# A Novel Three-Phase Transmission and Unbalance Distribution Co-Simulation Power Flow Model For Long Term Voltage Stability Margin Assessment

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***Abstract*—The recent increasing deployment of distributed energy resources (DERs) is affecting many aspects of current power system operations including the coupling between distribution and transmission networks especially due to the dynamic and intermittent nature of DERs. An integrated grid modeling approach combining transmission and distribution (T&D) can help capture these unprecedented interactions between distribution and transmission systems. This paper proposes for the first time, a three-phase integrated T&D co-simulation power flow model and investigates various other co-simulation approaches for analyzing voltage stability of the power grid considering unbalances, and various load types. Results show that the proposed co-simulation approach can provide a weak phase voltage stability margin and thus provide the overall system stability considering each phase in the integrated T&D network.**

***Index Terms*—Transmission-Distribution Co-Simulation, Voltage Stability Analysis, Distributed Energy Resources.**

## I. Introduction

POWER grid modernization significantly changes the grid operation and develops new challenges. Even though smart grid technologies like DERs, energy storage, smart appliances, and demand response mechanisms, are deployed at the distribution level, most of the benefits are assessed at the transmission level. In general, by representing the distributed smart grid assets using reduced-order models, their impacts on the bulk power system with respect to stability and reliability could be analyzed. However, with active distribution networks, it is important to conduct transmission system analysis considering distribution level variations and changes. To capture these transmission and distribution (T&D) interactions, an integrated grid modeling may be required. Furthermore, the impacts of distribution level changes on transmission systems can be analyzed with an integrated T&D model.

Several efforts have been proposed recently to model an integrated T&D system for power flow and stability assessment such as in [1] and [2]. In most of these approaches, the transmission system (with transposed lines and balanced loads) is modeled based on a positive sequence framework and the distribution system (with untransposed lines and unbalanced loads) in three-phase representation. The importance of detailed modeling of an unbalanced distribution system and the importance of T&D co-simulation to accurately assess voltage stability of the grid are discussed in [3]. The significance of T&D co-simulation for voltage stability analysis is discussed in [4], [5]. A power to voltage (PV) curve superimposition approach was developed in [4] to analyze the voltage stability of the T&D system and a phasor measurement-based method to assess voltage stability was developed in [5]. A continuation power flow model for a three-phase unbalanced system is developed in [6] to assess the voltage stability in the presence of an unbalanced network and loads in the system. Some of the main challenges in T&D modeling are with: a) the difficulty in the representation of multiple distribution systems at the transmission interconnection points, b) the difficulty in the representation of the unbalanced nature of loads at the interconnection points, c) the difficulty in developing power flow models that can solve both T&D, and d) the difficulty in the representation of PV curves considering each phase. The unbalanced nature of the distribution system is increasing with high penetration of DERs especially due to the non-symmetrical placement of single-phase Photo-Voltaic (PV) farms. So a per phase analysis or a positive sequence analysis assuming a balanced system would not be sufficient.

Thus, for analysis of the power grid with large penetration of PV farms, three-phase modeling of the transmission system in an integrated T&D framework would be useful. The transposed lines in the transmission system give us the flexibility to perform three-phase power flow by decoupling three phases and performing computations of each phase. The steady-state response of power systems (power flow analysis) using integrated T&D is challenging because numerical methods used for power flow analysis which are stable for transmission networks may not work as required for distribution networks. A Current Injection (CI) based power flow can solve weakly meshed systems as well as radial systems effectively [7], [8]. Being derived from the Newton-based method, the CI method has quadratic convergence compared to Gauss Implicit methods and can also include all network components such as voltage regulators, transformers, and single-phase laterals compared to network topology based direct methods [9].

Previously, we have investigated decoupled and unified approaches for solving integrated T&D system [10]. In this paper, a multi-period three-phase integrated T&D power flow is developed which is used to obtain the voltage stability margin (VSM) of the system. The voltage stability of a system is expressed in terms of VSM which is defined as the difference between the critical loadability limit and the

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current operating load level. In this approach, the transmission and distribution systems are modeled considering all the three-phases including the distribution system unbalances and all load types. With this, the effect of the distribution system net load unbalance on transmission system voltages, and the effect of resultant unbalanced transmission system voltages on the distribution system can be accurately captured with dependable insights on voltage stability margin. Also, we have investigated other approaches that can be used for plotting the P-V curves of the integrated T&D system.

The main advantage of the proposed method are as follows: a) First, this approach provides a framework for power flow considering both transmission and distribution systems. b) Second, the methodology described here has quadratic convergence compared to other methods such as Gauss Implicit methods (GI). Third, the bus admittance matrix in the proposed method includes all network components such as voltage regulators, transformers, and single-phase laterals and does not require additional modifications to includes each of the network components as in forward-backward sweep methods. For solving power flow with multiple periods (such as 8760 analysis), the time series load profile is used as the input and the Jacobian is calculated from the bus admittance matrix. Also, the tertiary controllers such as transformer/regulator taps and the capacitor values are included from the previous time-step. Then for each time, the voltages for the buses are initialized from the previous time step and the power flow for each time is calculated. The rest of the paper is structured as follows. Section II discusses the mathematical modeling of the power flow for an integrated T&D system, Section III discusses the integrated T&D simulation results, and section IV presents the main conclusions drawn from this work.

## II. Proposed Integrated T & D Power Flow Model

Consider an $N$ bus distribution system with the nodal current expressed according to the following matrix form

$$I_{bus} = Y_{bus} V_{bus} = \sum_{i=1}^{N} \sum_{j=1}^{N} Y_{ij} V_j \tag{1}$$

where $I_{bus}$, $V_{bus}$, and $Y_{bus}$ represent nodal injection current vector, bus voltage vector, and bus admittance matrix of the network, respectively. Formulation of $Y_{bus}$ considers various power grid elements including distribution lines, voltage regulators, and transformers. The $Y_{bus}$ of the three phase distribution line can be written as

$$Y^{dl}_{abc} = \begin{bmatrix} Z^{ser}_{abc}(l)^{-1} + \frac{1}{2}B^{sh}_{abc}(l) & -Z^{ser}_{abc}(l)^{-1} \\ -Z^{ser}_{abc}(l)^{-1} & \frac{1}{2}B^{sh}_{abc}(l) + Z^{ser}_{abc}(l)^{-1} \end{bmatrix} \tag{2}$$

where $Z^{ser}_{abc}(l)$ is three phase series impedance and $B^{sh}_{abc}(l)$ is three phase shunt admittance matrix of the line $l$. The $Y_{bus}$ of a three phase Yg-Yg type load tap changer in series with a distribution line with admittance $Y_{abc}(r)$ is given by

$$Y^{reg}_{abc} = \begin{bmatrix} RY_{abc}(r)R^T & -RY_{abc}(r) \\ -Y_{abc}(r)R^T & Y_{abc}(r) \end{bmatrix} \tag{3}$$

where $R = \begin{bmatrix} \frac{1}{a^a_t} & 0 & 0 \\ 0 & \frac{1}{a^b_t} & 0 \\ 0 & 0 & \frac{1}{a^c_t} \end{bmatrix}$ and $a_t = 1 \mp d_V\, tp$ where $tp$ is the tap setting and $d_V$ is the per unit voltage change per tap. The $Y_{bus}$ for a transformer connected between node $m$ and $n$ with $\alpha$, $\beta$ being off-nominal tap ratios on the primary and second sides respectively can be represented as

$$Y^{trf}_{abc} = \begin{bmatrix} \frac{Y_{nn}}{\alpha^2} & \frac{Y_{nm}}{\alpha\beta} \\ \frac{Y_{mn}}{\alpha\beta} & \frac{Y_{mm}}{\beta^2} \end{bmatrix} \tag{4}$$

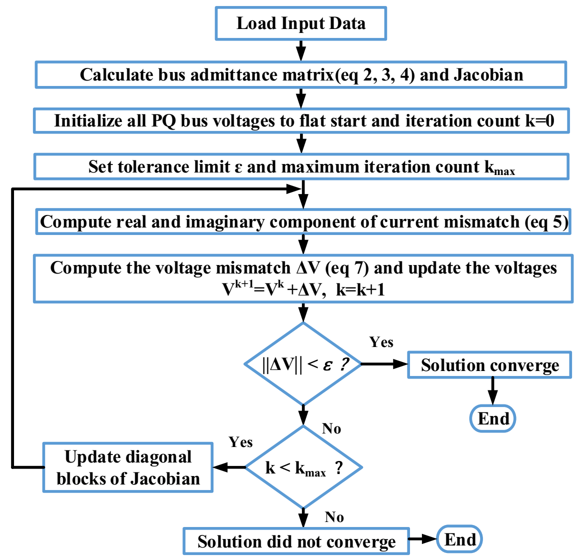


Fig. 1. Current Injection power flow.

### A. Treatment of Distribution System for power Flow

In current injection based power flow, the complex current injection equations are expressed in terms of rectangular coordinates. The Jacobian matrix is formed from the bus admittance matrix where, each $3 \times 3$ matrix in bus admittance matrix is replaced with $6 \times 6$ blocks [8]. The off-diagonal blocks of Jacobian are fixed over iterations and diagonal blocks are updated based on type of load model connected. The three phase complex current mismatch at a bus $i$ for phase $s$, $s \in \{a, b, c\}$ can then be written as

$$\Delta(I_i)^s = (I^{sp}_i)^s - \sum_{i=1}^{N} \sum_{j=1}^{N} Y_{ij} V_j \tag{5}$$

The voltage sensitive loads can be modeled as a combination of constant power, constant current, and constant impedance loads (ZIP) [11]. With ZIP load modeling, the net specified nodal current injection is given by

$$(I^{sp}_i)^s = -\left[ \left( \frac{|S^s_i| \angle \theta^s}{|V^s_i| \angle \delta^s} \right)^* + \left( \frac{|S^s_i|}{|V^s_{0i}|} \right) \angle(\delta^s - \theta^s) + \left( \frac{V^s_i}{Z^s_i} \right) \right] \tag{6}$$

where $S^s_i$ is scheduled power, $\delta$ is the voltage angle, $\theta$ is the power factor angle, $V_{0i}$ is the nominal voltage and $Z_i$ is impedance load. The current mismatch in (5) can be represented in terms of real($I_r$) and imaginary($I_m$) component and the power flow formulation using current injections can be solved as

$$\begin{bmatrix} \Delta V_{r1}^{abc} \\ \Delta V_{m1}^{abc} \\ \vdots \\ \Delta V_{rN}^{abc} \\ \Delta V_{mN}^{abc} \end{bmatrix} = \begin{bmatrix} \frac{\partial I_{m1}^{abc}}{\partial V_{r1}} & \frac{\partial I_{m1}^{abc}}{\partial V_{m1}} & \cdots & \frac{\partial I_{m1}^{abc}}{\partial V_{rN}} & \frac{\partial I_{m1}^{abc}}{\partial V_{mN}} \\ \frac{\partial I_{r1}^{abc}}{\partial V_{r1}} & \frac{\partial I_{r1}^{abc}}{\partial V_{m1}} & \cdots & \frac{\partial I_{r1}^{abc}}{\partial V_{rN}} & \frac{\partial I_{r1}^{abc}}{\partial V_{mN}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial I_{mN}^{abc}}{\partial V_{r1}} & \frac{\partial I_{mN}^{abc}}{\partial V_{m1}} & \cdots & \frac{\partial I_{mN}^{abc}}{\partial V_{rN}} & \frac{\partial I_{mN}^{abc}}{\partial V_{mN}} \\ \frac{\partial I_{rN}^{abc}}{\partial V_{r1}} & \frac{\partial I_{rN}^{abc}}{\partial V_{m1}} & \cdots & \frac{\partial I_{rN}^{abc}}{\partial V_{rN}} & \frac{\partial I_{rN}^{abc}}{\partial V_{mN}} \end{bmatrix}^{-1} \begin{bmatrix} \Delta I_{m1}^{abc} \\ \Delta I_{r1}^{abc} \\ \vdots \\ \Delta I_{mN}^{abc} \\ \Delta I_{rN}^{abc} \end{bmatrix} \quad (7)$$

The updated voltage can be obtained as

$$[V]^{k+1} = [V]^{k} + [\Delta V] \quad (8)$$

A flowchart of the CI based load flow is shown in Fig. 1. Two power flow cases are performed on IEEE 123 bus distribution system where loads are varied using a loading factor ($\alpha$) from baseload ($\alpha = 0$) to a loading where power flow diverged($\alpha = \alpha_{max}$). In case 1, the swing bus voltage was held at a fixed value (1.05pu) and in case 2 the swing bus voltage is slightly decreased with an increase in loading. The voltage profile of phase a of node 1 is shown in Fig. 2. For the same set of load variations, it can be concluded that the divergence occurs at a lower loading when the swing bus voltage was lower. It is also observed that when swing bus voltages are unbalanced, ($\alpha_{max}$) is decreased more compared to the case with balanced swing bus voltages. This is because with the increase in loading factor, the load unbalances between phases will increase, making the weak phase (phase with the highest load) weaker. In addition to this, the unbalanced source bus voltage in distribution power flow would lead to unbalanced losses with the most heavily loaded phase having the highest loss. These two factors would lead to faster divergence of distribution system power flow when swing bus voltages are unbalanced.

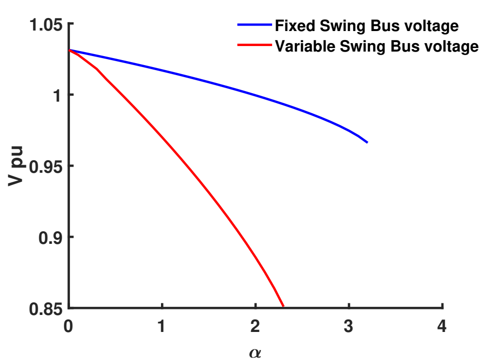


Fig. 2. Distribution power flow with different swing bus voltage.

## B. Treatment of Transmission System for power Flow

The same approach of current injection in its single-phase version is used for transmission system power flow. Since transmission system lines are considered to be transposed lines, the mutual coupling between them is negligible and a three-phase power flow solution can be obtained by solving 3 phases independently. The Jacobian matrix is formed from the bus admittance matrix where each element in the bus admittance matrix is replaced with $2 \times 2$ blocks [7]. The diagonal blocks are updated based on the type of load connected to that bus.

## C. Proposed Integrated T&D Power Flow Model

The three-phase integrated T&D power flow is based on a master-slave approach [1] where transmission and distribution systems are decoupled at a boundary bus and solved independently. The process is initiated with a transmission power flow where three phases are solved independently. For each boundary bus(B), the voltage magnitude and angle of three phases are passed to the corresponding distribution system solvers. The per-phase net power injections obtained after solving all distribution system power flow are passed back to the transmission system solver. When the voltage magnitude at the boundary bus between consecutive iterations is less than a tolerance threshold value, T&D power flow is converged. The iteration count(k) is reset and T&D power flow with the next loading factor($\alpha(p)$) is initiated. If T&D iterations are going beyond a maximum iteration count($k_{max}$), the T&D power flow is assumed to be diverging. This iterative process is stopped when either transmission or distribution or T&D power flow diverges. The corresponding loading factor ($\alpha_{max}$) is taken as the VSM of the system. The voltage regulator taps are fixed for all loading conditions in this process. A detailed flowchart of the proposed T&D power flow for VSM assessment is shown in Fig. 3.

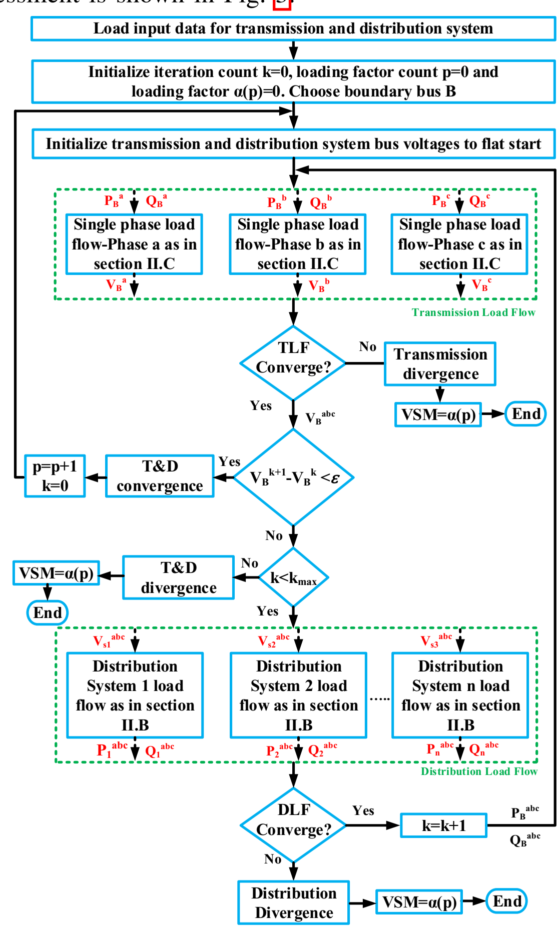


Fig. 3. Proposed Integrated T&D power flow.

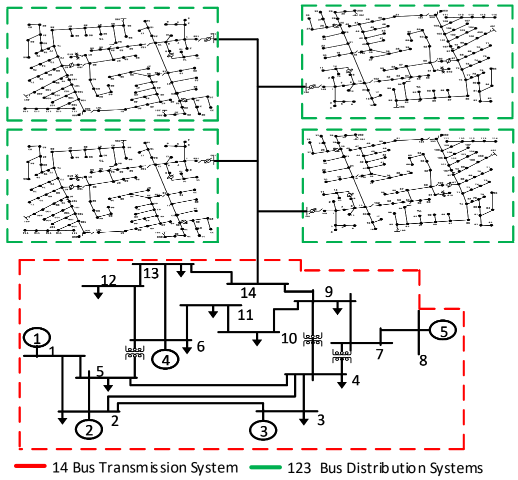


Fig. 4. One line diagram of T&D system.

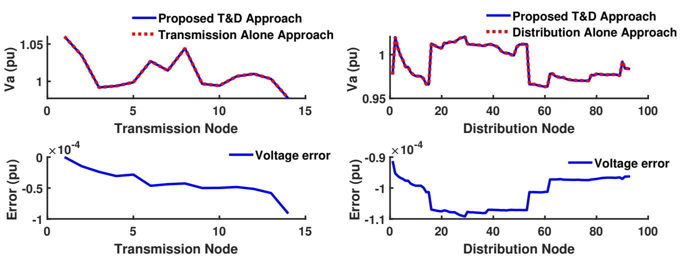


Fig. 5. Validation of Proposed Approach a) Transmission System b) Distribution System.

## III. Simulation Results

An integrated power grid model is developed that includes the IEEE 14 bus for the transmission side and four IEEE 123 bus test distribution feeder on the distribution side connected to the bus# 14 of the transmission system as shown in Fig. 4. Modified versions of the IEEE 123 bus system is developed where the loads on all 3 phases are modified such that the combination of all four feeders would result in approximately the same loading on each phase of load at bus 14 of the transmission system. This is referred to as a balanced distribution system in further discussions. For validating the proposed approach, with baseload, the transmission system model is solved with the spot load approach, where all the loads on distribution feeders are lumped at a corresponding transmission bus. The distribution system is solved with a substation voltage of the respective transmission bus. It is observed that the voltage variations are very close to each other as shown in Fig. 5. Also, since a phasor-based benchmark to validate T&D decoupled approach is lacking in the literature, a unified T&D power flow model that combines and solves transmission and distribution systems as one unit was developed [10] to validate the proposed approach.

### A. VSM Comparison with existing methods

The VSM obtained using the proposed method is compared with VSM obtained using a.) Transmission system with spot loads, b.) Transmission system with equivalent distribution system and c.) Integrated T&D system with the transmission system modeled in a positive sequence. For the transmission system with spot loads, the spot loads represent the total load and losses of distribution systems connected. These loads are varied using loading factor $\alpha$. The load at bus $i$ with base power $S_{0i}$ can be represented by

$$S_{Li} = S_{0i}(1+\alpha) \quad \forall\, \alpha \in 0, 1, ..\alpha_{max} \tag{9}$$

To model losses in a better way, the equivalent distribution feeder method is used where the distribution system is modeled as a distribution line connected to a spot load representing the net load of the distribution system. To find the parameters of equivalent distribution feeder ($R_{eq}$, $X_{eq}$), a distribution system power flow is performed, and the net losses ($S_{loss}$) and injected current ($I_{sub}$) at the substation are used. The $R_{eq}$, $X_{eq}$ parameters should be computed for each loading level of the system.

$$R_{eq} + jX_{eq} = \frac{S_{loss}}{I_{sub}^2} \tag{10}$$

An integrated T&D system with a transmission system modeled in a positive sequence for VSM assessment is discussed in [3]. The active and reactive power injection at each phase of the substation bus obtained after distribution system power flow is added to represent a three-phase load value for the boundary bus which is then used for positive sequence transmission power flow in the next iteration.

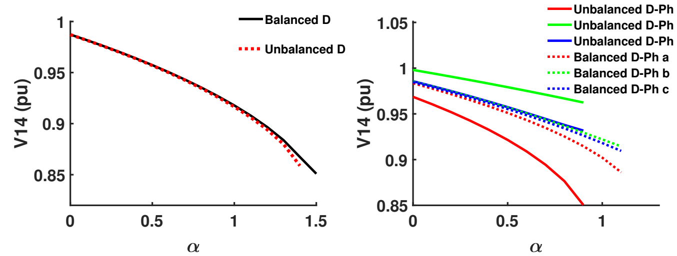


Fig. 6. PV Curves using a) Positive Sequence T&D method b) Proposed Three Phase T&D method.

The VSM obtained with positive sequence T&D for a balanced and unbalanced distribution system is depicted in Fig.6a and proposed approach is depicted in Fig.6b. With the proposed approach, the PV curves for each phases are different with phase a being the most vulnerable. It can be observed that when balanced distribution system is replaced with unbalanced system, the reduction in $\alpha_{max}$ was from 1.2 to 0.9 when a three-phase transmission power flow is done whereas the reduction was from 1.5 to 1.4 with positive sequence transmission power flow. The effect of distribution system unbalance on stability margin is captured prominently in the proposed approach compared to positive sequence T&D approach. Thus it can be proved that the proposed architecture is extremely critical for evaluating the long-term voltage stability margin especially with unbalance load and proliferation of multi-phase DERs in distribution network.

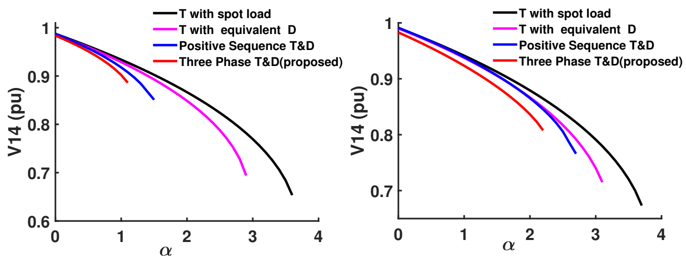


Fig. 7. PV curves of T&D system with a) Constant power load model b) ZIP load model.

TABLE I
AVERAGE COMPUTATIONAL TIME FOR CONVERGENCE

| $\alpha$ | Avg Time(s) Positive Sequence T&D (Existing) | Avg Time(s) Three phase T&D (Proposed) | % Increase in Time |
|---|---|---|---|
| 0 | 0.1995 | 0.2069 | 3.709273 |
| 0.3 | 0.2195 | 0.2319 | 5.654626 |
| 0.5 | 0.2621 | 0.2795 | 6.638688 |
| 0.8 | 0.2693 | 0.2892 | 7.342891 |
| 1 | 0.2705 | 0.2912 | 7.652495 |
| 1.2 | 0.3274 | 0.3532 | 7.882147 |
| 1.5 | 0.3763 | 0.4097 | 8.875897 |

### *B. Analysis of PV curves With Different Load Types*

The comparison of all methods for a balanced distribution system with constant power loads is depicted in Fig. 7a and ZIP loads are depicted in Fig. 7b. It can be seen from Fig. 7a that $\alpha_{max}$ decreases drastically from 3.6 to 1.5 when spot load is replaced by distribution system in 3 phase detail while transmission system still represented in positive sequence. Furthermore, even with an approximately balanced net load at the boundary bus, the $\alpha_{max}$ is further reduced to 1.1 when the transmission system is represented in 3 phase detail. The distribution system consists of untransposed lines and a large number of single-phase laterals. This will lead to different line losses in each phase which results in unbalanced substation power even if total loads in all the phases are approximately the same. These slight unbalance in power when used for transmission power flow would lead to unbalance boundary bus voltage. When this is used as a source bus voltage in distribution power flow, it would lead to more unbalance in losses and leads to faster divergence as loading is increased. The $\alpha_{max}$ when all loads considered as constant power loads are lower than when ZIP loads are considered. This is because, in the case of constant power loads, the lower bus voltage during heavy loading will lead to higher current flow to maintain constant power drawn by the load. This higher current leads to higher line losses and hence lower $\alpha_{max}$ or faster divergence. A detailed comparison of $\alpha_{max}$ for all the cases are depicted in Fig. 8. The average time required for convergence of proposed T&D power flow for different values of $\alpha$ is shown in Table I. The computational load when the three-phase T&D method is used is very close to the positive sequence T&D method. It is also observed that, as loading is increased, the number of T&D iteration required in the three-phase T&D method is more, which is also the reason for higher values of computational time.

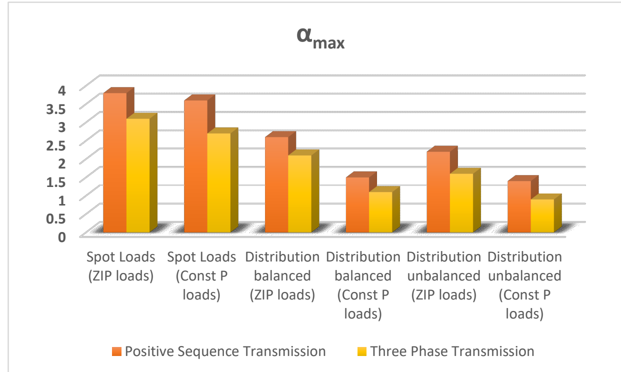


Fig. 8. Maximum loading factor.

## IV. CONCLUSION AND FUTURE WORK

In the work, a new model for T&D co-simulation that is suitable for power flow and developing PV curves is presented. The proposed modeling framework can include all the unbalances in the power distribution system and its effect on the transmission network. The model can be used for voltage stability studies by plotting PV curves on any transmission system nodes (each phase for all three phases) and thus can be used for finding the weak node in the system. It is shown that the stability margin for a given system is reduced as a three-phase transmission power flow is performed instead of a positive sequence load flow. Future works include extending the proposed method to study the impact of DERs present in the distribution system on VSM and also study the effect of voltage regulator tap operation on the stability of large scale T&D systems including 8500 node distribution system.

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