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Timely Gossip

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***Abstract*—A source node forwards fresh status updates as a point process to a network of observer nodes. Within the network of observers, these updates are forwarded as point processes from node to node. Each node wishes its knowledge of the source to be as timely as possible. In this network, timeliness at each node is measured by an age of information metric: how old is the timestamp of the freshest received update. This work extends a method for evaluating the average age at each node in the network when nodes forward updates using a memoryless gossip protocol. This method is then demonstrated by age analysis for a simple network.**

I. Introduction

Gossip is a popular mechanism to convey status information in a distributed systems and networks. The efficacy of gossip mechanisms for distributed computation [1], [2] and message dissemination [3] is well known. While it is also known that gossip mechanisms can be inefficient relative to more complex or application-specific algorithms, it is recognized that gossip remains an attractive option in settings when protocols need to be simple or the network topology or connectivity is time-varying [4]. For example, gossip protocols could be a good choice for low latency vehicular safety messaging. And yet, while vehicular message exchange was the early motivation for age of information (AoI) research [5], [6], there has been little (if any) effort to examine AoI for gossip protocols.

This work is a companion paper to [7] which initiated a re-examination of gossip from an age-of-information (AoI) perspective [8], [9]. In particular, [7] extended AoI analysis to a class of status sampling networks, a networking paradigm that is consistent with gossip models in that short messages, representing samples of a node's status update process, are delivered as point processes to neighbor nodes. This "zero service time" model may be useful in a high speed network in which updates represent small amounts of protocol information (requiring negligible time for transmission) that are given priority over data traffic. This model has also been widely used in the age analysis of energy harvesting updaters [10]–[16] where updating rates are constrained by energy rather than bandwidth. While the transmission of a single update may be negligible, the update rates are limited so that protocol information in the aggregate does not consume an excessive fraction of network resources.

Prior work [17], [18] on status sampling networks analyzed age in line networks where node i only received updates from node $i-1$. The key advance of [7] and this work is the development of an average age analysis method for monitors that receive updates via multiple network paths. Specifically, a source wishes to share its status update messages with a network of n nodes. These nodes, which can be viewed monitors of the source, employ gossip to randomly forward these update messages amongst themselves in order that all nodes have timely knowledge of the state of the source.

In [7], timeliness is measured by a discrete form of age of information: the age at a node is how many versions out of date is its most recent update from the source. In this work, age is measured continuously in the units of time and the parallel results derived here highlight how the two age formulations are essentially interchangeable.

II. System Model and Summary of Results

Status updates of a source node 0 are shared via a network with a set of nodes $\mathcal{N} = \{1, 2, \dots, n\}$. Timeliness at each node is measured by the average age of information. If the current update at a node j has timestamp u at time t, then the age at node j is $X_j(t) = t - u$. The source node 0 maintains the current (fresh) version of its status and starting at time $t = 0$, node 0 sends fresh status updates to each node $j \in \mathcal{N}$ as a rate λ_{0j} Poisson process $N_0(t)$, With each received update from the source, the age at node j is reset to zero. In addition, network nodes forward updates using gossip. Node i sends its most recent (but not necessarily fresh) update to node j as a rate λ_{ij} Poisson process. If node i sends its update to node j at time t, the age at node j becomes

$$X_j'(t) = \min[X_i(t), X_j(t)]. \tag{1}$$

With these updating rules, the age at each node is sawtooth process, as depicted in Figure 1.

This work develops a method for evaluating the limiting average age $\lim_{t\to\infty} \mathrm{E}[X_i(t)]$, referred to as the AoI at node i. Building on prior work [18], [19], this paper employs the stochastic hybrid system (SHS) methodology to analyze the convergence of the expected age. The SHS approach is to develop a set of ordinary differential equations for $\mathrm{E}[X_i(t)]$ that enables the evaluation of the limiting age $\lim_{t\to\infty} \mathrm{E}[X_i(t)]$. As shown in (1), this will require the characterization of age variables such as $X_{\{i,j\}}(t) \equiv \min(X_i(t), X_j(t))$. More generally, for arbitrary subsets $S \subseteq \mathcal{N}$, the analysis will need to track the age

$$X_S(t) \equiv \min_{j\in S} X_j(t) \tag{2a}$$

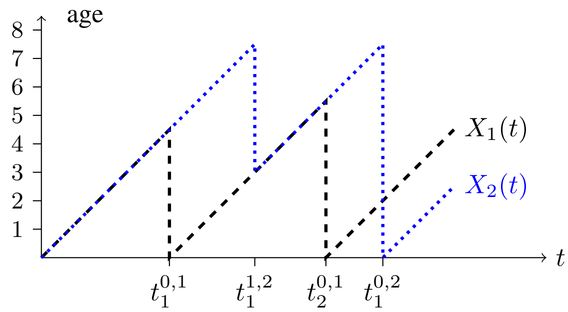

Fig. 1. Fresh updates from source node 0 pass through the network as point processes; $t_n^{i,j}$ marks the nth update sent on link (i,j). Node 1 gets updates from the source. Node 2 gets updates from both the source and node 1.

and its expected value

$$v_S(t) \equiv \mathrm{E}[X_S(t)]. \tag{2b}$$

The process $X_S(t)$ can be interpreted as the status age of an observer of updates arriving at any node in S and it is convenient to call $X_S(t)$ simply the age of subset S.

The main contribution of the paper is the development of a system of linear equations for the calculation of the limiting average age $\bar{v}_S = \lim_{t\to\infty} \mathrm{E}[X_S(t)]$. To describe this system of equations, define the update rate of node i into set S as

$$\lambda_i(S) \equiv \begin{cases} \sum_{j\in S} \lambda_{ij} & i \notin S, \\ 0 & i \in S, \end{cases} \tag{3}$$

and the set of updating neighbors of S as

$$N(S) \equiv \{i \in \mathcal{N} \colon \lambda_i(S) > 0\}. \tag{4}$$

With this notation, here is the main result.

Theorem 1: The expected status age $v_S(t) = \mathrm{E}[X_S(t)]$ of an observer of node set S converges to $\bar{v}_S = \lim_{t\to\infty} v_S(t)$ satisfying

$$\bar{v}_S = \frac{1 + \sum_{i\in N(S)} \lambda_i(S) \bar{v}_{S\cup\{i\}}}{\lambda_0(S) + \sum_{i\in N(S)} \lambda_i(S)}. \tag{5}$$

Note that Theorem 1 is almost identical to [7, Theorem 1] which measures age at a network node by how many update versions the node is lagging the source. The only difference being that the 1 in the numerator of the right side of (5) is replaced in [7, Theorem 1] by λ_{00}, the Poisson rate at which the source node 0 generates new versions of its status. In short, this work shows that it makes no difference in these settings whether age is measured in absolute time or in version changes with a unit rate Poisson clock. Proof of Theorem 1 is presented in Section V-B since it differs in various (perhaps non-obvious) details from the corresponding proof in [7].

Because of this parallelism in the theorems, version age results in [7] with $\lambda_{00} = 1$ hold for the continuous age model in this work. For example, consider the n node symmetric gossip network on a complete graph, as depicted in Figure 2 for $n = 6$ nodes. In the complete graph, $\lambda_{ij} = \lambda/(n-1)$ for all node pairs $i, j \in \mathcal{N}$. This corresponds to each node $i \in \mathcal{N}$ randomly sending its current updates to each of the other $n-1$ nodes as a rate $\lambda/(n-1)$ Poisson process. In addition, the source sends symmetrically to each node $j \in \mathcal{N}$ with Poisson rate $\lambda_{0j} = \lambda/n$. By exploiting the symmetry of the complete graph, [7, Theorem 2] verified that the average age at a node grows as $\log n$. Here this result is restated for the traditional continuous time age metric.

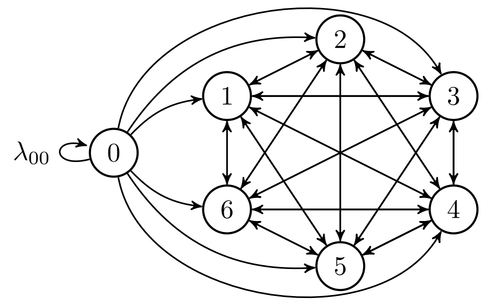

Fig. 2. Nodes in $\mathcal{N} = \{1, \dots, 6\}$ form complete graph. Node 0 sends fresh updates to each node $j \in \mathcal{N}$ at rate $\lambda/6$. Each node $i \in \mathcal{N}$ send updates to every other node j at rate $\lambda/5$.

Theorem 2: For the symmetric complete gossip network with the source sending updates to each node $i \in \mathcal{N}$ at rate λ/n, the age $X_i(t)$ at each node i satisfies

$$\frac{1}{\lambda}\left[\frac{n-1}{n}\sum_{k=1}^{n-1}\frac{1}{k} + \frac{1}{n}\right] \le \lim_{t\to\infty} \mathrm{E}[X_i(t)] \le \frac{1}{\lambda}\sum_{k=1}^{n}\frac{1}{k}. \tag{6}$$

Hence, as the network size n grows, the average age at each node only grows logarithmically in n. Although the communication models are different in various small ways, this average result is analogous to [3, Theorem 3.1] in which the ϵ-dissemination time, i.e. the time until the probability a source message has not reached all nodes is less than ϵ, is shown to grow as $O(\log n)$.

III. Related Work

AoI analysis of updating systems started with the analyses of status age in single-source single-server first-come first-served (FCFS) queues [8], the M/M/1 last-come first-served (LCFS) queue with preemption in service [20], and the M/M/1 FCFS system with multiple sources [21]. This section examines AoI contributions relating to networks carrying the updates of a single source, as in this work. A more extensive overview of AoI research can be found in [9].

To evaluate AoI for a single source sending updates through a network cloud [22] or through an M/M/m server [23]–[25], out-of-order packet delivery was the key analytical challenge. The first evaluation of the average AoI over multihop network routes [26] employed a discrete-time form of the status sampling network also employed in [17], [18]. These works obtained simple AoI results because the updates followed a single path to a destination monitor. This avoided the complexity of multiple paths and the consequent accounting for repeated and out-of-order update message deliveries.

When multiple sources employ wireless networks subject to interference constraints, AoI has been analyzed under a

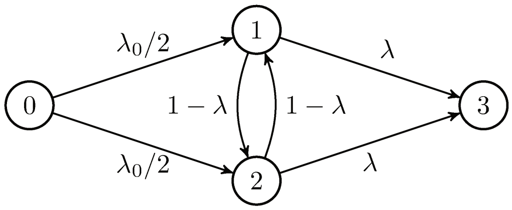

Fig. 3. Updates generated at node 0 are forwarded to nodes 1 and 2. Nodes 1 and 2 exchange updates and also forward updates to node 3.

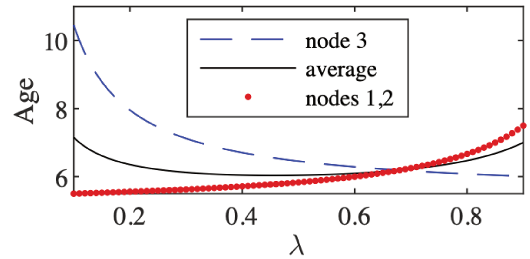

Fig. 4. Average ages: $\bar{v}_{\{1\}} = \bar{v}_{\{2\}}$ at nodes $1, 2$, $\bar{v}_{\{3\}}$ at node 3, and the average age $(\bar{v}_{\{1\}} + \bar{v}_{\{2\}} + \bar{v}_{\{3\}})/3$ with $\lambda_0 = 0.2$.

variety of link scheduling strategies [27]–[35]. Age bounds were developed from graph connectivity properties [36] when each node needs to update every other node. For DSRC-based vehicular networks, update piggybacking strategies were developed and evaluated [6].

When update transmission times over network links are exponentially distributed, sample path arguments were used [37]–[39] to show that a preemptive Last-Generated, First-Served (LGFS) policy results in smaller age processes at all nodes of the network than any other causal policy. Note that the status sampling network model in this work can also be viewed as a network of preemptive LGFS servers; see [18] for details. As this work introduces a new tool for AoI analysis in networks, it may be useful in the characterization of AoI scaling laws [40], [41].

IV. An Example of Theorem 1

To utilize Theorem 1 to calculate the average age at node n, the first step starts with $S = \{n\}$ and generates an equation for $\bar{v}_{\{n\}}$ in terms of the variables $\bar{v}_{\{i,n\}}$ for neighbor nodes i such that $\lambda_{i,n} > 0$. For each such node i, the second step applies (5) recursively with $S = \{i, n\}$. This generates an equation for each $\bar{v}_{\{i,n\}}$ in terms of variables $\bar{v}_{i,j,n}$ for each node j that sends updates to one or both nodes in $\{i, n\}$.

In general, step k constructs equations for $\bar{v}_S$ for sets S with size $|S| = k$ in terms of variables $\bar{v}_{S'}$ such that each S' has size $|S'| = k+1$. In the worst case, this procedure terminates at stage $k = n$ when $S = \mathcal{N}$. For a fully connected graph, this procedure generates equations for all $2^n - 1$ non-empty subsets of $\mathcal{N}$. On the other hand, when the network graph is sparse, substantially fewer equations may be generated.

Here Theorem 1 is demonstrated with the three-node network of Figure 3. In this network, node 0 forwards updates to node $i \in \{1, 2\}$ at rate $\lambda_0/2$. In addition, each node $i \in \{1, 2\}$ is constrained to forward updates at unit rate. That is, for $\lambda \in (0, 1]$, node 1 sends updates to node 3 at rate λ and to node 2 at rate $1 - \lambda$. In a symmetric fashion, node 2 sends updates to node 3 at rate λ and to node 1 at rate $1-\lambda$. Thus $\lambda \in (0, 1]$ can be selected to balance timeliness at nodes 1 and 2 against timeliness at node 3. To characterize this tension, Theorem 1 is employed to solve for the average ages $\bar{v}_{\{1\}}, \bar{v}_{\{2\}}$ and $\bar{v}_{\{3\}}$ at nodes 1, 2 and 3.

For the network in Figure 3, the recursive application of (5) with $S = \{3\}$, and then $S = \{1\}$ and $S = \{2\}$ yields

$$\bar{v}_{\{3\}} = \frac{1 + \lambda\bar{v}_{\{1,3\}} + \lambda\bar{v}_{\{2,3\}}}{2\lambda}, \tag{7a}$$

$$\bar{v}_{\{1\}} = \bar{v}_{\{2\}} = \frac{1 + (1-\lambda)\bar{v}_{\{1,2\}}}{\lambda_0/2 + 1 - \lambda}. \tag{7b}$$

Next, with $S = \{1, 3\}$ and $S = \{2, 3\}$, (5) implies

$$\bar{v}_{\{1,3\}} = \bar{v}_{\{2,3\}} = \frac{1 + (\lambda + 1 - \lambda)\bar{v}_{\{1,2,3\}}}{\lambda_0/2 + 1}. \tag{7c}$$

Finally, with $S = \{1, 2\}$ and $S = \{1, 2, 3\}$, it follows from (5) that

$$\bar{v}_{\{1,2\}} = \bar{v}_{\{1,2,3\}} = 1/\lambda_0. \tag{7d}$$

Note that for $S = \{1, 2, 3\} = \mathcal{N}$, (7d) is an example of the general result that $\bar{v}_{\mathcal{N}} = 1/\lambda_0(\mathcal{N})$. For this network, it follows from (7) that nodes 1, 2 and 3 have average ages

$$\bar{v}_{\{1\}} = \bar{v}_{\{2\}} = \frac{2}{\lambda_0}\left[\frac{1 + (1-\lambda)/\lambda_0}{1 + 2(1-\lambda)/\lambda_0}\right], \tag{8a}$$

$$\bar{v}_{\{3\}} = \frac{1}{2\lambda} + \frac{2}{\lambda_0}\left[\frac{1 + 1/\lambda_0}{1 + 2/\lambda_0}\right]. \tag{8b}$$

For $\lambda_0 = 0.2$, Figure 4 plots the average ages $\bar{v}_{\{1\}}$, $\bar{v}_{\{2\}}$ and $\bar{v}_{\{3\}}$ at nodes 1, 2, and 3, as well as the the average age over the nodes $\bar{v}_{\text{avg}} = (\bar{v}_{\{1\}} + \bar{v}_{\{2\}} + \bar{v}_{\{3\}})/3$. Equation (8) shows that increasing λ increases the arrival rate of updates at node 3; thus the average age at node 3 is decreasing in λ. However, increasing λ results in node 1 and 2 exchanging fewer updates and thus the average ages at nodes 1 and 2 are increasing in λ. It can be verified that for $\lambda_0 \le 2\sqrt{2}$, $\bar{v}_{\text{avg}}$ is minimized at $\lambda = (1 + \lambda_0/2)/(1 + \sqrt{2})$.

V. Stochastic Hybrid Systems for AoI Analysis

In this section, a stochastic hybrid system (SHS) model is used to derive Theorem 1. While there are many SHS variations [42], this work follows [18], [19], which employ the model and notation in [43]. In general, the SHS is described by a discrete state $q(t) \in \mathcal{Q} = \{0, 1, \ldots, q_{\max}\}$ that evolves as a point process, a continuous component $\mathbf{X}(t) \in \mathbb{R}^n$ described by a stochastic differential equation in each state $q \in \mathcal{Q}$, and a set $\mathcal{L}$ of transition/reset maps that correspond to both changes in the discrete state and jumps in the continuous state.

A. AoI for gossip networks as an SHS

In this work, the operation of the gossip network is memoryless; each node i sends its current update to node j as a Poisson process of rate λ_{ij}. Hence, the SHS discrete state space is the trivial set $\mathcal{Q} = \{0\}$. Moreover, in the absence of an update delivery, the age at each node grows at unit rate; the stochastic differential equation of the SHS is $\dot{\mathbf{X}}(t) = \mathbf{1}_n$.

The remaining component of the SHS model is the set $\mathcal{L}$ of discrete transition/reset maps. In the gossip network, $\mathcal{L}$ corresponds to the set of directed edges (i,j) over which node i updates node j. However, because of the special role of node 0 as the source, there are two kinds of transitions. First, the $(i,j) = (0,j)$ transition corresponds to the source node generating a fresh update and sending it to node j so that the age at node j is reset to zero. In the second type, a gossiping node i forwards its current update to node j; node j accepts the update if it is a fresher than its current update. To summarize, the set of transitions is

$$\mathcal{L} = \{(0,j)\colon j \in \mathcal{N}\} \cup \{(i,j)\colon i,j \in \mathcal{N}\} \tag{9}$$

and transition (i,j) occurs at rate $\lambda_{i,j}$. In that transition, the age vector becomes $\phi_{i,j}(\mathbf{X}) = \begin{bmatrix} X_1' & \cdots & X_n' \end{bmatrix}$ such that

$$X_k' = \begin{cases} 0 & i=0, k=j \in \mathcal{N}, \\ \min(X_i, X_j) & i \in \mathcal{N}, k=j \in \mathcal{N}, \\ X_k & \text{otherwise.} \end{cases} \tag{10a}$$

Because of the generality and power of the SHS model, complete characterization of the $\mathbf{X}(t)$ process is often impossible. The approach in [43] is to define test functions $\psi(q, \mathbf{X}, t)$ whose expected values $\mathrm{E}[\psi(q(t), \mathbf{X}(t), t)]$ are performance measures of interest that can be evaluated as functions of time; see [43], [44], and the survey [42] for additional background.

Since the simplified SHS for the gossip network is time invariant and has a trivial discrete state, it is sufficient to employ the time invariant test functions $\psi_S(\mathbf{X}) = X_S$, $S \subseteq \mathcal{N}$. These test functions yield the processes

$$\psi_S(\mathbf{X}(t)) = X_S(t), \tag{11}$$

which have expected values

$$\mathrm{E}\big[\psi_S(\mathbf{X}(t))\big] = \mathrm{E}\big[X_S(t)\big] \equiv v_S(t). \tag{12}$$

The objective here is to use the SHS framework to derive a system of differential equations for the $v_S(t)$. To do so, the SHS mapping $\psi \to L\psi$ known as the extended generator is applied to every test function $\psi(\mathbf{X})$. The extended generator $L\psi$ is simply the function whose expected value is the expected rate of change of the test function ψ. Specifically, a test function $\psi(\mathbf{X}(t))$ has an extended generator $(L\psi)(\mathbf{X}(t))$ that satisfies Dynkin's formula

$$\frac{d\,\mathrm{E}[\psi(\mathbf{X}(t))]}{dt} = \mathrm{E}[(L\psi)(\mathbf{X}(t))]. \tag{13}$$

For each test function $\psi(\mathbf{X})$, (13) yields a differential equation for $\mathrm{E}[\psi(\mathbf{X}(t))]$.

From [43, Theorem 1], it follows from the trivial discrete state, the time invariance of $\psi_S(\mathbf{X})$ in (11) and the trivial stochastic differential equation $\dot{X}_S(t) = 1$ for each S that the extended generator of a piecewise linear SHS is given by

$$(L\psi_S)(\mathbf{X}) = 1 + \sum_{(i,j)\in\mathcal{L}} \lambda_{ij}[\psi_S(\phi_{i,j}(\mathbf{X})) - \psi_S(\mathbf{X})]. \tag{14}$$

B. Proof of Theorem 1

In (14), it follows from (2a), (10), and (11) that the effect on the test function of transition (i,j) is

$$\psi_S(\phi_{i,j}(\mathbf{X})) = X_S' = \min_{k\in S} X_k'. \tag{15}$$

Evaluation of (15) depends on the transition type (i,j), as given in (10). In transition $(0,j)$, the source sends a fresh update to node j, resetting the age at node j to zero. If $j \in S$, then $X_S = \min_{k\in S} X_k$ is reset to zero; otherwise X_S is unchanged. Similarly, for other transitions (i,j), only the age X_j at node j is changed. Thus if $j \in S$,

$$\begin{aligned} X_S' = \min_{k\in S} X_k' &= \min(\min(X_i, X_j), \min_{k\in S\setminus\{j\}} X_k) \\ &= \min_{k\in S\cup\{i\}} X_k = X_{S\cup\{i\}}; \end{aligned} \tag{16}$$

otherwise, X_S is unchanged. Note that if $i \in S$, then $S\cup\{i\} = S$ and $X_S' = X_S$. That is, an update sent by a node in S cannot reduce the age X_S.

Based on these transitions, it follows from (14) that

$$\begin{aligned} (L\psi_S)(\mathbf{X}) = 1 &+ \sum_{j\in S} \lambda_{0j}[0 - X_S] \\ &+ \sum_{\substack{i\in\mathcal{N} \\ i\notin S}} \sum_{j\in S} \lambda_{ij}\big[X_{S\cup\{i\}} - X_S\big]. \end{aligned} \tag{17}$$

In taking the expectation of (17), the left side yields $\mathrm{E}[(L\psi_S)(\mathbf{X}(t))] = \dot{v}_S(t)$ by Dynkin's formula (13). Since $\mathbf{X}$, X_S, and $X_{S\cup\{i\}}$ in (17) refer to the age processes $\mathbf{X}(t)$, $X_S(t)$ and $X_{S\cup\{i\}}(t)$, applying the expectation to the right side yields $\mathrm{E}[X_S(t)] = v_S(t)$ and $\mathrm{E}\big[X_{S\cup\{i\}}(t)\big] = v_{S\cup\{i\}}(t)$ for all i. With these substitutions, (17) becomes

$$\dot{v}_S(t) = 1 - v_S(t)\sum_{j\in S}\lambda_{0j} + \sum_{\substack{i\in\mathcal{N} \\ i\notin S}}\sum_{j\in S}\lambda_{ij}[v_{S\cup\{i\}}(t) - v_S(t)].$$

Employing the definitions (3) and (4) of the update rate $\lambda_i(S)$ of node i into S and the neighbor set $N(S)$ yields

$$\dot{v}_S(t) = 1 - v_S(t)\Big[\lambda_0(S) + \sum_{i\in N(S)}\lambda_i(S)\Big] + \sum_{i\in N(S)}\lambda_i(S) v_{S\cup\{i\}}(t).$$

Setting the derivative $\dot{v}_S(t) = 0$ generates a linear equation for the time average age $\bar{v}_S = \lim_{t\to\infty} v_S(t)$ in terms of the necessary $\bar{v}_{S\cup\{i\}}$. This yields (5).

VI. Conclusion

This work has extended the AoI analysis tools for gossip algorithms introduced in [7] to continuous-time age metrics with traditional sawtooth age functions. In particular, Theorem 1 provides a set of linear equations for the computation of average age at any node in a gossip network described by an arbitrary graph. Although the general solution has exponential complexity in the number of nodes, symmetry properties can simplify the calculations in special cases. When this method is applied to the n node complete graph, the average age at each node grows as $\log n$. This promising result suggests that gossip networks may indeed be suitable for low latency measurement dissemination, particularly in sensor network settings.

As age analysis for gossip networks is new, considerable work remains. For example, the analysis of the simple network of Figure 3 showed that age depends on how nodes gossip and an optimized updating policy will depend on the update rate of the source. Age analysis of gossip for energy harvesting sensors would also be another obvious area of interest. It may also be possible to derive distributional properties of the age in a gossip network by extending the moment generating function (MGF) approach to age analysis introduced in [18].

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