

A Generalized Method for Calculating Atmospheric Ionization by Energetic Electron Precipitation

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Key Points:

- We tabulate the atmospheric ionization response to monoenergetic beams of precipitating electrons with different pitch angles
- We report a method to derive the ionization profile under any Earth atmosphere condition by any electron energy and pitch angle distribution
- This method provides a reliable means to convert space measurements of precipitation into ionization input in atmospheric chemistry modeling

14 **Abstract**

15 Accurate specification of ionization production by energetic electron precipitation
 16 is critical for atmospheric chemistry models to assess the resultant atmospheric effects.
 17 Recent model-observation comparison studies have increasingly highlighted the impor-
 18 tance of considering precipitation fluxes in the full range of electron energy and pitch an-
 19 gle. However, previous parameterization methods were mostly proposed for isotropically-
 20 precipitation electrons with energies up to 1 MeV and the pitch angle dependence has not
 21 yet been parameterized. In this paper, we first characterize and tabulate the atmospheric
 22 ionization response to monoenergetic electrons with different pitch angles and energies be-
 23 tween \sim 3 keV and \sim 33 MeV. A generalized method that fully accounts for the dependence
 24 of ionization production on background atmospheric conditions, electron energy, and pitch
 25 angle has been developed based on the parameterization method of *Fang et al.* [2010].
 26 Moreover, we validate this method using 100 random atmospheric profiles and precipita-
 27 tion fluxes with monoenergetic and exponential energy distributions, and isotropic and sine
 28 pitch angle distributions. In a suite of 6,100 validation tests, the error in peak ionization
 29 altitude is found to be within 1 km in 91% of all the tests with a mean error of 2.7% in
 30 peak ionization rate, and 1.9% in total ionization. This method therefore provides a reli-
 31 able means to convert space-measured precipitation energy and pitch angle distributions
 32 into ionization inputs for atmospheric chemistry models.

33 **1 Introduction**

34 Energetic particle precipitation (EPP) causes significant disturbances to the entire
 35 magnetosphere-ionosphere-atmosphere system, including the dynamics of the radiation
 36 belts [e.g., *Lyons and Thorne*, 1973] and the thermal, electrical [e.g., *Mironova et al.*, 2015],
 37 and chemical [e.g., *Sinnhuber et al.*, 2012] properties of the Earth's atmosphere [e.g., *Mar-*
 38 *shall and Cully*, 2020]. Of particular relevance to our living environment is the direct in-
 39 fluence on ozone concentration in the stratosphere and mesosphere [e.g., *Thorne*, 1980;
 40 *Randall et al.*, 2007; *Sinnhuber et al.*, 2012]. Through efficient ionization interactions,
 41 EPP results in production of reactive odd nitrogen (NO_x) [e.g., *Rusch et al.*, 1981] and
 42 odd hydrogen (HO_x) [e.g., *Solomon et al.*, 1981] in the atmosphere. The short-lived HO_x
 43 compounds can locally deplete the ozone concentration by as large as 90% [*Andersson*
 44 *et al.*, 2013] in the mesosphere. NO_x compounds in the absence of sunlight can have a
 45 lifetime of months. If trapped inside the winter polar vortex, the NO_x gas will be trans-
 46 ported downward from the lower thermosphere to the stratosphere by the residual circu-
 47 lation which has a descending branch over the winter pole, thereby accelerating the cat-
 48 alytic ozone destruction cycle [*Callis et al.*, 1998; *Randall et al.*, 2007]. The ozone losses
 49 induced by EPP have indirect implications for radiative balance, cloud formation, atmo-
 50 spheric electricity, and circulation of the upper and middle atmosphere [e.g., *Sinnhuber*
 51 *et al.*, 2012; *Rozanov et al.*, 2012; *Mironova et al.*, 2015].

52 In spite of extensive theoretical and observational efforts, the atmospheric chemistry
 53 effects caused by EPP are still not fully understood [e.g., *Sinnhuber et al.*, 2012; *Mironova*
 54 *et al.*, 2015]. Precipitating auroral electrons are often isotropic in pitch angle, have ener-
 55 gies ranging from a fraction to a few tens of keV, and deposit most of their energies in
 56 the lower thermosphere in the auroral oval region. The resultant chemical changes are
 57 found to be positively correlated with the intensity of geomagnetic activity [e.g., *Baker*
 58 *et al.*, 2001; *Barth et al.*, 2003; *Marsh et al.*, 2007]. On the other hand, the chemical ef-
 59 fects produced by more energetic electron precipitation (EEP) are less well understood
 60 [e.g., *Sinnhuber et al.*, 2012; *Mironova et al.*, 2015]. This is partially due to the variability
 61 of precipitation fluxes in electron energy and pitch angle, as caused by repetitive parti-
 62 cle acceleration/deceleration and pitch-angle scattering during wave-particle interactions.
 63 Therefore, while evaluating EEP-induced effects, it is of essential importance to convert

64 these highly variable precipitation fluxes into an accurate ionization source in atmospheric
65 chemistry models.

66 Accurate specification of ionization production requires consideration of precipitation
67 fluxes in the full range of electron energy and pitch angle, a point that has been re-
68 peatedly raised in recent model-observation comparison studies [e.g., *Randall et al.*, 2015;
69 *Nesse Tyssøy et al.*, 2016; *Smith-Johnsen et al.*, 2018; *Pettit et al.*, 2019]. In particular,
70 *Randall et al.* [2015] analyzed the NO_x enhancements produced by an intense EPP event
71 during the 2003–2004 Arctic winter. However, numerical simulations using the Whole At-
72 mosphere Community Climate Model with Specified Dynamics (SD-WACCM) provided
73 inconsistent results with satellite measurements; modeling results underestimated the NO_x
74 fluxes by at least a factor of four. The reason, as suggested by *Randall et al.* [2015], was
75 the inaccurate transport rate adopted in SD-WACCM, as well as not including high-energy
76 precipitating electrons.

77 The studies of *Nesse Tyssøy et al.* [2016] and *Pettit et al.* [2019] not only confirmed
78 this finding, but also highlighted the role of pitch angle distribution. By fitting the pitch
79 angle distribution from the theory of wave-particle interactions, *Nesse Tyssøy et al.* [2016]
80 combined measurements from the two telescopes (0° telescope: pointing close to the zenith;
81 90° telescope: pointing close to the horizon) of the Medium Energy Proton and Electron
82 Detector (MEPED) onboard the Polar Orbiting Environmental Satellites (POES), and de-
83 rived a complete distribution of loss cone fluxes in both energy and pitch angle. The au-
84 thors have further verified this distribution using OH measurements by the Microwave
85 Limb Sounder onboard the Aura satellite, and revealed that the comparison with obser-
86 vational data achieved good agreements only after taking the full pitch angle distribution
87 into account.

88 *Pettit et al.* [2019] performed detailed comparison between NO_x measurements dur-
89 ing the Austral winter in 2003 and WACCM simulations using two different sets of precip-
90 itation data: one set of the data only included particle measurements from the 0° telescope
91 of MEPED, while the other one incorporated measurements from both telescopes. It has
92 been clearly demonstrated that including more information of precipitation pitch angle, as
93 in the latter set of simulations, greatly improves the agreements with NO_x observation at
94 middle latitudes. Despite these findings, there does not exist to date a reliable method to
95 convert precipitation distributions into atmospheric ionization rate profiles: previous pa-
96 rameterization methods were mostly proposed for precipitation electrons with energies up
97 to 1 MeV [*Fang et al.*, 2008, 2010], and the dependence on pitch angle has been largely
98 overlooked.

99 Numerous methods have been established for atmospheric ionization production by
100 EEP using range calculations [*Spencer*, 1959; *Lazarev*, 1967; *Roble and Ridley*, 1987;
101 *Lummerzheim*, 1992]. The methods of *Spencer* [1959] and *Lazarev* [1967] were devel-
102 oped by scaling laboratory measurements of electron energy absorption function to the
103 atmosphere. These methods have been extended by *Roble and Ridley* [1987] to calculate
104 the ionization of Maxwellian-distributed precipitating electrons, but only for the pitch an-
105 gle of zero degree, and later by *Lummerzheim* [1992] to calculate the ionization for sev-
106 eral predefined pitch angle distributions. Nevertheless, these methods were derived us-
107 ing simplified range calculations and the accuracy is limited at low energies and highly
108 anisotropic pitch angles [e.g., *Solomon*, 2001], therefore no longer adequate in recent
109 model-observation comparison studies.

110 More rigorous parameterization methods have been proposed using physics-based
111 models for a Maxwellian energy distribution [*Fang et al.*, 2008], and for monoenergetic
112 electrons [*Fang et al.*, 2010]. Both methods have only parameterized precipitating elec-
113 trons with energies less than 1 MeV and with an isotropic pitch angle distribution; the
114 recently-emphasized pitch angle dependence was not considered. Using the Cosmic Ray
115 Atmospheric Cascade (CRAC) model [e.g., *Artamonov et al.*, 2016], *Artamonov et al.*

[2017] have performed detailed studies on the atmospheric ionization by electrons with non-vertical precipitation. However, the ionization yield function for non-vertical precipitation electrons was not obtained from first-principles approach and how the yield function can be applied to different background atmospheres was not explained. The necessity to fully characterize the dependence of ionization production on the energy and pitch angle of precipitating electrons, as well as the background atmospheric condition, motivate the present work to extend the parameterization method of *Fang et al.* [2010]. The goal is to establish a robust generalized method for the specification of ionization profiles in any Earth atmosphere by precipitating electrons with any distribution in energy and pitch angle, with a view of providing a reliable ionization source to the modeling of EEP-induced atmospheric effects.

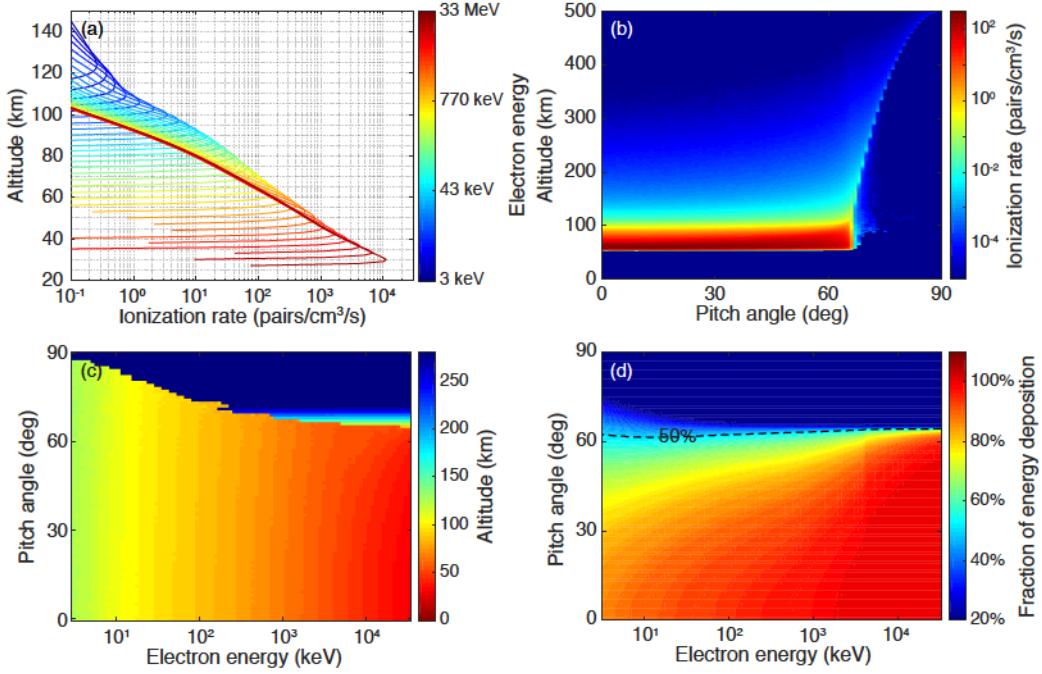
2 Model and Methodology

In this work, we calculate the impact ionization from precipitating energetic electrons using a first-principles model: the Energetic Precipitation Monte Carlo (EPMC) model, originally developed by *Lehtinen et al.* [1999] and modified by our group at CU Boulder over the past few years. A brief description of this model is given in section 2.1. In section 3.2, we introduce a lookup table, i.e., atmospheric response functions in terms of ionization production to monoenergetic electrons with discrete energies and pitch angles. We elucidate, in section 2.3, how to convert this lookup table from its default background atmosphere to a new atmosphere, and, in section 2.4, how to calculate the altitude profile of ionization production by precipitation electrons with arbitrary distribution in energy and pitch angle. In section 2.5, we explain the numerical error determined by validation testing. Moreover, we validate this method, in section 3, through its application to monoenergetic and exponential energy distributions in 100 different atmospheres, and isotropic and sine pitch angle distributions.

2.1 Energetic Precipitation Monte Carlo Model

The EPMC model was originally developed by *Lehtinen et al.* [1999] for studies of energetic radiation from thunderstorm activity, and has been recently adapted to simulate EEP effects [e.g., *Marshall et al.*, 2014; *Xu et al.*, 2018; *Marshall and Bortnik*, 2018] since the underlying electron/photon collisional processes are similar. The details of this model have been described earlier [e.g., *Lehtinen et al.*, 1999; *Marshall et al.*, 2014]. In short, this model explicitly solves the equation of electron motion at the microscopic level using the stopping power and Möller cross section. The angular diffusion is mostly due to elastic scattering by ambient neutral species and the resultant change in momentum direction and magnitude is modeled using the method of small-angle collisions [*Lehtinen*, 2000, pp. 15–18]. As a built-in feature, this model outputs height-resolved energy deposition, and ionization production is then derived from energy deposition by assuming that an average energy of ~ 35 eV is needed to produce an ion-electron pair [*Rees*, 1989, p. 40]. We emphasize that this model has been validated in the past few decades in a variety of studies, including gamma-ray emissions produced by lightning discharge [*Lehtinen et al.*, 1999], interaction of a beam of relativistic electrons with the atmosphere [*Marshall et al.*, 2014], bremsstrahlung effects in EEP [*Xu et al.*, 2018; *Xu and Marshall*, 2019], and lightning-induced electron precipitation [*Marshall et al.*, 2019].

This model can adopt arbitrary background mass density profile and magnetic field as input. In this work, the background magnetic field is assumed to be vertical with a magnitude typical of that at around 700 km altitude at Poker Flat, Alaska at nighttime (41,528 nT). Magnetic mirroring due to the magnetic gradient force is included in the model [*Lehtinen*, 2000, pp. 108–109]. The background profiles of neutral atmosphere are obtained from the NRLMSISE-00 model [*Picone et al.*, 2002]. A total of 100 MSIS atmospheric profiles are generated (see Figure 3e) and used in validation testing (section 3).



169 **Figure 1.** (a) Ionization production at altitudes between 20 and 150 km by monoenergetic beams of pre-
 170 precipitating electrons with an incident pitch angle of 0° , and with energies between ~ 3 keV and ~ 33 MeV. The
 171 number flux of source electrons used in this set of simulations is assumed to be 10^4 el/cm 2 /s. (b) Ionization
 172 production by 1 MeV electrons with pitch angles (at 500 km altitude) varying from 0° to 90° . (c) Altitude
 173 of peak ionization production by monoenergetic beams of precipitating electrons with different energies
 174 (3 keV–33 MeV) and pitch angles (0° – 90°). (d) The fraction of total precipitation energy deposited in the
 175 atmosphere.

166 These profiles are obtained for years between 1990 and 2019 with random combinations
 167 of latitude and longitude, and with a broad coverage of $F_{10.7}$ (50–300 sfu) and A_p values
 168 (0–40 nT).

176 **2.2 Lookup Table: Atmospheric Ionization by Monoenergetic Electrons with Dif-
 177 ferent Pitch Angles**

178 The EPMC model is employed to calculate a lookup table of ionization rate as a
 179 function of altitude produced by monoenergetic electrons with discrete pitch angles. Specif-
 180 ically, monoenergetic beams of energetic electrons are assumed to precipitate into the up-
 181 per atmosphere from an altitude of 500 km. This altitude is chosen from the following
 182 considerations: 1) it is far above the interaction region between precipitation electrons and
 183 air molecules under most atmospheric conditions; 2) computation time; 3) it is close to
 184 that of a future EEP-observing CubeSat mission: the Atmospheric Effects of Precipitation
 185 through Energetic X-rays (AEPEX) mission [Marshall *et al.*, 2020]. The background at-
 186 mosphere utilized in the lookup-table calculation (black curve in Figure 3e) is calculated
 187 using the NRLMSISE-00 model with $F_{10.7} = 200$ sfu and $A_p = 15$ nT. The electron ener-
 188 gies are roughly uniformly distributed in the logarithmic space; 91 energy values spanning
 189 from 3 keV to 33 MeV are chosen (see Figure 1); the specific choice of electron energies
 190 is not critical as long as sufficient energy values are used to provide a good resolution of
 191 ionization production at altitudes between 30 and 120 km; the number flux of precipitat-

192 ing electrons is assumed to be 10^4 el/cm²/s in the downward hemisphere [e.g., *Whittaker et al.*, 2013]. For the sake of consistency, this flux number is used throughout this work
193 for source electrons in Monte Carlo simulations.
194

195 For each energy, we vary the input pitch angle (at 500 km altitude) from 0° to 90°
196 with 1° step. This range of electron energy and pitch angle is chosen based on two
197 considerations. First, it ensures a good resolution of ionization rate at altitudes of interest
198 in studies of atmospheric chemical changes. Second, it provides a broad coverage over
199 the detection capability of present space-borne instruments. Ionization production is the-
200oretically governed by the background atmospheric conditions, precipitation energy and
201 pitch angle, and the background magnetic field. Due to the complexity of this problem,
202 the dependence on background magnetic field is not considered in the present study. *Cotts*
203 [2011] have specifically investigated the dependence of ionization production on geomag-
204 netic field dip angle and found that the effects are not significant (not larger than 10%).
205 The monoenergetic simulation results are summarized in Figure 1.

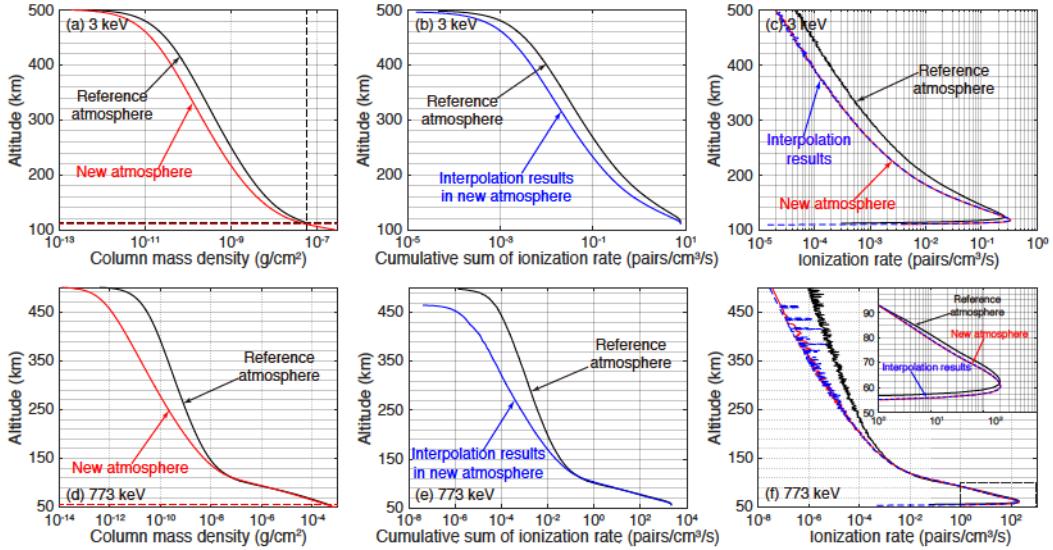
206 Figure 1a shows the ionization production at altitudes between 20 and 150 km by
207 monoenergetic electrons with energies between ~3 keV and ~33 MeV when vertically in-
208 cident on the atmosphere. It is clear that the penetration depth increases with precipita-
209 tion energy, roughly linearly with the logarithmic value of the incident energy. Figure 1b
210 shows the ionization production by 1 MeV electrons with pitch angles varying from 0°
211 to 90°. With an initial altitude of 500 km, the nominal loss cone angle is ~66° based on
212 a loss cone defined by a mirror altitude of 100 km. Electrons with larger pitch angle are
213 mirrored before reaching 100 km altitude and only a small portion of precipitation energy
214 is dissipated (see Figure 1b).

215 Figure 1c shows the altitude of peak ionization production for different precipitation
216 energies and pitch angles, while Figure 1d shows the fraction of total precipitation energy
217 deposited in the atmosphere. As the electron energy increases from 3 keV to 33 MeV, the
218 fraction of energy deposition gradually increases since electrons become more penetrating
219 and can thus interact with denser atmosphere at relatively lower altitudes. The sharp edge
220 in Figure 1d around 65° (dashed line) roughly depicts how the loss cone angle changes
221 with electron energy, which is roughly consistent with the results reported in *Marshall and*
222 *Bortnik* [2018]. Note that this line does not specifically show the bounce loss cone angle.

234 2.3 Converting the Lookup Table to A New Atmosphere

235 To infer the ionization profile in a new independent atmosphere (denoted as the
236 new atmosphere hereafter), we convert the lookup table using its background atmosphere
237 (denoted as the reference atmosphere hereafter, black line in Figure 3e) in four steps.
238 First, we cumulatively sum the mass density and ionization rate in the reference atmo-
239 sphere from the starting altitude (500 km) down to the altitude at which electrons are
240 completely absorbed by the atmosphere. This step is largely motivated by the parame-
241 terization scheme described in *Fang et al.* [2010] as the authors transformed ionization
242 results into two normalized quantities: energy dissipation versus a power law of column
243 mass density. Figure 2 shows two examples of how to convert the lookup table from its
244 default background atmosphere to a new atmosphere, for 3 keV (the minimum energy in
245 our lookup table) and 773 keV electrons. In the following, we use the 3-keV case as an
246 example to explain this conversion method. The black curve in Figure 2a shows the col-
247 umn mass density that 3 keV electrons propagate through in the reference atmosphere,
248 while the black curve in Figure 2b shows the cumulative sum of ionization rate versus al-
249 titude.

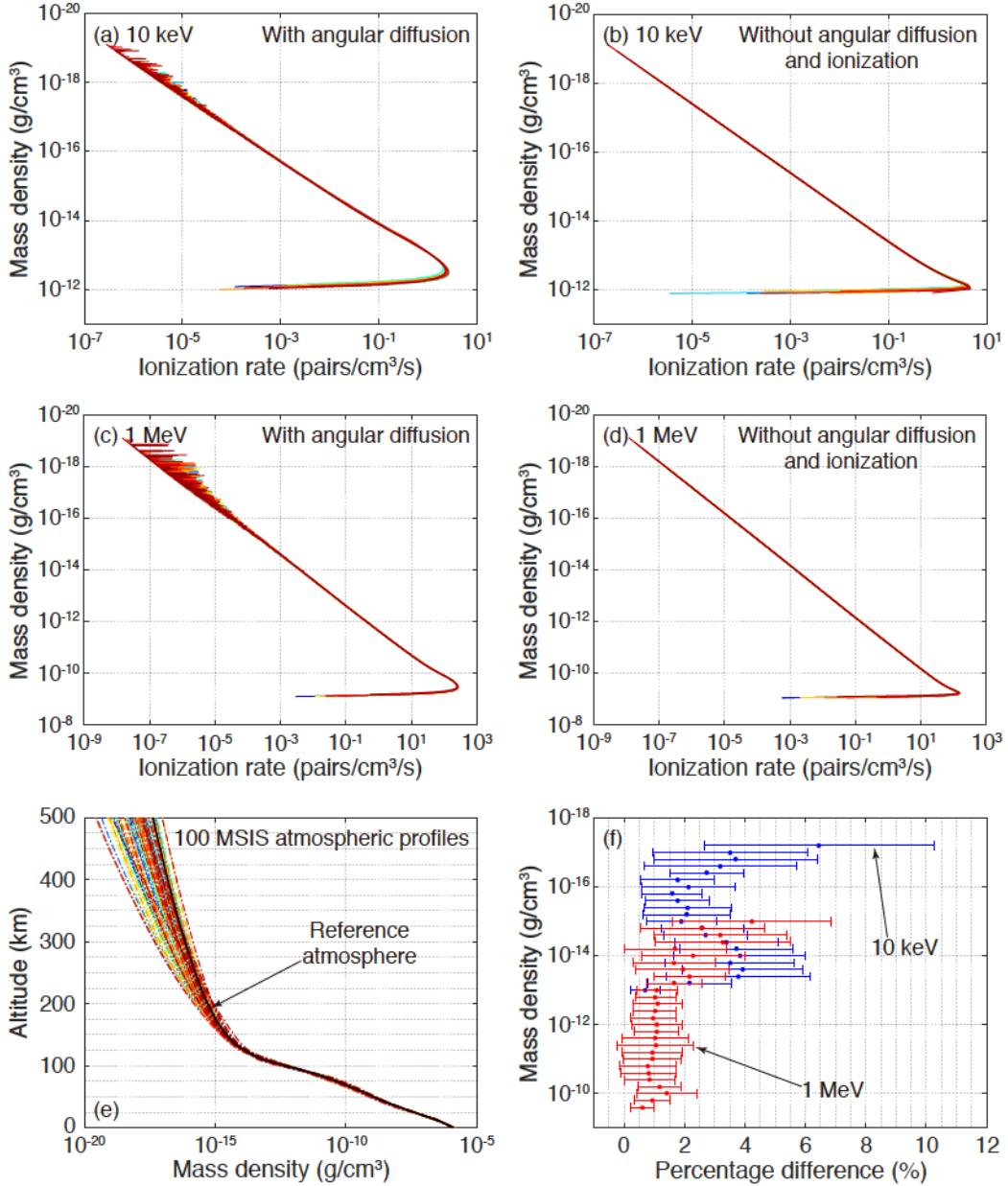
250 Second, we determine the lowest altitude of ionization in the new atmosphere by
251 finding out the altitude above which the column mass density is similar to what electrons
252 traverse in the reference atmosphere. For example, in the reference atmosphere, a 3-keV
253 electron is absorbed at ~112 km altitude by a column mass density of 5.9×10^{-8} g/cm²



223 **Figure 2.** (a) Column mass density that 3 keV electrons with a pitch angle of 0° propagate through. The
 224 horizontal dashed lines delineate the lowest altitude of ionization production. (b) Cumulative sum of ionization
 225 rate in the reference atmosphere (black curve) and interpolation results of cumulative ionization in
 226 the new atmosphere (blue curve). (c) Comparison of ionization profile produced by 3 keV electrons between
 227 direct Monte Carlo results (red curve) and interpolation results (blue curve). The bottom panels show similar
 228 results, but for 773 keV electrons.

229 (black dashed line in Figure 2a), corresponding to the total mass density above ~ 110 km
 230 in the new atmosphere (red dashed line in Figure 2a). Thus, a 3-keV electron would most
 231 likely stop around 110 km altitude in the new atmosphere and this is the lowest altitude
 232 of ionization production. However, this is not the “perfect” lowest altitude: because of the
 233 variation in background density and resolution in altitude, we cannot find an altitude above
 234 which the column mass density is exactly the same as that in the reference atmosphere.
 235 Instead, the first altitude grid point (starting from the upper boundary) above which the
 236 column mass density is larger than that in reference atmosphere is chosen to be the low-
 237 est ionization altitude. As will be explained later, this approximation introduces a small
 238 numerical error in our validation test. This error can be reduced by using a better method
 239 to determine the lowest ionization altitude in new atmosphere or using finer grid cells in
 240 altitude. The development of a better method/algorithm is left for future work.

241 Using the column mass density in the reference and new atmospheres (black and
 242 red curves in Figure 2a), we further interpolate the cumulative sum of ionization rate from
 243 the upper boundary down to the lowest ionization altitude in reference atmosphere (black
 244 curve in Figure 2b); the cumulative sum of ionization rate versus altitude in the new at-
 245 mosphere is obtained (blue curve in Figure 2b). Finally, we differentiate the interpolation
 246 results and calculate the ionization rate at each altitude; the results are then altitude pro-
 247 file of ionization rate, and shown as the blue curve in Figure 2c. The new atmospheres in
 248 Figures 2c and 2f are randomly chosen from the 100 MSIS atmospheres (Figure 3e) used
 249 in validation test. Note that the column mass density exhibits great variation at altitudes
 250 above ~ 200 km in different atmospheres (see Figure 3e). Since we interpolate in column
 251 mass density, the interpolation results of cumulative ionization in new atmosphere usu-
 252 ally do not start from the same altitude of reference atmosphere (blue and black curves in
 253 Figure 2e); in this case, we extrapolate the ionization profile up to 500 km altitude.



229 **Figure 3.** Altitude profile of ionization production in 20 MSIS atmospheres by 10 keV electrons with a
 230 pitch angle of 0° (a) with and (b) without considering angular diffusion and ionization production in Monte
 231 Carlo simulations. The source precipitation flux is $10^4 \text{ el/cm}^2/\text{s}$. Panels (c) and (d) show similar results,
 232 but for 1 MeV electrons. (e) Mass density versus altitude for 100 MSIS atmospheric profiles. (f) Percentage
 233 difference from the mean value of ionization rate versus mass density for 10 keV and 1 MeV electrons.

279 To validate this interpolation method, we have directly calculated the true ionization
 280 in the new atmosphere using a direct Monte Carlo simulation in the new atmosphere, and
 281 these results are shown as red curves in Figures 2c and 2f. Clearly, one sees that the in-
 282 terpolated profiles are extremely close to the true ionization for both energies. The error
 283 between direct Monte Carlo results (red curve) and interpolation results (blue curve) is,
 284 on average, 3.9% and 22% for the precipitation energy of 3 keV (Figure 2c) and 773 keV
 285 (Figure 2f), respectively. Particularly for the energy of 773 keV, this method not only re-
 286 produces the ionization over an altitude range of 450 km, but the altitude and magnitude
 287 of peak ionization as well (see the inset of Figure 2f).

288 2.4 Atmospheric Ionization by Arbitrary Distribution in Electron Energy and 289 Pitch Angle

290 Having obtained the lookup table in the new atmosphere, the ionization profile pro-
 291 duced by an arbitrary distribution of electron energy and pitch angle can be calculated
 292 as a weighted sum of the ionization contribution from each energy and pitch angle com-
 293 ponent [e.g., *Fang et al.*, 2010; *Berger and Seltzer*, 1972]. Specifically, we construct the
 294 ionization profile using the following formula:

$$I = \sum_{\varepsilon_{\min}}^{\varepsilon_{\max}} \sum_{0^\circ}^{90^\circ} I_0(\varepsilon, \alpha) f(\varepsilon, \alpha) \Delta\alpha \Delta\varepsilon \quad (1)$$

295 where $I_0(\varepsilon, \alpha)$ is the interpolated lookup table in the new atmosphere, which contains the
 296 ionization rate versus altitude produced by electrons with an energy ε and a pitch angle
 297 α , ε_{\min} and ε_{\max} are the lowest and highest electron energy, $\Delta\alpha$ is the width of pitch an-
 298 gle bins, and $\Delta\varepsilon$ is the width of energy bins, and $f(\varepsilon, \alpha)$ is the differential number flux
 299 of precipitation electrons in energy and pitch angle. In the following validation test (sec-
 300 tion 3), we assume that the energy and pitch angle distributions are not coupled; but this
 301 assumption is unnecessary in the analysis of real measurements and the ionization profile
 302 can be explicitly calculated using equation 1, where $f(\varepsilon, \alpha)$ may define a coupled energy-
 303 pitch angle dependence.

304 2.5 Numerical Error in Validation Test

305 We validate the above-mentioned lookup table and interpolation method in sec-
 306 tion 3. In this section, we explain the numerical error involved in validation testing. As
 307 the foundation of the electron transport model, the stopping power describes the effect of
 308 the medium in slowing down the projectile and is usually expressed in energy loss per
 309 unit distance or mass traversed [e.g., *Carron*, 2006]. For the same precipitation energy,
 310 within the continuous slowing down range, energy deposition scales in theory as the mass
 311 density that the electron encounters, so is the rate of ionization interaction. However, the
 312 input of atmospheric chemistry models is usually in the form of ionization production
 313 per unit altitude of travel, which is not the same as the distance traversed by a precipi-
 314 tating electron due to its non-zero pitch angle. Hence, the ionization profile does not scale
 315 strictly with the background mass density.

316 The altitude profile of ionization rate is determined as much by ambient mass den-
 317 sity as by angular diffusion (defined herein as the change in direction of propagation due
 318 to collisions between electrons and air molecules, e.g., elastic scattering, ionization col-
 319 lision, and excitation collision), especially at low energies. A single electron does not al-
 320 ways follow the same gyromotion before stopping power brings it to rest and its trajectory
 321 is likely tortuous. Collisions with air molecules result in a change of electron energy and
 322 random scattering results in a change of pitch angle; both interrupt the gyromotion and
 323 the amount of mass density that the electron traverses in unit altitude. The rate of energy
 324 absorption and ionization production would be accordingly enhanced or suppressed. This

statement also holds for a limited number of electrons propagating in low-density region, for example, present Monte Carlo simulations at high altitudes, wherein angular diffusion by air molecules is infrequent and the resultant change in ionization rate is highly non-linear and unpredictable. This is the main cause of variation in the ionization profiles at relatively high altitudes (>150 km), as evidenced by the results shown in Figure 3.

Figure 3a shows the ionization profiles produced by 10-keV electrons in 20 different MSIS atmospheres versus the corresponding mass density above the ionization peak. These atmospheres are the first 20 of the 100 MSIS profiles used in validation test (Figure 3e). Figure 3b shows similar results, but obtained by turning off the angular diffusion and ionization production term in Monte Carlo simulations. The main difference between these two plots is that the ionization curves in Figure 3b lie nearly on top of the darkest red curve at densities below 6×10^{-13} g/cm³ (lower density corresponds to higher altitude). Without angular diffusion and ionization production, the ionization profiles are much less spread, as being roughly proportional to the mass density. It is thus conceivable that, if ionization profiles in different atmospheres are well normalized to the mass density, like those in Figure 3b, we can simply pick any of these curves as reference, interpolate in mass density, and precisely derive the ionization profile in other atmospheres. However, in Monte Carlo simulations with a limited number of particles, angular diffusion and ionization production adds some randomness to these otherwise well-normalized ionization curves (Figure 3b) and this is the origin of numerical error in our validation test (Section 3).

These randomness effects are less pronounced at higher precipitation energies. Similar results for 1 MeV electrons are presented in Figures 3c and 3d. Contrary to the 10 keV results, the 1-MeV ionization curves lie close together at densities greater than $\sim 10^{-14}$ g/cm³. The reason is that angular diffusion at high-density altitudes is frequent and, for an ensemble of electrons, its impact on ionization rate becomes more deterministic. A better illustration of this point is shown in Figure 3f. For both 10 keV and 1 MeV electrons, we first simulate the ionization profiles in 100 different atmospheres; the mean value of ionization rate in these atmospheres is derived as a function of background density; we then compute the percentage difference of each ionization profile from the mean value. The average value of percentage difference versus mass density is presented in Figure 3f with error bars showing one standard deviation. Above the ionization peak, the percentage difference becomes smaller with increasing mass density and is notably larger for 10 keV electrons than 1 MeV. This figure shows a quantitative measure of the variation and represents the inherent error in our validation test.

3 Validation and Error Analysis

We have performed a total of 6,100 tests using 100 MSIS atmospheric profiles (Figure 3e) in order to examine the robustness of our method. These tests are categorized into three different sets of energy and pitch angle distributions at the initial altitude of EPMC simulations (500 km): 1) monoenergetic beams of electrons with a pitch angle of 0°; 2) isotropic distributions in pitch angles between 0° and 90°, and exponential energy distributions in the range from 10 keV to 10 MeV: $f(\varepsilon) \propto \exp(-\varepsilon/\varepsilon_0)$, where ε_0 is the characteristic energy of electron distribution; and 3) exponential distributions in energy and sine distributions in pitch angle: $f(\alpha) \propto \sin(\alpha)$, where α is the pitch angle (between 0° and 90°; 0° means propagating vertically downward) at 500 km altitude. The first set of monoenergetic tests is conducted mainly to verify the above-mentioned interpolation method. Space-borne measurements commonly reveal an exponential energy distribution for EEP [e.g., *Whittaker et al.*, 2013; *Breneman et al.*, 2017] and this is the main focus of our validation test. The last set of energy and pitch angle distributions is more realistic and the second set is utilized for the sake of completeness and for the comparison with *Fang et al.* [2010].

376 Note that it is commonly assumed that the pitch angle distribution at the equator
 377 follows the Vampola's equation [Vampola, 1997]: precipitation flux varies with the pitch
 378 angle as sine to the power of n . The specific choice of n value in Vampola's equation is
 379 not critical in present study and we opt to assess the performance of this lookup table in
 380 the simplest scenario ($n = 1$). As in EPMC simulations, we do not consider the change of
 381 pitch angle distribution due to change in background geomagnetic field. However, one can
 382 always map the pitch angle distribution of satellite-measured precipitation fluxes from its
 383 location to the initial altitude of present lookup table using the background geomagnetic
 384 field.

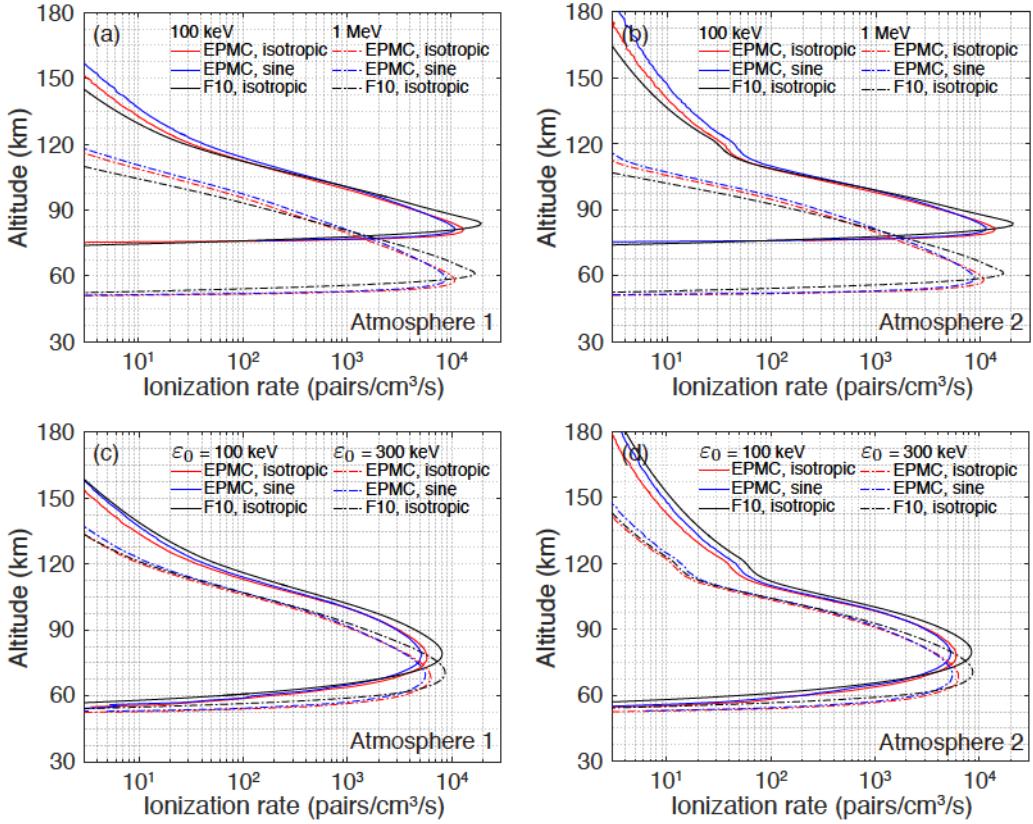
385 In the monoenergetic test, we use 19 electron energies that are uniformly distributed
 386 in the logarithmic scale between 3 keV and 33 MeV. Typical space-measured ε_0 values
 387 are in the range of 70–500 keV [e.g., Whittaker *et al.*, 2013; Crew *et al.*, 2016; Breneman
 388 *et al.*, 2017]. Therefore, for the exponential distribution, 21 ε_0 values uniformly spaced in
 389 the logarithmic scale from 10 keV to 1 MeV are utilized. We conduct 100 Monte Carlo
 390 runs for each ε_0 value and each monoenergetic test using 100 MSIS profiles as the back-
 391 ground atmosphere. The ionization results of these runs are employed as truth data against
 392 which the ionization derived from the lookup table can be evaluated. The ideal scenario
 393 is to verify the lookup-table calculation using real measurements, but the resolution of
 394 precipitation measurements and the lack of reliable observational data prevent us from per-
 395 forming such comparisons.

396 By measuring the difference from the truth, we estimate the error of derived ioniza-
 397 tion profile mainly in three aspects: peak ionization altitude, peak ionization rate, and total
 398 ionization, similar to those in Fang *et al.* [2010]. Note that the lookup table and truth data
 399 in validation tests are both calculated using the EPMC model, which has been validated
 400 in a variety of studies. In the following, we validate this model again through comparison
 401 with Fang *et al.* [2010] in section 3.1, and evaluate the performance of above-mentioned
 402 lookup table in sections 3.2, 3.3, and 3.4.

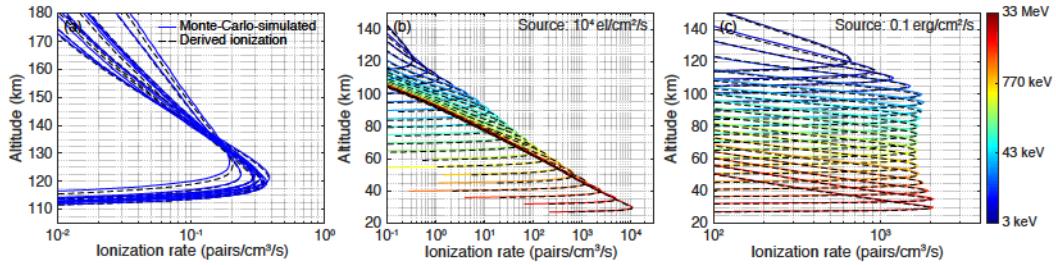
412 3.1 Comparison with the Parameterization Method of Fang *et al.* [2010]

413 Before testing the lookup table in different atmospheres, we first compare the sim-
 414 ulation results of EPMC with the method of Fang *et al.* [2010]. Figures 4a and 4b show
 415 the comparison of ionization profiles produced by monoenergetic beams of 100 keV and
 416 1 MeV electrons with an isotropic pitch angle distribution. This comparison is performed
 417 for the two MSIS atmospheres utilized in Fang *et al.* [2010]: atmosphere 1 with $F_{10.7} =$
 418 50 sfu and $A_p = 5$ nT, and atmosphere 2 with $F_{10.7} = 300$ sfu and $A_p = 65$ nT. The ioniza-
 419 tion profiles produced by 100-keV and 1-MeV electrons, but with a sine pitch angle dis-
 420 tribution, are plotted as blue solid and dashed lines. The precipitation source is assumed
 421 to have a total energy flux of 1 erg/cm²/s. Figures 4c and 4d show similar results, but for
 422 precipitation electrons with an exponential energy distribution with the characteristic en-
 423 ergy ε_0 being 100 keV or 300 keV.

424 For the sake of direct comparison, the magnetic mirroring force is turned off in this
 425 set of simulations since it is not considered in Fang *et al.* [2010]. As shown in Figure 4,
 426 the altitude of peak ionization calculated using the EPMC model is close to the parame-
 427 terization method of Fang *et al.* [2010]. However, EPMC simulation tends to predict more
 428 ionization production at relatively higher altitudes. The difference in total ionization be-
 429 tween these two methods is, on average, $\sim 19\%$ for monoenergetic beams and 21% for the
 430 exponential distribution. The difference for monoenergetic beams is similar to what we
 431 reported in Xu *et al.* [2018]. This discrepancy is not unacceptable since different electron
 432 transport models are employed; the stopping power is different; and different approaches
 433 are used to describe the angular diffusion of precipitation beam: Fang *et al.* [2010] used
 434 a two-stream model for electrons with energies above 50 keV and another multi-stream



403 **Figure 4.** Comparison of ionization profiles produced by monoenergetic beam of 100 keV and 1 MeV
 404 electrons with an isotropic pitch angle distribution between present Monte Carlo simulations (denoted as
 405 EPMC in the legend) and the parameterization method of *Fang et al.* [2010] (denoted as F10 in the legend).
 406 This comparison is performed for two different MSIS atmospheres: (a) atmosphere 1 with $F_{10.7} = 50$ sfu
 407 and $A_p = 5$ nT; (b) atmosphere 2 with $F_{10.7} = 300$ sfu and $A_p = 65$ nT. The ionization profiles produced
 408 by 100-keV and 1-MeV electrons with a sine pitch angle distribution are plotted as blue solid and dashed
 409 lines, respectively. The precipitation source is assumed to have a total energy flux of $1 \text{ erg/cm}^2/\text{s}$. The bottom
 410 panels show similar results, but for precipitation electrons with an exponential energy distribution with the
 411 characteristic energy ε_0 being 100 keV or 300 keV.



454 **Figure 5.** Comparison between direct Monte Carlo results and ionization profiles derived from the lookup
 455 table. (a) Ionization production by 3 keV electrons in 10 different MSIS atmospheric profiles. (b) Ioniza-
 456 tion production by 19 precipitation energies in a fixed background atmosphere. (c) Similar to panel (b), but
 457 normalized so that the source precipitation flux is $0.1 \text{ erg/cm}^2/\text{s}$.

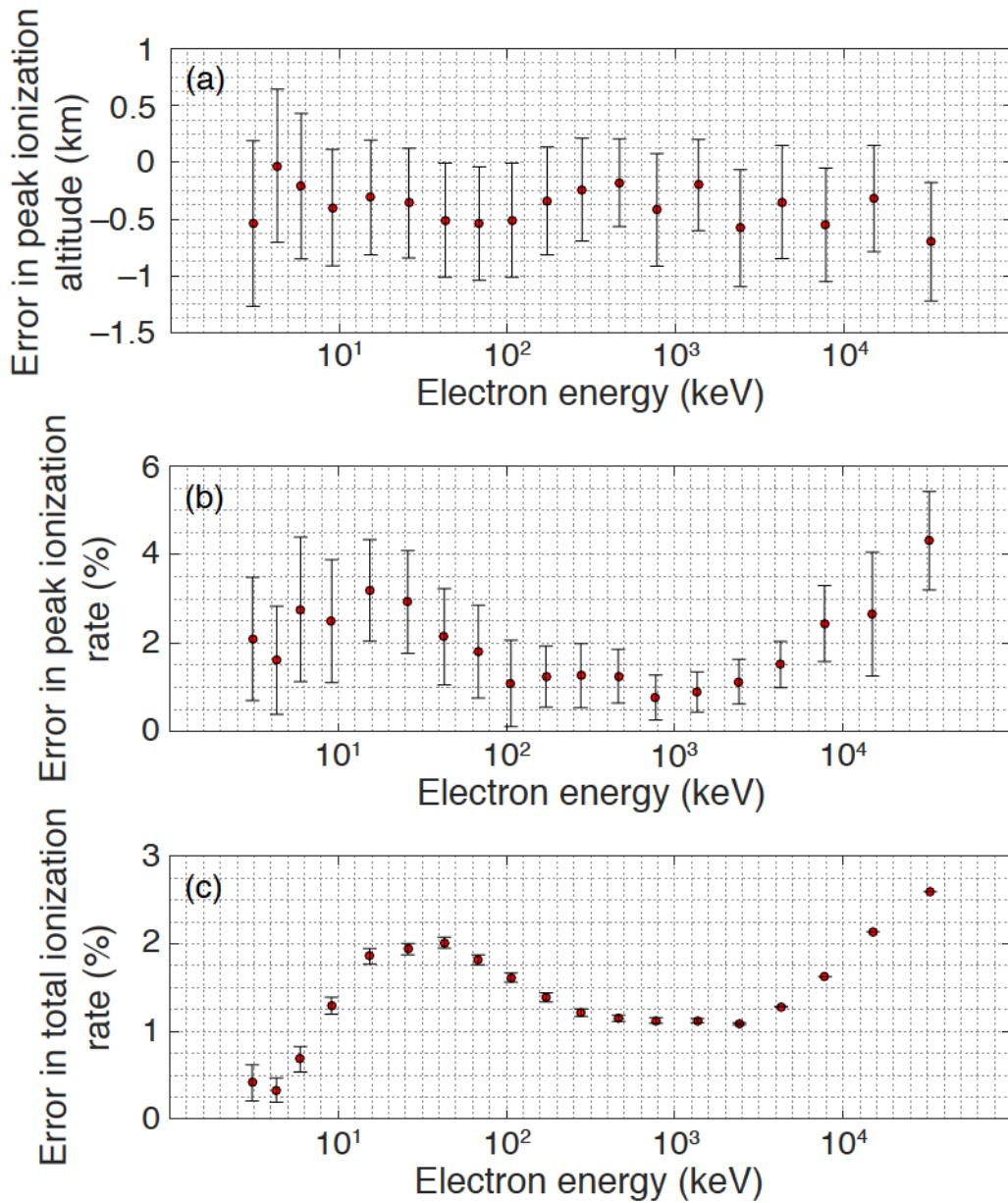
435 model for energies below 50 keV, whereas EPMC uses the method of small-angle colli-
 436 sions consistently for all energies [Lehtinen, 2000, pp. 15–18].

437 We have also calculated the ionization profiles produced by a sine pitch angle dis-
 438 tribution, and computed the difference from the isotropic distribution. A sine pitch angle
 439 distribution contains a smaller portion of electrons within the loss cone and, thus, precip-
 440 itating electrons deposit less energy in the atmosphere, especially at lower altitudes. For
 441 monoenergetic beams, when compared to the sine distribution, an isotropic distribution
 442 leads to 22% more ionization around the peak, and 10% more ionization in total. The dif-
 443 ference in the altitude of peak ionization is 1 km. For the exponential energy distribution,
 444 the difference between these two pitch angle distributions is 13% and 11% in peak and
 445 total ionization, respectively. These differences roughly represent the uncertainty of NO_x
 446 and HO_x production in chemistry simulation due to the pitch angle distribution. It is im-
 447 portant to note that we have only examined several sets of background atmosphere and
 448 precipitation distribution here; the difference mentioned above could be notably enhanced
 449 under certain precipitation conditions. For example, in a separate test, we found that, if
 450 one only considers the data recorded by the 0° telescope onboard POES as the precipita-
 451 tion flux and if the true pitch angle distribution is sine, this could underestimate the peak
 452 ionization altitude by ~ 2 km and the peak ionization rate by almost one order of magni-
 453 tude. Similar conclusions have also been found by Tyssøy *et al.* [2019].

461 3.2 Monoenergetic Electrons

462 The validation results of monoenergetic tests are summarized in Figures 5 and 6.
 463 Figure 5a shows the comparison of ionization profile produced by 3 keV electrons in 10
 464 different MSIS atmospheres between Monte-Carlo-simulated and derived results, while
 465 Figure 5b shows the comparison for 19 electron energies in a fixed background atmo-
 466 sphere. These 10 profiles are randomly chosen from the 100 MSIS atmospheres (Fig-
 467 ure 3e) used in validation test. To better compare the ionization rate around the peak, the
 468 ionization curves are also normalized so that the source precipitation flux is $0.1 \text{ erg/cm}^2/\text{s}$
 469 and shown in Figure 5c.

470 Due to the variation in background atmospheric density, the altitude profiles of ion-
 471 ization production by 3 keV electrons vary dramatically in different runs as shown in Fig-
 472 ure 5a. Nevertheless, our method very well captures this range of variation and the largest
 473 difference in peak ionization altitude is 1 km. For the other 18 energies, the derived pen-
 474 etration depth is also consistent with the truth as evident in Figures 5b and 5c. The pre-
 475 cipitation energy of 3 keV presented in Figure 5a is among the lowest energy of interest



458 **Figure 6.** Error in (a) peak ionization altitude, (b) peak ionization rate, and (c) total ionization rate esti-
 459 mated using monoenergetic beams of precipitating electrons and 100 MSIS atmospheric profiles. The error
 460 bars show one standard deviation.

476 for chemistry studies [e.g., *Mironova et al.*, 2015] and represents the worst-case scenario in
477 error analysis.

478 The estimated error in peak ionization altitude, peak ionization rate, and total ionization
479 rate is presented in Figure 6. These results are obtained using 100 Monte-Carlo runs
480 for each electron energy; each run uses a background MSIS atmosphere and yields an esti-
481 mate of the error; we compute the difference in peak altitude, and percentage difference in
482 peak rate and total ionization; the mean errors are denoted as red points in Figure 6; the
483 error bars show one standard deviation. The largest error in peak altitude is -0.7 km at
484 the electron energy of ~ 33 MeV with a standard deviation of ~ 0.52 km. In 58% of 1,900
485 tests (19 electron energies and 100 MSIS atmospheres), the peak altitude of the derived
486 ionization profile turns out to be the same as the truth. Of note, the resolution of ioniza-
487 tion profiles is 1 km in altitude and, in majority of these tests, our method can pinpoint
488 the true peak within one grid cell.

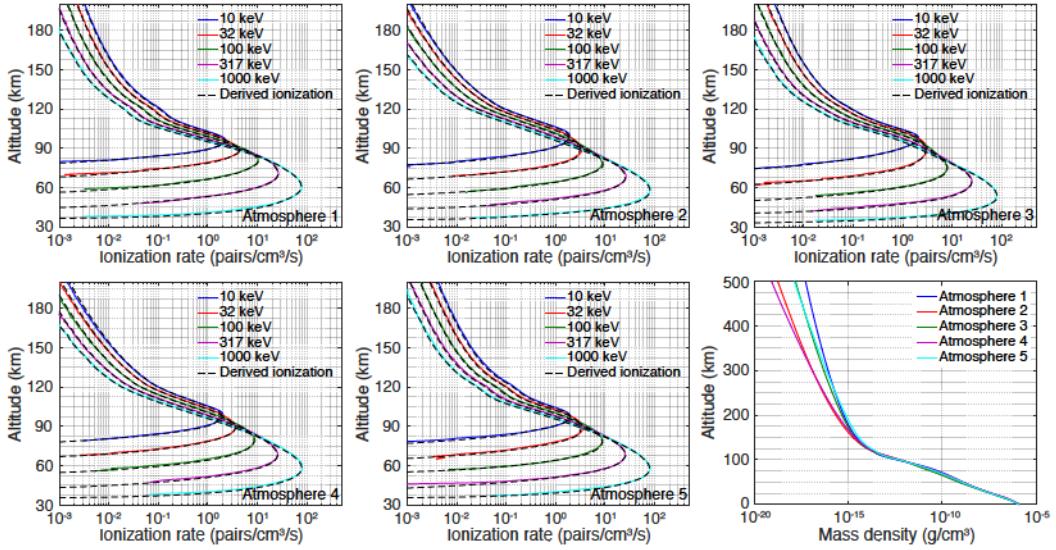
489 The largest error is approximately 4.3% in peak ionization rate, and 2.6% in total
490 ionization rate, both at the energy of 33 MeV. As noted, we predetermine the lowest alt-
491 iude of ionization before performing interpolation and this allows a good estimation of
492 total ionization in the new atmosphere. For this reason, the errors in total ionization are
493 overall smaller than 3%. However, as explained above, the column mass density above the
494 predetermined lowest altitude of ionization in the new atmosphere is slightly larger than
495 that in the reference atmosphere. In this regard, the altitude of peak ionization in the de-
496 rived ionization profile is found to be lower than the truth (Figure 6a). The errors in the
497 total ionization are related to how much larger the column mass density in the new atmo-
498 sphere is than that in the reference atmosphere. The standard deviation of the total ioniza-
499 tion is controlled by how variable the column mass density is around the lowest ionization
500 altitudes. For example, the column mass density above 50 km does not vary significantly
501 for the 100 background atmospheres and this is why the error bars are much smaller at
502 energies greater than 2 MeV in Figure 6c. It is expected that, with a finer altitude resolu-
503 tion, the errors shown in Figure 6 could become even smaller.

514 3.3 Exponential Energy Distribution and Isotropic Pitch Angle Distribution

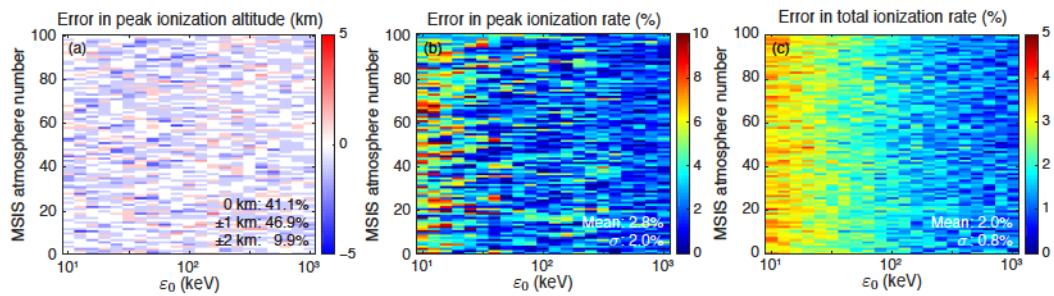
515 In addition to monoenergetic beams, we have tested exponential energy distributions.
516 Figure 7 shows the comparison between direct Monte Carlo results and ionization pro-
517 files derived from the lookup table. For context, source precipitating electrons in this set
518 of tests are assumed to have an exponential distribution in energy and an isotropic distri-
519 bution in pitch angle; a total of 2,100 validation tests are performed using 21 ε_0 values
520 and 100 MSIS atmospheric profiles. To illustrate the effectiveness of our method, we plot
521 the ionization profiles in 5 randomly-picked atmospheres; the background mass density
522 profiles of these atmospheres are shown in Figure 7f. For each atmosphere, the 5 ε_0 val-
523 ues that are equally spaced in logarithmic scale between 10 keV and 1 MeV are shown
524 (10 keV, 32 keV, 100 keV, 317 keV, and 1 MeV).

525 For different combinations of MSIS atmosphere and ε_0 value, the derived ioniza-
526 tion profiles show excellent agreements with the truth data in terms of the overall trend
527 and ionization peak. The slight difference at altitudes above ~ 160 km is likely due to the
528 numerical error that we explain in section 2.5. We emphasize that the choice of the most
529 energetic beam (ε_{\max}) in equation 1 is critical in this comparison in that it controls the
530 magnitude of ionization rate at the lowest altitudes. Although we assume an energy range
531 of 10 keV to 10 MeV, due to the limited number of particles, the highest energy of elec-
532 trons generated in Monte Carlo runs is usually lower than 10 MeV. In these comparisons,
533 ε_{\max} is specifically set to be the highest energy of source electrons in Monte Carlo runs.

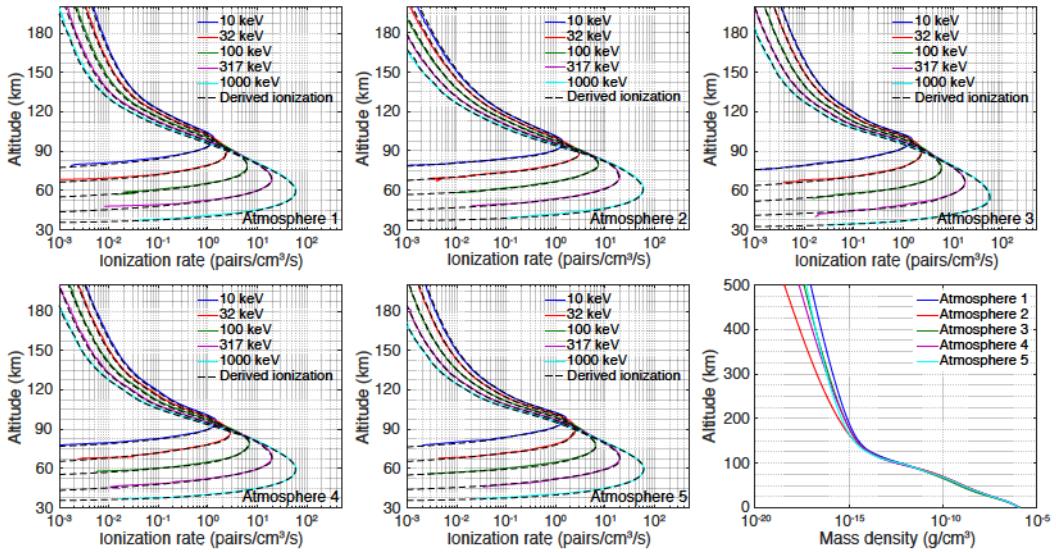
534 Figure 8 shows the corresponding error in peak ionization altitude, peak ionization
535 rate, and total ionization for different ε_0 values (x axis) and MSIS atmosphere (y axis).
536 The errors shown in Figures 8b and 8c are the absolute value of percentage difference be-



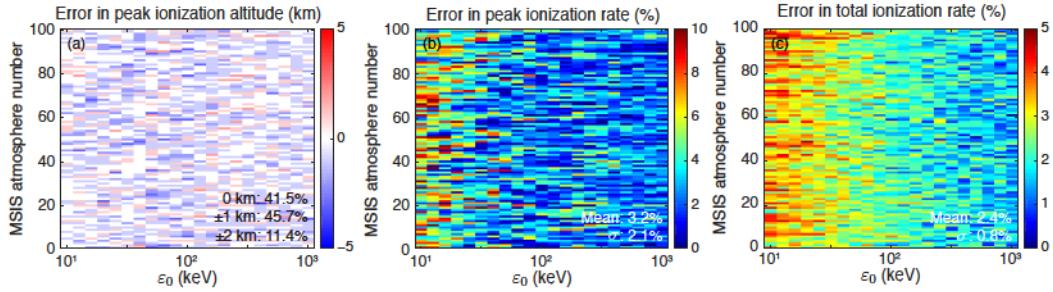
504 **Figure 7.** Comparison between direct Monte Carlo results and ionization profiles derived from the lookup
 505 table. Source precipitating electrons are assumed to have an exponential distribution in energy and an
 506 isotropic distribution in pitch angle. 21 characteristic energies (ε_0) and 100 MSIS atmosphere are used in
 507 this set of tests. We show the comparison in 5 randomly-picked atmospheres to illustrate the effectiveness of
 508 our method. For each atmosphere, the 5 ε_0 values that are equally spaced in logarithmic scale between 10 keV
 509 and 1 MeV are shown. The bottom right panel shows the mass density of these atmospheres.



510 **Figure 8.** Error in (a) peak ionization altitude, (b) peak ionization rate, and (c) total ionization estimated
 511 using precipitating electrons with an exponential distribution in energy and an isotropic distribution in pitch
 512 angle, and 100 MSIS atmospheric profiles (y axis). The ε_0 values used for the exponential energy distribution
 513 are shown as the x axis.



554 **Figure 9.** Similar to Figure 7, but calculated using precipitating electrons with a sine distribution in pitch
555 angle.



556 **Figure 10.** Similar to Figure 8, but calculated using precipitating electrons with a sine distribution in pitch
557 angle.

537 between the derived and true ionization profile. These errors are reasonably small and con-
538 sistent with the percentage difference presented in Figure 3f, indicating that our method
539 has satisfactorily estimated the true ionization in 2,100 tests. In 41% of all the tests, the
540 peak altitude of derived ionization is the same as the truth to within 1 km resolution. The
541 difference in peak altitude is found to be ± 1 km in 47% of all the tests, and ± 2 km in
542 10% of all the tests. As noted above, because of the way we determine the lowest ion-
543 ization altitude, the ionization profile derived from the lookup table tends to bias towards
544 lower peak altitude and larger total ionization value.

545 The mean error and standard deviation are 2.8% and 2% for peak ionization rate,
546 and 2% and 0.8% for total ionization. The largest error in peak ionization rate and total
547 ionization is, respectively, $\sim 13\%$ and $\sim 4.3\%$. On average, these errors become smaller
548 with increasing ε_0 value because lower ε_0 value contains relatively more low-energy com-
549 ponent, corresponding to more ionization production at higher altitudes, where the ran-
550 domness effects (see section 2.5) are pronounced. If we define good estimation as an error
551 within 1 km in peak altitude and an error of less than 5% in peak ionization rate, the de-
552 rived profiles in 1,807 cases meet this criterion. Our method can accurately estimate the
553 true ionization in $\sim 86\%$ of all the tests.

558 **3.4 Exponential Energy Distribution and Sine Pitch Angle Distribution**

559 Figures 9 and 10 are similar to Figures 7 and 8 above, but the source precipitating
 560 electrons have a sine distribution in pitch angle. Satellite-measured pitch angle distribu-
 561 tions of precipitation fluxes are poorly resolved primarily due to instrumental challenges.
 562 Data from Van Allen Probes show no evidence of discontinuity in pitch angle distribu-
 563 tion [e.g., *Baker et al.*, 2014], and a sine function is believed to be more representative and
 564 thus adopted here.

565 This set of test results is comparable to those of the isotropic distribution (Figures 7
 566 and 8). We observe from Figure 9 that, for the sine distribution, the derived ionization
 567 profiles are also remarkably close to the truth. The largest error is 2 km in peak ionization
 568 altitude, ~13% in peak ionization rate, and ~5% in total ionization. For the peak ioniza-
 569 tion rate, the mean error is 3.2% with a fairly large deviation of 2.1%. These values are
 570 2.4% and 0.8% for the total ionization. Among 2,100 tests, the error in peak altitude is
 571 found to be 0 km in 42%, ± 1 km in 46%, and ± 2 km in 11% of all the tests. Our method
 572 achieves a good-estimation (≤ 1 km in peak altitude and $\leq 5\%$ in peak ionization rate) rate
 573 of 82% (1,721 cases). The errors obtained using the sine pitch angle distribution are, on
 574 average, larger than those of isotropic distribution because, as explained above, a sine dis-
 575 tribution causes more ionization production at relatively higher altitudes, where the numer-
 576 ical error is larger.

577 **4 Discussion and Conclusions**

578 In this study, using Monte Carlo simulations of EEP, we have tabulated the atmo-
 579 spheric ionization response to monoenergetic electrons with different pitch angles and en-
 580 ergies between ~ 3 keV and ~ 33 MeV. We have quantified the pitch angle dependence, and
 581 explained the randomness effects in Monte Carlo simulations at low-density altitudes, as
 582 well as the resultant uncertainty in ionization calculation. Based on the parameterization
 583 method of *Fang et al.* [2010], a robust method has been developed for the specification of
 584 ionization production in an arbitrary atmosphere by precipitating electrons with any distri-
 585 bution in electron energy and pitch angle. Moreover, we have validated this method using
 586 100 MSIS atmospheric profiles and different energy and pitch angle distributions.

587 Among traditional precipitation transport models, although the Monte Carlo tech-
 588 nique provides the most detailed simulation, it is usually regarded as computationally
 589 intense and unsuitable for routine calculation of ionization profiles in large-scale atmo-
 590 spheric modeling [e.g., *Solomon*, 2001]. However, tabulating the atmospheric ionization
 591 response to all possible combinations of electron energy and pitch angle can effectively
 592 remedy this deficiency. In this work, we have demonstrated the feasibility of this lookup
 593 table. Based on a total of 6,100 validation tests using monoenergetic and exponential en-
 594 ergy distributions, and isotropic and sine pitch angle distributions, this lookup table can
 595 satisfactorily estimate the atmospheric ionization produced by EEP, especially at the alti-
 596 tudes of peak ionization. In 91% of all the tests, the error in peak ionization altitude
 597 is found to be within 1 km with a mean error of 2.7% in peak ionization rate, and 1.9%
 598 in total ionization. In the validation using exponential energy distribution and sine pitch
 599 angle distribution, our method can accurately infer (≤ 1 km in peak altitude and $\leq 5\%$ in
 600 peak ionization rate) the ionization profile in 82% of all the tests. This rate rises to 86%
 601 if the pitch angle distribution is isotropic. However, we have only examined this method in
 602 the context of Monte Carlo simulations. Further comparison with other electron transport
 603 models and real data would be desirable to better calibrate this method.

604 This method is sufficiently accurate for atmospheric chemistry simulation. The changes
 605 in NO_x and HO_x production are roughly proportional to the peak ionization rate. In this
 606 regard, the errors found in our validation tests roughly represent the uncertainty in chem-
 607 istry simulations. In general, these errors are believed to be smaller than other uncertain-

ties in chemistry simulations, for example, reaction and transport rates, and the poorly-investigated abundance of minor species [e.g., *Verronen*, 2006]. We emphasize that this method is optimized to provide the best guess of ionization production around the peak (see section 2.3) because this part contributes most to chemical changes; the error could be as large as $\sim 20\%$ for ionization rate at altitudes above 200 km. If the main focus is ionization rate in the *E*- and *F*-region ionosphere, this method needs to be used with extra caution.

Different from prior parameterization schemes, this method fully accounts for the dependence of ionization production on the atmospheric conditions and on the electron energy and pitch angle distribution. It is applicable to any Earth's atmospheric condition and highly suitable to incorporate spacecraft measurements into atmospheric chemistry models with few limitations. Of note, the lookup table reported here is specifically constructed for electron pitch angles with respect to the background magnetic field, whereas space-borne instruments usually record fluxes from a given solid angle. This difference needs to be properly taken into account in future analysis. Moreover, the initial altitude of this lookup table may be different from many EEP-observing satellites, for example, POES. To directly apply this table to the analysis of POES data, one needs to first map the precipitation fluxes measured by POES from its location to the altitude of 500 km using the background geomagnetic field. The key factors to obtain an accurate ionization profile are the most energetic beam in equation 1, as noted above, and the smoothness of energy and pitch angle distribution. Space-borne measurements of precipitation fluxes usually have limited resolution in energy and pitch angle. If the input energy or/and pitch angle distribution is discontinuous at a given value, the derived ionization profile may exhibit some stepwise changes at the corresponding altitude.

Quantifying the atmospheric effects brought by EEP has been challenging for several decades and this is partially because of the accuracy in ionization source in chemistry modeling. Recent studies have gradually recognized the importance of considering precipitation fluxes in the full range of energy and pitch angle [Randall *et al.*, 2015; *Nesse Tyssøy et al.*, 2016; *Smith-Johnsen et al.*, 2018; *Pettit et al.*, 2019]. It is from this perspective that we extend the parameterization method of *Fang et al.* [2010] by expanding the energy range and including the pitch angle dependence, and establish a generalized method for the specification of EEP-produced ionization profile. Validation results show that this method provides a reliable means to convert space-borne measurements of precipitation fluxes into ionization input in atmospheric chemistry models, thereby assisting model-observation comparison and better quantification of atmospheric effects induced by EEP.

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 648 The simulation data and analysis codes used to generate all figures and results in this pa-
 649 per, as well as the code developed for the calculation of atmospheric ionization, are avail-
 650 able at <https://doi.org/10.5281/zenodo.3945306>.

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