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Scale-free protocol design for delayed regulated synchronization of multi-agent systems subject to unknown, nonuniform, and arbitrarily large communication delays

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Abstract

In this article, we study scale-free delayed regulated state/output synchronization for homogeneous and heterogeneous networks of multi-agent systems (MAS) subject to unknown, nonuniform, and arbitrarily large communication delays. A delay transformation is utilized to transform the original MAS to a new system without delayed states. The proposed *scale-free* dynamic protocols are developed for non-introspective homogeneous and introspective heterogeneous MAS. The protocol design utilizes localized information exchange with neighbors and is solely based on the knowledge of agent models. In other words, the scale-free protocol design is independent of information about the communication network or the size of the network.

KEYWORDS

delayed synchronization, homogeneous and heterogeneous MAS, scale-free protocol design, unknown communication delay

1 | INTRODUCTION

Cooperative control of multi-agent systems (MAS) such as synchronization, consensus, swarming, flocking, has become a hot topic among researchers because of its broad application in various areas such as biological systems, sensor networks, automotive vehicle control, robotic cooperation teams, and so on. See, for example, the books.[1-4]

State synchronization inherently requires homogeneous networks. Most works have focused on state synchronization where each agent has access to a linear combination of its own state relative to that of the neighboring agents, which is called full-state coupling; see References 5-10. A more realistic scenario which is partial-state coupling (i.e., agents share part of their information over the network) is studied in References 11-14. On the other hand, for heterogeneous network it is more reasonable to consider output synchronization since the dimensions of states and their physical interpretation may be different. For heterogeneous MAS with non-introspective agents,[*] it is well known that one needs to regulate outputs of the agents to a priori given trajectory generated by a so-called exosystem (see References 16 and 17). Other works on synchronization of MAS with non-introspective agents can be found in the literature.[15,18] Most of the literature for heterogeneous MAS with introspective agents are based on modifying the agent dynamics via local feedback to achieve some form of homogeneity. There have been many results for synchronization of heterogeneous networks with introspective agents, see, for instance, References 19-24.

[*]Agents are said to be introspective when they have access to either exact or estimates of their states, otherwise they are called non-introspective.[15]

In real applications, networks may be subject to delays. Time delays may affect system performance and can even lead to instability. As discussed in Reference 25, two kinds of delays have been considered in the literature: input delays and communication delays. Input delays encapsulate the processing time to execute an input for each agent, whereas communication delays can be considered as the time it takes to transmit information from an agent to its destination. Many works have been focused on dealing with input delays, specifically with the objective of deriving an upper bound on the input delays such that agents can still achieve synchronization. See, for example, References 7,26-29. Some research has been done for networks subject to communication delays. Fundamentally, there are two approaches in the literature for dealing with MAS subject to communication delays.

1. Standard state/output synchronization where the objective is to regulate the output to a constant trajectory.
2. Delayed state/output synchronization.

Both of these approaches preserve diffusiveness of the couplings (i.e., ensuring the invariance of the consensus manifold). Also, the notion of delayed output synchronization coincides with standard regulated output synchronization if the regulated output is required to be a constant trajectory. As such delayed synchronization can be viewed as the generalization of standard synchronization in the context of MAS subject to communication delay.

The majority of research on MAS subject to communication delay has been focused on achieving standard output synchronization by regulating the output to constant trajectory (see References 25, 28, 30, and 31 and references therein). It is worth noting that in all of the aforementioned papers, design of protocols requires at least some knowledge about the graph or the size of the network. We should also point out that References 32 and 33 give consensus conditions for networks with higher-order but require SISO dynamics. The paper[34] considers second-order dynamics, but the communication delays are assumed to be known. More recently, the notion of delayed synchronization was introduced in Reference 35 for MAS with passive agents and strongly connected and balanced graphs where it is assumed that there exists a unique path between any two distinct nodes. Then, the authors extended their results in References 36 and 37 when they allowed multiple paths between two agents in strongly connected communication graphs. Although the synchronized trajectory in these papers is constant and standard definition of synchronization can be utilized, the authors motivation for utilizing delayed synchronization is exploring the possible existence of delayed-induced periodicity in synchronized trajectory of coupled systems. These solutions, provided they exist, can be valuable in several applications, as clarified in, for example, References 38 and 39. It is worth to note that the protocol design in these papers does not need knowledge of the graph, since they are restricted to passive agents. An interesting line of research utilizing delayed synchronization formulation was introduced recently in References 40 and 41. These papers considered a *dynamic* synchronized trajectory (i.e., nonconstant synchronized trajectory) and designed protocols to achieve regulated delayed state/output synchronization in the presence of communication delays under the condition that the communication graph was a spanning tree. However, the protocol design required knowledge of the graph and size of the network.

In this article, we extend our earlier results of delayed synchronization by developing a *scale-free* framework utilizing localized information exchange for homogeneous and heterogeneous MAS subject to unknown, nonuniform, and arbitrarily large communication delays to achieve delayed regulated synchronization when the synchronized trajectory is a *dynamic* signal generated by a so-called exosystem. The associated graphs to the communication networks are assumed to be a directed spanning tree (i.e., they have one root node and the other non-root nodes have in-degree one). We achieve scale-free delayed regulated state synchronization for homogeneous MAS with non-introspective agents, and scale-free delayed regulated output synchronization for heterogeneous MAS with introspective agents. Our proposed design methodologies are scale-free, namely,

- The design is independent of information about the communication network such as the spectrum of the associated Laplacian matrix or the size of the network.
- The collaborative protocols will work for any network with associated directed spanning tree, and can tolerate any unknown, nonuniform, and arbitrarily large communication delays.

1.1 Notations and definitions

Given a matrix $A \in \mathbb{R}^{m\times n}$, A^{T} denotes the transpose of A. Let $\mathbf{j}$ denote $\sqrt{-1}$. A square matrix A is Hurwitz stable if all its eigenvalues are in the open left half complex plane. A linear system characterized by (A, B, C) is at most weakly unstable if all eigenvalues of A are in the closed left half plane. It should be noted that the set of at most weakly unstable agents contains stable agents, neutrally stable agents as well as weakly unstable agents.

$A \otimes B$ indicates the Kronecker product between A and B. I_n denotes the n-dimensional identity matrix and 0_n denotes $n \times n$ zero matrix; when the dimension is clear from the context, we drop the subscript. For linear time-invariant systems, we recall the following definitions.

Definition 1. A linear time-invariant system (A, B, C) is right-invertible if, given a smooth reference output y_r, there exists an initial condition $x(0)$ and an input u that ensures $y(t) = y_r(t)$ for all $t \geq 0$. For single-input-single-output system, a system is right-invertible if and only if its transfer function is nonzero.

Definition 2. The invariant zeros of a linear system (A, B, C) are those points $\lambda \in \mathbb{C}$ for which

$$\operatorname{rank}\begin{pmatrix} \lambda I - A & -B \\ C & 0 \end{pmatrix} < \operatorname{normrank}\begin{pmatrix} sI - A & -B \\ C & 0 \end{pmatrix},$$

where by *normrank* we mean the rank of a matrix with entries in the field of rational functions.

We describe the topology of the network by an associated graph. The *weighted graph* $\mathcal{G}$ of order N is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, \dots, N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix with non-negative elements a_{ij}. Each pair in $\mathcal{E}$ is called an *edge*, where $a_{ij} > 0$ denotes an edge $(j, i) \in \mathcal{E}$ from node j to node i with weight a_{ij}. Moreover, $a_{ij} = 0$ if there is no edge from node j to node i. We assume there are no self-loops, that is, we have $a_{ii} = 0$. A *directed path* is a sequence of nodes $\{i_1, \dots, i_k\}$ in a directed graph such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \dots, k-1$. A *directed tree* is a subgraph (subset of nodes and edges) in which every node has exactly one parent node except for one node, called the *root*, which has no parent node. A directed graph has a *directed spanning tree* if there exists at least one node that has a directed path to all the other nodes. The *Laplacian matrix* with respect to the weighted graph $\mathcal{G}$ is $L = [\ell_{ij}]$ with $\ell_{ii} = \sum_{k=1}^{N} a_{ik}$ and $\ell_{ij} = -a_{ij}, i \neq j$. If the graph contains a directed spanning tree, the Laplacian matrix L has a single eigenvalue at the origin and all other eigenvalues are located in the open right-half complex plane.[1]

2 | HOMOGENEOUS MAS WITH NON-INTROSPECTIVE AGENTS

Consider a MAS consists of N identical linear agents

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i, \\ y_i = Cx_i, \end{cases} \tag{1}$$

for $i \in \{1, \dots, N\}$, where $x_i \in \mathbb{R}^n$, $y_i \in \mathbb{R}^p$, and $u_i \in \mathbb{R}^m$ are the state, output, and the input of agent i, respectively.

We make the following assumption on agent models.

Assumption 1. For agent models

1. Agents are at most weakly unstable.
2. (A, B) is stabilizable and (A, C) is detectable.

The network provides agent i with the following information

$$\zeta_i(t) = \sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t - \tau_{ij})), \tag{2}$$

where $\tau_{ij} \in \mathbb{R}_{\geq 0}$ represents an unknown communication delay from agent j to agent i where we assume that $\tau_{ii} = 0$. In the above $a_{ij} \geq 0$. This communication topology of the network, presented in (2), can be associated to a weighted graph $\mathcal{G}$ with each node indicating an agent in the network and the weight of an edge is given by the coefficient a_{ij}. The communication delay implies that it took τ_{ij} seconds for agent j to transfer its state information to agent i.

In terms of the coefficients of the associated Laplacian matrix L, $\zeta_i(t)$ can be represented as

$$\zeta_i(t) = \sum_{j=1}^{N} \ell_{ij} y_j(t - \tau_{ij}), \tag{3}$$

where $\tau_{ii} = 0$. We refer to (3) as partial-state coupling since only part of the states are communicated over the network. When $C = I$, it means all states are shared over the network and we call it full-state coupling. Then, the original agents are expressed as

$$\dot{x}_i = Ax_i + Bu_i \tag{4}$$

and $\zeta_i(t)$ is written as

$$\zeta_i(t) = \sum_{j=1}^{N} \ell_{ij} x_j(t - \tau_{ij}). \tag{5}$$

In this article, we need an assumption on communication graph which is formulated in the following definition.

Definition 3. Let $\mathbb{G}^N$ denote the set of directed spanning tree graphs with N nodes for which the corresponding Laplacian matrix L is lower triangular. The corresponding Laplacian matrix L has the property that the entries of the first row are equal to zero and $\ell_{ii} > 0$ for $i = 2, \dots, N$. We consider agent 1 as the root agent.

Remark 1. Note that any graph, which is a directed spanning tree, has a possible lower triangular Laplacian matrix after reordering of the agents.

For the graph defined by Definition 3, we have

$$L = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ \ell_{21} & \ell_{22} & 0 & \dots & 0 \\ \ell_{31} & \ell_{32} & \ell_{33} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \ell_{N1} & \dots & \ell_{N,N-2} & \ell_{N,N-1} & \ell_{N,N} \end{pmatrix}.$$

Since the graph is equal to a directed spanning tree, in every row (except the first one) there are exactly two elements unequal to 0.

Our goal is to achieve delayed state synchronization among all agents while the synchronized state dynamics of each agent are equal to a time-shifted priori given trajectory generated by a so-called exosystem

$$\begin{aligned} \dot{x}_r &= Ax_r, \quad x_r(0) = x_{r0}, \\ y_r &= Cx_r, \end{aligned} \tag{6}$$

where $x_r \in \mathbb{R}^n$ and $y_r \in \mathbb{R}^p$. Clearly, we need some level of communication between the desired state trajectory and the agents. According to the structure of communication network, we assume that only agent 1 has access to y_r with delay τ_{1r}. Since the graph is a spanning tree, there is a unique path between agent i and the exosystem which is connected to agent 1. We define τ_{ir} as the sum of delays from agent i to the exosystem through this path. Then, we can define

$$\psi_i = \iota_i(y_i(t) - y_r(t - \tau_{ir})), \qquad \iota_i = \begin{cases} 1, & i = 1, \\ 0, & i = 2, \dots, N. \end{cases} \tag{7}$$

Therefore, the information available for agent $i \in \{1, \dots, N\}$ is given by

$$\bar{\zeta}_i(t) = \sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t - \tau_{ij})) + \iota_i(y_i(t) - y_r(t - \tau_{ir})). \tag{8}$$

We define the expanded Laplacian matrix for any graph $\mathbb{G}^N$, with the associated Laplacian matrix L, as

$$\overline{L} = L + \text{diag}\{\iota_i\} = [\bar{l}_{ij}]_{N\times N}. \tag{9}$$

Note that $\overline{L}$ is not a regular Laplacian matrix, since the sum of its rows need not be zero. Obviously, all the eigenvalues of $\overline{L}$, have positive real parts, that is, the matrix $\overline{L}$ is invertible. In terms of the coefficients of the matrix $\overline{L}$, Equation (8) can be rewritten as

$$\bar{\zeta}_i(t) = \sum_{j=1}^{N} \bar{\ell}_{ij}(y_j(t-\tau_{ij}) - y_r(t-\tau_{ir})) \tag{10}$$

and for full-state coupling case

$$\bar{\zeta}_i(t) = \sum_{j=1}^{N} \bar{\ell}_{ij}(x_j(t-\tau_{ij}) - x_r(t-\tau_{ir})). \tag{11}$$

We introduce the following definitions.

Definition 4. The agents of a MAS are said to achieve

- delayed state synchronization for all $i \in \{1, \dots, N\}$, if

$$\lim_{t\to\infty} \left[(x_i(t) - x_j(t-\tau_{ij})\right] = 0, \quad \text{for all } j \text{ such that } (j,i) \in \mathcal{E}, \tag{12}$$

- and delayed regulated state synchronization if

$$\lim_{t\to\infty} [(x_i(t) - x_r(t-\tau_{ir})] = 0, \quad \text{for all} \quad i \in \{1, \dots, N\}. \tag{13}$$

The goal of this article is to design scale-free protocols which can be achieved by utilizing localized information exchange among neighbors, as such each agent $i = 1, \dots, N$ also has access to localized information exchange denoted by $\hat{\zeta}_i$, of the form

$$\hat{\zeta}_i = \sum_{j=1}^{N} a_{ij}(\xi_i(t) - \xi_j(t-\hat{\tau}_{ij})), \tag{14}$$

where $\xi_j \in \mathbb{R}^n$ is a variable produced internally by agent j and to be defined in next sections while $\hat{\tau}_{ij} \in \mathbb{R}_{\geq 0}$ $(i \neq j)$ represents an unknown communication delay from agent j to agent i.

We formulate the following problem of scalable delayed regulated state synchronization for networks in presence of unknown, nonuniform, and arbitrarily large communication delay for the homogeneous networks as follows.

Problem 1. Consider a MAS described by (1) and (10) and the exosystem (6). Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 3. Then, the scalable delayed regulated state synchronization problem based on localized information exchange utilizing collaborative protocols for networks with unknown, nonuniform, and arbitrarily large communication delay is to find, if possible, a linear dynamic protocol for each agent $i \in \{1, \dots, N\}$, using only knowledge of agent model, that is, (A, B, C), of the form:

$$\begin{cases} \dot{x}_{c,i} = A_c x_{c,i} + B_{c1}\bar{\zeta}_i + B_{c2}\hat{\zeta}_i, \\ u_i = F_c x_{c,i}, \end{cases} \tag{15}$$

where $\hat{\zeta}_i$ is defined in (14) with $\xi_i = H_c x_{c,i}$ and $x_{c,i} \in \mathbb{R}^{n_c}$ such that for any N, any graph $\mathcal{G} \in \mathbb{G}^N$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ and $\hat{\tau}_{ij} \in \mathbb{R}_{\geq 0}$, we achieve delayed regulated state synchronization as stated by (13) in Definition 4.

2.1 | Protocol design

In this section, we provide our results for scalable delayed regulated state synchronization of MAS with full- and partial-state coupling.

2.1.1 | Full-state coupling

First, we consider MAS with full-state coupling, that is, with $C = I$.

Protocol 1. Homogeneous MAS with full-state coupling

We design collaborative protocols based on localized information exchanges for agents $i = 1, \ldots, N$ as

$$\begin{cases} \dot{\chi}_i = A\chi_i + Bu_i + \bar{\zeta}_i - \hat{\zeta}_i - \iota_i\chi_i, \\ u_i = -K\chi_i, \end{cases} \tag{16}$$

where $\bar{\zeta}_i$ is defined by (11) and $\hat{\zeta}_i$ is given by

$$\hat{\zeta}_i(t) = \sum_{j=1}^{N} a_{ij}(\chi_i(t) - \chi_j(t - \hat{\tau}_{ij})), \tag{17}$$

which means the agents communicate $\xi_i = \chi_i$. Matrix K is designed such that $A - BK$ is Hurwitz stable.

Then, we have the following theorem for scalable delayed regulated state synchronization of MAS with full-state coupling.

Theorem 1. *Consider a MAS consisting of N agents described by (4) and (11) and the associated exosystem (6) where the agents satisfy Assumption 1. Let $\mathbb{G}^N$ be the set of network graphs as defined by Definition 3. Then, the scalable delayed regulated state synchronization problem as defined in Problem 1 is solvable. In particular, the linear dynamic protocol (16) solves delayed regulated state synchronization problem for any N, any graph $\mathcal{G} \in \mathbb{G}^N$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ and $\hat{\tau}_{ij} \in \mathbb{R}_{\geq 0}$.*

Remark 2. It is worth to note that in the case that agents are introspective, that is, they have access to some knowledge about their own states (i.e., $z_i = C^m x_i$, where (C^m, A) is detectable), we do not have the restriction that the exosystem has the same model as the agents. In other words, we can then regulate to any arbitrary signal. After all, given that agents are introspective, one can reshape agent models via standard observer-based feedback, locally designed for each agent, to embed the desired modes of the exosystem in the agent models.

In the proof of Theorem 1, we need the following lemma from [42, Lemma 3].

Lemma 1. *Consider a linear time-delay system*

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{m} A_i x(t - \tau_i), \tag{18}$$

where $x(t) \in \mathbb{R}^n$ and $\tau_i \in \mathbb{R}_{\geq 0}$. Assume that $A + \sum_{i=1}^{m} A_i$ is Hurwitz stable. Then, (18) is asymptotically stable for $\tau_1, \ldots, \tau_N \in [0, \bar{\tau}]$ if

$$\det\left[\mathbf{j}\omega I - A - \sum_{i=1}^{m} e^{-\mathbf{j}\omega\tau_i} A_i\right] \neq 0, \tag{19}$$

for all $\omega \in \mathbb{R}$, and for all $\tau_1, \ldots, \tau_N \in [0, \bar{\tau}]$.

Proof of Theorem 1. First, we define

$$\bar{x}_i = x_i(t+\tau_{ir}) \text{ and } \bar{\chi}_i = \chi_i(t+\tau_{ir}),$$

where τ_{ir} denotes the sum of delays from agent i to the exosystem. Note that τ_{ir} is unique since the communication graph is spanning tree. Note that $\tau_{ij} = \tau_{ir} - \tau_{jr}$ if there is an edge from agent j to agent i (i.e., if $\bar{\ell}_{ij} \neq 0$), we have

$$\begin{aligned}\bar{\bar{\zeta}}_i &= \bar{\zeta}_i(t+\tau_{ir}) = \sum_{j=1}^{N} \bar{\ell}_{ij}(x_j(t+\tau_{ir}-\tau_{ij}) - x_r(t)) \\ &= \sum_{j=1}^{N} \bar{\ell}_{ij}(\bar{x}_j(t) - x_r(t))\end{aligned} \tag{20}$$

and

$$\begin{aligned}\bar{\hat{\zeta}}_i &= \hat{\zeta}_i(t+\tau_{ir}) = \sum_{j=1}^{N} \ell_{ij}(\chi_j(t+\tau_{ir}-\hat{\tau}_{ij})) \\ &= \sum_{j=1}^{N} \ell_{ij}\bar{\chi}_j(t+\tau_{ij}-\hat{\tau}_{ij}).\end{aligned} \tag{21}$$

Then, by defining $\tilde{x}_i(t) = \bar{x}_i(t) - x_r(t)$ and

$$\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_N \end{pmatrix}, \quad \bar{\chi} = \begin{pmatrix} \bar{\chi}_1 \\ \vdots \\ \bar{\chi}_N \end{pmatrix}$$

we have the following closed-loop system in frequency domain as

$$\begin{cases} \mathbf{j}\omega\tilde{x} = (I\otimes A)\tilde{x} - (I\otimes BK)\bar{\chi}, \\ \mathbf{j}\omega\bar{\chi} = (I\otimes(A-BK))\,\bar{\chi} + (\bar{L}\otimes I)\tilde{x} - (\bar{L}_{\mathbf{j}\omega}(\tau)\otimes I)\bar{\chi}, \end{cases}$$

where $\bar{L}_{\mathbf{j}\omega}(\tau) = L_{\mathbf{j}\omega}(\tau) + \operatorname{diag}\{\iota_i\}$ and

$$L_{\mathbf{j}\omega}(\tau) = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ \ell_{21}e^{-\mathbf{j}\omega(\hat{\tau}_{21}-\tau_{21})} & \ell_{22} & 0 & \dots & 0 \\ \vdots & \dots & \ddots & \ddots & \vdots \\ \ell_{N1}e^{-\mathbf{j}\omega(\hat{\tau}_{N1}-\tau_{N1})} & \ell_{N2}e^{-\mathbf{j}\omega(\hat{\tau}_{N2}-\tau_{N2})} & \dots & \dots & \ell_{NN} \end{pmatrix}.$$

Let $\delta = \tilde{x} - \bar{\chi}$. Then, we obtain,

$$\begin{cases} \mathbf{j}\omega\tilde{x} = (I\otimes(A-BK))\tilde{x} + (I\otimes BK)\delta, \\ \mathbf{j}\omega\delta = (I\otimes A - \bar{L}_{\mathbf{j}\omega}(\tau)\otimes I)\delta + \left((\bar{L}_{\mathbf{j}\omega}(\tau) - \bar{L})\otimes I\right)\tilde{x}. \end{cases} \tag{22}$$

Following Lemma 1, we prove the stability of (22) in two steps. In the first step, we prove the stability in the absence of communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ and $\hat{\tau}_{ij} \in \mathbb{R}_{\geq 0}$ and in the second step, we prove the stability of (22) by checking condition (19).

- In the absence of communication delays in the network, the stability of system (22) is equivalent to the stability of matrix

$$\begin{pmatrix} I\otimes(A-BK) & I\otimes BK \\ 0 & I\otimes A - \bar{L}\otimes I \end{pmatrix}. \tag{23}$$

Since we have that $\overline{\ell}_{ii}$ is positive for all i, we have that

$$I \otimes A - \hat{L} \otimes I$$

is an upper triangular matrix with $A - \overline{\ell}_{ii} I$ for $i = 1, \dots, N$, on the diagonal. Since all eigenvalues of A are in the closed left half plane, $A - \overline{\ell}_{ii} I$ is stable. Therefore, all eigenvalues of $I \otimes A - \overline{L} \otimes I$ have negative real part. Then, since we have that $A - BK$ and $I \otimes A - \overline{L} \otimes I$ are Hurwitz stable, we obtain that

$$\lim_{t \to \infty} \tilde{x}(t) \to 0$$

which implies that $\overline{x}_i \to x_r$.

- In the presence of communication delay, the closed-loop system (22) is asymptotically stable if

$$\det \left[\mathbf{j}\omega I - \begin{pmatrix} I \otimes (A - BK) & I \otimes BK \\ (\overline{L}_{\mathbf{j}\omega}(\tau) - \overline{L}) \otimes I & I \otimes A - \overline{L}_{\mathbf{j}\omega} \otimes I \end{pmatrix} \right] \neq 0 \tag{24}$$

for all $\omega \in \mathbb{R}$ and any communication delays $\tau_{ij} \in \mathbb{R}^+$ and $\hat{\tau}_{ij} \in \mathbb{R}^+$. Condition (24) is satisfied if matrix

$$\begin{pmatrix} I \otimes (A - BK) & I \otimes BK \\ (\overline{L}_{\mathbf{j}\omega}(\tau) - \overline{L}) \otimes I & I \otimes A - \overline{L}_{\mathbf{j}\omega} \otimes I \end{pmatrix} \tag{25}$$

has no eigenvalues on the imaginary axis for all $\omega \in \mathbb{R}$. That is to say it is sufficient to prove the stability of

$$\begin{cases} \mathbf{j}\omega\tilde{x} = (I \otimes (A - BK))\tilde{x} + (I \otimes BK)\delta, \\ \mathbf{j}\omega\delta = ((\overline{L}_{\mathbf{j}\omega}(\tau) - \overline{L}) \otimes I)\tilde{x} + (I \otimes A - \overline{L}_{\mathbf{j}\omega} \otimes I)\delta. \end{cases} \tag{26}$$

According to the structure of the expanded Laplacian matrix $\overline{L}$, (26) can be rewritten as

$$\begin{cases} \mathbf{j}\omega\tilde{x}_1 = (A - BK)\tilde{x}_1 + BK\delta_1 \\ \mathbf{j}\omega\delta_1 = (A - \overline{\ell}_{11} I)\delta_1 \end{cases} \tag{27}$$

and

$$\begin{cases} \mathbf{j}\omega\tilde{x}_i = (A - BK)\tilde{x}_i + BK\delta_i \\ \mathbf{j}\omega\delta_i = (A - \overline{\ell}_{ii})\delta_i - \sum_{j=1}^{i-1} \overline{\ell}_{ij} e^{\mathbf{j}\omega(\tau_{ij} - \hat{\tau}_{ij})}\delta_j + \sum_{j=1}^{i-1}(1 - e^{\mathbf{j}\omega(\tau_{ij} - \hat{\tau}_{ij})})\overline{\ell}_{ij}\tilde{x}_j \end{cases} \tag{28}$$

for $i = 2, \dots, N$.

Then for $i = 1$, since $\overline{\ell}_{11} > 0$, one can obtain that all eigenvalues of $A - \overline{\ell}_{11} I$ have negative real part, that is

$$\delta_1 \to 0 \text{ as } t \to \infty$$

then, given that $A - BK$ is Hurwitz stable, we have

$$\tilde{x}_1 \to 0 \text{ as } t \to \infty.$$

Therefore, the dynamics of $\tilde{x}_1$, and e_1 are asymptotically stable.

Then, for $i = 2$, we have

$$\begin{cases} \mathbf{j}\omega\tilde{x}_2 = (A - BK)\tilde{x}_2 + BK\delta_2, \\ \mathbf{j}\omega\delta_2 = (A - \overline{\ell}_{22} I)\delta_2 - \ell_{21} e^{\mathbf{j}\omega(\tau_{21} - \hat{\tau}_{21})}\delta_1 + (1 - e^{\mathbf{j}\omega(\tau_{21} - \hat{\tau}_{21})})\ell_{21}\tilde{x}_1. \end{cases} \tag{29}$$

Since the dynamics for $\tilde{x}_1$ and δ_1 are asymptotically stable, we just need to prove the stability of

$$\begin{cases} \mathbf{j}\omega\tilde{x}_2 = (A - BK)\tilde{x}_2 + BK\delta_2, \\ \mathbf{j}\omega\delta_2 = (A - \overline{\ell}_{22}I)\delta_2. \end{cases} \tag{30}$$

Similar to the analysis of the stability of system (27), since $\overline{\ell}_{22} > 0$, we have

$$\delta_2 \to 0, \quad \text{and} \quad \tilde{x}_2 \to 0,$$

as $t \to \infty$. Using a recursive argument, we can thus obtain that

$$\delta_i \to 0, \quad \text{and} \quad \tilde{x}_i \to 0, \quad \text{as} \quad t \to \infty$$

for $i = 2, \dots, N$, which is equivalent to the stability of system (26). In other words, condition (24) is satisfied. Therefore, based on Lemma 1, for all τ_{ij} and $\hat{\tau}_{ij}$,

$$\overline{x}_i \to x_r$$

as $t \to \infty$, which means that delayed synchronization (12) is achieved. ■

2.1.2 | Partial-state coupling

In this subsection, we consider MAS with partial-state coupling, that is, $C \neq I$.

Protocol 2. Homogeneous MAS with partial-state coupling

We design collaborative protocols based on localized information exchanges for agents $i = 1, \dots, N$ as

$$\begin{cases} \dot{\hat{x}}_i = A\hat{x}_i - BK\hat{\zeta}_i + H(\overline{\zeta}_i - C\hat{x}_i) + \iota_i Bu_i \\ \dot{\chi}_i = A\chi_i + Bu_i + \hat{x}_i - \hat{\zeta}_i - \iota_i\chi_i, \\ u_i = -K\chi_i, \end{cases} \tag{31}$$

where $\overline{\zeta}_i$ is defined by (10) and $\hat{\zeta}_i$ is given by

$$\hat{\zeta}_i(t) = \sum_{j=1}^{N} a_{ij}(\chi_i(t) - \chi_j(t - \hat{\tau}_{ij})), \tag{32}$$

which means the agents communicate $\xi_i = \chi_i$. Matrices K and H are designed such that $A - BK$ and $A - HC$ are Hurwitz stable.

Then, we have the following theorem for scalable delayed regulated state synchronization of MAS with partial-state coupling.

Theorem 2. *Consider a MAS consisting of N agents described by (1) and (10) and the associated exosystem (6) where the agents satisfy Assumption 1. Let $\mathbb{G}^N$ be the set of network graphs as defined by Definition 3. Then, the scalable delayed state synchronization problem as defined in Problem 1 is solvable. In particular, the linear dynamic protocol (31) solves delayed regulated state synchronization problem for any N, any graph $\mathcal{G} \in \mathbb{G}^N$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ and $\hat{\tau}_{ij} \in \mathbb{R}_{\geq 0}$.*

Proof of Theorem 2. Similar to the proof of Theorem 1 and by defining $\hat{\overline{x}}_i(t) = \hat{x}_i(t + \tau_{i,r})$ and $\hat{\overline{x}} = \left(\hat{\overline{x}}_1^{\mathrm{T}}, \dots, \hat{\overline{x}}_N^{\mathrm{T}}\right)^{\mathrm{T}}$, we have the following closed-loop system in frequency domain as

$$\begin{cases} \mathbf{j}\omega\tilde{x} = (I \otimes A)\tilde{x} - (I \otimes BK)\overline{\chi}, \\ \mathbf{j}\omega\overline{\chi} = (I \otimes (A - BK))\,\overline{\chi} + \hat{\overline{x}} - (\overline{L}_{\mathbf{j}\omega}(\tau) \otimes I)\overline{\chi}, \\ \mathbf{j}\omega\hat{\overline{x}} = (I \otimes (A - HC))\,\hat{\overline{x}} - (\overline{L}_{\mathbf{j}\omega}(\tau) \otimes BK)\overline{\chi} + (\overline{L} \otimes HC)\tilde{x}. \end{cases}$$

Then, by defining $\delta = \tilde{x} - \overline{\chi}$ and $\overline{\delta} = (\overline{L}_{\mathbf{j}\omega}(\tau) \otimes I)\tilde{x} - \hat{\overline{x}}$, we obtain

$$\begin{cases} \mathbf{j}\omega\tilde{x} = (I \otimes (A - BK))\tilde{x} + (I \otimes BK)\delta, \\ \mathbf{j}\omega\delta = \left(I \otimes A - \overline{L}_{\mathbf{j}\omega}(\tau) \otimes I\right)\delta + \overline{\delta}, \\ \mathbf{j}\omega\overline{\delta} = (I \otimes (A - HC))\,\overline{\delta} + ((\overline{L}_{\mathbf{j}\omega}(\tau) - \overline{L}) \otimes HC)\tilde{x}. \end{cases} \tag{33}$$

We prove (33) is asymptotically stable for all communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ and $\hat{\tau}_{ij} \in \mathbb{R}_{\geq 0}$. Similar to the proof of Theorem 1, following the critical Lemma 1, we first prove stability without communication delays τ_{ij} and $\hat{\tau}_{ij}$ and then we prove the stability of (33) by checking condition (19).

- In the absence of communication delays in the network, the stability of system (33) is equivalent to the stability of matrix

$$\begin{pmatrix} I \otimes (A - BK) & I \otimes BK & 0 \\ 0 & I \otimes A - \overline{L} \otimes I & I \\ 0 & 0 & I \otimes (A - HC) \end{pmatrix} \tag{34}$$

similar to the proof of Theorem 1, we have all eigenvalues of $I \otimes A - \overline{L} \otimes I$ have negative real part. Then, since we have that $A - BK$ and $I \otimes A - \overline{L} \otimes I$ are Hurwitz stable, we obtain that

$$\lim_{t \to \infty} \tilde{x} \to 0.$$

It implies that $\overline{x}_i \to x_r$.

- In the presence of communication delay, the closed-loop system (33) is asymptotically stable if

$$\det\left[\mathbf{j}\omega I - \begin{pmatrix} I \otimes (A - BK) & I \otimes BK & 0 \\ 0 & I \otimes A - \overline{L}_{\mathbf{j}\omega} \otimes I & I \\ (\overline{L}_{\mathbf{j}\omega}(\tau) - \overline{L}) \otimes HC & 0 & I \otimes (A - HC) \end{pmatrix}\right] \neq 0 \tag{35}$$

for all $\omega \in \mathbb{R}$ and any communication delays $\tau_{ij} \in \mathbb{R}^+$ and $\hat{\tau}_{ij} \in \mathbb{R}^+$. Condition (35) is satisfied if matrix

$$\begin{pmatrix} I \otimes (A - BK) & I \otimes BK & 0 \\ 0 & I \otimes A - \overline{L}_{\mathbf{j}\omega} \otimes I & I \\ (\overline{L}_{\mathbf{j}\omega}(\tau) - \overline{L}) \otimes HC & 0 & I \otimes (A - HC) \end{pmatrix} \tag{36}$$

has no eigenvalues on the imaginary axis for all $\omega \in \mathbb{R}$.

Then, according to the structure of the expanded Laplacian matrix $\overline{L}$, (33) can be rewritten as

$$\begin{cases} \mathbf{j}\omega\tilde{x}_1 = (A - BK)\tilde{x}_1 + BK\delta_1 \\ \mathbf{j}\omega\delta_1 = (A - \overline{\ell}_{11}I)\delta_1 + \overline{\delta}_1 \\ \mathbf{j}\omega\overline{\delta}_1 = (A - HC)\,\overline{\delta}_1 \end{cases} \tag{37}$$

and

$$\begin{cases} \mathbf{j}\omega\tilde{x}_i = (A - BK)\tilde{x}_i + BK\delta_i \\ \mathbf{j}\omega\delta_i = (A - \overline{\ell}_{ii}I)\delta_i - \sum_{j=1}^{i-1}\ell_{ij}e^{\mathbf{j}\omega(\tau_{ij} - \hat{\tau}_{ij})}\delta_j + \overline{\delta}_i \\ \mathbf{j}\omega\overline{\delta}_i = (A - HC)\,\overline{\delta}_i + \sum_{j=1}^{i-1}(1 - e^{\mathbf{j}\omega(\tau_{ij} - \hat{\tau}_{ij})})\ell_{ij}\tilde{x}_j \end{cases} \tag{38}$$

for $i = 2, \ldots, N$.

Then for $i = 1$, we have

$$\bar{\delta}_1 \to 0 \text{ as } t \to \infty$$

since $A - HC$ is Hurwitz stable. In the following, since $\bar{\ell}_{11} > 0$, one can obtain that all eigenvalues of $A - \bar{\ell}_{11} I$ have negative real part, that is

$$\delta_1 \to 0 \text{ as } t \to \infty$$

then, given that $A - BK$ is Hurwitz stable, we have

$$\tilde{x}_1 \to 0 \text{ as } t \to \infty.$$

Therefore, the dynamics of $\tilde{x}_1$, δ_1, and $\bar{\delta}_1$ are asymptotically stable.

Next, for $i = 2$, we have

$$\begin{cases} \mathbf{j}\omega\tilde{x}_2 = (A - BK)\tilde{x}_2 + BK\delta_2, \\ \mathbf{j}\omega\delta_2 = (A - \bar{\ell}_{22}I)\delta_2 - \ell_{21}e^{\mathbf{j}\omega(\tau_{21}-\hat{\tau}_{21})}\delta_1 + \bar{\delta}_2, \\ \mathbf{j}\omega\bar{\delta}_2 = (A - HC)\bar{\delta}_2 + (1 - e^{\mathbf{j}\omega(\tau_{21}-\hat{\tau}_{21})})\ell_{21}\tilde{x}_1. \end{cases} \tag{39}$$

Since we have that dynamics of $\tilde{x}_1$ and δ_1 are asymptotically stable, we just need to prove the stability of

$$\begin{cases} \mathbf{j}\omega\bar{x}_2 = (A - BK)\bar{x}_2 + BK\delta_2, \\ \mathbf{j}\omega\delta_2 = (A - \bar{\ell}_{22}I)\delta_2 + \bar{\delta}_2, \\ \mathbf{j}\omega\bar{\delta}_2 = (A - HC)\bar{\delta}_2. \end{cases} \tag{40}$$

Similar to the analysis of stability of system (37), since $\bar{\ell}_{22} > 0$, we have

$$\delta_2 \to 0, \bar{\delta}_2 \to 0, \text{ and } \tilde{x}_2 \to 0$$

as $t \to \infty$. We can then use a recursive argument to prove that

$$\delta_i \to 0, \bar{\delta}_i \to 0, \text{ and } \tilde{x}_i \to 0, \text{ as } t \to \infty$$

for $i = 1, \ldots, N$, which is equivalent to the stability of system (33). In other words, condition (35) is satisfied. Therefore, based on Lemma 1, for all τ_{ij} and $\hat{\tau}_{ij}$,

$$\bar{x}_i \to x_r$$

as $t \to \infty$, which means that delayed synchronization (12) is achieved.

■

3 | HETEROGENEOUS MAS WITH INTROSPECTIVE AGENTS

In this section, we study a heterogeneous MAS consisting of N nonidentical linear agents:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i, \\ y_i &= C_i x_i, \end{aligned} \tag{41}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, and $y_i \in \mathbb{R}^p$ are the state, input, output of agent i for $i = 1, \ldots, N$.

The agents are introspective, meaning that each agent has access to its own local information. Specifically, each agent has access to part of its state

$$z_i = C_i^m x_i, \tag{42}$$

where $z_i \in \mathbb{R}^{q_i}$.

Our goal is to achieve delayed output synchronization among all agents while the synchronized output dynamics of each agent are equal to a time-shifted priori given trajectory generated by the following exosystem†

$$\begin{aligned} \dot{x}_r &= A_r x_r, \quad x_r(0) = x_{r0}, \\ y_r &= C_r x_r, \end{aligned} \tag{43}$$

where $x_r \in \mathbb{R}^n$ and $y_r \in \mathbb{R}^p$. We make the following assumptions on agents and the exosystem.

Assumption 2. For agents $i \in \{1, \dots, N\}$,

1. (C_i, A_i, B_i) is stabilizable and detectable.
2. (C_i, A_i, B_i) is right-invertible.
3. (C_i^m, A_i) is detectable.

Assumption 3. For exosystem,

1. (C_r, A_r) is observable.
2. All the eigenvalues of A_r are on the imaginary axis.

Clearly, we need some level of communication between the desired output trajectory y_r and the agents. We assume that only agent 1 has access to y_r with delay τ_{1r}. Since the graph is spanning tree, there is a unique path between agent i and the exosystem which is connected to agent 1 as such similar to the previous section we define τ_{ir} as sum of the delays from agent i to the exosystem. Then, in light of (7) and (9), the available data for agent i, provided by the communication network can be written as

$$\bar{\zeta}_i(t) = \sum_{j=1}^{N} \bar{\ell}_{ij}(y_j(t - \tau_{ij}) - y_r(t - \tau_{ir})). \tag{44}$$

Next we introduce the following definitions.

Definition 5. The agents of a heterogeneous MAS are said to achieve

- delayed output synchronization for all $i \in \{1, \dots, N\}$, if

$$\lim_{t \to \infty} \left[(y_i(t) - y_j(t - \tau_{ij}) \right] = 0, \quad \text{for all } j \text{ such that } (j, i) \in \mathcal{E}, \tag{45}$$

- and delayed regulated output synchronization if

$$\lim_{t \to \infty} [(y_i(t) - y_r(t - \tau_{ir})] = 0, \quad \text{for all } i \in \{1, \dots, N\}. \tag{46}$$

We formulate the problem of scalable delayed regulated output synchronization in presence of unknown nonuniform and arbitrarily large communication delay for the heterogeneous networks as follows.

Problem 2. Consider a MAS describes by (41), (42), and (44) and the exosystem (6). Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 3. Then, the scalable delayed regulated output synchronization problem based on localized information exchange utilizing collaborative protocols for heterogeneous networks with unknown nonuniform and arbitrarily large communication delay is to find, if possible, a linear dynamic protocol for each agent $i \in \{1, \dots, N\}$, using only knowledge of agent models, that is, (C_i, A_i, B_i) of the form:

$$\begin{cases} \dot{x}_{i,c} = A_{i,c} x_{i,c} + B_{i,c} \bar{\zeta}_i + C_{i,c} \hat{\zeta}_i + D_{i,c} z_i, \\ u_i = E_{i,c} x_{i,c} + F_{i,c} \bar{\zeta}_i + G_{i,c} \hat{\zeta}_i + H_{i,c} z_i, \end{cases} \tag{47}$$

†Please note that the exosystem in this case is more general than the exosystem in homogeneous section (6).

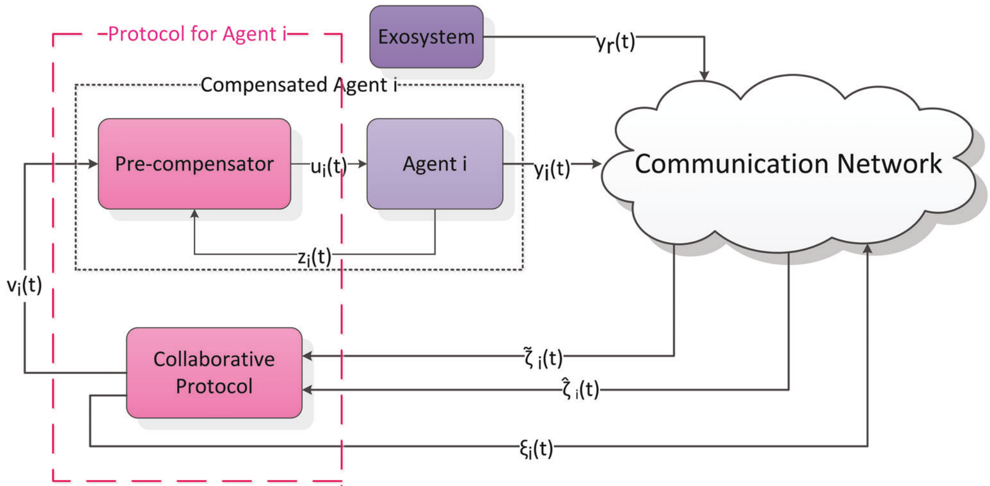

FIGURE 1 Architecture of scale-free protocols for output synchronization of heterogeneous networks

where $\hat{\zeta}_i$ is defined by (14) with $\xi_i = H_c x_{c,i}$ and $x_{c,i} \in \mathbb{R}^{n_c}$ such that for any N, any graph $\mathcal{G} \in \mathbb{G}^N$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ and $\hat{\tau}_{ij} \in \mathbb{R}_{\geq 0}$, we achieve delayed regulated output synchronization as stated by (46) in Definition 5.

We design scale-free protocols to solve scalable delayed regulated output synchronization problem as stated in Problem 2. After introducing the architecture of our protocol, we design the protocols through four steps.

3.1 | Architecture of the protocol

Our protocol has the structure shown below in Figure 1.

As seen in the figure, our design methodology consists of two major modules.

1. The first module is remodeling the exosystem to obtain the target model by designing pre-compensators following our previous results in Reference 20.
2. The second module is designing collaborate protocols for almost homogenized agents to achieve output and regulated output synchronization.

3.2 | Protocol design

To design our protocols, first we recall the following lemma.

Lemma 2 (20). *There exists another exosystem given by*

$$\begin{aligned} \dot{\check{x}}_r &= \check{A}_r \check{x}_r, \quad \check{x}_r(0) = \check{x}_{r0}, \\ y_r &= \check{C}_r \check{x}_r, \end{aligned} \tag{48}$$

such that for all $x_{r0} \in \mathbb{R}^r$, there exists $\check{x}_{r0} \in \mathbb{R}^{\check{r}}$ for which (48) generate exactly the same output y_r as the original exosystem (6). Furthermore, we can find a matrix $\check{B}_r$ such that the triple $(\check{C}_r, \check{A}_r, \check{B}_r)$ is invertible, of uniform rank n_q, and has no invariant zero, where n_q is an integer greater than or equal to maximal order of infinite zeros of $(C_i, A_i, B_i), i \in \{1, \dots, N\}$ and all the observability indices of (C_r, A_r). Note that the eigenvalues of $\check{A}_r$ consists of all eigenvalues of A_r and additional zero eigenvalues.

We design our protocols through the following four steps.

Then, we have the following theorem for scalable regulated output synchronization of heterogeneous MAS.

Theorem 3. *Consider a heterogeneous network of N agents described by (41) and (42) satisfying Assumption 2 and localized information exchange (44) and the associated exosystem (6) satisfying Assumption 3. Let $\mathbb{G}^N$ be the set of network graphs as defined by Definition 3. Then, the scalable delayed regulated output synchronization problem as defined in Problem 2 is*

Protocol 3. Heterogeneous MAS

Step 1: Remodeling the exosystem. First, we remodel the exosystem to arrive at suitable choice for the target model $(\check{C}_r, \check{A}_r, \check{B}_r)$ following the design procedure in Reference 20 summarized in Lemma 2.

Step 2: Designing pre-compensators. In this step, given the target model $(\check{C}_r, \check{A}_r, \check{B}_r)$, by utilizing the design methodology from [20, Appendix B], we design a pre-compensators for each agent $i \in \{1, \dots, N\}$ of the form

$$\begin{cases} \dot{\xi}_i = A_{i,h}\xi_i + B_{i,h}z_i + E_{i,h}v_i, \\ u_i = C_{i,h}\xi_i + D_{i,1h}z_i + D_{i,2h}v_i, \end{cases} \tag{49}$$

which yields the compensated agents as

$$\begin{aligned} \dot{x}_i^h &= \check{A}_r x_i^h + \check{B}_r(v_i + \rho_i), \\ y_i &= \check{C}_r x_i^h, \end{aligned} \tag{50}$$

where ρ_i is given by

$$\begin{aligned} \dot{\omega}_i &= A_{i,s}\omega_i, \\ \rho_i &= C_{i,s}\omega_i, \end{aligned} \tag{51}$$

and $A_{i,s}$ is Hurwitz stable. Note that the compensated agents are homogenized and have the target model $(\check{C}_r, \check{A}_r, \check{B}_r)$.

Step 3: Designing collaborative protocols for the compensated agents. Collaborative protocols based on localized information exchanges are designed for the compensated agents $i = 1, \dots, N$ as

$$\begin{cases} \dot{\hat{x}}_i = \check{A}_r\hat{x}_i - \check{B}_r K\hat{\zeta}_i + H(\bar{\zeta}_i - \check{C}_r\hat{x}_i) + \iota_i\check{B}_r v_i, \\ \dot{\chi}_i = \check{A}_r\chi_i + \check{B}_r v_i + \hat{x}_i - \hat{\zeta}_i - \iota_i\chi_i, \\ v_i = -K\chi_i, \end{cases} \tag{52}$$

where H and K are matrices such that $\check{A}_r - H\check{C}_r$ and $\check{A}_r - \check{B}_r K$ are Hurwitz stable. The exchanging information $\hat{\zeta}_i$ is defined as (14) and $\bar{\zeta}_i$ is defined as (44).

Step 4: Obtaining the protocols. The final protocol which is the combination of modules 1 and 2 is

$$\begin{cases} \dot{\xi}_i = A_{i,h}\xi_i + B_{i,h}z_i - E_{i,h}K\chi_i, \\ \dot{\hat{x}}_i = \check{A}_r\hat{x}_i - \check{B}_r K\hat{\zeta}_i + H(\bar{\zeta}_i - \check{C}_r\hat{x}_i) - \iota_i\check{B}_r K\chi_i, \\ \dot{\chi}_i = \check{A}_r\chi_i - \check{B}_r K\chi_i + \hat{x}_i - \hat{\zeta}_i - \iota_i\chi_i, \\ u_i = C_{i,h}\xi_i - D_{i,h}K\chi_i. \end{cases} \tag{53}$$

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solvable. In particular, the dynamic protocol (53) solves the scalable delayed regulated output synchronization problem for any N, any graph $\mathcal{G} \in \mathbb{G}^N$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ and $\hat{\tau}_{ij} \in \mathbb{R}_{\geq 0}$.

Proof of Theorem 3. Similar to the proof of Theorem 2 and by defining $\bar{x}_i(t) = x_i^h(t + \tau_{ir})$, $\bar{\rho}_i(t) = \rho_i(t + \tau_{ir})$, $\bar{\omega}_i(t) = \omega_i(t + \tau_{ir})$, $\tilde{x}_i = \bar{x}_i - \check{x}_r$, and

$$\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_N \end{pmatrix}, \hat{\bar{x}} = \begin{pmatrix} \hat{\bar{x}}_1 \\ \vdots \\ \hat{\bar{x}}_N \end{pmatrix}, \bar{\chi} = \begin{pmatrix} \bar{\chi}_1 \\ \vdots \\ \bar{\chi}_N \end{pmatrix}, \bar{\rho} = \begin{pmatrix} \bar{\rho}_1 \\ \vdots \\ \bar{\rho}_N \end{pmatrix}, \bar{\omega} = \begin{pmatrix} \bar{\omega}_1 \\ \vdots \\ \bar{\omega}_N \end{pmatrix}$$

then, we have the following closed-loop system in frequency domain

$$\begin{cases} \mathbf{j}\omega\tilde{x} = (I \otimes \check{A}_r)\tilde{x} - (I \otimes \check{B}_r K)\bar{\chi} + (I \otimes \check{B}_r)\bar{\rho}, \\ \mathbf{j}\omega\hat{\bar{x}} = (I \otimes (\check{A}_r - H\check{C}_r))\hat{\bar{x}} - (\bar{L}_{\mathbf{j}\omega}(\tau) \otimes \check{B}_r K)\bar{\chi} + (\bar{L} \otimes H\check{C}_r)\tilde{x}, \\ \mathbf{j}\omega\bar{\chi} = (I \otimes (\check{A}_r - \check{B}_r K))\bar{\chi} - (\bar{L}_{\mathbf{j}\omega}(\tau) \otimes I)\bar{\chi} + \hat{\bar{x}}. \end{cases} \tag{54}$$

By defining $\delta = \tilde{x} - \overline{\chi}$ and $\overline{\delta} = (\overline{L}_{\mathbf{j}\omega}(\tau) \otimes I)\tilde{x} - \hat{\overline{x}}$, we can obtain

$$\begin{aligned}
\mathbf{j}\omega\tilde{x} &= (I \otimes (\check{A}_r - \check{B}_r K))\tilde{x} + (I \otimes \check{B}_r K)\delta + (I \otimes \check{B}_r)C_s\overline{\varpi},\\
\mathbf{j}\omega\delta &= (I \otimes \check{A}_r - \overline{L}_{\mathbf{j}\omega}(\tau) \otimes I)\delta + \overline{\delta} + (I \otimes \check{B}_r)C_s\overline{\varpi},\\
\mathbf{j}\omega\overline{\delta} &= (I \otimes (\check{A}_r - H\check{C}_r))\overline{\delta} + ((\overline{L}_{\mathbf{j}\omega}(\tau) - \overline{L}) \otimes H\check{C}_r)\tilde{x} + (\overline{L}_{\mathbf{j}\omega}(\tau) \otimes \check{B}_r)C_s\overline{\varpi}.
\end{aligned} \tag{55}$$

Similar to the proofs of Theorem 1 and 2, we prove (55) is asymptotically stable for all communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ and $\hat{\tau}_{ij} \in \mathbb{R}_{\geq 0}$. Following the critical Lemma 1, we first prove stability without communication delays τ_{ij} and $\hat{\tau}_{ij}$ and then we prove the stability of (55) by checking condition (19).

- In the absence of communication delays in the network, the stability of system (55) is equivalent to the stability of matrix

$$\begin{pmatrix} I \otimes (\check{A}_r - \check{B}_r K) & I \otimes \check{B}_r K & 0 & (I \otimes \check{B}_r)C_s \\ 0 & I \otimes \check{A}_r - \overline{L} \otimes I & I & (I \otimes \check{B}_r)C_s \\ 0 & 0 & I \otimes (\check{A}_r - H\check{C}_r) & (\overline{L} \otimes \check{B}_r)C_s \\ 0 & 0 & 0 & A_s \end{pmatrix}, \tag{56}$$

  where $A_s = \text{diag}\{A_{i,s}\}$ for $i = 1, \ldots, N$. Similar to the proof of Theorem 2, we have all eigenvalues of $I \otimes A - \overline{L} \otimes I$ have negative real part. Then, since we have that $A - BK$ and $I \otimes A - \overline{L} \otimes I$ are Hurwitz stable, we obtain that

$$\lim_{t\to\infty} \tilde{x} \to 0.$$

  It implies that $\overline{x}_i \to x_r$.
- In the presence of communication delay, the closed-loop system (55) is asymptotically stable if

$$\det\left[\mathbf{j}\omega I - \begin{pmatrix} I \otimes (\check{A}_r - \check{B}_r K) & I \otimes \check{B}_r K & 0 & (I \otimes \check{B}_r)C_s \\ 0 & I \otimes \check{A}_r - \overline{L}_{\mathbf{j}\omega}(\tau) \otimes I & I & (I \otimes \check{B}_r)C_s \\ (\overline{L}_{\mathbf{j}\omega}(\tau) - \overline{L}) \otimes H\check{C}_r & 0 & I \otimes (\check{A}_r - H\check{C}_r) & (\overline{L}_{\mathbf{j}\omega}(\tau) \otimes \check{B}_r)C_s \\ 0 & 0 & 0 & A_s \end{pmatrix}\right] \neq 0 \tag{57}$$

  for all $\omega \in \mathbb{R}$ and any communication delays $\tau_{ij} \in \mathbb{R}^+$ and $\hat{\tau}_{ij} \in \mathbb{R}^+$. Condition (57) is satisfied if matrix

$$\begin{pmatrix} I \otimes (\check{A}_r - \check{B}_r K) & I \otimes \check{B}_r K & 0 & (I \otimes \check{B}_r)C_s \\ 0 & I \otimes \check{A}_r - \overline{L}_{\mathbf{j}\omega}(\tau) \otimes I & I & (I \otimes \check{B}_r)C_s \\ (\overline{L}_{\mathbf{j}\omega}(\tau) - \overline{L}) \otimes H\check{C}_r & 0 & I \otimes (\check{A}_r - H\check{C}_r) & (\overline{L}_{\mathbf{j}\omega}(\tau) \otimes \check{B}_r)C_s \\ 0 & 0 & 0 & A_s \end{pmatrix} \tag{58}$$

  has no eigenvalues on the imaginary axis for all $\omega \in \mathbb{R}$. Then, according to the structure of the expanded Laplacian matrix $\overline{L}$, and similar to the proof of Theorem 2 one can obtain that $\tilde{x}$ is asymptotically stable, that is, $\lim_{t\to\infty} \tilde{x}_i = 0$, which implies that $\lim_{t\to\infty} \tilde{y}_i = 0$, or $\overline{y}_i \to y_r$.

■

4 | NUMERICAL EXAMPLE

In this section, we will illustrate the feasibility of our scale-free linear protocols with numerical examples for delayed regulated state synchronization of homogeneous MAS with partial-state coupling and delayed regulated output synchronization for heterogeneous MAS when the communication networks are subject to communication delays.

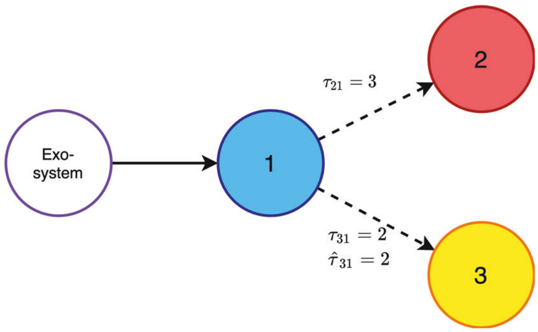

FIGURE 2 Communication graph of a network with 3 nodes

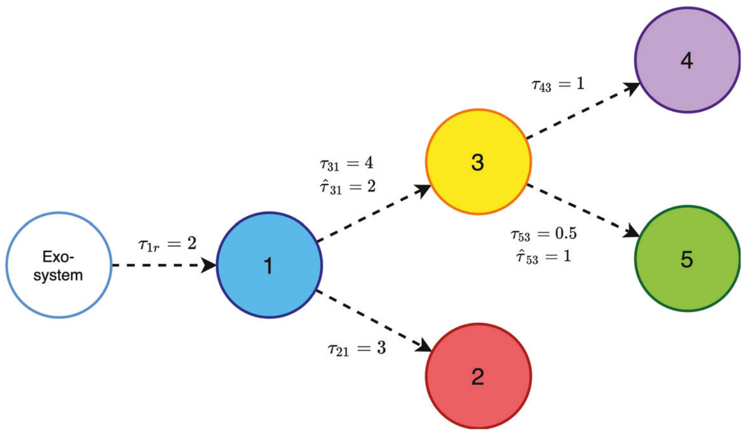

FIGURE 3 Communication graph of a network with 5 nodes

Example 1 (Homogeneous MAS). Consider identical agents models as

$$\begin{cases} \dot{x}_i = \begin{pmatrix} 0 & 0.25 \\ -0.25 & 0 \end{pmatrix} x_i + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_i, \\ y_i = \begin{pmatrix} 1 & 0 \end{pmatrix} x_i. \end{cases} \tag{59}$$

The goal is to achieve delayed regulated state synchronization when the reference nonconstant synchronized trajectory is generated by the following exosystem

$$\begin{cases} \dot{x}_r = \begin{pmatrix} 0 & 0.25 \\ -0.25 & 0 \end{pmatrix} x_r \\ y_r = \begin{pmatrix} 1 & 0 \end{pmatrix} x_r \end{cases} \tag{60}$$

with initial condition $x_r(0) = \begin{pmatrix} 0.3 & 0.1 \end{pmatrix}^{\mathrm{T}}$. We choose matrices $K = H^{\mathrm{T}} = \begin{pmatrix} 3 & 7.75 \end{pmatrix}$ such that $A - BK$ and $A - HC$ are Hurwitz stable. Therefore, we obtain the following protocol.

$$\begin{cases} \dot{\hat{x}}_i = \begin{pmatrix} -3 & 0.25 \\ -8 & 0 \end{pmatrix} \hat{x}_i - \begin{pmatrix} 0 & 0 \\ 3 & 7.75 \end{pmatrix} \hat{\zeta}_i + \begin{pmatrix} 3 \\ 7.75 \end{pmatrix} \bar{\zeta}_i + \iota_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_i \\ \dot{\chi}_i = \begin{pmatrix} 0 & 0.25 \\ -3.25 & -7.75 \end{pmatrix} \chi_i + \hat{x}_i - \hat{\zeta}_i - \iota_i \chi_i, \\ u_i = -\begin{pmatrix} 3 & 7.75 \end{pmatrix} \chi_i. \end{cases} \tag{61}$$

Note that protocol (61) is designed utilizing only the knowledge of agent models (59). In order to show the scalability of our protocols, we use our one-shot-designed protocol (61) for delayed regulated state synchronization of three different MAS with different communication networks and different number of agents as following cases.

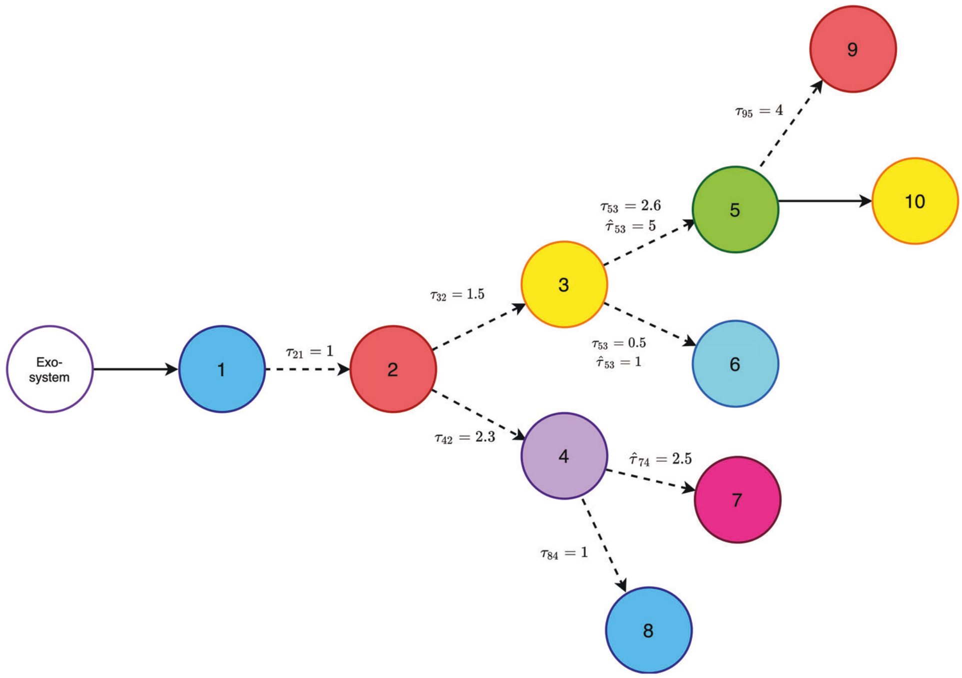

FIGURE 4 Communication graph of a network with 10 nodes

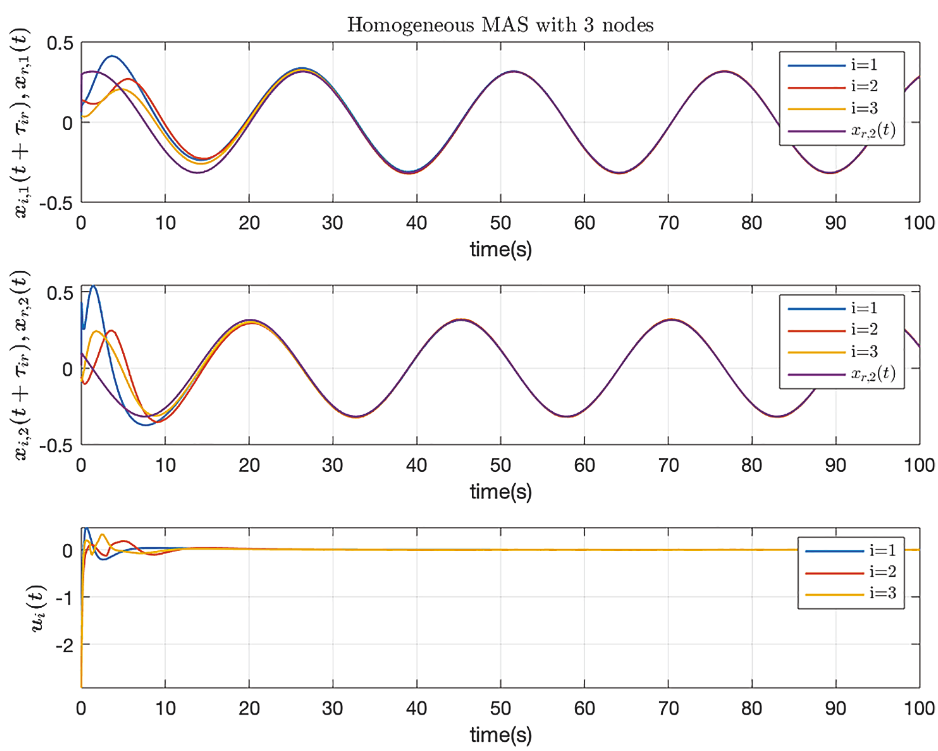

FIGURE 5 Scale-free delayed regulated state synchronization for homogeneous MAS with 3 nodes

Case 1: Consider a MAS consisting of three agents with agent models (59) and a tree communication topology shown in Figure 2 with associated adjacency matrix $\mathcal{A}_1$, where $a_{21} = a_{31} = 1$ and the rest of the entries are zero. The dashed links in the figures are subject to delay and the solid ones are delay-free. As it is shown in the figure, the communication delays are equal to $\tau_{21} = 3$, $\tau_{31} = 2$, and $\hat{\tau}_{31} = 2$. The exosystem provides $x_r(t)$ for agent 1. The simulation results are illustrated in Figure 5.

Case 2: Next, we consider another MAS consisting of five agents with agent models (59) and communication topology shown in Figure 3 with associated adjacency matrix $\mathcal{A}_2$, where $a_{21} = a_{32} = a_{43} = a_{53} = 1$ and rest of the entries are zero. The communication delays are equal to $\tau_{1r} = 2$, $\tau_{21} = 3$, $\tau_{31} = 4$, $\tau_{43} = 1$, $\tau_{53} = 0.5$, $\hat{\tau}_{31} = 2$, and $\hat{\tau}_{53} = 1$. By utilizing the same

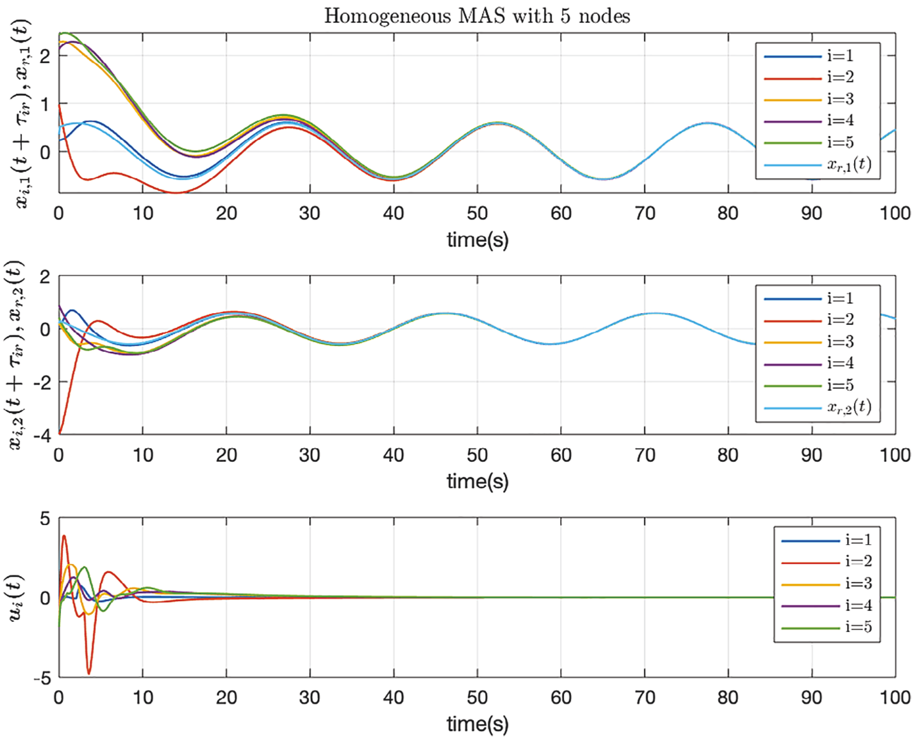

FIGURE 6 Scale-free delayed regulated state synchronization for homogeneous MAS with 5 nodes

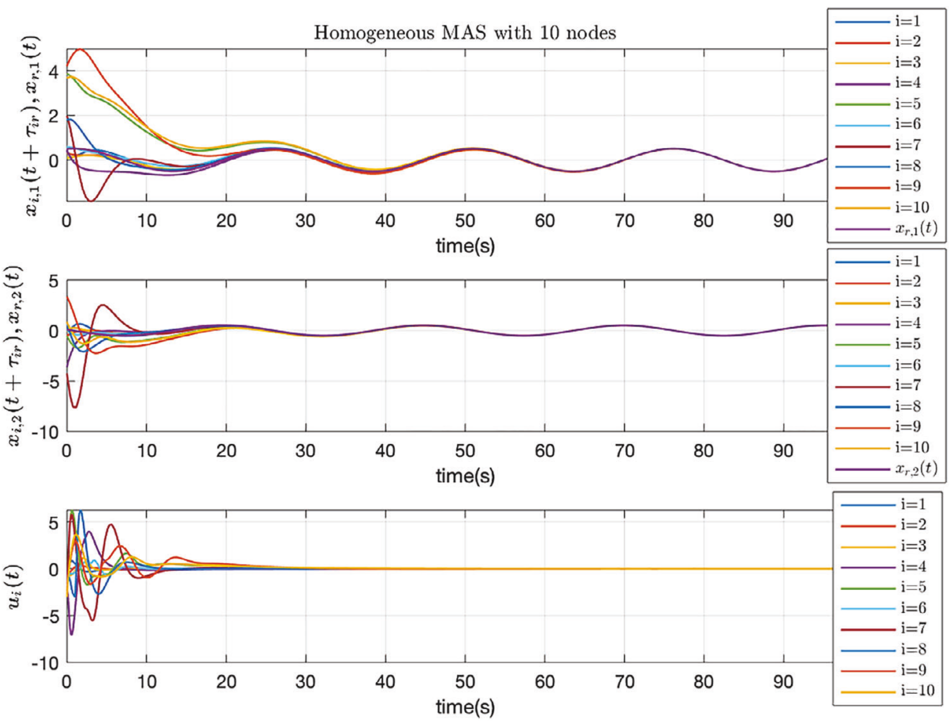

FIGURE 7 Scale-free delayed regulated state synchronization for homogeneous MAS with 10 nodes

protocol (61), we achieve delayed regulated state synchronization for MAS with communication network shown in (3). Figure 6 shows the simulation results for this MAS.

Case 3: Finally, consider a MAS consisting of 10 agents with agent models (59) and directed communication topology shown in Figure 4 with associated adjacency matrix $\mathcal{A}_3$, where $a_{21} = a_{32} = a_{42} = a_{53} = a_{63} = a_{74} = a_{84} = a_{95} = a_{10,5} = 1$ and rest of the entries are zero. The communication delays are equal to $\tau_{21} = 1$, $\tau_{32} = 1.5$, $\tau_{42} = 2.3$, $\tau_{53} = 2.6$, $\tau_{63} = 3$, $\tau_{84} = 1$, $\tau_{95} = 4$, $\hat{\tau}_{74} = 2.5$, and $\hat{\tau}_{53} = 5$. The exosystem provides x_r for agent 1. The simulation results for this MAS are presented in Figure 7.

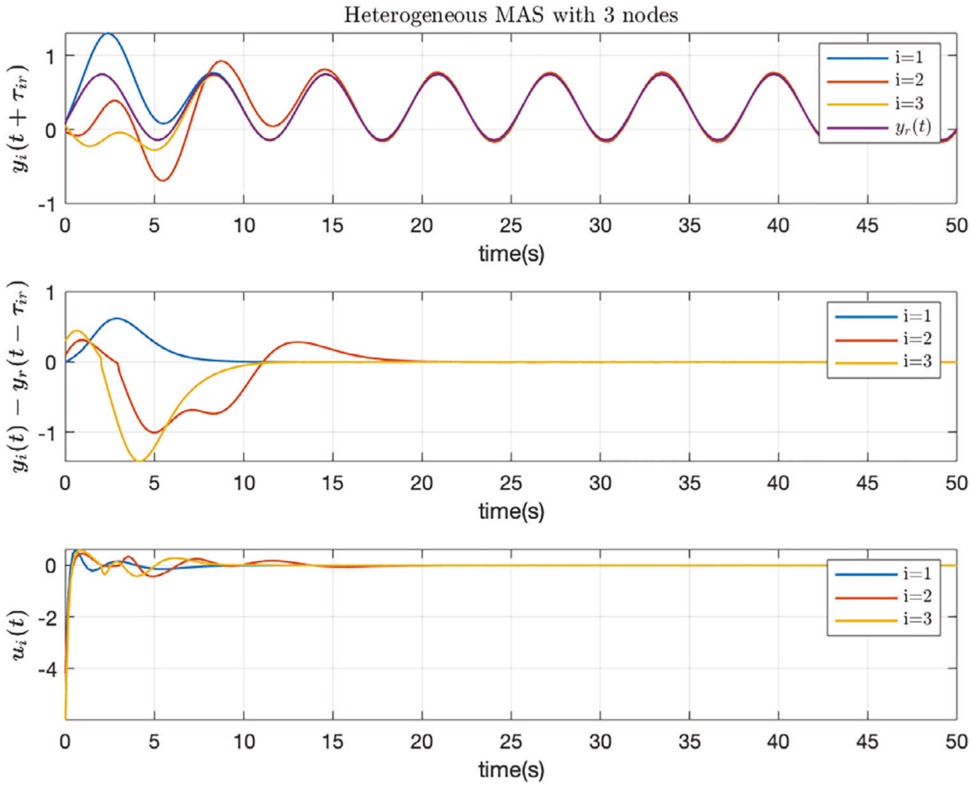

FIGURE 8 Scale-free delayed regulated output synchronization for heterogeneous MAS with 3 nodes

The simulation results show that our one-shot-design protocol (59) achieves delayed regulated state synchronization for any communication network with associated spanning tree graph and any size of the network. Moreover, the protocol can tolerate any unknown, nonuniform, and arbitrarily large communication delays.

Example 2 (Heterogeneous MAS). In this example, we consider numerical examples for delayed regulated output synchronization of heterogeneous MAS. We show that our protocol design Protocol 3 is scale-free and it works for any graph $\mathcal{G} \in \mathbb{G}^N$ with any number of agents. Consider the agents model (41) with

$$A_i = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, C_i = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}, C_i^m = I,$$

for $i = 1, 6$, and

$$A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C_i = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, C_i^m = I,$$

for $i = 2, 7$, and

$$A_i = \begin{pmatrix} 0 & -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, C_i = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix}, C_i^m = I,$$

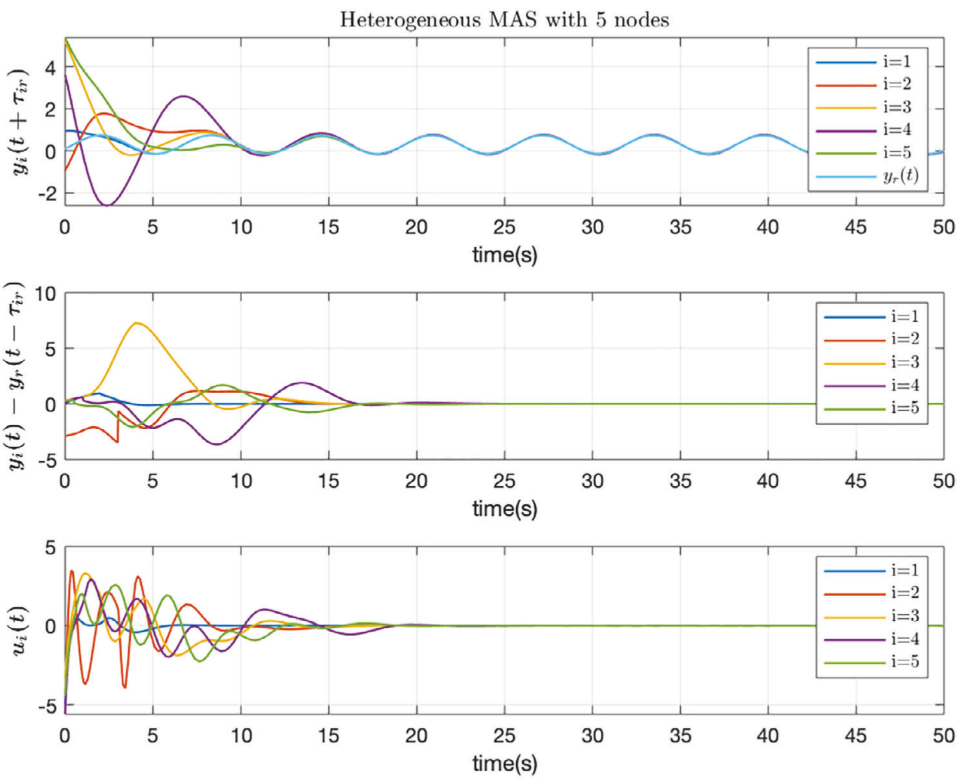

FIGURE 9 Scale-free delayed regulated output synchronization for heterogeneous MAS with 5 nodes

for $i = 3, 4, 8, 9$, and

$$A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C_i = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, C_i^m = I,$$

for $i = 5, 10$. Note that $\overline{n}_d = 3$, which is the degree of infinite zeros of (C_2, A_2, B_2). In this example, our goal is to achieve delayed regulated output synchronization when the nonconstant reference trajectory is generated by

$$\begin{cases} \dot{x}_r = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} x_r, \\ y_r = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x_r \end{cases}$$

with $x_r(0) = \begin{pmatrix} 0.1 & 0.4 & 0.2 \end{pmatrix}^{\mathrm{T}}$. According to Step 1 of Protocol 3 utilizing Lemma 2, we choose $(\check{C}_r, \check{A}_r, \check{B}_r)$ as

$$\check{A}_r = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad \check{B}_r = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \check{C}_r = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}.$$

Then as stated in Step 2, given the chosen target model, we homogenize the agents by designing pre-compensators for agent $i \in \{1, \ldots, 10\}$ as

$$u_i = \begin{pmatrix} 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} z_i + \begin{pmatrix} 1 \\ 0 \end{pmatrix} v_i$$

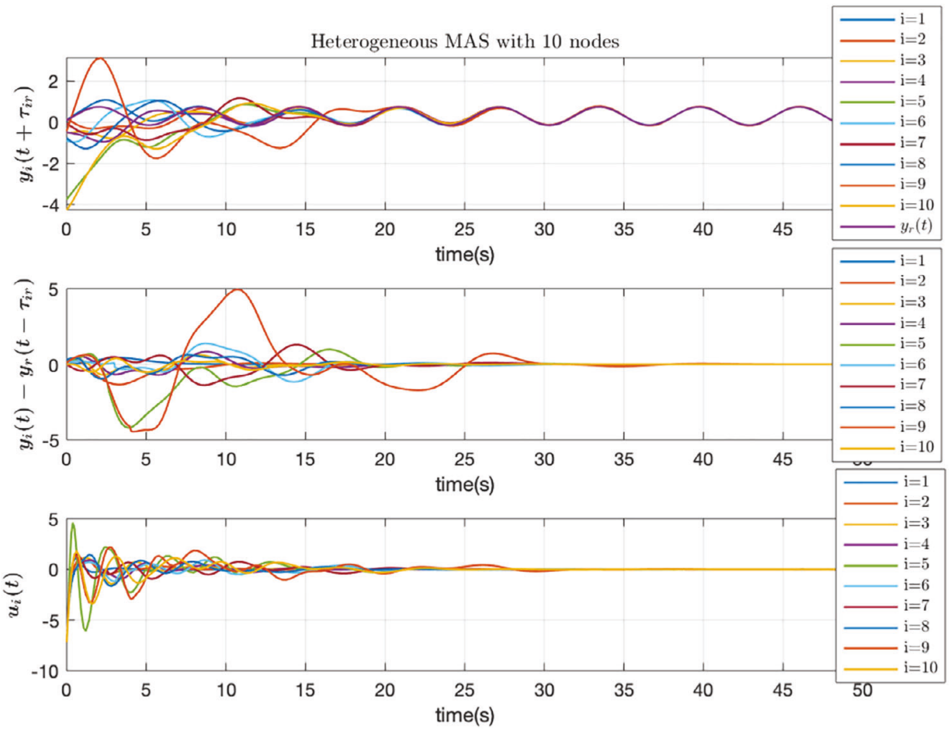

FIGURE 10 Scale-free delayed regulated output synchronization for heterogeneous MAS with 10 nodes

for $i = 1, 6$, and

$$u_i = \begin{pmatrix} 0 & -1 & 0 & 0 \end{pmatrix} z_i + v_i$$

for $i = 2, 7$, and

$$u_i = \begin{pmatrix} 0 & 0 & -1 & -1 & 0 \\ -1 & 1 & -2 & 0 & -1 \end{pmatrix} z_i + \begin{pmatrix} 1 \\ 0 \end{pmatrix} v_i$$

for $i = 3, 4, 8, 9$, and finally

$$u_i = \begin{pmatrix} -1 & -2 & 0 \end{pmatrix} z_i + v_i$$

for $i = 5, 10$.

The next step is designing collaborative protocols for the compensated agents. We choose $K = \begin{pmatrix} 6 & 10 & 6 \end{pmatrix}$ and $H = \begin{pmatrix} 6 & 10 & 0 \end{pmatrix}^{\mathrm{T}}$ such that $\check{A}_r - \check{B}_r K$ and $\check{A}_r - H\check{C}_r$ are Hurwitz stable. We obtain the collaborative protocols as

$$\begin{cases} \dot{\hat{x}}_i = \begin{pmatrix} -6 & 1 & 0 \\ -10 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \hat{x}_i - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 6 & 10 & 6 \end{pmatrix} \hat{\zeta}_i + \begin{pmatrix} 6 \\ 10 \\ 0 \end{pmatrix} \overline{\zeta}_i + \iota_i \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_i, \\ \dot{\chi}_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix} \chi_i + \hat{x}_i - \hat{\zeta}_i - \iota_i \chi_i, \\ u_i = -\begin{pmatrix} 6 & 10 & 6 \end{pmatrix} \chi_i. \end{cases} \tag{62}$$

To show the scalability of our protocols, similar to Example 1, we consider three heterogeneous MAS with different number of agents and different communication topologies.

Case 1: Consider a MAS with three agents with agent models (C_i, A_i, B_i) for $i \in \{1, \ldots, 3\}$, and directed communication topology shown in Figure 2. Values of communication delays are same as Example 1, Case 1. The simulation results are illustrated in Figure 8.

Case 2: In this case, we consider a MAS with five agents and agent models (C_i, A_i, B_i) for $i \in \{1, \dots, 5\}$ and directed communication topology shown in Figure 3. Values of communication delays are same as Example 1, Case 2. The simulation results for this MAS are presented in Figure 9.

Case 3: Finally, we consider a MAS with 10 agents and agent models (C_i, A_i, B_i) for $i \in \{1, \dots, 10\}$ and directed communication topology, shown in Figure 4. Values of communication delays are same as Example 1, Case 3. The simulation results are shown in Figure 10.

Case 3: Finally, we consider a MAS with 10 agents and agent models (C_i, A_i, B_i) for $i \in \{1, \dots, 10\}$ and directed communication topology, shown in Figure 4. Values of communication delays are same as Example 1, Case 3. The simulation results are shown in Figure 10.

We observe that our one-shot-design protocols work for any MAS with any communication networks $\mathcal{G} \in \mathbb{G}^N$ and any number of agents N.

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interests.

DATA AVAILABILITY STATEMENT

Data sharing not applicable—No data generated.

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