Analysis of Local and Centralized Control of PV Inverters for Voltage Support in Distribution Feeders

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Abstract—Higher photovoltaic penetration on distribution system brings operational challenges including overvoltage issues. With smart inverters, efficient voltage control can be achieved through adjusting active/reactive powers of inverters. Moreover, reactive power may not be as effective as active power in regulating voltage due to high R/X ratio of distribution networks. Thus, one may have to apply active power curtailment (APC) techniques, in distribution networks in coordination with reactive power control. This study aims at evaluating performance of a sensitivity based method and an optimal power flow (OPF) based centralized method of reactive power control (in coordination with APC) from inverters in managing voltage profile on distribution networks. Based on the case studies, we observed that: *a)* sensitivity based method is not always able to solve overvoltage issues and energy curtailments are high, and *b)* OPF-based method can ensure that voltage remains within the operational bound with significantly less energy curtailment.

Index Terms—Photovoltaic, Power curtailment, Distribution grid, Over-voltage, Reactive power, Inverter, Voltage control.

I. NOMENCLATURE

$\Delta\mathbf{t}$	Time interval.
$\Delta\mathbf{V}$	Voltage change on a node.
$\Delta\mathbf{V}/\Delta\mathbf{P}$	Voltage sensitivity with respect to active power.
$\Delta\mathbf{V}/\Delta\mathbf{Q}$	Voltage sensitivity with respect to reactive power.
$\mathbf{e}$	End node of a lateral.
$\mathbf{E^{cur}}$	Curtailed energy.
$\mathbf{I}$	Nodal current injection.
$\mathbf{j, k}$	Set of nodes, $\mathbf{j, k} \in \{1, 2, .., N\}$.
$\mathbf{l}$	Set of laterals $\mathbf{l} \in \{1, 2, 3, .., L\}$.
$\mathbf{min/max}$	Minimum and maximum operational limit.
$\mathbf{m, n}$	set of nodes with PVs. $\mathbf{m, n} \in \{1, 2, 3, .., M\}$
$\mathbf{P^{cur}}$	Active power curtailment of PV.
$\mathbf{P^L}$	Active power of load.
$\mathbf{P^{PV}}$	PV generation.
$\mathbf{Q^c}$	Available reactive power capability.
$\mathbf{Q^{PV}}$	Reactive power of PV.
$\mathbf{Q^L}$	Reactive Power of load.
$\mathbf{S}$	Sensitivity matrix.
$\mathbf{S^{PV}}$	Inverter Rating.
$\mathbf{t}$	Time index, $\mathbf{t} \in \{1, 2, 3, .., T\}$.
$\mathbf{V}$	Nodal voltage.
$\Delta\mathbf{V^{Rq}}$	Required voltage change.
$\Delta\mathbf{V^{Rm}}$	Required voltage change by APC.
$\mathbf{Y}$	Y-bus matrix.

II. INTRODUCTION

Distributed solar photovoltaic (PV) systems are the fastest growing renewable energy resources being integrated onto distribution grids [1]. This has some obvious advantages including less carbon emissions and economic benefits. Significantly increasing penetration of rooftop PVs has raised number of concerns to utilities. High PV penetration causes several operational challenges to the distribution grids including overvoltage [2] and power quality [3]. A detailed review to overcome method for mitigating overvoltage problems including active power curtailment (APC) and reactive power absorption is provided in [4]. Though APC of PVs is more effective on managing distribution grid voltage due to high R/X ratio of feeders, the reactive power control should also be considered to reduce unnecessary energy curtailment resulting from the APC based approach alone [5].

There are several studies focused on solving overvoltage problems in distribution systems with PVs. The methods are mainly centralized or distributed types for APC and dispatch of reactive power. The centralized approaches solve optimal power flow (OPF) or its variants to find the dispatch of active and/or reactive power from the PVs. The centralized approaches demand communication infrastructure. However, local approaches are droop-based and avoid the need of communication.

The authors of [6] uses droop-based active power curtailment taking sensitivity calculations into consideration to overcome over-voltage problem in LV networks with high PV penetration. [7] uses a combination of a droop-based reactive power control and an active power curtailment algorithm, by using only local measurements. In [8], when the voltage of a node with PV reaches critical value (upper bound, i.e., 1.05 pu) all other PVs are informed to curtail and a PV capping algorithm is used. [9] uses sensitivity based approach to control voltage during ramp-rate events using PV's reactive power on MV feeder. [10] proposes reactive power control based on sensitivity analysis and assigns a location-dependent power factor for each inverter. Also, the method in [10] combines the two droop functions that are inherited from standard cosφ(P) and Q(U) strategies. Local droop-based control techniques don't allow coordinated operation leading to non-optimal APC.

Centralized and local voltage control methods were combined in [11], where local control is based on droop, and the droops are adjusted by the central controllers to minimize the errors caused by the droop-based control. In [12], another combination of centralized and decentralized control approach is proposed, where local control acts fast considering linear V-Q relation, and the corrections are made through a centralized optimization model. Another centralized/decentralized strategy is provided in [13] to determine reactive power references for PV inverters connected to a LV radial network. In [14], a decentralized reactive power control approach is used, and it is shown that centralized approach may provide better voltage profile compared to a decentralized approach. In [15], a centralized approach that uses different timescales for voltage control devices is proposed, where. slow acting devices such as tap changers and capacitor banks are dispatched at a slower timescale and inverter reactive power are dispatched at faster time scale.

This paper proposes a sensitivity-based approach, different from the existing approaches such as in [16], that includes APC in coordination with reactive power control for voltage control in distribution networks, and compares its performance with an OPF-based method. We evaluate the performance using a practical-sized 730-node MV/LV with voltage levels of 12.66 kV and 240 V.

Rest of the paper is organized as following. Section II provides the details of sensitivity based method. In Section III, mathematical model of an OPF based method is provided. Section IV provides detailed information about the test feeder used and simulation setup. In Section V, simulation results are discussed. Finally, we conclude the paper in Section VI.

III. SENSITIVITY BASED APPROACH

Fig. 1 shows a schematic of inverter control based on proposed sensitivity-based approach. We distribute the control actions at each lateral level, and using nodal power flow sensitivities computed offline, and based on available real-time nodal voltage measurements, we dispatch the active and reactive power control of the PV inverters. The sensitivity matrix can be defined as [17],

$$S = \begin{bmatrix} 1 & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} \frac{\partial P}{\partial \delta} & V\frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & V\frac{\partial Q}{\partial V} \end{bmatrix}^{-1} \approx \begin{bmatrix} \frac{\Delta \delta}{\Delta P} & \frac{\Delta \delta}{\Delta Q} \\ \frac{\Delta V}{\Delta P} & \frac{\Delta V}{\Delta Q} \end{bmatrix} \quad (1)$$

For APC, $\Delta V/\Delta P$ are used. For reactive power based voltage control $\Delta V/\Delta Q$ is used. The coordination among multiple inverters, and APC and reactive power control is achieved as following and further detailed are provided in Algorithm 1.

- If a voltage magnitude of the end node of a lateral is within the allowed maximum and minimum ranges, no new control action is required.
- If the voltage magnitude of the end node of a lateral is outside the maximum/minimum allowed voltage magnitude, all the PVs on the lateral provide active power output information to the lateral controller, which is then used to find reactive power capability of a PV
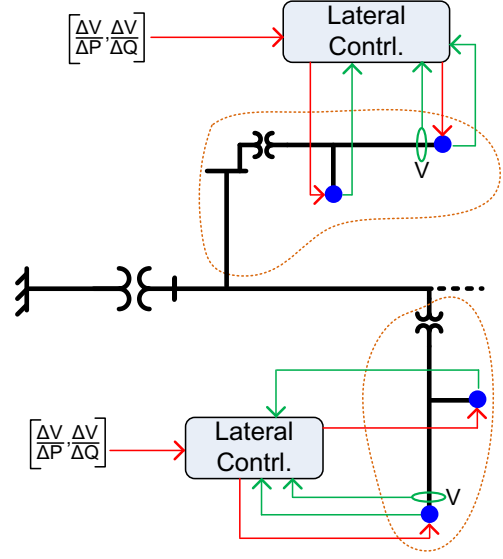

Fig. 1. Schematic showing control based on sensitivity based approach.

Algorithm 1: Sensitivity-based Method

while $t < T$ **do**
  Run Power Flow
  while $l < L$ **do**
    if $V_e < V^{min}$ **then**
      sort $\Delta V_m = Q^c_m \frac{\Delta V_e}{\Delta Q_m}$
      while $V_e + \Delta V_m < V^{min}$ **do**
        $Q_i = Q_i + Q^c_i$
      end
    else if $V_e > V^{max}$ **then**
      sort $\Delta V_m = Q^c_m \frac{\Delta V_e}{\Delta Q_m}$
      while $V_e - \Delta V_m > V^{max}$ **do**
        $Q_i = Q_i - Q^c_i$
      end
      if $V_e < V_{max}$ **then**
        $\Delta V_e^{Rm} = V^{max} - V_e$
        $P_n^{cur} = \frac{\Delta V_e^{Rm}}{\frac{1}{M}\sum_m \frac{\Delta V_e}{\Delta Q_m}}$
      end
    else
    end
    $l = l + 1$
  end
  $t = t + 1$
end

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using $Q^c_m = \pm\sqrt{S^{PV}_m{}^2 - P^{PV}_m{}^2}$. Also, the sensor on the end node of the lateral provides the voltage information, which is then used by the controller to compute voltage increment using the reactive power capability and sensitivity $\Delta V/\Delta Q$ as following, and sorted in a descending order.

$$\Delta V_m = Q^c_{\ m} \frac{\Delta V_e}{\Delta Q_m}. \quad (2)$$

- The difference of measured voltage magnitude at the end

node of a lateral and the maximum/minimum allowed limits is calculated, i.e., $\Delta \mathbf{V_e^{Rq}}$. Then, the first **r** sorted reactive capability information that is sufficient to bring the voltage magnitude within the limits are selected, and dispatch signals are sent to the corresponding inverter controllers.

- If the reactive power capability of the inverters are not enough in maintaining the voltage, then we apply APC based on $\Delta \mathbf{V}/\Delta \mathbf{P}$ to compensate the remaining voltage difference, i.e., $\Delta \mathbf{V_e^{Rm}}$. To be fair to the PV owners, we curtail PV active power equally from all inverters using average sensitivity values as,

$$\mathbf{P_n^{cur}} = \frac{\Delta \mathbf{V_e^{Rm}}}{\frac{1}{\mathbf{M}} \sum_{\mathbf{m}} \frac{\Delta \mathbf{V_e}}{\Delta \mathbf{Q_m}}} \qquad (3)$$

IV. OPTIMAL POWER FLOW BASED CONTROL

For the centralized approach, OPF model can be solved to dispatch inverters reactive power while minimizing APC and maintaining voltages within the operating limits. The control schema for an OPF-based method for voltage control is provided in Fig. 2. A generic multi-period OPF model for this purpose can be formulated as following,
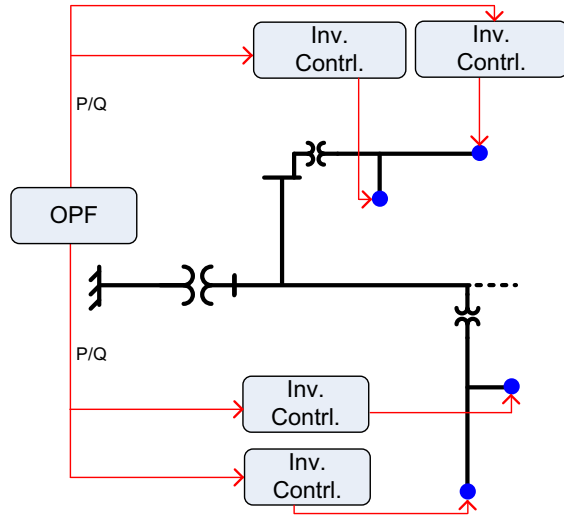

Fig. 2. Schematic showing OPF-based centralized control of inverters.

$$\text{Min:} \quad \mathbf{E^{cur}} = \sum_{\mathbf{m,t}} \mathbf{P_{m,t}^{cur}} \, \Delta \mathbf{t} \qquad (4)$$

subject to:

$$\mathbf{I_{j,t}} = \sum_{\mathbf{k} \in \mathbf{N}} \mathbf{Y_{j,k}} \mathbf{V_{k,t}} \quad \forall \mathbf{j, t} \qquad (5)$$

$$\mathbf{P_{j,t}^{PV}} - \mathbf{P_{j,t}^{cur}} - \mathbf{P_{j,t}^{L}} = \mathbf{Real}\left(\mathbf{V_{j,t}} \, \mathbf{I_{j,t}^*}\right) \quad \forall \mathbf{j, t} \qquad (6)$$

$$\mathbf{Q_{j,t}^{PV}} - \mathbf{Q_{j,t}^{L}} = \mathbf{Imag}\left(\mathbf{V_{j,t}} \, \mathbf{I_{j,t}^*}\right) \quad \forall \mathbf{j, t} \qquad (7)$$

$$|\mathbf{V_{m,t}}| \leq \mathbf{V^{max}} \ \forall \mathbf{m, t} \qquad (8)$$

$$\mathbf{Q_{m,t}^{PV}} \leq \sqrt{\left(\mathbf{S_{m,t}^{PV}}\right)^2 - \left(\mathbf{P_{m,t}^{PV}} - \mathbf{P_{m,t}^{cur}}\right)^2} \ \forall \mathbf{m, t} \qquad (9)$$

$$\mathbf{Q_{m,t}^{PV}} \geq -\sqrt{\left(\mathbf{S_{m,t}^{PV}}\right)^2 - \left(\mathbf{P_{m,t}^{PV}} - \mathbf{P_{m,t}^{cur}}\right)^2} \ \forall \mathbf{m, t} \qquad (10)$$

where objective function (4) defines the energy curtailment over study horizon for all PVs. Eq. (5) defines the power flow equations in current injection form. Eq. (6) and (7) define load and PV power models. Eq. (8) ensures overvoltage limit is enforced. Eq. (9) and (10) model the reactive power capability of PV inverters based on active power dispatch and inverters' rating. The resulting OPF formulation is non-linear programming (NLP) in nature.

V. TEST SYSTEM AND SETUP

A modified Baran and Wu system is considered [18] for testing the proposed method. The 12.66 kV MV circuit in [18] is modified by adding 43 LV laterals (0.24 kV), which resulted in 730-node MV/LV system as shown in Fig. 3 [19]. We consider 70% and 100% penetration levels, which correspond to 306 and 436 number of inverters, respectively. Each inverter is rated 8 kW.
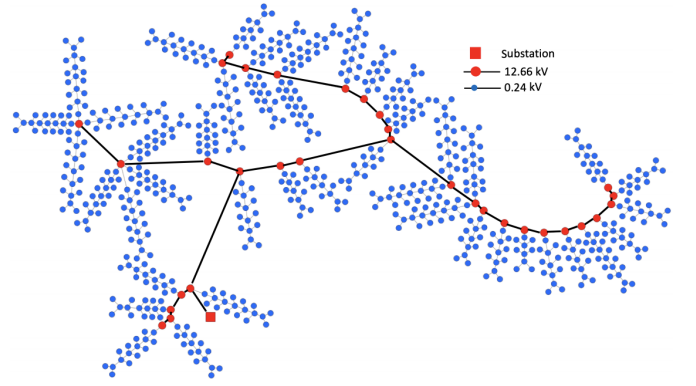

Fig. 3. 730-node MV/LV feeder used for the case studies [19].

The load profiles for each node are different, which represents realistic scenario. Fig. 4 shows net active and reactive power loads with 1-minute resolution for a typical day. We used two PV profile as shown in Fig. 5: one corresponds to a sunny day and the second profile corresponds to a cloudy day. We used Newton-Raphson based method to perform daily simulations on a minute based resolution for the sensitivity-based approach. For OPF-based approach, we modelled using GAMS and solved using KNITRO solver.
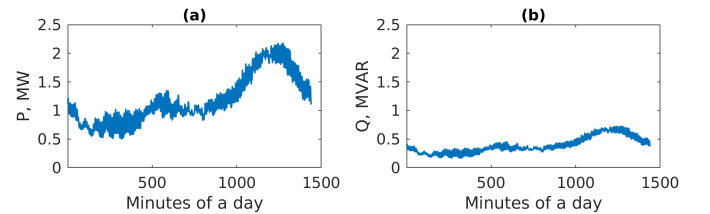

Fig. 4. Total loads: a) active load profile, b) reactive load profile.

VI. NUMERICAL SIMULATIONS

Next, we study performance of the sensitivity-based and OPF-based approach in terms of voltage profile and energy curtailment of PVs.
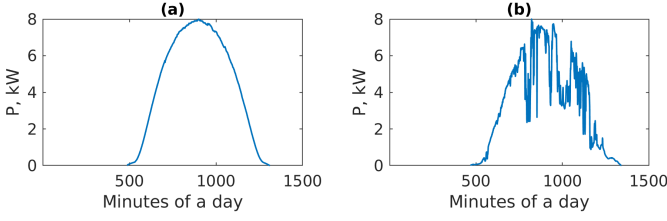
Fig. 5. PV profiles: a) PV-1 (sunny day), b) PV-2 (cloudy day).

A. Sensitivity-Based Approach

A series of daily simulations are carried out with 1-minute time resolution, and the voltage profiles (maximum voltage on the feeder) obtained from the base case (non-dispatchable PVs) simulation and with control of PVs are compared in Fig. 6.
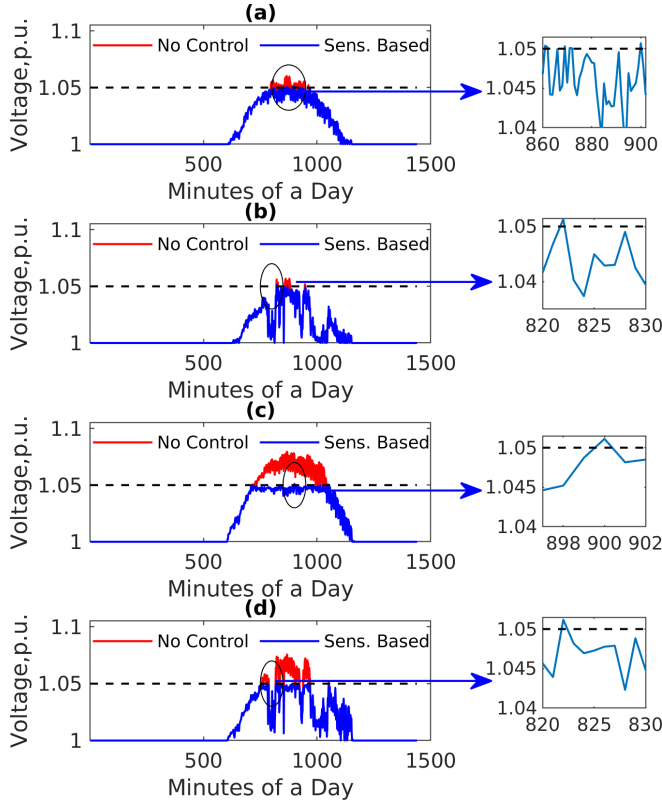

Fig. 6. Max. voltage with base case vs. sensitivity based method: a) 70% PV with PV-1, b) 70% PV with PV-2, c) 100% PV with PV-1, and d) 100% PV with PV-2.

It can be observed that with sensitivity-based method, overvoltage problems can be generally solved. However, there are very few instances where overvoltage still persists. The overvoltage cases are shown in the insets of Fig. 6, and are very close to upper limit of 1.05 p.u. The overvoltage cases are observed around the time when PV outputs are at their maximum. Since, we assumed not all node voltage measurements are available at the lateral level controller, the proposed approach can lead to instances of overvoltage. Another reason could be attributed to the constant sensitivity, which are computed offline, and leads to error due to approximation.

APC of PVs using the sensitivity-based method is shown in Fig. 7. Non-zero APC means the available reactive power

capability alone is not sufficient to mitigate the overvoltage issue. It can be seen from the Fig. 7, with higher PV penetration level, the controller needs to curtail higher power/energy from PV in order to maintain the feeder voltage profile.
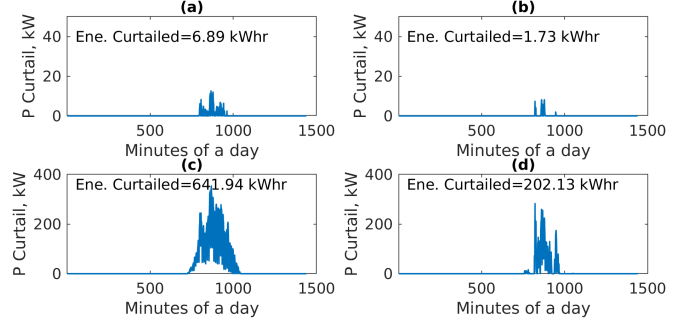

Fig. 7. APC from OPF-based method: a) 70% PV with PV-1, b) 70% PV with PV-2, c) 100% PV with PV-1, and d) 100% PV with PV-2.

B. OPF-based Approach

The maximum feeder voltage profile obtained using OPF-based method is compared with the base case in Fig. 8. From the case studies, it can be seen that the OPF-based approach can completely eliminate the overvoltage issues. To compare the voltage performances of two methods, we define voltage violation index as: $\mathbf{VVI} = \sum_j (\mathbf{V^{max}} - \mathbf{V_j})^2$ if $\mathbf{V_j} > \mathbf{V^{max}}$, and the numerical values are compared in the Table I. Results show that even though the sensitivity-based approach may not completely mitigate the over-voltage issue, the actual values of over-voltage are so small that it may not be concerning.

TABLE I
COMPARISON OF VOLTAGE VIOLATION (SENSITIVITY-BASED VS. OPF).

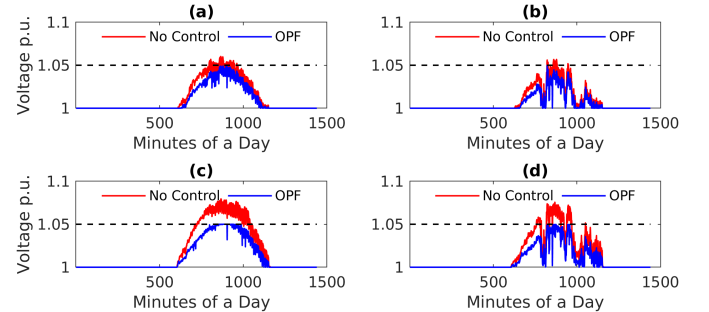| Pen. Level	PV-1		PV-2	
	Sens.	OPF	Sens.	OPF
70% PV	7.94e-07	0	1.72e-06	0
100% PV	1.25e-06	0	1.41e-06	0

Fig. 8. Max. voltage with base case vs. OPF-based method: a) 70% PV with PV-1, b) 70% PV with PV-2, c) 100% PV with PV-1, and d) 100% PV with PV-2.

APC obtained from OPF-based method is shown in Fig. 9. For 70% PV penetration level, OPF-based method led upto 36.72% less energy curtailed compared to sensitivity-base approach. In 100% PV penetration level, we observed

substantially less energy curtailment from OPF-based method. In a full sunny day, the energy curtailment could be reduced upto 99% compared to sensitivity-based approach.
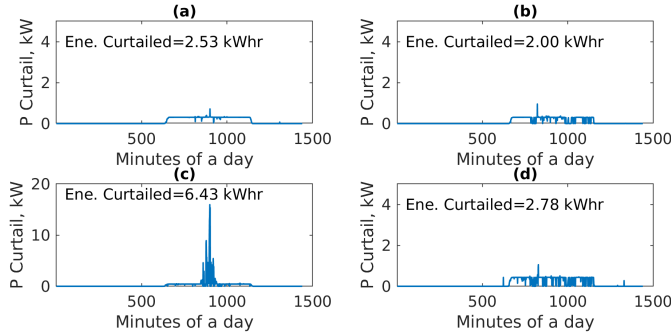

Fig. 9. APC from OPF-based method: a) 70% PV with PV-1, b) 70% PV with PV-2, c) 100% PV with PV-1, and d) 100% PV with PV-2.

C. Monte Carlo Simulation

We also performed Monte Carlo simulation by varying PV penetration level to evaluate the average performance of the two approaches. For each penetration level (randomized 25%, 50%, 75% and 100% by varying PV location) 1,000 different simulations were run for base case, sensitivity-based method, and OPF-based method. The minimum and maximum of the maximum feeder voltage obtained from the 1,000 runs are shown in Fig. 10 along with the upper voltage bound of 1.05 p.u. The feeder exhibits over-voltage issues above 50% penetration level without any control. OPF-based approach is able to solve over-voltage issue for any penetration level. On the other hand, sensitivity-based method may not mitigate over-voltage issue completely as slight over-voltage above 1.05 p.u are observed occasionally.
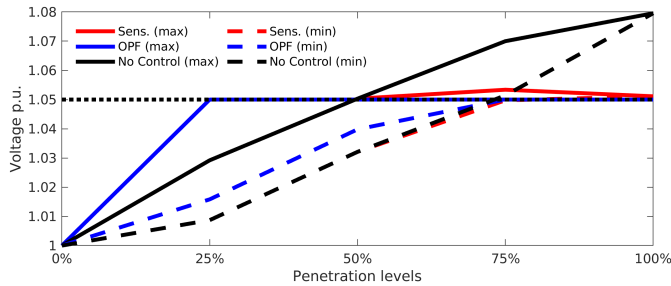

Fig. 10. Voltage performance obtained form Mote Carlo simulation by varying PV penetration level.

VII. Conclusion

This paper presented a sensitivity-based method for controlling active and reactive power of PV inverters to maintain voltage profile on distribution feeders, and the performance is compared with a centralized OPF-based approach. The simulation results on a 730-node MV/LV with hundreds of PV inverters show that sensitivity-based approach is not fully able to solve over-voltage problems; however, the over-voltage is not that significant as the maximum nodal voltages lie close to the upper bound. Given the extensive communication need of centralized-OPF and its computational complexity, a sensitivity-based method similar to the one proposed here could still be a viable alternative to regulate voltage on distribution feeders with high penetration of PVs.

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