A Comparative Study on State Estimation Algorithms for Power Systems

Yuting Chen
Department of Electrical and Computer Engineering
Binghamton University
Binghamton, NY, USA
ychen411@binghamton.edu

Ning Zhou
Department of Electrical and Computer Engineering
Binghamton University
Binghamton, NY, USA
ningzhou@binghamton.edu

***Abstract*--The state estimation (SE) has been widely used in power system control centers to optimally estimate the states of the power grid in real time. Using different objective functions, many SE algorithms have been proposed to filter out measurement noise in different ways. In this paper, three widely-used SE algorithms, i.e., the weighted least squares (WLS), least absolute value (LAV), and projection statistics (PS) based algorithms, are compared in their estimation accuracy and computation time. The comparison was made using the simulation data generated from the IEEE 14-bus system and IEEE 118-bus system through the Monte-Carlo method. It is found that when the measurement noise is reasonably small and follows the independent Gaussian distribution, the WLS algorithm has the best accuracy and shortest computation time. When some measurements at leverage points were compromised by outliers, the PS based algorithm is the most robust among the three methods. The study results can be used to assist control centers in choosing the right SE algorithm based on the features of the measurement noise and setup.**

***Index Terms*--Least absolute value (LAV), projection statistics based algorithm, state estimation (SE), weighted least squares (WLS).**

I. Introduction

As the modern power grid becomes more complicated and interconnected than ever, timely and accurately monitoring its operating conditions is essential for its reliable and efficient operation. To address the needs, a Supervisory Control and Data Acquisition (SCADA) system collects measurements of the power grid using remote terminal units and maintains a power flow model in a control center. However, a SCADA system still faces the following three challenges in monitoring operating conditions: i) The accuracy of measurements is often limited because of noise and limited instrument accuracy; ii) The number of direct measurements is often limited to save the cost; iii) Gross errors (a.k.a. outliers) may occur in some particular cases like damaged sensors and unexpected electromagnetic interference. These challenges limit the capability of the SCADA system in providing accurate and complete operating conditions of the power system.

To address these challenges, state estimation (SE) has been used to filter out noise and estimate variables that are not directly measured [1] by fusing information from direct measurements and power flow models. A SE algorithm often consists of two steps: First, bus voltage phasors are estimated by solving over-determined algebraic equations constructed using the power flow models and direct measurements. Second, other related variables such as transmission-line power flows and bus power injections are derived by substituting the estimated bus voltage phasors into the power flow models. Note that the first step is the focus of the SE algorithms because it often needs to solve nonlinear equations constructed from noisy measurements for the bus voltage phasors, which are also known as (a.k.a.) the power system states. Leveraging the measurement redundancy in a power flow model, SE can accurately and efficiently monitor operational constraints on quantities such as bus voltage magnitudes and transmission line loading.

Several algorithms have been developed to estimate the power system states [2]-[4] using different objective functions. Because each algorithm has its advantages and disadvantages, a systematic comparison of different algorithms' performance is necessary. Thus, the purpose of this paper is to compare some widely-used SE algorithms in their estimation accuracy and computation time under the same conditions. Special attention is given to how robust each SE algorithm is when outliers show up at leverage points because these outliers often have small residuals and thus are very difficult to detect [6]. The SE algorithms under study include the weighted least squares (WLS) [1], least absolute value (LAV) [5], and projection statistics (PS) based algorithm [6].

The rest of the paper is structured as follows. Section II reviews the theoretical background as well as the analytical aspects related to the WLS, LAV, and PS based SE methods.

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Yuting Chen and Ning Zhou are with the Electrical and Computer Engineering Department, Binghamton University, State University of New York, Binghamton, NY, 13902, USA (email: ychen411@binghamton.edu; ningzhou@binghamton.edu).

Section III compares the performance of the SE methods through case studies. The conclusion is drawn in Section IV.

II. Review on the SE Problem and Algorithms

To make this paper self-contained, the SE problem and three SE algorithms are briefly reviewed in this section. For technical details, readers are referred to [1] for the WLS, [5] for the LAV, and [6] for the PS algorithm.

A. SE Problem

Given measurements z and power flow model $h(\cdot)$, the objective of SE is to estimate the states x that fits the measurement. For a power system with n buses and m measurements, the measurements can be written as the nonlinear algebraic function of the state x shown in (1) [1].

$$z = h(x) + r \tag{1}$$

Here, z is the $m \times 1$ measurement vector, which often consists of measured real/reactive power injections, line flows, bus voltage magnitude. Symbol $h(x)$ is for the m nonlinear measurement equations, which relate the states to the measurements. Symbol r is the $m \times 1$ measurement noise vector. Symbol x is the $(2n\text{-}1) \times 1$ vector of bus voltage magnitudes and angles, which need be estimated. Because the voltage angle of the slack bus is often used as the reference angle, there are $2n\text{-}1$ unknown states and m equations in the SE problem. Normally, $m>2n\text{-}1$ within an observable island so that one has an over-determined problem. Different SE algorithms propose different objective functions to estimate the states that provide the "best" fit.

B. WLS Algorithm

In the WLS algorithm, the SE problem is formulated as an optimization problem defined by (2).

$$\min_x J(x) = [z - h(x)]^{\mathrm{T}} R^{-1} [z - h(x)] \tag{2}$$

where R is the covariance matrix of the measurement noise.

At the solution point, the first-order derivative of object function $J(x)$ will be equal to 0 as described in (3).

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x) R^{-1} [z - h(x)] = 0 \tag{3}$$

Here $H(x) = [\frac{\partial h(x)}{\partial x}]$ is the first-order derivative of measurement function with respect to state x. Using the Taylor series to expand the non-linear function $g(x)$ around the current state x_k and neglect the higher-order terms, one may obtain (4).

$$x^{k+1} - x^k = -[G(x^k)]^{-1} \cdot g(x^k) \tag{4}$$

where $G(x^k) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k) \cdot R^{-1} \cdot H(x^k)$ and k in the superscript is the iteration index.

As $g(\cdot)$ is linearized through the Taylor expansion, the Gauss-Newton method can be used to solve the WLS problem. The iterative step of the Gauss-Newton solution is outlined as follows:

1. Initialize the state vector x;
2. Calculate Δx^k;
3. Terminate the iterations if the Δx^k is smaller than the preset threshold;
4. Use Δx^k to update the state vector x_{k+1} and go to step 2.

C. LAV Algorithm

In the LAV algorithm, the SE problem is formulated as an optimization problem defined by (5).

$$\min_x J(x) = \sum_{i=1}^{m} |z_i - h_i(x)| \tag{5}$$

The optimal solution of (5) can be obtained by iteratively solving the linear programming (LP) problem defined by (6)-(8).

$$\min_{u,v,\Delta x} \sum_{i=1}^{m} ({u_i}^k + {v_i}^k) \tag{6}$$

subject to $\quad {u^k}_i - {v^k}_i = \Delta z^k - H(x^k) \cdot \Delta x^k \quad (7)$

$${u^k}_i,\ {v^k}_i > 0 \tag{8}$$

where u, v are the non-negative slack vectors.

By introducing the formulation (6)-(8), the LAV solve the SE problem through an iterative procedure which consists of a successive set of the LP problems. The iterative step is outlined as follows:

1. Initialize the state vector x;
2. Use the simplex LP to get ${u^k}_i$, ${v^k}_i$ and Δx^k;
3. Terminate the iterations if Δx^k is smaller than the preset threshold;
4. Use Δx^k to update the state vector x and go to step 2

D. *PS based Algorithm*

In the PS based algorithm, the SE problem is formulated as an optimization problem defined by (9).

$$\min_x J(x) = [z - h(x)]^{\mathrm{T}} W_{PS} R^{-1} [z - h(x)] \tag{9}$$

Here, W_{PS} is the matrix used to down-weight the measurements at leverage points. In order to identify the leverage points, PS is introduced, which is defined by (10) (11)[10][11][12][13].

$$PS_i = \max_v \frac{|l_i^T v|}{S_m} \tag{10}$$

$$S_m{}' = 1.1926 \ \underset{i}{lomed} \ \underset{j \neq i}{lomed} \ \left| l_i^T v + l_j^T v \right| \tag{11}$$

Here, the *lomed* denotes a low median operator, which is defined as the $[(m+1)/2]^{\text{th}}$ order statistic out of m numbers. Symbol l_i is the i^{th} row of $(R^{-1/2}H)$ and $v = l_j, j = 1, \ldots, m$. When the PS value of a measurement exceeds the preselected threshold, it will be down-weighted in W_{PS} [6].

The iterative step of the PS based SE algorithm is outlined as follows:

1. Initialize the state vector x;
2. Calculate the PSs for all the measurements;
3. Identify a leverage point by checking if its PS exceeds the preset threshold;
4. Apply (4) with an extra weight matrix which down-weights all the leverage points to get Δx^k;
5. Terminate the iterations if the Δx^k is smaller than the preset threshold;
6. Use Δx^k to update the state vector x and go to step 2

E. Total Vector Error (TVE) and Mean Squared Error(MSE)

To evaluate the estimation accuracy of different SE algorithms, metrics are defined in (12) for the TVE and (13) for the MSE.

$$TVE = \frac{\sum_{i=1}^{n} \left| V_i^{estimated} - V_i^{true} \right|}{\sum_{i=1}^{n} \left| V_i^{true} \right|} \tag{12}$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} \left| V_i^{estimated} - V_i^{true} \right|^2 \tag{13}$$

F. Monte Carlo Simulation Method

The Monte Carlo simulation method is used to model the probability that cannot be easily predicted due to the intervention of random variables. It was invented in the late 1940s by Stanislaw Ulam, and the widely used in mathematical areas, especially for optimization, numerical integration, and generation of draws from a probability distribution problem.

The Monte Carlo simulation is an appropriate option to assess the statistical performance of the three algorithms. In the study, a large number of repeated samples of noisy measurements are generated using (1) to mimic possible outcomes from random noise. These simulated measurements will be applied to assess the estimation accuracy of the three SE algorithms using the MSE, TVE.

III. Case Studies

To support well-informed decision making in a control center, one should select a SE algorithm that can accurately and timely estimate the states. To identify the right SE algorithm, the three SE algorithms are compared in this section to reveal their computation needs as well as estimation accuracy under different scenarios. Considering the randomness of the measurement noise, the comparison is made under a statistical framework using the Monte-Carlo method to focus on the distribution and average of performance metrics under many different instances of measurement noise.

A. Studies using the IEEE 14-bus System

As it is shown in Fig. 1, the IEEE 14-bus system has 20 branches and 14 buses. It is assumed that the system has 30 measurements, including 14 injection power measurements and 16 power flow measurements, which are marked out in Fig. 1. Two categories of test cases are set up: the ones with outliers and the ones without outliers. For the test cases without outliers, all the measurement errors are assumed to have independent Gaussian distribution with zero mean and standard deviation (σ) of 0.01. In the test cases with outliers, three measurements are assumed to be contaminated by additional outliers which follow the Gaussian distribution with mean of zero and standard deviation of 0.4. All the measurements with outliers are placed at leverage points because the outliers at leverage points have small residuals and thus are difficult to detect.

To account for the randomness of measurement noise, the Monte Carlo method is used to generate 500 instances of measurement noises to evaluate the statistical performance. For each instance of the test case, the estimation errors are quantified using the TVEs and MSEs defined by (12) and (13).

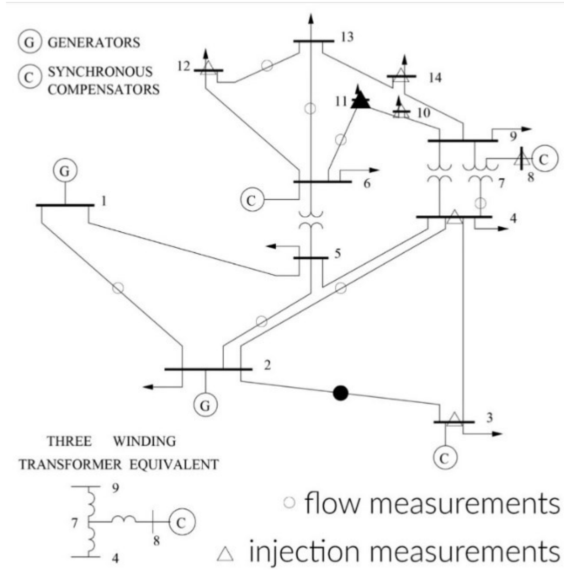

Figure 1. Topology and measurement setup of the IEEE 14-bus system [14].

The WLS, LAV and PS algorithms are applied to the 500 test cases without outliers and the 500 test cases with outliers to estimate the states. The estimation accuracy of the three algorithms are compared in Fig. 2 and Fig. 3 using TVEs and MSEs, respectively. Note that the boxplots in the figures reveal the distribution of the TVEs and MSEs in terms of their minimum, maximum, median, first and third quartiles as well as outliers.

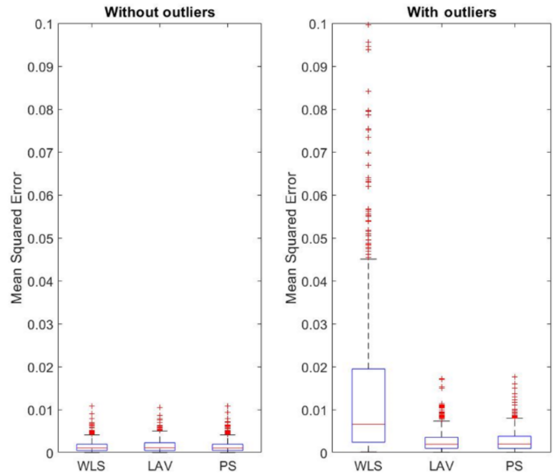

Figure 2. MSEs of the SE results of the IEEE 14-bus system for the 500 Monte-Carlo simulations.

From Fig. 2 and Fig. 3, it can be observed that for the cases without outliers, the median TVEs and MSEs of the three algorithms are similar. Also, the WLS has lower dispersion in both TVEs and MSEs. After the outliers are added, it can be observed that the estimation errors become larger. Also, Fig. 2 shows that the median TVE of the WLS approximately equals to 0.075 while the median TVE of the LAV and PS approximately equals to 0.05. Fig. 3 shows that the median MSE of WLS approximately equals 0.02 and the median MSE of LAV and PS approximately equal 0.01. The TVE and MSE of the WLS also have longer outlier tail than the other two algorithms. These observations indicate that when the system only has small measurement errors, the three SE algorithms have similar accuracy and the WLS algorithm has a bit advantage in its accuracy. But in the cases with outliers at the leverage points, the LAV and PS algorithms give more accurate estimates, which suggests that they can handle the outliers much better than the WLS algorithm.

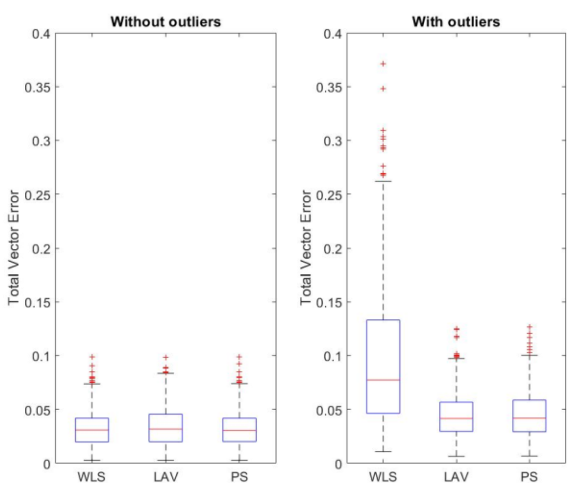

Figure 3. TVEs of the SE results of the IEEE 14-bus system for the 500 Monte-Carlo simulations.

B. Studies using the IEEE 118-bus System

The IEEE 118-bus system, which has 152 branches and 118 buses, is used in this subsection to evaluate the scalability of the SE algorithms when the problem size increases. It is assumed that the system has 300 measurements, including 78 injection measurements and 222 power flow measurements. Similar to the studies for the IEEE 14-bus system, two categories of study cases are generated: the ones without outliers and the ones with outliers. For the test cases without outliers, all the measurement errors are assumed to have independent Gaussian distribution with zero mean and standard deviation of 0.01. For the test cases with outliers, up to six measurements at leverage points are assumed to be contaminated by additional gross noise, which has Gaussian distribution with zero mean and standard deviation of 1.0.

The WLS, LAV and PS algorithms are applied to the 500 test cases without outliers and the 500 test cases with outliers to estimate the states. The estimation accuracy are compared in Fig. 4 through Fig. 6 in terms of TVEs and MSEs using boxplots.

From Fig. 4, it can be observed that in the cases without outliers, the median TVE and median MSE of the WLS algorithm are lower than those of the other two algorithms. Among the three algorithms, the median TVE and median MSE of the PS based algorithm is the highest. From Fig. 5 and Fig. 6, it can be observed that when the outliers are introduced at the leverage points, the median TVE and median MSE of the PS based algorithm are much lower than those of the other two algorithms. The median TVE and median MSE of the PS based algorithm are lower than those of the WLS algorithm. The observations indicate that the WLS algorithm achieves the highest accuracy when the measurements only carry small noise (without outliers), which follows Gaussian distribution. When facing the challenge of the outliers at leverage points, the PS based algorithm is the most robust one among the three algorithms. Also, the LAV algorithm is more robust against the outliers than the WLS algorithm.

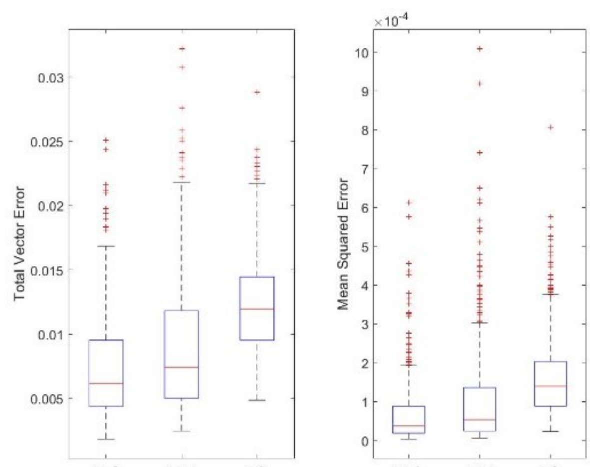

Figure 4. TVEs and MSEs of the SE results of the IEEE 118-bus system for 500 Monte-Carlo simulations ***without*** outliers.

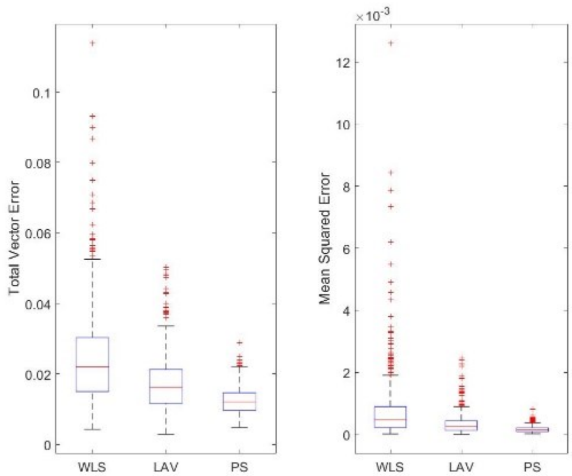

Figure 5. TVEs and MSEs of the SE results of the 118-bus system for the 500 Monte-Carlo simulations ***with three outliers***.

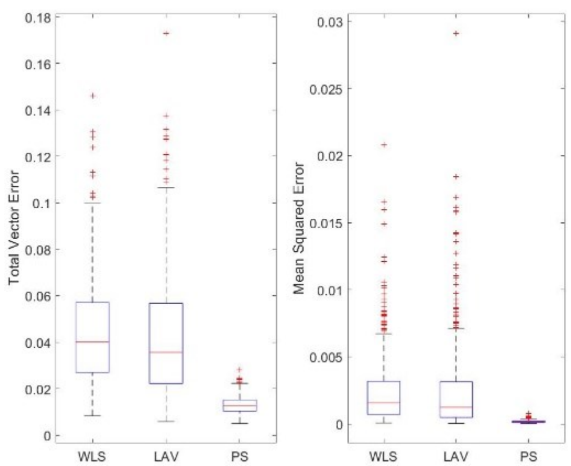

Figure 6. TVEs and MSEs of the SE results of the 118-bus system for the 500 Monte-Carlo simulations ***with six outliers***.

C. *Comparison of Computational Time*

Power system operators depend on the SE to gain situational awareness of the system status in real time. The computation time of SE contributes to the time delay between the time instance that an event actually happens and the time instance that it shows up on the screens at the control center. When violations occur, the long-time delay of SE will incur delayed remedial reactions. Typically, the SE runs every 1-5 minutes. As such, the computational time of a SE algorithm should be much less than 1 minute.

TABLE I. COMPUTATION TIME OF THE SE ALGORITHMS IN THE CASE STUDIES USING THE IEEE 14-BUS SYSTEM

SE Algorithms	Computation Time (in seconds/case)	
	Without outliers	With 3 outliers
WLS	0.0009	0.0008
LAV	0.0237	0.0263
PS	0.0071	0.0071

TABLE II. COMPUTATION TIME OF THE SE ALGORITHMS IN THE CASE STUDIES USING THE IEEE 118-BUS SYSTEM

SE Algorithms	Computation Time (in seconds/case)		
	Without outliers	With 3 outliers	With 6 outliers
WLS	0.031	0.036	0.037
LAV	0.107	0.235	0.205
PS	5.806	5.883	6.267

The three SE algorithms are implemented using MATLAB™ and run on a computer with 6 cores @3.2 GHz, 16 GB memory. The computation time is counted and summarized in Table I and II. The tables show that the WLS algorithm uses the least computation time in the SE for both the IEEE 14-bus system and the IEEE 118-bus system. The PS based algorithm uses less computation time than the LAV algorithm in the IEEE 14-bus system, but more computation time in the IEEE 118-bus system. The observation indicates that the computation time of the PS based algorithm increases rapidly with the increase of the system complexity. In most cases, the computation time increases when the outliers are introduced. For all the studied cases, the computation time is much shorter than one minute and therefore shall not be a major concern for causing time delay. For a larger system, the computation time of the PS based algorithm may cause some noticeable time delay, which could fail the real-time monitoring purpose of the state estimation when a power grid is subject to fast changes.

IV. CONCLUSIONS

Based on the studies carried out above, it can be concluded that when the measurements only have low-level noise following the Gaussian distribution, the WLS algorithm should be used for SE because it has the highest accuracy and shortest computation time. When outliers are introduced to the measurements at leverage points, the PS based algorithm has a built-in strategy in dealing with the outliers and therefore can estimate the states with the highest accuracy. However, the computation time of the PS based estimation algorithm is the longest one for a large system, which could be a drawback, especially when the system is subject to quick changes. If outliers present at leverage points and the PS based estimation could not be finished within the allocated time for a large system, the LAV algorithm can be a good option, because it has shorter computation time than the PS based algorithm and is more robust against the outliers than the WLS algorithm.

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