Terms*—Cooperative game theory, distributed slack buses, and Shapley value.**

NOMENCLATURE

$\mathcal{N}$	set of players of a cooperative game
V, W	characteristic functions
S	a coalition that is subset of $\mathcal{N}$
$2^{\mathcal{N}}$	possible set of coalitions
α	payoff vector
$\{i\}$	singleton or unit set of player j
$S\backslash\{i\}$	coalition set without player i
ψ_i	Shapley value of player i
n	number of players or generators
N	number of buses
P_i	active power injection at bus i
Q_i	reactive power injection at bus i
$\tilde{V}_k$	the phasor voltage at bus k
V_i	the voltage magnitude at bus i
C	the total generation cost function
C_j	the generation cost function of generator j
P_{gj}	the active power of generator j
$\alpha_j, \beta_j, \gamma_j$	the cost coefficients of generator j
P_{loss}	the total active power loss of the system
θ_{ik}	the voltage angle difference between bus i and k
G_{ik}	the conductance of branch ik
B_{ik}	the susceptance of branch ik
ψ_j^{eqv}	equivalent Shapley value of jth generator
π_j	participation factor of jth generator

I. INTRODUCTION

A. Motivation and Background

During the formulation of power flow problems in conventional power systems, buses are classified into PV, PQ, and slack buses based on known variables of the bus and the practical conditions of the power system operation. For a PV bus, also referred to as a generator bus or a voltage controlled bus, the bus active power P and the voltage magnitude V are known. For a PQ bus, also referred to as load bus, the bus active power P and reactive power Q are known. The slack bus, also referred to as swing bus or reference bus, serves as a reference for the calculation of voltage angles and compensates the difference between the net power injected into the system at other buses and the total system load and losses [1]. In conventional power systems, slack bus has merely been regarded as a mathematical entity without linking it with the physical system [2] and hence a single slack bus has been utilized for the aforementioned purpose. When the power mismatch is taken solely by a single slack bus, it may result in higher generation cost, higher power loss, and higher loading of the slack bus generator. Distributed slack buses, which are used to distribute power mismatch among different voltage controlled buses, can help distribute the slack power among different generators so as to reduce the cost of generation and power loss. Also, distributed slack buses can help in a more realistic quantification of economic impacts of generators on the power system [3]. Therefore, the participation factors need to be calculated to distribute the slack powers among the participating generators.

B. Relevant Literature

Different approaches have been presented in the literature to compute participation factors of distributed slack buses. The determination of participation factors is affected by various parameters including machine inertia, droop characteristic of governor, and frequency control participation factors [2]. An

978-1-7281-7660-4/21/$31.00 ©2021 IEEE

approach to compute the participation factors utilizing the concept of generator domains has been presented in [4]. In [5], the concept of participation factors based on penalty factors and network sensitivities has been presented. In [6], a technique has been proposed to determine the proportionate share of each generator to the slack power of power systems. In [7], the participation factors of distributed slack buses have been computed by performing perturbation analysis in classical economic load dispatch problems. The work in [7] also modifies load-flow formulation to obtain a participation factor load-flow, which can be used to solve economic load dispatch problems. In [8], an economic approach has been proposed for the computation of distributed slack bus participation factors. Different from the approaches presented in the existing literature, the cooperative game theoretic approach proposed in this paper determines participation factors of distributed slack buses without changing the existing network formulation.

C. Contributions and Organization

Game theory-based approaches (both cooperative and non-cooperative) have been successfully applied in various fields of power systems. These applications include power system reliability enhancement, loss allocation, and transmission expansion planning [9]. A cooperative game theory-based approach has been proposed in [10] for loss reduction allocation of distributed generations using the Shapley values. In [11], the gaming problem of incentive-based demand response program has been addressed using a probabilistic approach. A cooperative game theory-based approach has been proposed in [12] for under frequency load shedding control. Motivated by these works, this paper proposes a cooperative game theoretic two-stage approach to compute participation factors that are used to distribute the slack active power among different participating generators. In the first stage, the worth or value of each coalition of participating generators is computed. For computing the worth of individual participation generators and their coalitions, the total generation cost of participating generators and active power loss of each coalition are calculated. In the second stage, participation factors of individual participating generators are computed based on the Shapley values.

The rest of the paper is organized as follows. Section II explains the concept of cooperative game theory along with the Shapley value. Section III presents the formulation of the cooperative game model, which is essential for the computation of participation factors of distributed slack buses. Section IV describes the proposed approach of calculating participation factors of distributed slack buses. Section V provides the evaluation of the proposed approach through case studies on the IEEE 14- and 30-bus systems. Finally conclusion is provided in section VI.

II. Shapley Value in Cooperative Game Theory

In game theory, a cooperative game (or coalitional game) refers to a special class of games in which players are allowed to compete with each other by forming alliances (coalitions) among themselves. Mathematically, a cooperative game is defined through specifying a certain value for each coalition. A cooperative game has the following components:

- a finite set $\mathcal{N}$, and
- a real-valued set function V, called characteristic function, defined on all subsets of $\mathcal{N}$ that satisfies $V(\phi) = 0$.

In the game theory terms, $\mathcal{N}$ is defined as the player set, and $V(S) : 2^{\mathcal{N}} \to \mathbb{R}$ is defined as the "worth" or "value" of coalition S, or the total utility that members of S can acquire if a coalition is formed among themselves and the game is played without any assistance from other players.

A. The Core of a Cooperative Game

In game theory, the core refers to the set of feasible allocations that cannot be further improved through any other coalitions. Generally, outcomes of a cooperative game are specified as n-tuples of utility: $\alpha = \{\alpha^i : i \in \mathcal{N}\}$, called payoff vectors that are measured in a common unit of money [13]. The core of a game is defined as the set of payoff vectors that are feasible and coalitionally rational. In other words, the core is the set of imputations under which no coalition has a value greater than the sum of its members' payoffs. In other words, the core is the set of imputations under which all sets of coalition have values less than or equal to the sum of its members' payoffs. Thus, α is core if and only if [13],

$$\alpha.e^S \geq V(S), \forall S \subset \mathcal{N} \tag{1}$$

$$\alpha.e^{\mathcal{N}} = V(\mathcal{N}) \tag{2}$$

where e^S denotes the vector of size n with $e_i^S = 1$ if $i \in S$ and $e_i^S = 0$ if $i \in \mathcal{N} - S$.

B. The Shapley Value

Shapley value, which is one solution concept of cooperative game theory, assigns a unique payoff vector that is efficient, stable, symmetric, and satisfies monotonicity [14]. The Shapley value allocates the payoffs in such a way that is fair for cooperative solutions. The Shapley value of a cooperative game is given as follows [15].

$$\psi_i(V) = \sum_{S \in 2^{\mathcal{N}}, i \in S} \frac{(|S|-1)!(n-|S|)!}{n!}[V(S) - V(S\backslash\{i\})] \tag{3}$$

where $n = |\mathcal{N}|$ is the total number of players.

The Shapley value satisfies the following axioms [15]:

1) *Efficiency:* The efficiency axiom states that the sum of the Shapley values of all players is equal to the worth of grand coalition, so that all the gain is allocated among the players, i.e., $\sum_{i \in \mathcal{N}} \psi_i(V) = V(\mathcal{N})$.
2) *Individual Rationality:* This axiom states that the Shapley value of each player should be greater than or equal to its individual worth, i.e., $\psi_i(V) \geq V(\{i\}), \forall i \in \mathcal{N}$.
3) *Symmetry:* This axiom states that the players contributing the same amount in every coalition should have the same Shapley values. If j and k are such that

$V(S \cup \{j\}) = V(S \cup \{k\})$ for every coalition S not containing j and k, then $\psi_j(V) = \psi_k(V)$.
4) *Dummy Axiom:* If j is such that $V(S) = V(S \cup \{j\})$ for every coalition S not containing j, then $\psi_j(V) = 0$.
5) *Additivity:* If V and W are characteristic functions, then $\psi(V + W) = \psi(V) + \psi(W)$.

III. Cooperative Game Model

In this paper, the task of computing participation factors of generators in distributed slack buses is regarded as a game. Since all the generators should work in coordinated manner for the determination of efficient participation factors, the game is a cooperative game. As explained in Section II, a cooperative game is defined with a finite set of players and characteristic functions, which are essential to determine Shapley values of players. In this paper, two types of characteristic functions are implemented for the cooperative model. The first characteristic function is the total generation cost of participating generators. The second characteristic function is the total active power loss. Using each of these characteristic functions, two Shapley values are computed for each participating generator using (3) and the equivalent Shapley values are determined by taking their average.

The generator cooperative model formulation for the proposed approach can be enumerated as follows.

1) Collect system data including generation data, transmission line data, load data, etc., which serve as input to the cooperative game model.
2) Generate the list of all possible coalitions of generators. For example, if three generators (G_1, G_2, and G_3) are participating in the process of computation of participation factors of distributed slack buses, the set of all possible coalitions, denoted by $2^{\mathcal{N}}$, is as follows.
$2^{\mathcal{N}} = \{\phi, \{G_1\}, \{G_2\}, \{G_3\}, \{G_1, G_2\}, \{G_1, G_3\}, \{G_2, G_3\}, \{G_1, G_2, G_3\}\}$,
where ϕ denotes an empty set.
3) For each participating generator and its possible coalitions, compute total cost of generation and total active power loss. These values serve as the worth of each generator and their coalitions.
4) Compute two Shapley values, ψ_i^1 and ψ_i^2, of each generator, G_i, using the characteristic functions determined in step 3 using (3).
5) Determine the equivalent Shapley value, ψ_i^{eqv}, of each participating generator, G_i, taking the average of two Shapley values computed in step 4. The equivalent Shapley values are computed by taking the average of two Shapley values to both types of characteristic functions.

$$\psi_i^{eqv} = \frac{\psi_i^1 + \psi_i^2}{2} \tag{4}$$

The generator cooperative model formulated by utilizing the procedure explained in the above steps is essential for determining participation factors of distributed slack buses.

IV. The Proposed Approach

The proposed approach for the determination of participation factors of distributed slack buses is implemented in the following two steps:

1) Computation of characteristic functions of the game which maps every coalition of players to a payoff.
2) Determination of participation factors on the basis of the Shapley values of each participating generator.

In the first stage i.e., the computation of characteristic functions of the game, the power flow analysis is performed. The general form of power flow equation can be expressed as follows:

$$P_i - jQ_i = \sum_{k=1}^{N} Y_{ik} \tilde{V}_k, \tag{5}$$

The power flow equation (5) can be solved using any method such as Gauss-Seidel or Newton-Raphson. After performing power flow analysis, the total cost of generation and active power loss are computed for each set of coalitions using (6) and (7), respectively.

$$\mathcal{V}_1 : C = \sum_{j=1}^{n} C_j(P_{gj}) = \sum_{j=1}^{n} \alpha_j P_{gj}^2 + \beta_j P_{gj} + \gamma_j, \tag{6}$$

$$\mathcal{V}_2 : P_{loss} = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} \sum_{k=1}^{N} V_i V_k (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}), \tag{7}$$

where $\mathcal{V}_1$ is the first characteristic function and $\mathcal{V}_1$ is the second characteristic function.

In the second stage, the Shapley values are computed for each candidate location using (3), the equivalent Shapley values are computed using (4), and the participation factor, π_j, of the j^{th} generator is determined using (8).

$$\pi_j = \psi_j^{eqv} \Big/ \sum_{j=1}^{n} \psi_j^{eqv}, \tag{8}$$

where ψ_j^{eqv} is the equivalent Shapley value of the j^{th} generator.

Now, for the total mismatch power of ΔP, the change in active power of j^{th} generator can be calculated as follows.

$$\Delta P_{gj} = \pi_j \times \Delta P. \tag{9}$$

The proposed approach or the solution algorithm to determine the generator participation factors of distributed slack buses can be summarized follows.

1) Provide system data related to lines, loads, transformers, and generators.
2) Enumerate all possible coalitions of participating generators and compute their characteristic functions.
3) Compute the equivalent Shapley value of each generator based on (4) and the respective participation factor using (8).

The flowchart of the proposed solution algorithm is shown in Fig. 1.

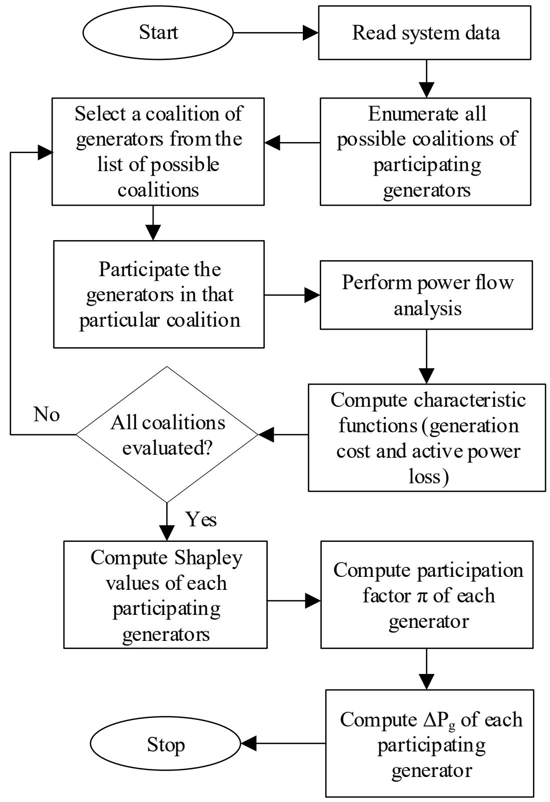

Fig. 1. Flowchart of the proposed approach

V. Case Studies and Discussions

A. Case Study Parameters

This section presents the evaluation of the proposed approach through case studies on the IEEE 14- and 30-bus systems. The IEEE 14-bus system consists of 14 buses, 5 generators, and 11 loads with a total generation capacity of 772.4 MW and total peak load of 259 MW. The IEEE 30-bus system consists of 30 buses, 6 generators, and 21 loads with a total generation capacity of 900.2 MW and total peak load of 283.4 MW. For the detailed data of IEEE 14- and 30-bus systems, the readers are referred to reference [16].

B. Results

The proposed approach starts by enumerating all possible coalitions of participating generators. There are 5 and 6 generators, respectively, in case of the IEEE 14- and 30-bus systems. All these generators are treated as players of the cooperative game. This results in a total 31 and 63 sets of coalitions, except an empty set, respectively, in case of the IEEE 14- and 30-bus systems. All the sets of possible coalitions of generators, except an empty set, in case of IEEE 14-bus system are shown in the first column of Table I.

For each set of the coalitions, the generation cost due to participating generators and active power loss are com-

TABLE I
Characteristic Functions of Possible Coalitions for IEEE 14-bus System

Coalitions of generator locations (buses)	Generation cost ($\$/hr$)	Active power loss (MW)
1	1108	15.92
2	1625	12.90
3	2025	9.79
6	2025	11.73
8	2025	10.71
1, 2	2733	12.89
1, 3	3133	9.76
1, 6	3133	11.71
1, 8	3133	10.69
2, 3	3650	7.56
2, 6	3650	9.31
2, 8	3650	8.36
3, 6	4050	6.50
3, 8	4050	5.68
6, 8	4050	7.43
1, 2, 3	4758	7.55
1, 2, 6	4758	9.30
1, 2, 8	4758	8.34
1, 3, 6	5158	6.49
1, 3, 8	5158	5.67
1, 6, 8	5158	7.41
2, 3, 6	5675	4.88
2, 3, 8	5675	4.12
2, 6, 8	5675	5.67
3, 6, 8	6075	3.29
1, 2, 3, 6	6783	4.88
1, 2, 3, 8	6783	4.11
1, 2, 6, 8	6783	5.66
1, 3, 6, 8	7183	3.28
2, 3, 6, 8	7700	2.31
1, 2, 3, 6, 8	8808	2.30

TABLE II
Cost Function Parameters of Generators for IEEE 14-bus System

Generator number	Location (bus)	α ($\$/MW^2hr$)	β ($\$/MWhr$)	γ ($\$/hr$)
1	1	0.043	20	0
2	2	0.250	20	0
3	3	0.010	40	0
4	6	0.010	40	0
5	8	0.010	40	0

TABLE III
Cost Function Parameters of Generators for IEEE 30-bus System

Generator number	Location (bus)	α ($\$/MW^2hr$)	β ($\$/MWhr$)	γ ($\$/hr$)
1	1	0.0200	2.00	0
2	2	0.0175	1.75	0
3	5	0.0625	1.00	0
4	8	0.0083	3.25	0
5	11	0.0250	3.00	0
6	13	0.0250	3.00	0

TABLE IV
SHAPLEY VALUES AND PARTICIPATION FACTORS OF GENERATORS FOR IEEE 14-BUS SYSTEM

Generator Locations (buses)	Equivalent Shapley values	Participation Factors (proposed)	Participation Factors (conventional)
Bus 1	555.4	0.1261	0.4303
Bus 2	813.1	0.1846	0.1813
Bus 3	1011.7	0.2297	0.1295
Bus 6	1012.6	0.2299	0.1295
Bus 8	1012.2	0.2298	0.1295

puted, which serve as worth (or characteristic function) of the coalition. If only generator connected to bus 1 is allowed to participate in the game, its generation cost would be 1108 $/hr and the active power loss would be 15.92 MW. If only generator connected to bus 2 is allowed to participate in the game, its generation cost would be 1625 $/hr and the active power loss would be 12.90 MW. If the generators connected to buses 1 and 2 are allowed to participate in the game, its generation cost would be 2733 $/hr and the active power loss would be 12.89 MW. In this way, the characteristic functions for all the coalitions can be computed. Table I shows the characteristic functions of possible coalitions for the IEEE 14-bus system. Similarly, the characteristic functions of possible coalitions for the IEEE 30-bus system can be obtained. Based on the characteristic functions, Shapley values of each generator locations can be computed using (3) and the equivalent Shapley values can be computed by taking their average. In case of IEEE 14-bus system, the equivalent Shapley values obtained for distributed slack bus locations 1, 2, 3, 6, and 8 are 555.4, 813.1, 1011.7, 1012.6, and 1012.2, respectively. In case of IEEE 30-bus system, the equivalent Shapley values obtained for distributed slack bus locations 1, 2, 5, 8, 11 and 13 are 76.7, 66.3, 102.3, 91.5, 106.1, and 106.5, respectively. The second columns of Table IV and Table VI show the equivalent Shapley values of each generator in case of the IEEE 14- and 30-bus systems, respectively. The participation factors of the generators can then be calculated using (8). In case of IEEE 14-bus system, participation factors obtained for distributed slack bus locations 1, 2, 3, 6, and 8 using the proposed approach are 0.1261, 0.1846, 0.2297, 0.2299, and 0.2298, respectively. In case of IEEE 30-bus system, participation factors obtained for distributed slack bus locations 1, 2, 5, 8, 11 and 13 using the proposed approach are 0.1396, 0.1207, 0.1862, 0.1666, 0.1931, and 0.1938, respectively. The third columns of Table IV and Table VI show participation factors of each generator in case of the IEEE 14- and 30-bus systems, respectively.

C. Comparison

In order to check the effectiveness of the proposed approach, three cases are formulated and the generator cost, the active power loss, and the voltage profile for the three cases are compared. First case is the case of single slack bus, second case is the case of distributed slack buses (conventional approach) in which participation factors are chosen based on generator capacities, and third case is the case of distributed slack buses in which participation factors are computed based on the proposed approach. For the IEEE 14-bus system, a power mismatch (here, increment in load) of 100 MW is simulated. In the case of a single slack bus, the power mismatch is taken by a single generator, whereas in the case of distributed slack buses (both conventional and proposed), the power mismatch is shared among all the generators according to participation factors shown in Table IV. The total generation cost obtained using single slack bus, distributed slack buses (conventional), and distributed slack buses (proposed) are, respectively, 13299 $/hr, 12628 $/hr, and 12382 $/hr. The total active power loss obtained using single slack bus, distributed slack buses (conventional), and distributed slack buses (proposed) are, respectively, 27.55 MW, 19.98 MW, and 15.88 MW. These values are shown in Table V.

TABLE V
COST OF GENERATION AND ACTIVE POWER LOSS IN CASE OF IEEE 14-BUS SYSTEM FOR POWER MISMATCH OF +100 MW

Cases	Cost of generation	Active power loss
Single slack bus	13299 $/hr	27.55 MW
Distributed slack buses (conventional)	12628 $/hr	19.98 MW
Distributed slack buses (proposed)	12382 $/hr	15.88 MW

TABLE VI
SHAPLEY VALUES AND PARTICIPATION FACTORS OF GENERATORS FOR IEEE 30-BUS SYSTEM

Generator Locations (buses)	Equivalent Shapley values	Participation Factors (proposed)	Participation Factors (conventional)
Bus 1	76.7	0.1396	0.4001
Bus 2	66.3	0.1207	0.1555
Bus 5	102.3	0.1862	0.1111
Bus 8	91.5	0.1666	0.1111
Bus 11	106.1	0.1931	0.1111
Bus 13	106.5	0.1938	0.1111

For the IEEE 30-bus system, a power mismatch (here, increment in load) of 50 MW is simulated. In the case of a single slack bus, the power mismatch is taken by a single generator, whereas in the case of distributed slack buses (both conventional and proposed), the power mismatch is shared among all the generators according to participation factors shown in Table VI. The total generation cost obtained using single slack bus, distributed slack buses (conventional), and distributed slack buses (proposed) are, respectively, 2764.8 $/hr, 2372.3 $/hr, and 2214.5 $/hr. The total active power loss obtained using single slack bus, distributed slack buses (conventional), and distributed slack buses (proposed) are, respectively, 25.16 MW, 21.12 MW, and 19.13 MW. These values are shown in Table VII.

The comparison of the results in Table V and Table VII shows that the generation cost and the active power loss are reduced in the case of distributed slack buses (proposed). Also, Fig. 3 and Fig. 2 show the improvement of voltage profile for IEEE 14- and 30-bus systems, respectively.

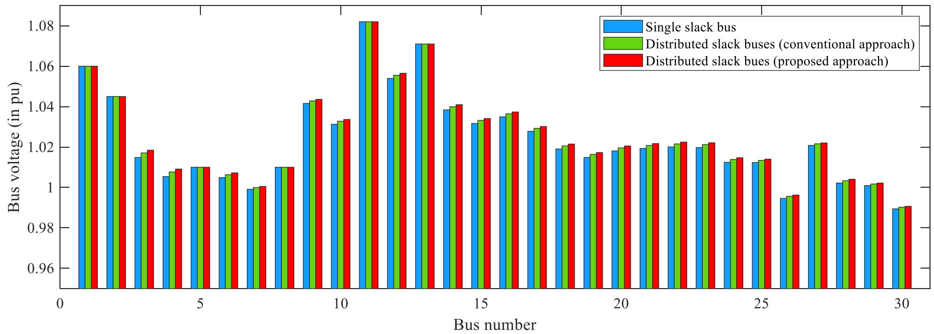

Fig. 2. Voltage profile for IEEE-30 bus system

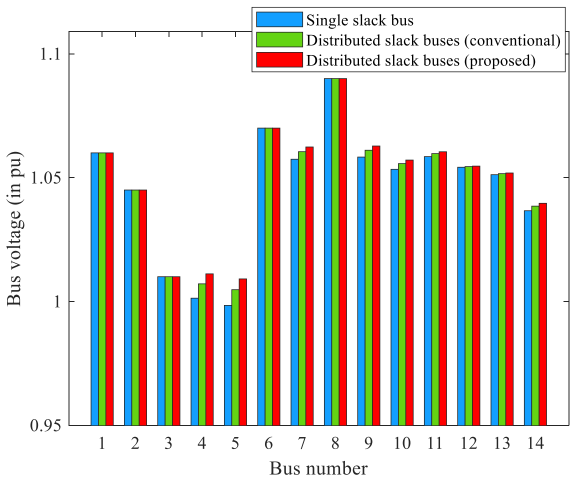

Fig. 3. Voltage profile for IEEE-14 bus system

TABLE VII
COST OF GENERATION AND ACTIVE POWER LOSS IN CASE OF IEEE 30-BUS SYSTEM FOR POWER MISMATCH OF +50 MW

Cases	Cost of generation	Active power loss
Single slack bus	2764.8 \$/hr	25.16 MW
Distributed slack buses (conventional)	2372.3 \$/hr	21.12 MW
Distributed slack buses (proposed)	2214.5 \$/hr	19.13 MW

VI. CONCLUSION

In this paper, a cooperative game theoretic two-stage approach has been proposed to calculate the participation factors of distributed slack buses. In the first stage, the generation cost and active power loss were calculated, which served as the characteristic functions of the cooperative game. In the second stage, the participation of distributed slack buses were calculated using the equivalent Shapley values. The proposed approach can calculate the participation factors of distributed slack buses without changing the existing network formulation. The proposed approach was implemented in the IEEE 14- and 30-bus systems. The comparison of the results obtained using a single slack bus and the distributed slack buses showed that the use of distributed slack buses can reduce generation cost and power loss and improve the voltage profile.

ACKNOWLEDGEMENT

This work was supported by the U.S. National Science Foundation (NSF) under Grant NSF 1847578.

REFERENCES

[1] S. Dhople, C. Chen, A. Al-Digs, and A. Dominguez-Garcia, "Reexamining the distributed slack bus," *IEEE Transactions on Power Systems*, 2020.

[2] A. G. Expósito, J. L. M. Ramos, and J. R. Santos, "Slack bus selection to minimize the system power imbalance in load-flow studies," *IEEE Transactions on power systems*, vol. 19, no. 2, pp. 987–995, 2004.

[3] S. Tong, M. Kleinberg, and K. Miu, "A distributed slack bus model and its impact on distribution system application techniques," in *2005 IEEE International Symposium on Circuits and Systems*. IEEE, 2005, pp. 4743–4746.

[4] S. Tong and K. N. Miu, "A participation factor model for slack buses in distribution systems with dgs," in *2003 IEEE PES Transmission and Distribution Conference and Exposition (IEEE Cat. No. 03CH37495)*, vol. 1. IEEE, 2003, pp. 242–244.

[5] S. Tong *et al.*, "Participation factor studies for distributed slack bus models in three-phase distribution power flow analysis," in *2005/2006 IEEE/PES Transmission and Distribution Conference and Exhibition*. IEEE, 2006, pp. 92–96.

[6] D. Kirschen, R. Allan, and G. Strbac, "Contributions of individual generators to loads and flows," *IEEE Transactions on power systems*, vol. 12, no. 1, pp. 52–60, 1997.

[7] J. Meisel, "System incremental cost calculations using the participation factor load-flow formulation," *IEEE transactions on power systems*, vol. 8, no. 1, pp. 357–363, 1993.

[8] A. Zobian and M. D. Ilic, "Unbundling of transmission and ancillary services. i. technical issues," *IEEE Transactions on Power Systems*, vol. 12, no. 2, pp. 539–548, 1997.

[9] S. Abapour, M. Nazari-Heris, B. Mohammadi-Ivatloo, and M. T. Hagh, "Game theory approaches for the solution of power system problems: A comprehensive review," *Archives of Computational Methods in Engineering*, vol. 27, no. 1, pp. 81–103, 2020.

[10] K. Shaloudegi, N. Madinehi, S. Hosseinian, and H. A. Abyaneh, "A novel policy for locational marginal price calculation in distribution systems based on loss reduction allocation using game theory," *IEEE transactions on power systems*, vol. 27, no. 2, pp. 811–820, 2012.

[11] J. Vuelvas, F. Ruiz, and G. Gruosso, "Limiting gaming opportunities on incentive-based demand response programs," *Applied Energy*, vol. 225, pp. 668–681, 2018.

[12] M. Gautam, N. Bhusal, and M. Benidris, "A cooperative game theory-based approach to under-frequency load shedding control," in *2021 IEEE Power & Energy Society General Meeting (PESGM)*. IEEE, 2021, pp. 1–5.

[13] L. S. Shapley and M. Shubik, "Competitive outcomes in the cores of market games," *International Journal of Game Theory*, vol. 4, no. 4, pp. 229–237, 1975.

[14] A. E. Roth, *The Shapley value: essays in honor of Lloyd S. Shapley*. Cambridge University Press, 1988.

[15] I. Curiel, *Cooperative game theory and applications: cooperative games arising from combinatorial optimization problems*. Springer Science & Business Media, 2013, vol. 16.

[16] U. of Washington, *Power Systems Test Case Archive*, 1993.