A Two-Stage Combined UC-OPF Model Using Mixed Integer and Semi-Definite Programming

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***Abstract*—A two-staged combined unit-commitment (UC) and Optimal power flow (OPF) architecture are proposed in this paper based on mixed-integer semi-definite programming (MISDP). The MISDP problem is then divided into a mixed-integer linear programming problem and a semidefinite programming problem to achieve the solution. Different cases have been studied on 6 bus, IEEE 14 bus, and IEEE 118 bus systems to test the efficacy of the proposed method. It has been observed that the proposed two-staged combined UC gives very close results (2% differences in solution differences in comparison with unified approach) for smaller systems while scalable and feasible for IEEE 118 bus system where the unified approach fails. The optimal operating point derived from the two-staged approach is comparable with the unified approach but the proposed approach performs better in terms of scalability.**

***Index Terms*—Unit commitment (UC), AC OPF (ACOPF), Convex Optimization, Mixed Integer Programming (MIP), Semidefinite Programming (SDP), Two-staged Optimization.**

NOMENCLATURE

$\mathcal{G}$ Graph of transmission system
$r_{g,t}$ Spinning reserve for generator at bus g at time period t; $g \in N_G$
E Set of edges (branches) in G
G_{ij},B_{ij} Conductance and susceptance of transmission line between buses i and j; $(i,j) \in N$
N_G Set of generator buses (nodes) in G
N Set of buses (nodes) in G
$P^G_{g,t}$,$Q^G_{g,t}$ Active and reactive power generation at generator bus g at time period t; $g \in N_G$, $t \in T$
P^{min}_g,P^{max}_g Upper and lower bound of active power generation at bus g; g $\in N_G$
P^D_n,Q^D_n Active and reactive power demand of bus n; $n \in N$
Q^{min}_g,Q^{max}_g Upper and lower bound of reactive power generation at bus g; $g \in N_G$
R_t Spinning reserve for the system at time period t; $t \in T$
RU_g,RD_g Ramp up and ramp down limit for generator at bus g; $g \in N_G$
SU_g Startup cost for generator at bus g at time period t; $g \in N_G$
T Set of hourly time periods
t Time period index; $t \in T$
$u_{g,t}$ binary variable for generator status at bus g at time period t; $g \in N_G, t \in T$
UT_g,UD_g Minimum up and down time limit for generator at bus g; $g \in N_G$
V^{min},V^{max} Upper and lower bound of bus voltage magnitude
$v_{g,t}$ binary variable for generator startup command at bus g at time period t; g $\in N_G, t \in T$
V_n Voltage magnitude at bus n; $n \in N$
$w_{g,t}$ binary variable for generator shutdown command at bus g at time period t; g $\in N_G, t \in T$
Y_{ij} Admittance of transmission line between buses i an j; (i,j) $\in N$

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I. INTRODUCTION

UNIT Commitment (UC) is an important model in the power system to optimally schedule the generating resources over a horizon of time considering the load changes and various other factors. UC is a non-convex problem, which is also discrete. Since the beginning of UC formulation [1], many types of research have been explored different paths to formulate UC as a Mixed Integer Linear Programming (MILP) problem without network constraints [2]–[4]. Various researches have been conducted over time for the formulation of this problem that represents the power network in DC form with or without considering active power losses [5], [6]. Generator scheduling using such models ignore reactive power dispatch, which should be considered. Various methodologies have been applied to solve UC problems such as Dynamic Programming [7], [8], Branch & Bound (B&B) method [9], and Lagrangian Relaxation Method [10]. Each of these approaches has its drawbacks, such as B&B and genetic algorithm approaches are not computationally efficient.

One of the basic properties of the UC problem formulation is that it considers mostly linear constraints. Also, it overlooks the losses in the system and other line constraints. Those constraints are very crucial to get the correct optimal solution. OPF is another important model for power grid operations that consider the power flow and power balance constraints for specific nodes along with other line constraints. However, OPF is another non-convex, non-linear problem and NP-hard in nature [11]–[13]. As a result, the combined formulation of UC with OPF is extremely difficult to solve and poses higher stress for the solver [14]. There are few works where the UC-OPF problem is solved in MINLP form [15]–[17]. Albeit, the formulation for the smaller system may be possible but the scalability of the MINLP version is an issue. Nasri et al. [18] and Fu et al. [16] did extensive work on UC formulation including AC network constraints and security constraints using Bender's Decomposition method. To convexify the non-convex OPF problem various relaxation methods are utilized. It has been studied that SDP relaxations provide more exact solutions for mesh networks in transmission systems than the second-order cone programming (SOCP) relaxation. Though SDP relaxed problem puts an additional computational burden on the solver than the SOCP problems, one major advantage is that SDP relaxed model contains the bus voltage angle while SOCP models mostly do not. SDP relaxed OPF formulations include rectangular representation of power flow equations [13], [19] or polar representation of the bus voltages [20].

In this paper, a two-stage approach of UC-OPF formulation is proposed as a combination of the MILP UC problem and SDP OPF formulation. Comparisons with unified MISDP UC-OPF formulation have been presented to show the advantage of the two-staged approach. The contributions of this paper are fourfold. The approach develops a combined UC-OPF model a) without leveraging the rounding of the binary variables as done in the unified formulation b) Includes the active power loss of the network for power balance constraint in UC c) Provides close to global solutions and scalable. The rest of the paper is organized as follows. Section II discusses UC-OPF preliminaries. Conventional unified and proposed two-staged UC-OPF formulation is described in Section III. The numerical studies and comparison are showcased in Section IV and conclusions and future work is discussed in section V.

II. UC-OPF Preliminaries

A. UC Constraints

The objective of UC is to determine a day-ahead schedule to minimize the power system operation cost while supplying the demand and satisfying other constraints. The UC constraints are briefly explained next.

1) Power Balance: The power balance equation without considering losses can be represented as

$$\sum_{g=1}^{N_G} P_{g,t}^G - \sum_{n=1}^{N} P_{n,t}^D = 0 \tag{1}$$

2) Spinning Reserve: The spinning reserve constraint is

$$\begin{aligned} & r_{g,t} \leq RU_g \\ & \sum_{g=1}^{G} r_{g,t} = R_t \\ & \sum_{n=1}^{N} P_{n,t}^D + R_t - \sum_{g=1}^{G} u_{g,t} P_g^{max} = 0 \end{aligned} \tag{2}$$

3) Minimum start-up and shut-down time of units: The minimum up and down time can be formulated as [21].

$$\sum_{i=t-UT_g+1}^{t} v_{g,i} \leq u_{g,t}; \forall g \in N_G, \forall t \in [UT_g+1, T] \tag{3}$$

$$\sum_{i=t-DT_g+1}^{t} w_{g,i} \leq 1 - u_{g,t}; \forall g \in N_G, \forall t \in [DT_g+1, T] \tag{4}$$

4) Ramping up and Ramping Down: Further, the ramp-rate constraints can be represented as

$$P_{g,t} - P_{g,t-1} \leq RU_g; \forall g \in N_G \tag{5}$$

$$P_{g,t-1} - P_{g,t} \leq RD_g; \forall g \in N_G \tag{6}$$

5) Active and Reactive Power Generation Limit: The active and reactive power generation of the generating units are constrained by following boundaries,

$$P_g^{min} \leq P_{g,t} \leq P_g^{max}; \forall g \in N_G \tag{7}$$

$$Q_g^{min} \leq Q_{g,t} \leq Q_g^{max}; \forall g \in N_G \tag{8}$$

B. Power Flow Constraints

Let us assume, G = (N, E) represents a undirected graph as the power transmission network where N is the set of buses and E is the set of branches. Let, V_i is the voltage of bus $i \in N$. The power balance of the power network represents the equality of total incoming power and outgoing power. If, P_i^G, Q_i^G, P_i^D, Q_i^D denotes the active and reactive power generation and active and reactive power demand of bus $i \in N_G$ and y_{ij} denotes the admittance of line between bus i and j, then the power balance for the bus i can be written as shown below:

$$P_i^G - P_i^D = \sum_{i \neq j}^{N} Re[V_i(V_i - V_j)^* y_{ij}^*] \tag{9}$$

$$Q_i^G - Q_i^D = \sum_{i \neq j}^{N} Im[V_i(V_i - V_j)^* y_{ij}^*] \tag{10}$$

Here, * denotes the complex conjugate of the parameter. Let $Y \in \mathbb{C}^{N \times N}$ is the admittance matrix of the network, where y_{ij} represents the admittance for the line segment between bus i and j. Here, $Y_{ij} = G_{ij} + iB_{ij}$ where, G and B represents the conductance and susceptance matrices. Also, $G_{ii} = g_{ii} - \sum_{i \neq j} G_{ij}$ and $B_{ii} = b_{ijj} - \sum_{i \neq j} B_{ij}$ where, g_{ii} and b_{ii} are shunt conductance and susceptance of bus i. Now, the bus voltage, V_i can be written in it's rectangular form as, $V_i = a_i + ib_i$ and similarly, $|V_i|^2 = a_i^2 + b_i^2$ represents the voltage magnitude squared for that specific bus. With these notations, the power balance equations can be written as follows:

$$\begin{aligned} P_i^G - P_i^G = G_{ii}(a_i^2 + b_i^2) + \sum [G_{ij}(a_i a_j + b_i b_j) \\ - B_{ij}(a_i b_j - a_j b_i)] \end{aligned} \tag{11}$$

$$\begin{aligned} Q_i^G - Q_i^D = -B_{ii}(a_i^2 + b_i^2) + \sum [-B_{ij}(a_i a_j + b_i b_j) \\ - G_{ij}(a_i b_j - a_j b_i)] \end{aligned} \tag{12}$$

Here, this rectangular formulation of power balance equation formulates the OPF as a non-linear and non-convex problem. Non-linearity is coming in the following expressions of variables, $(a_i^2 + b_i^2)$, $(a_i a_j + b_i b_j)$ and $(a_i b_j - a_j b_i)$. To get rid of this non-linearity, following new variables are introduced as, $c_{ii} = (a_i^2 + b_i^2)$, $c_{ij} = (a_i a_j + b_i b_j)$ and $d_{ij} = (a_i b_j - a_j b_i)$. The newly introduced variables are related to each other through the following equation, $c_{ij}^2 + d_{ij}^2 = c_{ii} c_{jj}$. The updated formulation of power balance constraints then becomes

$$P_i^G - P_i^D = G_{ii} c_{ii} + \sum [G_{ij} c_{ij} - B_{ij} d_{ij}] \tag{13}$$

$$Q_i^D - Q_i^D = -B_{ii} c_{ii} + \sum [-B_{ij} c_{ij} - G_{ij} d_{ij}] \tag{14}$$

where the matrix variables c_{ii}, c_{ij} and d_{ij} are related to each other as $c_{ij} = c_{ji}, d_{ij} = -d_{ji}, c_{ij}^2 + d_{ij}^2 = c_{ii} c_{jj}$. If a Hermitian matrix Z is introduced such as, $Z = VV^*$, then all the variables c_{ii}, c_{ij} and d_{ij} can be mapped in to Z as shown below

$$Z = \begin{bmatrix} c_{ii} & (c_{ij} + id_{ij}) \\ (c_{ij} - id_{ij}) & c_{jj} \end{bmatrix} \tag{15}$$

III. UC-OPF Formulations

A. Unified UC-OPF Formulation

Combined UC-OPF formulation can be written in the MISDP form as in (2)-(10), (13)-(15). In this approach, the problem consists of both mixed integer problem and convex optimization problem. Since, currently there isn't much mature MISDP solvers that can solve large scale complex MISDP problems, that's why in this unified approach the binary variables are initialized as continuous variables and once the problem is solved then with the help of rounding the values of unit-commitment variables, the ultimate solution is achieved. The formulation of the unified UC-OPF problem is as follows:

$$\text{Min}: \sum_{t=1}^{T}\sum_{g=1}^{N_G}(u_{g,t}f(P^G_{g,t}) + v_{g,t}SU_g) \tag{16}$$

Here, $f(P^G_g)$ represents the generating cost function. The other cost associated is the start up cost of the generator SU_g. The constraints are,

$$P^G_{i,t} - P^D_{i,t} = G_{ii}c_{ii,t} + \sum[G_{ij}c_{ij,t} - B_{ij}d_{ij,t}] \tag{17}$$

$$Q^G_{i,t} - Q^D_{i,t} = -B_{ii}c_{ii,t} + \sum[-B_{ij}c_{ij,t} - G_{ij}d_{ij,t}] \tag{18}$$

$$u_{i,t}P^{min}_i \leq P^G_{i,t} \leq u_{i,t}P^{max}_i; \forall i \in G \tag{19}$$

$$u_{i,t}Q^{min}_i \leq Q^G_{i,t} \leq u_{i,t}Q^{max}_i; \forall i \in G \tag{20}$$

$$\sum_{n=1}^{N} P^D_n + R_t - \sum_{g=1}^{N_G} u_{g,t}P^{max}_g = 0 \tag{21}$$

$$P^G_{g,t} - P^G_{g,t-1} \leq RU_g; \forall g \in N_G \tag{22}$$

$$P^G_{g,t-1} - P^G_{g,t} \leq RD_g; \forall g \in N_G \tag{23}$$

$$\sum_{i=t-UT_g+1}^{t} v_{g,i} \leq u_{g,t}; \forall g \in N_G, \forall t \in [UT_g + 1, T] \tag{24}$$

$$\sum_{i=t-DT_g+1}^{t} w_{g,i} \leq 1 - u_{g,t}; \forall g \in N_G, \forall t \in [UD_g + 1, T] \tag{25}$$

$$u_{i,t}, v_{i,t}, w_{i,t} \in \{0, 1\} \tag{26}$$

$$c_{ij} = c_{ji} \tag{27}$$

$$d_{ij} = -d_{ji} \tag{28}$$

$$Z = \begin{bmatrix} c_{ii} & (c_{ij} + id_{ij}) \\ (c_{ij} - id_{ij}) & c_{jj} \end{bmatrix} \tag{29}$$

$$Z \succcurlyeq 0 \tag{30}$$

$$(V^{min})^2 \leq c_{ii} \leq (V^{max})^2; \forall i \in N \tag{31}$$

In the MISDP UC problem, the variables u, v, w are binary variables, but since there are not enough mature solvers that can solve large scale MISDP problems, so the binary variables are relaxed to continuous variables. Then, a rounding-off approach is applied to obtain an integer solution. When the problem (16)-(30) is solved, the values of the variable u, v, and w are converted to binary value using the rounding operation. Then, with those binary values, the OPF problem is solved to get the generation set-points.

B. Two-staged UC-OPF Formulation

To solve integer recovery as mentioned above, in this paper, initially, the value of $Ploss_t$ is an estimated system loss. Once the OPF problem is solved for the given generator status, and actual power loss is calculated. In the next iteration, while the UC problem is to be formulated, that loss is updated in the power balance equation. This iterative process is continued until the generator commitment status remains same for two successive iterations. The MILP UC problem in the two-staged

Algorithm 1 Proposed Two-staged UC-OPF

Step: 1 Initialize network parameters.
Step: 2 Initialize $P_{loss,1}$ as $x\%$ of P_D.
Step: 3 Use eq^n 32, 19-26, 33 to formulate UC problem.
Step: 4 From the solution use generator status value to identify active generator
Step: 5 Use eq^n 34, 17-20, 27-30 to formulate OPF problem.
Step: 6 After convergence calculate $P_loss, 2$.
if $P_{loss,1} = P_{loss,2}$ **then**
  Update the solution to dispatch generator)
else
  Update $P^{k+1}_{loss,1} = P^k_{loss,2}$

approach can be formulated as,

$$\text{Min}: \sum_{t=1}^{T}\sum_{g=1}^{N_G}(u_{g,t}f(P^G_{g,t}) + v_{g,t}SU_g) \tag{32}$$

$$\text{Subject to}:$$

$$\text{Constraints}: (19) - (26)$$

$$\sum_{n=1}^{N} P^D_n - \sum_{g=1}^{N_G} P^G_{g,t} + Ploss_t = 0; \forall g \in N_G \tag{33}$$

If $u^*_{g,t}$ is obtained from UC solution, then using $u^*_{g,t}$ as parameter, OPF in two-stage formulation is modeled as,

$$\text{Min}: \sum_{t=1}^{T}\sum_{g=1}^{N_G} u^*_{g,t}f(P^G_{g,t}) \tag{34}$$

$$\text{Subject to}:$$

$$\text{Constraints}: (17) - (20), (27) - (30)$$

IV. Numerical Case Studies

The proposed two-stage approach to solve combined UC-OPF problem is implemented in YALMIP, an optimization toolbox for MATLAB. The simulations are conducted on modified IEEE 6 bus network, IEEE 14 bus, and IEEE 118 bus test networks. The simulation is performed on a Dell laptop with a 2.5GHz Core i5 processor and 16 GB RAM, running a 64bit Windows-10 operating system. To test the approach test systems of three different sizes were selected. For IEEE 6 bus system, there are 3 generators at bus 1, 2 and 3 of capacity 200MW, 150MW, 800MW respectively and 3 load buses. A load profile is generated for 24 hours and used to solve the problem. The maximum capacity of the generation is 1150 MW. IEEE 14 bus network contains 5 generators and 11 loads. A 24-hour load profile is also generated based on standard benchmark load conditions. The base voltage of the system is 230 kV. IEEE 118 bus network consists of 19 generators, 35 synchronous condensers, 177 lines, 9 transformers, and 91 loads.

A. UC-OPF for 6 bus system

The UC-OPF problem for 6 bus system is solved using both unified and two-staged approach. The generators' parameters

TABLE I
UC PARAMETERS LIMITS FOR 6 BUS SYSTEM

Constraints	Gen 1	Gen 2	Gen 3
Ramping Up (MW)	55	50	20
Ramping Down (MW)	55	50	20
Minimum Up Time (Hr)	4	2	1
Minimum Down Time (Hr)	4	3	1

TABLE II
UCOPF SOLUTION FOR 6 BUS SYSTEM

Parameters	Unified MISDP	Two-staged MISDP
Total Pgen (MW)	5174.474	5174.127
Total Ploss (MW)	22.0738	21.7274
Total Cost	84795.32	86602.04

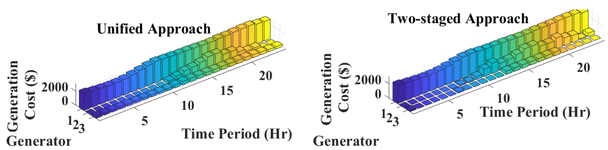

Fig. 1. Generator status comparison of 6 bus system for unified and two-staged approaches.

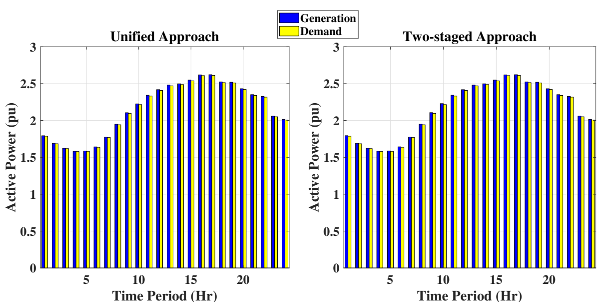

Fig. 2. Total demand and total generation comparison of 6 bus system for 24-hr time horizon.

are given in Table I. In a unified approach, the binary variables are initially defined as continuous variables. Once the problem is solved, the value of those variables is compared with a threshold value to perform the rounding off operation. Then the feasibility is checked by solving the OPF problem. The total cost from this approach for 1-day is $84,795.32. For the two-stage approach, the total generation cost for the day is $86,602.04 which can be seen higher than the unified approach. However, the total active power loss in the two-staged approach is 21.73 MW, which is lower than the 22.07 MW from the unified approach (see Table II). The generators' status comparison from both approaches are shown in Fig. 1. The voltage profile comparison between the two approaches for the time of maximum and minimum loading is shown in Fig. 5. The total demand and generation comparison on an hourly basis is shown in Fig. 2. The total generation from the proposed approach for each time was compared with the same from the unified approach. The maximum error was 0.038%.

B. UC-OPF for IEEE 14 bus system

In case of IEEE 14 bus system, both the total generation cost and system active power loss is less in unified approach than the two-staged approach. The comparison is given in Table IV. The generator UC parameter data are given in Table III. The generators' status in Fig. 3, in unified approach all the generators have been committed, as the value of the generator status variable was higher than the threshold value for all instances, While in two-staged approach, the cheap generators (e.g., G1, G2) have been committed for all the time and costly generators (e.g., G3, G4 and G5) are offline for some periods following the minimum up time. The voltage profile comparison for the maximum and minimum loading time is shown in Fig. 5. The demand and generation profile comparison for the test case is shown in Fig. 4. The maximum error for total active power generation comparison between the approaches was 0.014%.

TABLE III
UC PARAMETERS LIMIT VALUE FOR IEEE 14 BUS SYSTEM

Constraints	Gen 1	Gen 2	Gen 3	Gen 4	Gen 5
Ramping Up (MW)	55	50	50	40	30
Ramping Down (MW)	55	50	50	40	30
Minimum Up Time (Hr)	4	2	1	2	1
Minimum Down Time (Hr)	4	2	1	2	1

TABLE IV
UCOPF SOLUTION FOR IEEE14 BUS SYSTEM

Parameters	Unified MISDP	Two-staged MISDP
Total Pgen (MW)	3898.189	3898.48
Total Ploss (MW)	90.8890	91.183
Total Cost	77963.775	77969.67

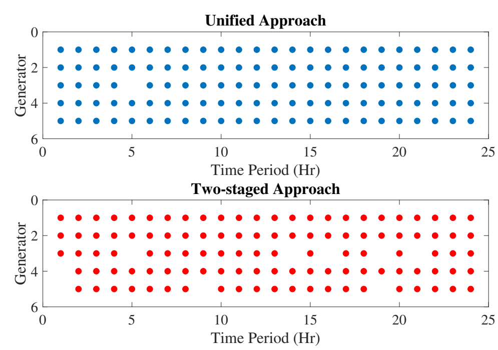

Fig. 3. Generator status comparison of IEEE 14 bus system for unified and two-staged approaches.

C. UC-OPF for IEEE 118 bus system

For a large system like modified IEEE 118 bus network, the unified approach was not solvable as the number of constraints and variables are large. So, here only the solution from two-staged approach is presented. This problem is also formulated for a 24-hr time horizon with a maximum load of around 6,800 MW and the solver could easily solve the problem. The generators' cost coefficients for the the system are available in [22]. The total demand and total generation profile for the whole time horizon of the network is shown in Fig. 6. The solution has Pgen (MW) = 132396.31, Ploss (MW) = 4135.41 and Generation cost ($) is 4135.41.

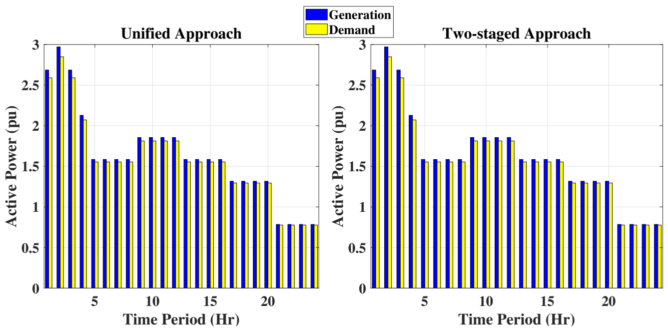

Fig. 4. Total demand and total generation comparison of IEEE 14 bus system for 24-hr time horizon.

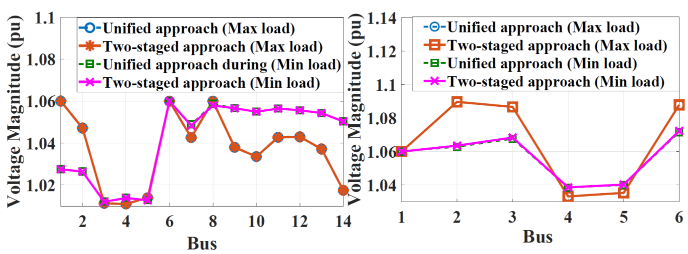

Fig. 5. Voltage profile comparison for maximum and minimum loading hours in 6 bus and IEEE 14 bus system.

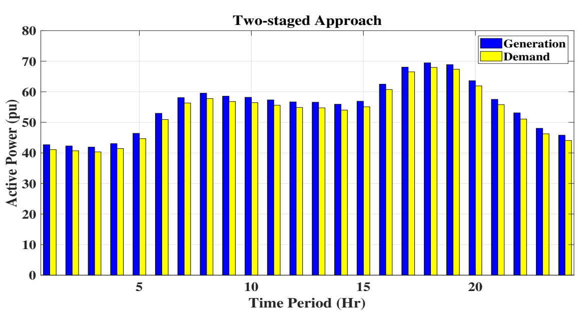

Fig. 6. Total demand and total generation comparison of IEEE 118 bus system for 24-hr time horizon.

V. Conclusions and Future Work

In this paper, a two-stage approach for UC-OPF formulation is proposed. The approach is scalable, accurate with respect to optimal solutions, and feasible. For example, due to the lack of availability of mature solvers, the unified UC-OPF problem in the MISDP form cannot be solved for larger systems, i.e., IEEE 118 bus network where the two-staged approach was able to solve and is scalable for larger networks. The solution from the two-staged approach may not be the most economic (we have seen up to 2% difference when compared to the unified approach) but the scheduling of the generating units is feasible. Future work includes extending to integrating contingency scenarios and tighter network constraints. Also, the computational time can be reduced significantly by leveraging the matrix sparsity for larger networks.

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