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Next Directions in Measurement of the Home Mathematics Environment: An International and Interdisciplinary Perspective

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Abstract

This article synthesizes findings from an international virtual conference, funded by the National Science Foundation (NSF), focused on the home mathematics environment (HME). In light of inconsistencies and gaps in research investigating relations between the HME and children's outcomes, the purpose of the conference was to discuss actionable steps and considerations for future work. The conference was composed of international researchers with a wide range of expertise and backgrounds. Presentations and discussions during the conference centered broadly on the need to better operationalize and measure the HME as a construct – focusing on issues related to child, family, and community factors, country and cultural factors, and the cognitive and affective characteristics of caregivers and children. Results of the conference and a subsequent writing workshop include a synthesis of core questions and key considerations for the field of research on the HME. Findings highlight the need for the field at large to use multi-method measurement approaches to capture nuances in the HME, and to do so with increased international and interdisciplinary collaboration, open science practices, and communication among scholars.

Keywords

home mathematics environment; mathematics; research agenda; measurement; early childhood

Early mathematics skills are an essential foundation for later academic outcomes and career success (Evan et al., 2006; Geary, 1994; Jordan et al., 2003; Watts et al., 2014), and this success is a driver of economic and social prosperity (Harding et al., 2012; National Mathematics Advisory Panel [NMAP], 2008). Evidence demonstrates that caregiver¹-child mathematical interactions in the home environment (the home mathematics environment [HME]) are related to children's mathematical understanding (Daucourt et al., 2020; Elliott & Bachman, 2018; Mutaf-Yıldız et al., 2020). The breadth of evidence suggests a positive relation between the HME and mathematics performance (Kleemans et al., 2012; Skwarchuk

¹We use the term caregiver to include children's primary caregivers in the home (e.g., parent, legal guardian, grandparent). This is distinct from a formal childcare provider

et al., 2014). However, some findings are contradictory showing both positive and negative relations (Blevins-Knabe & Musun-Miller, 1996; Skwarchuk, 2009), and other studies do not demonstrate significant associations (Blevins-Knabe et al., 2000; DeFlorio & Beliakoff, 2015; De Keyser et al., 2020; Missall et al., 2015—see also Blevins-Knabe, 2016 for a review).

Historically, researchers studying the HME predominantly focused on numeracy, as compared to other subdomains of mathematics (Blevins-Knabe et al., 2000), which was in alignment with the broader field's focus on numeracy assessment and development (Ginsburg & Baroody, 2003; Sarama & Clements, 2004). Given that, over time, the conceptualization of mathematics itself within the literature has been extended (Sarama & Clements, 2009; Starkey et al., 2004), there has been a parallel, though delayed, focus on broadening the HME to include other aspects of mathematics (e.g., measurement, geometry, patterning, and spatial activities; De Keyser et al., 2020; Hart et al., 2016; Purpura et al., 2020; Zippert & Rittle-Johnson, 2020)². As such, though much of the work in the field uses the terminology of “Home Numeracy Environment,” the term “Home Mathematics Environment” is used throughout this document, similar to Hart et al. (2016) and Sonnenschein et al. (2012), for consistency and to acknowledge that this construct may comprise more than just numeracy.

One serious challenge to understanding the relation between the HME and children's mathematics development is variability in how the HME is measured across studies. Measures of the HME are often different across studies, which may begin to explain inconsistencies in findings within the literature. Further, although existing measures of the HME provide a basis for understanding the quantity of mathematics activities in the home, their composite nature may insufficiently capture nuanced relations with children's outcomes, such as varied item difficulty and content that may differentially promote early mathematics skills (Purpura et al., 2019).

Beyond inconsistencies and gaps in measurement of the HME, discrepancies in findings linking the HME to mathematics outcomes likely result from contextual factors that may influence children's experiences in the HME such as child factors (e.g., gender, age; Chang et al., 2011; Thompson et al., 2017), family factors (e.g., socioeconomic status; Levine et al., 2010), community factors (e.g., caregiver-educator communication; Lin et al., 2019), and country and cultural³ differences (e.g., differences in caregiver or child beliefs about mathematics across countries or cultural groups; del Río et al., 2017; LeFevre et al., 2010; Musun-Miller & Blevins-Knabe, 1998; Susperreguy et al., 2020b). There is also a need to understand the breadth of activities that constitute mathematics (Zippert & Rittle-Johnson, 2020) and how individual-level characteristics of caregivers and children might

²It should be noted that there was disagreement among authors on what skills constitute “mathematics” during this age period. In particular, there were discussions surrounding the inclusion of patterning and spatial skills within this broader construct. These differences in definitions of mathematics were based on both empirical literature and differential emphasis of what is included in early mathematics curricula and standards across different countries. For the purposes of this document, we have elected to use a broad definition of the construct of mathematics and believe that this is an important question to address through future discussions and research.

³We acknowledge that the term culture can be interpreted in multiple ways. Here we use the term culture to refer to groups with shared experiences, values, and norms that may vary both within and across countries. We also recognize that the experiences, values, and norms of some individuals may span across multiple cultural groups.

explain variations in the HME. For example, caregivers' and children's attitudes, beliefs, and expectations (Davis-Kean & Sexton, 2009; del Río et al. 2019), mathematics anxiety (Maloney et al., 2015), and resources in the home (Melhuish et al., 2008) may influence the engagement of children and their caregivers around mathematics and the effects of that engagement on early mathematics development.

Given inconsistencies in prior work and gaps in knowledge about the HME, a collaborative and interdisciplinary discussion and integration of research on the HME was needed to provide direction for future work in this area. Alcock and colleagues (2016) published a *Challenges in Mathematical Cognition* article based on a systematic process with 16 experts in mathematical cognition, in which they set forth a series of goals and future directions to guide the field forward. Within the six major topic areas identified for the field, they identified 26 more specific questions as important next directions for mathematical cognition research. In particular, in the section on *The Development of Valid and Reliable Measures* (major question F), question 25 was "How can we measure the variability in children's mathematical experiences outside of school?" (Alcock et al., 2016; p. 29). Given the growth in research in this area, and the need to understand the contextual influences on children's mathematics development, in this document we focus on identifying core questions and key considerations for future research on the HME.

To articulate a clear and actionable set of next directions, a group of international scholars representing a wide range of disciplines and backgrounds were invited to a conference to discuss next directions in measurement of the HME. The approach to generating these next directions was modeled after the Sutherland et al. (2012) and Alcock et al. (2016) processes, but was more qualitative in design. Core questions (i.e., big questions for the field that may be answered through programmatic lines of work) and key considerations (i.e., recommendations for how to approach answering the core questions) in the HME were identified and refined. Moreover, we sought to incorporate a diverse set of voices from both within the field of HME research and outside (e.g., home literacy environment, language and culture, mathematical cognition) to ensure that the process was not overly inward focused. Through a virtual conference, asynchronous poster session, and subsequent writing workshops, we synthesized knowledge in this domain and formulated questions for the field of research on the HME to generate this document (see method section for more details). In the results section, we highlight the core questions and key considerations in measurement of the HME and the relevant background. Last, we provide a synthesis of these next directions and a broad overview of how researchers examining the HME may attain these goals moving forward.

To advance research on the HME, there is a significant need to increase collaboration and communication on the measurement and conceptualization of the HME. Thus, researchers in the conference focused on raising and addressing questions and key considerations central to research on the HME, which resulted in this document. By identifying the next directions for research on measurement of the HME, we intend that the results of this work will facilitate future efforts in producing high-quality research that examines the HME and its links to children's mathematics outcomes.

Method

A virtual conference, funded by the United States National Science Foundation (grant # DRL-1920479), *Refining our Understanding of the Home Math Environment in the Early Years: An Interdisciplinary Working Conference*, was held in Fall 2020. The purpose of this conference was to bring together an international and interdisciplinary group of scholars to identify and discuss next directions in research on the measurement of the HME.

The next directions presented in the results section originated from discussions among conference participants during three phases. *Phase One* included three virtual conference meetings (one 2-hour meeting each week across three weeks). *Phase Two* was a week-long asynchronous interactive virtual poster session. *Phase Three* consisted of a series of five writing workshops (supplemented by pre- and post-process work) to synthesize the ideas raised by the participants in the conference into a manuscript draft.

Participants

Twelve conference organizers (including the principal investigator [PI], five co-PIs, and six members of the PI's lab) and nine advisory board members provided input into the design of the conference (including scholars from four different countries with a breadth of expertise). All conference organizers, and all but two of the advisory board members, were able to attend the conference. The conference organizers invited researchers from around the world who have impacted their fields (similar to the process used by Alcock et al., 2016) and also early-career scholars who will likely be engaged in their fields for many years to come. In total, 56 participants were invited from 23 different countries (33 attended from 13 countries), and they had varied areas of expertise in mathematical cognition and learning, language and literacy, home learning environments, developmental contexts, culture, and cognitive and affective characteristics of caregivers and/or children. Conference speakers were selected based on similar criteria. In addition, five program officers (from the agency that funded the conference) attended one or more conference sessions in Phase 1. In total (including the conference organizers, advisory board, program officers, and invited participants), 57 scholars participated in Phase 1, and there was representation from 14 different countries. The conference was conducted in English.

The asynchronous poster session (Phase 2) was advertised broadly over relevant listservs, social media, and through professional connections, given that the asynchronous virtual format allowed for greater participation. In total, there were 45 posters from researchers representing various career stages from 12 countries (four of which were not represented in Phase 1), and there were 189 unique visitors to the poster session.

The writing workshops (Phase 3) involved a subset ($n = 39$) of the conference participants that was representative of the original participant group. The conference organizers, advisory board, and conference speakers were all invited to participate. Additional participants were invited based on their interest and availability, areas of expertise, varied career stages, and recent work in this domain, keeping the overall number of participants manageable for collaborative writing sessions. The conference organizers also aimed to ensure representation of international perspectives by including participants from outside

North America and Europe. Table 1 includes all authors in the manuscript writing process as well as their roles, current institution and location, and research foci.

Procedure

Overview—The approach to proposing next directions in research on the HME was based on the method used by Sutherland et al. (2012) and Alcock et al. (2016). Sutherland et al. had participants representing a range of disciplines pose a list of unanswered questions and rate them in terms of priority in the science policy field. Alcock et al. generated a list of research questions as future directions for the field of numerical cognition through a similar narrowing and refining process. Alcock et al. began with a large list of generated questions and participants scored these questions in terms of their importance, resulting in the reduction of questions prior to additional review and refinement of specific questions. The current model took a more qualitative approach than the Alcock et al. method by incorporating discussion and consensus approaches in small groups to both generate new questions and narrow down and refine the final list of questions to convey the nature of the conference discussions. In addition, more international and interdisciplinary participation was used to address limitations raised about the Alcock et al. process, such as representation across research foci (Berch, 2016).

Phase One—The three virtual conference meetings each included guiding presentations from invited speakers, interactive breakout room discussions focused on questions related to the presentations (see Table 2 for an outline of topics and example discussion questions), and conference-wide debriefs. Participants recorded ideas generated from breakout sessions within the virtual conference platform.

Phase Two—The week-long asynchronous poster session included both static posters and pre-recorded lightning talks (organized into categories related to topics in Phase One). Participants could interact by commenting and/or asking questions within the conference platform.

Phase Three—The writing workshop series consisted of five 90-minute sessions across four weeks. In the first three sessions, participants focused on the results sections included below by synthesizing the notes taken in the conference platform during Phase One. The document writing and editing process took place using a shared electronic document that all participants could access in real-time. During the first session, participants were assigned a section based on their expertise. One section leader (conference organizer) as well as at least one content expert stayed with their assigned section during each of the first three sessions to ensure continuity in the ideas generated across sessions within a section. During the second and third sessions, participants rotated to two other groups to review the document for comprehensiveness and continuity. During the fourth session, participants edited results sections and contributed to the formulation of the discussion. Finally, in the fifth session, all participants reviewed and edited content across the entire document. After the writing workshop series, the first and last author worked together to consolidate suggestions and comments from other authors and created a complete online document for all authors to review.

Process

The results of the conversations during the conference and synthesis of activities during the writing workshop sessions are presented below. These topics are grouped into five sections: (a) Operationalizing and Measuring the HME, (b) Child, Family, and Community Factors, (c) Country and Cultural Factors, (d) Numeracy and Other Mathematical Domains, and (e) Cognitive and Affective Characteristics of Caregivers and Children. Each of these sections include both core questions and key considerations. The core questions focus on larger long-term issues that are not likely to be addressed through a single research study, but rather iteratively and collaboratively over time. The key considerations include the more qualitative feedback surrounding the discussions of how to attain the broader goals of operationalizing and measuring the HME, which must be kept at the forefront while attending to the core questions. Broad next directions for the field to consider are listed first in the operationalizing and measuring the HME section, which may encompass the other areas noted in later sections.

Results

A. Operationalizing and Measuring the HME

Accumulated research on the HME does not indicate what features are critical to the HME, whether and how it is related to children's mathematical development, or which aspects may be universal (if any) versus contextual. Addressing these limitations will require clarity of constructs (both in definition and measurement) because, at present, findings across studies cannot easily be compared and synthesized into a consistent literature base.

Core Questions

1. How should researchers define and operationalize the HME?
2. How does the HME fit into the broader structure of the home learning environment?
3. How should researchers measure the qualitative dimensions of the HME?
4. How does the HME change across development (within the child) and across time (societal changes)?
5. What aspects of the HME may be universal (if any) and what aspects may be dependent on specific contexts including family, community, country, and cultural factors?

Key Considerations

- a. The conceptualization of what is being measured in regards to HME activities (e.g., breadth of mathematics-related activities, total time engaged in activities, depth/quality of focus within a specific activity or content area) needs to be refined.
- b. As a field, researchers use a range of semi-distinct, but overlapping, terminology (e.g., informal/indirect versus formal/direct; Skwarchuk et al., 2014; Hart et

al., 2016) or use alternative conceptualizations (e.g., basic versus advanced, foundational versus advanced, or mapping versus operational) for categorizing activities (Elliott & Bachman, 2018; LeFevre et al., 2009; Skwarchuk, 2009; Susperreguy et al., 2020a; 2020b; Zippert & Ramani, 2017). There is a need to develop clear operational definitions by determining which HME terms and components are necessary and sufficient for broad use in the field (Cahoon et al., 2017).

- c. Researchers studying the HME need to understand how the importance of different aspects of the HME may vary for children's mathematical development across contexts.
- d. A multi-method approach (e.g., combinations of surveys, observations, focus groups, interviews, time diaries; Bachman et al., 2020; Cannon & Ginsburg, 2008; Elliott et al., 2020; Saxe et al., 1987; Skwarchuk, 2009; Thippana et al., 2020; Tudge & Doucet, 2004) may be necessary to capture the range of the HME as well as to better understand both quantitative and qualitative dimensions of the HME.
 - i. Additional work needs to be conducted to identify what families are doing in the home environment that researchers are not currently measuring in existing assessment surveys (Zippert & Rittle-Johnson, 2020).
 - ii. It remains unclear whether naturalistic/observational and survey methods are measuring the same or different facets of the HME (Elliott et al., 2020; Missall et al., 2017; Mutaf-Yildiz et al., 2018).
- e. There is a need to examine under what circumstances HME activities need to be done intentionally with the goal of promoting children's mathematics skills versus when incidental engagement is sufficient or operates differently to influence children's mathematics learning (e.g., Eason & Ramani, 2020).
- f. Researchers need to understand what quality means when discussing HME activities, whether and how the quality of engagement in HME activities influences mathematics learning, and if some activities are more important than others (Elliott & Bachman, 2018).
- g. Researchers need to consider how the HME (and the broader home learning environment) has changed and may continue to change over time in relation to societal shifts (e.g., technology, global pandemics, public service campaigns; Davis-Kean & Tang, 2015; Mazzocco, 2016; Rosenfeld et al., 2020; Yoshikawa et al., 2020).

The core questions and key considerations above are cross-cutting themes throughout the results section. Research on the HME includes discrepant findings about relations between the HME and children's outcomes (DeFlorio & Beliakoff, 2015; LeFevre et al., 2009, 2010; Manolitsis et al., 2013; Thompson et al., 2017), which may, in part, be due to differing approaches to operationalizing and measuring the HME and children's outcomes (as well as differing samples due to child age, etc.). Ultimately, these core questions and key

considerations indicate a need to develop a theoretical and testable framework for the HME in which alternative plausible models are explicitly contrasted. It is evident, however, that the framing of these questions may depend on topics raised in subsequent sections of this article, as such a model may have both universal and context-dependent components, if any universal components exist. In addition, questions and considerations are placed within their primary topic due to overlap among the four sections detailed below.

B. Child, Family, and Community Factors

Core questions that span across areas of child, family, and community factors are provided first, and then key considerations for each area are listed in separate subsections. These factors do not operate independently of each other. Thus, they must be considered in relation to one another and within their cultural contexts (see section on Country and Cultural Factors; also Bronfenbrenner & Morris, 2007). In the consideration of community factors, intersections with child and family factors are also addressed (e.g., partnerships with families and schools). Note that the discussion here is focused on adult-child interactions more broadly, and the cognitive and affective characteristics of caregivers and children (e.g., abilities, beliefs, attitudes, anxiety) are reserved for the final results section (see Cognitive and Affective Characteristics of Caregivers and Children) because those characteristics are typically viewed as broader, multidimensional aspects of the home learning environment (Melhuish et al., 2008).

Core Questions

1. Who are the people involved in a child's HME? Is the HME restricted to caregiver-child interactions, or does it extend to the broader family such as siblings, extended family and kin, and neighbors?
2. What constitutes the child's HME (e.g., is the HME only within the home or does it extend to contexts outside of the home such as museums and parks)?
3. How do various child, family, and community factors contribute to the structure and development of the HME?
4. How do the interactions among child, family, and community factors influence the HME and vice versa?
5. How do experiences in out-of-home learning environments, such as childcare/school settings and other community contexts, interact with the HME?

Key Considerations

Child

- a. Children's age and cognitive development need to be considered within the measurement of the HME, in terms of identification of which items and/or measures may be relatively more important at particular ages or developmental levels (Thompson et al., 2017).

- b. Attention needs to be paid to the types and modes of activities relevant and accessible to children with disabilities (Kritzer & Pagliaro, 2013; Louden et al., 2000).
- c. Researchers need to consider the amount of language input and number of languages to which children are exposed.
- d. Given that results about gender differences in children's mathematical skills are mixed (Bakker et al., 2018; Hutchison et al., 2019), additional research is needed to determine how the HME may or may not contribute to potential performance differences on school- or research-based measures (De Keyser et al., 2020; Dearing et al., 2012; Chang et al., 2011; Clerkin & Gilligan, 2018; Thippana et al., 2020).
- e. Researchers typically measure the HME in ways that focus on caregiver-initiated activities (e.g., how often do you do [specific activity] with your child?). However, it is important to also understand how child-initiated activities or interactions may contribute to the HME and recognize that caregivers may in fact be responding to children's initiations of mathematics activities.

Family

- a. Researchers must identify and understand the family practices that provide early mathematics learning opportunities for children (LeFevre et al., 2009; Niklas & Schneider, 2014; Verdine et al., 2014).
- b. Researchers need to consider the family structure (e.g., diverse family compositions and arrangements) and who engages in mathematical activities with the child at home. Accordingly, researchers need to move beyond single-caregiver surveys when there are multiple caregivers in the home (del Río et al., 2017).
- c. Family socioeconomic status (SES) is a proxy for a range of characteristics (e.g., education, material and social resources; Bradley & Corwyn, 2002; Duncan et al., 2015; Foster, 2002). Researchers need to consider more deeply which family characteristics that are typically subsumed within SES are directly connected to various aspects of the HME (Davis-Kean et al., 2020; Dearing & Taylor, 2007; DeFlorio & Beliakoff, 2015; Gaylord et al., 2020; Levine et al., 2005; Votruba-Drzal, 2003).
- d. Family members may engage in distinct types and amounts of activities in the HME depending on individual's characteristics (e.g., child age, child gender, caregiver gender, presence of child learning difficulties or other aspects of developmental differences), and researchers need to account for this variation (Chang et al., 2011; Thompson et al., 2017). The way in which the presence and actions of siblings affect HME engagements also must be considered.

Community

- a. Researchers should identify characteristics of the surrounding community (e.g., material and social resources, access to libraries or community centers, community parks, digital mathematics programs) that may influence the HME or provide alternative opportunities for mathematics learning beyond the HME. Moreover, consideration should be given to family engagement with these mathematics learning opportunities that exist within communities, and to the barriers that may exist (Eason et al., 2020).
- b. Researchers need to document the type of, and access to, preschool education (De Keyser et al., 2020), the age at which children enter childcare/school settings, the amount of time children spend in those settings, as well as educator training and competence. Furthermore, researchers should address how the instruction/learning opportunities in those settings relate to the role of the HME, given that time in childcare/school settings is time not at home (Anders et al., 2012; Melhuish et al., 2008).
- c. Researchers must consider the potential partnership, or lack thereof, between childcare/school settings and the HME (e.g., caregiver-educator communication, Lin et al., 2019).

These core questions and key considerations suggest that there are significant gaps in knowledge related to how child, family, and community factors are connected to the HME and how activities are enacted to foster children's mathematics skills. They also address possible interactions among these factors and the mechanisms by which these factors are connected. Further, the child, family, and community factors may shape the methodological tools used to assess the HME; thus, researchers should consider using survey items that are inclusive of a broad range of children, families, and communities. Finally, these factors and their interactions must be considered with regard to the broader context in which they are situated. In the subsequent section, country and cultural factors are addressed.

C. Country and Cultural Factors

Research with **Western Educated Industrialized Rich and Democratic**, or WEIRD, participants (Henrich et al., 2010) comprises most of the field's knowledge of the HME. However, the HME may be culturally grounded and embedded within a larger context that is complex and evolves over time (Bronfenbrenner & Morris, 2007). Specifically, the HME may look very different across diverse contexts (LeFevre et al., 2010; Susperreguy et al., 2020b). The homogeneity of existing research points to a need for international studies that include both cross-cultural and within-cultural work. In particular, researchers need to understand what aspects of the HME may be universal (if any) and what aspects may vary even within the country or culture (or cultures) in which families reside.

By conducting analyses across and within countries and cultures, researchers will be able to better understand the diversity of children's home contexts. Adopting an approach that considers contextual variability may be more generative than setting one group as the norm and evaluating other groups in comparison to the normative group. As there is variability

that occurs both within countries as well as between broadly defined groups, it is important to conduct studies within countries including within-group samples of diverse populations (e.g., within-group studies of African American children and families in the United States without comparison to White children and families) to examine normative processes within groups, as well as the diversity within those groups (e.g., Tudge & Doucet, 2004). In this way, the field can understand what is shared across countries and cultures and what is unique to specific cultural groups. Thus, we can describe the diversity of human experiences and how families across cultures contribute to children's mathematical learning.

Examining broader contextual aspects of an environment requires consideration of factors such as 1) cultural roles and norms, 2) national educational practice and policy factors, and 3) language(s), which should be recognized and taken into account when measuring the HME. First, core questions that span these factors are described, and then key considerations are listed.

Core Questions

1. How do country and cultural norms and societal expectations related to child rearing, gender, and age affect the HME and children's opportunities to engage in mathematical activities?
2. What constitutes "mathematics" in various cultures, and which activities are interpreted as mathematics-related or not?
3. What is the level of influence of the different country and cultural factors on the various components of the HME (e.g., structure of the HME, frequency and methods of engagement in mathematical activities)?
4. What is the role of language(s) in the HME related to children who experience differences between home and school languages and/or multilingualism?

Key Considerations

Cultural Roles and Norms

- a. Researchers need to consider cultural variations in societal expectations of mathematics performance (LeFevre et al., 2010; Galindo et al., 2019), stereotypes (Nollenberger et al., 2016; Schroeder & Bámaca-Colbert, 2019), beliefs about children's general behavior and development at various ages, beliefs about the ways in which children learn and develop, beliefs about child rearing, and how each may affect the HME.
- b. The relation between children's mathematics learning and caregivers' larger socialization goals for their children dynamically affects what knowledge the caregivers try to foster, and this relation shifts as children age (Bronfenbrenner & Morris, 2007).
- c. Caregiver intentionality about home mathematics support may differ across and within cultural groups. Researchers also need to attend to cultural variation in the focus of mathematics activities, in terms of the emphasis on achievement versus enjoyment (Kovas et al., 2015).

- d. Roles of different family members in the home may differ according to cultural groups, and these roles may influence who provides mathematics activities and which activities they engage in with children (Huntsinger & Jose, 2009; Kvalsvig et al. 1991).
- e. Researchers need to consider cultural differences in school-family connections, particularly, caregiver expectations of who should be providing educational experiences (teachers, caregivers, others; Sonnenschein et al., 2005). Caregivers' beliefs, attitudes, and anxiety may differ across cultures and can affect engagement in mathematics activities and how caregivers educate their children at home.

National Educational Practice and Policy Factors

- a. Timing of mandatory and/or formal school entry, universal provision of early childcare and primary education policies, length of time spent daily in formal schooling, and access to early educational opportunities outside of the home (including program enrollment eligibility) as well as the quality of those opportunities differs across countries (De Keyser et al. 2020; Niklas et al., 2018). These factors may influence the relations between the HME and mathematics development.
- b. Parental leave policies and employment rates differ across countries (Budig et al., 2012; Morrissey, 2017) and may affect how caregivers view their role in the education of their children and how intentional caregivers are with providing home learning activities.
- c. Explicit governmental emphasis on the importance of education (and mathematics education, specifically) and curricular requirements may vary across countries, which may affect what caregivers choose to focus on for supporting their children's learning.
- d. Some countries may provide specific programs designed to connect the home and school environments (e.g., Centros de Desarrollo Infantil in Colombia, Head Start in the United States, Programa Educa a tu Hijo in Cuba) or may have specific mandates for schools to engage caregivers. These programs or mandates may provide caregivers with resources to engage their children in HME activities (Belsky et al., 2006; Boyce et al., 2010; Ford et al., 2009; Starkey & Klein, 2000). Moreover, programs and resources available for children with disabilities may also vary.
- e. In some countries such as the United States, the importance of home literacy practices for children's language outcomes has long been endorsed by policymakers (Council on Early Childhood, 2014; National Reading Panel [U.S.], 2000) and pediatricians (Klass et al., 2009) alike. In contrast, the importance of engaging children in numeracy activities at home has only received attention in the past decade, which likely contributes to parents privileging literacy activities over mathematics activities (Mazzocco, 2016). This may, in part, be due to challenges in creating a singular message for a

national campaign about promoting mathematics in the home. Researchers need to understand how different national policies might influence public opinion of mathematics and the HME, and how these policies might shift over time.

Language(s)

- a. The structure of various languages (e.g., how languages encode number and mathematical language in their structure) may inherently support mathematics learning differently (Comrie, 2005; Kung et al., 2019; Purpura et al., 2018; Zuber et al., 2009) and may elicit different amounts and/or types of mathematics supports at home.
- b. Home versus school language may differ both within and across countries and communities (e.g., in Canada and New Guinea; Matang, 2005; Matang & Owens, 2014) and this may affect children's mathematical interactions across settings.
- c. The context and frequency in which multilingual and immigrant families do mathematics in specific languages may differ (Huntsinger et al., 1997), which may also affect the influence of the HME on children's mathematical development.

These core questions and key considerations indicate the multifaceted ways that culture and country can affect the nature of the HME, whether it is in regards to the roles and norms of the culture (Kovas et al., 2015; LeFevre et al., 2010; Nollenberger et al., 2016; Sonnenschein et al., 2005; Sonnenschein & Galindo, 2019; Schroeder & Bámaca-Colbert, 2019), national policies (Belsky et al., 2006; Boyce et al., 2010; Ford et al., 2009; Starkey & Klein, 2000), or language contexts (Comrie, 2005; Huntsinger et al., 1997; Kung et al., 2019; Purpura et al., 2018; Zuber et al., 2009). Culture shapes different aspects of the HME, including caregiver expectations (LeFevre et al., 2010; Sonnenschein & Galindo, 2019), the nature of the activities (LeFevre et al., 2018), as well as the support children have for learning mathematics (Pan et al., 2006). In order to draw more concrete conclusions about what aspects of the HME may be universal (if any) and what aspects may be context-specific, the field has a need for a more diverse research base beyond WEIRD samples. In addition to contextual factors, variability in content factors also needs to be considered. Thus, the following section presents the consideration of HME research beyond only numeracy.

D. Numeracy and Other Mathematical Domains

Most research examining mathematical activities in the home focuses exclusively within the domain of numeracy (e.g., counting, numeral identification, comparison, addition; Daucourt et al., 2020; Elliott & Bachman, 2018; Muta-Yıldız et al., 2020). However, early mathematics development encompasses multiple subcomponents, including measurement, geometry, patterning, and spatial domains (Milburn et al., 2019; NMAP, 2008). Only a few studies have investigated activities in the home that may support the development of mathematics skills other than numeracy (Dearing et al., 2012; Hart et al., 2016; Purpura et al., 2020; Zippert & Rittle-Johnson, 2020; Zippert et al., 2020; Zippert et al., 2020), in line with a broad conceptualization of the mathematics skills children learn during the

early years (Milburn et al., 2019). Further work is needed to understand the activities that comprise the broader HME (including numeracy and non-numeracy domains), the frequency and nature of experiences children have in these contexts, and their effect on children's mathematics outcomes. In this section, core questions and key considerations spanning all mathematical domains are presented.

Core Questions

1. What are the mathematical domains that should be measured within the HME, and what are the relations among them?
2. How should researchers measure each of these mathematical domains within the HME validly and reliably?
3. How do caregivers' views of what constitutes mathematics affect the activities they intentionally or unintentionally engage in with their children?
4. How does caregiver support (e.g., affordance of opportunities or interactions) of each of these mathematical domains individually relate to children's mathematics skill development specific to each domain and/or more broadly across domains?

Key Considerations

- a. More factor analysis work is needed to understand the structure of the HME and of the relevant mathematical domains (Hart et al., 2016; Purpura et al., 2020).
- b. There is the potential for activities to overlap in their influence on multiple mathematical domains and their elicitation of parental support of these domains, and survey items may need to include concrete examples to differentiate among ways that caregivers may engage with children in particular activities (e.g., Zippert et al., in press).
- c. It is possible that the existing survey items for non-numeracy activities are not able to appropriately capture what caregivers are doing within the HME. Caregivers may be engaging in activities with children which promote mathematical thinking and broader critical analysis skills that are not captured by existing measures, because those activities may not be viewed by caregivers as "mathematical." Thus, the field needs more descriptive work on the frequency and characteristics of mathematical activities and activities related to mathematics that go beyond numeracy, and how these might support the development of children's early mathematics skills.
- d. Caregivers may have an inaccurate perception of children's non-numeracy skills (e.g., ratings of children's patterning and spatial skills mismatched with children's measured skills, Zippert & Rittle-Johnson, 2020), and thus caregivers may not be providing activities for children at the appropriate level of support.
- e. Family-centered qualitative work is needed to better identify the breadth of activities in which families engage across the different domains of mathematics and how they change with children's development (e.g. Cahoon et al., 2017).

The core questions and key considerations above expand on a question posed by Alcock et al. (2016), which remains relevant: “How can we measure informal numeracy experiences validly and reliably?” (question 24, p. 29). Specifically, we extend this to “How can we measure mathematics experiences broadly (including numeracy and non-numeracy activities), validly, and reliably?” The field of research on the HME needs better measures of these experiences, as well as the non-numeracy domains themselves (see question 26 in Alcock et al. [2016], p. 29). Although the field of research on the HME is limited by available measures of the mathematical domains within the HME, Zippert and Rittle-Johnson (2020) found that caregivers report engaging more in numeracy activities than in spatial and patterning activities at home with their children, similar to results found by Hart et al. (2016) indicating that caregivers engaged in more numeracy activities than spatial activities. These findings were replicated in observations of parent-preschooler play (Zippert et al., 2020). Caregivers also report infrequent engagement in home measurement activities (LeFevre et al., 2010; Ramani et al., 2015), and to date there are no data specifically linking home measurement activities to children’s mathematics skills. These findings suggest that the frequency and quality of caregiver-child engagement in mathematics-related activities may vary, which may differentially influence the effects of the home environment on facets of mathematics learning. There is a clear need to identify the range of mathematical activities that occur naturally in the home environment that may support children’s early mathematics development. Moreover, how the efficacy and frequency of these experiences may be affected by cognitive and affective characteristics of caregivers and children needs to be better examined, which is discussed in the next section.

E. Cognitive and Affective Characteristics of Caregivers and Children

Although the bulk of research on the HME has focused on the activities that children experience in the home environment, some research indicates that characteristics of caregivers and children play a role in the nature of mathematics exposure in the home. As such, these cognitive and affective characteristics should be included in the conceptualization and measurement of the HME. Some caregiver characteristics that could affect caregiver-child engagement in the HME and the relation between the HME and children’s mathematics outcomes include caregivers’ mathematics abilities, self-concept, attitudes, stereotypes, anxieties, and values regarding mathematics, as well as beliefs about the importance of mathematics for their child’s development and learning (Blevins-Knabe et al., 2000; Borriello et al., 2020; Missall et al., 2015; Pan et al., 2018; Sari & Hunt, 2020; Sonnenschein et al., 2012; Zippert & Rittle-Johnson, 2020). Potential child characteristics that may also need to be accounted for include child abilities, interests, beliefs, attitudes, and anxieties surrounding mathematics (e.g., Cheung et al., 2018; 2020; del Río et al., 2019; 2020; Gunderson et al., 2012; LeFevre et al., 2009; Kleemans et al., 2012; Rathé et al., 2020; Skwarchuk et al., 2014). Core questions that span a range of cognitive and affective characteristics of both caregivers and children are described, followed by key considerations for research on the HME in regard to these characteristics.

Core Questions

1. What are individual caregiver characteristics that are relevant to understanding the HME and its relations with children’s mathematics outcomes?

2. Most research has focused on characteristics of caregivers that lead to caregiver-initiated HME activities, but what cognitive and affective characteristics of children are relevant to the HME and how do these contribute to child-initiated HME activities (Lukie et al., 2014)?
3. How should researchers measure each cognitive and affective caregiver and child characteristic validly and reliably?
4. How do caregivers' and children's cognitive and affective characteristics interact with caregiver-child engagement in mathematical activities and children's mathematics outcomes, both concurrently and longitudinally?
5. How do caregivers' beliefs about learning and education in general (e.g., academic expectations, value of education, stereotypes) and the home learning environment, as well as math-specific attitudes and beliefs, work in tandem with one another to influence individual caregiver and child characteristics, caregiver-child interactions, and children's mathematics outcomes (Crowley et al., 2001)?
6. To what degree are the associations between the HME and children's mathematics development driven by causal environmental factors versus genetic or environmental confounds (Hart et al., 2021)?

Key Considerations

- a. The ways in which caregiver and child characteristics (e.g., abilities, beliefs, attitudes, and anxiety) regarding mathematics are conceptualized and operationalized vary across studies. There is a need for more explicit and clear definitions within and across studies.
- b. The field needs measures of the individual caregiver and child characteristics related to mathematics (e.g., anxiety, attitudes, expectations, self-concept/-efficacy, confidence, beliefs; e.g., Loughlin-Presnal & Bierman, 2017) that are valid, reliable, and distinct from one another, with items that are intentionally designed for these constructs, rather than necessarily borrowed from other related domains. Many measures of mathematics anxiety address adults' experiences in mathematics courses or with mathematics exams, for example (Suinn & Winston, 2003), disregarding mathematical experiences that may occur in more real-world settings. The development of measures specific to caregivers' mathematics anxiety when engaging children with mathematics would help move the field forward.
- c. Researchers need to better understand changes in individual caregiver and child characteristics across time and development, and how certain characteristics may be more salient at different time points. In particular, these constructs may change in how they are measured over time and in how they relate to the HME and to children's mathematics development as children age.

Individual caregiver and child characteristics including abilities, attitudes, beliefs, expectations, and anxieties are important for understanding links between the HME and children's mathematics outcomes (Berkowitz et al., 2015; del Río et al., 2019; 2020;

Davis-Kean & Sexton, 2009; Cheung et al., 2020; Elliott & Bachman, 2018; Elliott et al., 2019; Maloney et al., 2015; Niklas & Schneider, 2014; Niklas et al., 2016; Serpell et al., 2005; Sonnenschein et al., 2016; Silver et al., 2020). Within the area of the HME, most work has focused on activities in which children and caregivers engage (e.g., Hart et al., 2016; Skwarchuk et al., 2014). Some work has focused on the other cognitive and affective characteristics, but there is a need for stronger conceptualization of these constructs by intentionally developing targeted measures. For example, there is existing research on mathematics anxiety, but this research is primarily focused on older children and teachers (e.g., Carey et al., 2017; Ganley et al., 2019; Núñez-Peña et al., 2013). It is recognized that there is significant overlap among these constructs, and further measurement work will be necessary for evaluating the structure of these components within the broader HME. It should also be noted that the constructs in this section may be affected considerably by child, family, community, country, and cultural factors.

Discussion

The core questions and key considerations described above are a result of a conference designed to advance research on the HME, specifically focused on measurement. Empirical work suggests that the HME is likely supportive of early mathematics development and achievement in mathematics at school entry and beyond (e.g., Anders et al., 2012; Mutaf-Yıldız et al., 2020; Napoli, 2019; Niklas et al., 2016; Skwarchuk et al., 2014). However, the literature base examining associations between the HME and children's mathematics outcomes is relatively small, and more research is needed to identify the best methods of characterizing how caregivers engage children in mathematics content in early childhood. Critically, many of the factors highlighted earlier are overlapping processes or contexts, and each may interact as they influence the measurement of the HME. As the quantity of research on the HME increases in the fields of educational and developmental sciences, researchers must increase their efforts to identify and measure valid and reliable indicators of the HME. Researchers should also aim to identify contextual factors (e.g., child, family, community, country, and culture), and other characteristics (both individual and dyadic) that may contribute to variations in the HME in ways that inform measurement. Furthermore, advanced methodological designs (e.g., panel longitudinal designs, behavioral genetics methods) and multi-method approaches should be used to support the development of tools for measuring the HME. The goal of the conference was to move the field forward by bringing together experts on the HME and related areas (see Table 1) to discuss these issues. Ultimately, this resulted in a list of next directions for future research on the HME.

Cross-Cutting Themes

Although most researchers have considered whether and how the home mathematics environment is related to early numeracy skills, the field has broadened its conceptualization of mathematics (Milburn et al., 2019; Sarama & Clements, 2009). Accordingly, the measurement of the HME should focus on aspects of mathematics including numeracy and potentially those that extend beyond numeracy, such as measurement, geometry, patterning, and spatial domains (Zippert & Rittle-Johnson, 2020). It is also recognized that HME researchers can learn from other areas of the home learning environment

that have seen an expansion beyond simply focusing on caregiver-child activities (e.g., home literacy environment; e.g., Niklas, Wirth et al. 2020). Similarly, the view of the HME needs to be broadened beyond caregiver-child activities to include cognitive and affective characteristics of caregivers and children. Specifically, researchers need to have clear operational definitions of the components of the HME within testable theoretical frameworks such as those included in other areas (e.g., academic socialization; Taylor et al., 2004) in which caregiver behaviors and caregiver characteristics are clearly differentiated. These theoretical frameworks should also be developed in a manner that reflects both field-specific and more general theoretical developmental models (e.g., Bronfenbrenner's Ecological Theory and Eccles' Expectancy-Value Theory; Bronfenbrenner & Morris, 2007; Eccles, 1983; Lauermann et al., 2017).

In conjunction with testing theoretical frameworks of children's mathematics ability and the HME, researchers must also understand how these frameworks vary across context, country, and culture. To develop these frameworks, the next step would be to conduct research using multi-method approaches (e.g., qualitative, naturalistic, and survey methods) to enhance the field's understanding of the breadth of the HME. Moreover, researchers need to better understand if some aspects of the HME may be universal or context-specific (e.g., Would an overarching structure of direct vs. indirect activities be universally appropriate, though the specific items within each of those classifications vary across contexts?). Using a measure with a cultural group or in a country apart from where it originates may be problematic, as there could be critical components of differences in practices that result in variability of measurement. For example, an item asking about reading mathematics storybooks in places where books for young children are more scarce may not be a relevant item for assessing the HME in those countries. Using measures developed in one country or context should be carefully evaluated through both qualitative and quantitative methods before they are applied more broadly to ensure that HME measures are culturally appropriate. Along these lines, it is important to consider varying meanings of behaviors and interactions across cultures. This points back to the need for clear operational definitions of the components of the HME (as well as in the measurement of children's mathematical skills) and specifically shared terminology and aims, as the field seeks to conduct more international research.

Ways the Field Can Move Forward

To accomplish the aforementioned goals and better measure the HME, there are several avenues forward. However, we intentionally do not provide specific recommendations for what should be addressed first, because many of these core questions and key considerations are interrelated and require diverse perspectives to appropriately address. Rather, our recommendations for moving the field forward center on broader approaches to interdisciplinary and sustainable collaboration.

First, the field must expand opportunities and funding for cross-lab and international collaborations (e.g., the ManyNumbers Project, a large scale collaborative effort to replicate and extend foundational effects in numerical cognition, <https://osf.io/e4xb7/>). Thus, we encourage researchers to seek collaborative experiences, implore research societies to foster opportunities that build collaborations, and lobby research funding agencies to provide and

expand mechanisms that enhance the feasibility and sustainability of these collaborations. It is with this collaborative attitude from the ground up that researchers can make progress (e.g., moving from the individual lab level up to higher-level collaborations across labs; using similar measures across labs; Sheskin et al., 2020). Collaborative experiences allow researchers to better understand multiple perspectives of development, and fostering these experiences is especially important for labs and countries that do not have as many resources to support research partnerships. Sharing of knowledge across collaborators should be reciprocal with the goals of partnering collaborators learning from one another, irrespective of language and cultural differences. Moreover, building and supporting these collaborations will allow researchers to ask and effectively answer the transformative questions in this area.

One method for encouraging collaboration across researchers is moving towards an open science framework, where hypotheses, measures, and findings are shared with the broader field prior to publication (Davis-Kean & Ellis, 2019; Gennetian et al., 2020). Through online repositories of pre-registered studies as well as presentations of pre-registered work at conferences, researchers can solicit feedback from others in the field with diverse perspectives in a way that is often not possible once a study is completed. Several efforts are already underway on this front within the broader field, including a repository of measures of the HME that is publicly available (<https://osf.io/rdpzm/>), as well as increased opportunities for presenting pre-registered projects through conferences (e.g., the annual conference of the Mathematical Cognition and Learning Society) and making field-specific open data more readily accessible (Hart et al., 2020).

Next, researchers should combine a top-down and bottom-up approach, not just using theory as a guide, but involving families in the process (e.g., using interviews and focus groups; Aldoney & Cabrera, 2016). In particular, it is important to understand how caregivers consider their role in teaching mathematics and how that might relate to their behavior in the home. Caregivers may see mathematics as the responsibility of schools and not of families (Cannon & Ginsburg, 2008; DeFlorio & Beliakoff, 2015; Starkey & Klein, 2000). Decades ago, reading interventions made caregivers a primary target for introducing early reading into the home environment (e.g., “Reading is Fundamental” [Boldovici, 1971]; “Reach Out and Read”; Klass et al., 2009). However, programs targeting mathematics at home (e.g., Kotsopoulos & Lee, 2014) are less common, as the changing nature of both children’s skill levels and accompanying home activities make the endorsement of universal messages more difficult than those embedded within literacy campaigns. To make progress in the field and determine the relative role of caregivers and teachers in children’s mathematics learning, researchers should understand the frequency of the use of math talk and behaviors that are done in the naturalistic environment (e.g., non-contrived). Some work has been done by using digital recorders in the home to capture the naturalistic mathematics environment (Susperreguy & Davis-Kean, 2015; Susperreguy & Davis-Kean, 2016) and by using mobile sensing to assess the frequency and duration of mathematics learning applications usage on tablets in the home (Niklas, Annac & Wirth, 2020). This work needs to be expanded to include more diverse families than the convenience samples commonly used by education scientists and psychologists. Furthermore, the field needs to move beyond only single-caregiver ratings (typically of mothers) that are generalized to represent children’s overall HME, and instead aim to capture the true nature of the HME.

Finally, continued emphasis on international and interdisciplinary working conferences is necessary to advance ideas, practices, and collaborations in this field. These spaces are critical for open discussions to explore ideas and identify shared goals and processes among research groups. This collaboration includes ensuring that diverse viewpoints are communicated when discussing concepts that may not be universally defined, but are still recognized as broadly important.

Limitations

We aimed to ensure that participants in this conference represented a broad range of countries, disciplinary perspectives (e.g., developmental, cognitive, and educational psychology, early childhood education, human development and family studies), and content foci (e.g., mathematics, literacy and language, home environment, culture, dual language learners). However, the ideas generated at the conference, which are described here, as well as the literature reviewed and cited, are limited by the expertise and knowledge of the participants involved, as well as the types of research published in the literature. Despite broader invitations for participation, the final participant list was heavily centered on individuals from North America and Europe and limited to participants who spoke English. We also acknowledge that the virtual conference and writing sessions were held during the COVID-19 global pandemic, which may have limited participation from some individuals. The pandemic may have also influenced participants' individual experiences of mathematics in the home environment as well as the focus of conference discussions.

We also recognize that the overall scope of this work needed to be constrained within the bounds of the topic of measurement of the HME, as the conference format necessitated a more narrow focus. Similarly, although many additional ideas were raised and discussed throughout this process, we aimed to synthesize these ideas to present them in a cohesive manner for readers, and thus not all generated ideas are presented here. Finally, we note that the primary focus of these next directions is on measurement of the HME, rather than on the development of interventions, because measurement lays the foundation for effective intervention (Raudenbush et al., 2020).

Conclusion

Whether intentional or incidental, mathematical activities (e.g., counting, number words, extending patterns, manipulating shapes) are naturally a part of children's lives throughout the world. Thus, children experience and engage in mathematics beyond school in their homes and communities, and in their daily activities, such as cooking, helping do chores, or organizing their toys. The nature of the HME that each child experiences, however, is heterogeneous and diverse, and this could be related to a variety of contextual factors (e.g., child, family, community, country, and culture). Furthermore, researchers need to study families from a broad range of countries and contexts. Because many studies in this area have been conducted within Western countries (Susperreguy et al., 2020b), measures within the field tend to take on a Western-specific lens. In summary, we have formulated a list of focused next directions for the field to help build an accurate and comprehensive knowledge

base about children's home mathematics environments and their relations to children's mathematics development.

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Table 1

Participants' Role, Institution, and Research Foci

Participant	Role	Institution and Location	Research Foci
David Purpura	PI	Purdue University, USA	- Early mathematics development - Home mathematics environment - Intervention
Caroline Hornburg	Co-PI	Virginia Tech, USA	- Individual differences in early mathematics skills - Mathematics learning environments - Middle childhood education and development
Giulia Borriello	Co-PI	Indiana University, USA	- Individual differences in mathematics and cognitive skills - Gene-environment interplay - Contextual contributions to development
Melody Kung	Co-PI	Georgia State University, USA	- Early childhood education and development - Cultural contributions to development - Early language, literacy, and mathematics development
Joyce Lin	Co-PI	California State University, Fullerton, USA	- Contextual contributions to development - Home learning environments - Early childhood education and development
Ellen Litkowski	Co-PI	Society for Research in Child Development Policy Fellow, USA	- Home learning environments - Early childhood education and development - Contextual contributions to development
Jimena Cosso	Conference Organizer	Purdue University, USA	- Home mathematics environment - Cultural and contextual contributions to development - Measurement
Alexa Ellis	Conference Organizer	Purdue University, USA	- Development of early mathematics - Measurement of early skills - Contextual contributions to mathematics
Yeminah King	Conference Organizer	Purdue University, USA	- Early language and mathematics development - Home learning environments - Parent-child interaction
Erica Zippert	Conference Organizer, Conference Speaker	Purdue University, USA	- Home mathematics environment - Parent-child and peer interaction - Development of early mathematics skills
Natasha Cabrera	Advisory Board Member; Conference Speaker	University of Maryland, College Park, USA	- Early home learning environment - Mother and father contributions to children's development - Cultural context of learning and interventions
Pamela Davis-Kean	Advisory Board Member	University of Michigan, USA	- Home environment - Socioeconomic status - Longitudinal mathematics development
Sarah Eason	Advisory Board Member	Purdue University, USA	- Home mathematics environment - Parent-child mathematics interactions - Family engagement in learning

Participant	Role	Institution and Location	Research Foci
Sara Hart	Advisory Board Member	Florida State University, USA	- Etiology of mathematics development - Contextual influences on development - Co-development of cognitive skills
Iheoma Iruka	Conference Speaker	University of North Carolina at Chapel Hill	- Parent-child interactions and parenting practices - Ethnic variation in parenting and engagement - Culturally grounded family-centered policies
Jo-Anne LeFevre	Advisory Board Member	Carleton University, Canada	- Mathematical cognition - Home learning environments for mathematics and literacy - Individual differences in mathematical processes
Victoria Simms	Advisory Board Member; Conference Speaker	Ulster University, Northern Ireland, United Kingdom	- Mathematical development - Cognitive development and educational attainment in children born preterm - Evidence synthesis
María Inés Susperreguy	Advisory Board Member; Conference Speaker	Pontificia Universidad Católica de Chile, Chile	- Development of early mathematics - Home mathematics environment - Cultural and contextual contributions to development
Abbie Cahoon	Participant	Ulster University, Northern Ireland, United Kingdom	- Home mathematics environment - Measurement of HME - Longitudinal development of early mathematical skills
Winnie Wai Lan Chan	Participant	University of Hong Kong, Hong Kong SAR, China	- Early mathematical development - Home mathematics environment - Mathematical learning difficulties
Sum Kwing Cheung	Participant	The Education University of Hong Kong, Hong Kong SAR, China	- Children's mathematical development - Home learning environments - Early childhood mathematics education
Marie Coppola	Participant	University of Connecticut, USA	- Language and number concept development - Number development in deaf and hard of hearing children - Early support/intervention
Bert De Smedt	Participant	KU Leuven, University of Leuven, Belgium	- Typical and atypical mathematical development - Mathematical learning disabilities - dyscalculia - Educational neuroscience
Leanne Elliott	Participant	University of Pittsburgh, USA	- Individual differences in parents' engagement in children's learning - Development of early mathematics skills - Socioeconomic status and family processes
Nancy Estévez-Pérez	Participant	Cuban Neurosciences Center, Cuba	- Neurocognitive foundations of typical and atypical mathematical development - Specific learning disabilities - dyscalculia, dyslexia - Early childhood development and education
Thomas Gallagher-Mitchell	Participant	Liverpool Hope University, United Kingdom	- Cognitive factors influencing mathematics performance - Mathematics anxiety - Spatial-numerical associations across the lifespan
Nicole Gardner-Neblett	Participant	University of Michigan, USA	- Early language and literacy development - Early childhood care and education - Young African-American children and families

Participant	Role	Institution and Location	Research Foci
Camilla Gilmore	Participant	Loughborough University, United Kingdom	- Early number skills - Cognitive and environmental predictors of mathematics development - Conceptual understanding of mathematics
Diana Leyva	Participant	University of Pittsburgh, USA	- Early numeracy development - Family practices - Low-income and ethnically diverse communities
Erin Maloney	Participant	University of Ottawa, Canada	- Mathematics anxiety - Academic achievement and well-being - Numerical and spatial representation
George Manolitsis	Participant	University of Crete, Greece	- Home learning environment - Early literacy and mathematics development - Cross-cultural differences on literacy and home learning environment
Gigliana Melzi	Participant	New York University, USA	- Early language and mathematics learning environments - Cultural influences on early learning environments - Home-school connections for culturally and linguistically minoritized children
Belde Murat-Yildiz	Participant	Marmara University, Turkey	- Home mathematics environment - Early mathematics development - Cultural differences in home mathematics environment
Gena Nelson	Participant	Boise State University, USA	- Early numeracy intervention - Mathematics learning disabilities - Home mathematics environment
Frank Niklas	Participant	LMU Munich, Germany	- Home learning environments - Language, literacy, and mathematics development - Family interventions
Yuejuan Pan	Participant	Beijing Normal University, China	- Early mathematics education - Early childhood education quality and rating - Play and kindergarten curriculum
Geetha Ramani	Participant	University of Maryland, College Park, USA	- Early mathematical development - Home mathematics environment - Playful learning
Sheri-Lynn Skwarchuk	Participant	The University of Winnipeg, Canada	- Home learning environment - Early childhood development - Caregiver/child interventions and education
Susan Sonnenschein	Participant	University of Maryland, Baltimore County, USA	- Language, literacy, and mathematics development - Home learning environments - Parent-child interactions

Note. PI = Principal Investigator.

Table 2

Home Mathematics Environment Virtual Conference Topics, Keynotes, and Example Discussion Questions

Session and Breakout	Main Topic and Keynote Title	Examples Questions from Breakout Groups
Session 1, Breakout 1	Home Learning Environment Overview <i>Critical Examination of Family Engagement: Pathway to Excellence for Black Children?</i>	- What are the current gaps in our understanding of the HME? - What are key future directions for the field?
Session 1, Breakout 2	What We Know About the HME <i>The Home Math Environment and Math Achievement: A Meta Analysis</i>	- Why are the relations between the HME and children's outcomes so small? - How can we improve measurement?
Session 2, Breakout 1	Child, Family and Community Factors Associated with the HME <i>Thinking Outside the Box: Assessing Math in Ethnically Diverse Families</i>	- What is/what do we expect to be qualitatively/quantitatively different about the HME across developmental time points? - How might the HME look different among families of different socioeconomic backgrounds?
Session 2, Breakout 2	Country and Cultural Factors related to the HME <i>Can We Take a Global Perspective on the HME?</i>	- What are cultural considerations that we need to take into account when considering the HME of diverse children? - What HME activities are culturally-unique? - What HME activities are culturally-universal?
Session 3, Breakout 1	Moving Beyond Only Measuring Numeracy Content <i>Beyond Numeracy: Examining the HME More Broadly</i>	- What revisions or additions should be made to measuring this aspect of the HME (e.g., Numeracy, Patterning, Geometry, Spatial)? - What are some ways we can best categorize individual activities/concepts? How can we better measure the quality of these types of activities?
Session 3, Breakout 2	Measurement Beyond Caregiver-Child Activities <i>Challenges of Measuring the Broader Home Math Environment</i>	What revisions or additions should be made in measurement (e.g., Caregiver Beliefs/Attitudes/Expectations, Caregiver Mathematics Anxiety, Resources)?

Note. During session 2, community factors were discussed in the second breakout session alongside cultural and country factors, but given the overlap of discussions that occurred during the event and the present consolidation of next directions, community factors are included alongside family factors. In advance of session 3, participants were sent a file of survey items measuring aspects of the HME and other characteristics of caregivers and children beyond caregiver-child activities, to review in preparation for their discussions.