Comparison of Parameter Conditioning in Output Error and Equation Error Approaches in Speed and Parameter Estimation in Induction Machines[1]

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***Abstract*— Equation Error and Output Error are common formulations used in speed and parameter estimation for induction machines. This paper presents a study of the quality of the estimated speed and parameters using local sensitivity analysis. We studied parameter conditioning as a function of input signals and estimation methodology at nominal speed. Simulation results are used to show that output error formulation is better conditioned than equation error for speed and parameter estimation using PWM and six-step voltage inputs.**

I. INTRODUCTION

Many speed and parameter estimation schemes used in adaptive control, sensorless control, or self-commissioning of induction motor drives are based on equation error or output error methodologies for system identification. To develop robust methods for speed and parameter estimation, it is important to quantify the information content about rotor speed and machine parameters on measured signals. This is of particular importance when we are limited only to electrical terminal quantities such as stator voltages and currents, and because of the natural tradeoff between input richness and low harmonic content required for efficiency in electric drives.

Output Error (OE) and Equation Error (EE) are the most used formulations for parameter estimation in dynamical systems [1]. Many speed and parameter estimation techniques for induction machine described in the literature are based on OE and EE approaches and minimization of a least squares cost function [2]-[8]. Here we present results of our work on local sensitivity analysis for speed and parameter estimates obtained using OE and EE approaches. The analysis results are based on local sensitivity analysis of the mapping relating the measured data to the parameter estimates.

II. INDUCTION MACHINE MODELING

Different induction machine state variable representations are possible depending on the selected state variables. The selection of state variables and corresponding model parameterization is of great importance when performing parameter estimation since the sensitivity of the system output (stator current) to different parameterization can vary significantly [10]. A state variable representation of the electrical equations using stator and rotor currents as state variable is given by [9]:

$$\frac{d\bar{\mathbf{x}}(t)}{dt} = \mathbf{A}(\boldsymbol{\phi})\bar{\mathbf{x}}(t) + \mathbf{B}(\boldsymbol{\phi})\bar{\mathbf{u}}(t) \tag{1}$$

$$\bar{\mathbf{y}}(t) = \mathbf{C}(\boldsymbol{\phi})\bar{\mathbf{x}}(t) \tag{2}$$

where

$$\mathbf{A}(\ddot{\mathbf{o}}) = \begin{bmatrix} -\frac{R_s}{L_s\sigma} - j\frac{(1-\sigma)}{\sigma}\omega_r & \frac{(1-\sigma)}{T_r\sigma} - j\frac{(1-\sigma)}{\sigma}\omega_r \\ \frac{R_s}{L_s\sigma} + j\frac{1}{\sigma}\omega_r & -\frac{1}{T_r\sigma} + j\frac{1}{\sigma}\omega_r \end{bmatrix}$$

$$\mathbf{B}(\phi) = \begin{bmatrix} \frac{1}{L_s\sigma} & -\frac{1}{L_s\sigma} \end{bmatrix}^T, \quad \mathbf{C}(\phi) = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$\bar{\mathbf{x}}(t) = \begin{bmatrix} \bar{i}_s(t) & \bar{i}_r(t) \end{bmatrix}^T, \quad \bar{\mathbf{u}}(t) = \bar{v}_S,$$

$\boldsymbol{\phi} = [\sigma \ L_S \ T_r \ R_S \ \omega_r]^T$, $\bar{y}(t) = \bar{i}_S$, and $\sigma = (L_sL_r - M^2)/L_s$ is the leakage coefficient, $L_S = L_{ls} + M$ is the stator inductance, $T_r = L_r/R_r$ is the rotor time constant, R_S is the stator resistance, and ω_r is the rotor speed. We selected this parameterization to compare our results with other presented in literature .

III. SPEED AND PARAMETER ESTIMATION FOR INDUCTION MACHINES

In the output error approach, as shown in Fig. 1, the model (1)-(2) is run in parallel with the system and the parameters

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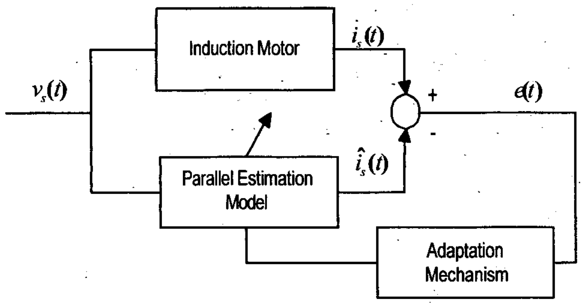

Fig. 1: Output Error Approach for System Identification.

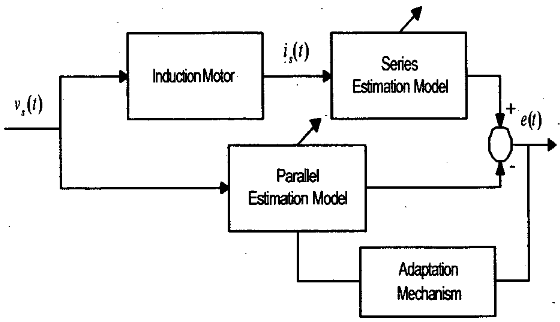

Fig. 2: Equation Error Approach for System Identification.

are estimated by minimizing a measure of the model prediction error

$$\mathbf{e}(\phi) = \hat{\mathbf{y}}(\phi) - \mathbf{y} \tag{3}$$

where $\hat{\mathbf{y}}(\phi)$ is the N-vector of model predictions for the measurements, given by

$$\hat{y}(\phi) = \begin{bmatrix} \hat{y}(t_1/\phi) \\ \vdots \\ \hat{y}(t_N/\phi) \end{bmatrix}$$

and $\mathbf{y}$ is the N-vector of actual measurements. Notice that prediction error $\mathbf{e}(\phi)$ is a nonlinear function of ϕ. The estimate of ϕ can be found by

$$\hat{\phi} = \underset{\phi}{argmin}\ V(\phi) \tag{4}$$

where $V(\phi)$ is a measure of model fit typically least squares

$$V(\phi) = \frac{1}{2}\left\|\mathbf{e}(\phi)\right\|^2 = \frac{1}{2}\sum_{k=1}^{N} e_k^2(\phi)$$

For batch least squares estimation, the Gauss Newton Method is typically used to compute the estimate $\hat{\phi}$.

To introduce the equation error approach, we need to introduce the transfer function between the stator voltage space vector and the current space vector. This is easily derived using standard LTI systems theory applied to (1)-(2) and is given by:

$$\frac{\bar{I}_{dqs}(s)}{\bar{V}_{dqs}(s)} = \frac{s+\frac{1}{T_r}-j\omega_r}{\sigma L_s s^2 + \left(R_s + \frac{L_s}{T_r} - j\sigma L_s \omega_r\right)s + R_s\left(\frac{1}{T_r} - j\sigma L_s \omega_r\right)}$$

This transfer function has an associated differential equation given by:

$$\sigma L_s \frac{d^2\bar{i}_s}{dt^2} + \left(R_s + \frac{L_s}{T_r} - j\sigma L_s \omega_r\right)\frac{d\bar{i}_s}{dt} + R_s\left(\frac{1}{T_r} - j\omega_r\right)\bar{i}_s = \frac{d\bar{v}_s}{dt} + \left(\frac{1}{T_r} - j\omega_r\right)\bar{v}_s \tag{5}$$

In the equation error approach, parameters on both sides of (5) are adjusted until their difference is minimized. Fig. 2 shows a block diagram for the equation error identifier, the left-hand side of (5) is the parallel model and the right-hand side is the series model. Differentiation of stator voltages and currents is avoided by means of state variable filters [1].

To derive the associated optimization problem, (5) can be re-written in a regression format as follows

$$\bar{\mathbf{y}}(t) = \mathbf{c}^{\mathbf{T}}(t)\,\mathbf{f}(\phi) \tag{6}$$

where:

$$\bar{y}(t) = \frac{d\bar{v}_s}{dt},$$

$$c(t) = \left[\frac{d^2\bar{i}_s}{dt^2} \quad \frac{d\bar{i}_s}{dt} \quad -j\frac{d\bar{i}(t)_s}{dt} \quad \bar{i}_s \quad -j\bar{i}_s \quad -\bar{v}_s \quad j\bar{v}_s\right],$$

$$\mathbf{f}(\phi) = \left[\sigma L_s \quad R_s + \frac{L_s}{T_s} \quad \sigma L_s \omega_r \quad \frac{R_s}{T_s} \quad R_s \omega_r \quad \frac{1}{T_s} \quad \omega_r\right]^T$$

The equation error batch estimation problem is given by the following nonlinear least squares optimization problem

$$\hat{\phi} = \arg\min_{\phi}\left\|\mathbf{y}_N - \mathbf{C}_N \mathbf{f}(\phi)\right\|^2 \tag{7}$$

where

$$\mathbf{C}_N = \begin{bmatrix} \mathbf{c}(t_1) \\ \vdots \\ \mathbf{c}(t_N) \end{bmatrix} \text{ and } \mathbf{y}_N = \begin{bmatrix} \mathbf{y}(t_1) \\ \vdots \\ \mathbf{y}(t_N) \end{bmatrix}$$

Again, Gauss-Newton is the method of choice for batch optimization.

IV. Sensitivity Analysis

In our context, sensitivity analysis refers to the methodology of quantifying changes in the solution of a problem by perturbations in the problem data [11]. One methodology to quantify sensitivity is to view the relation between problem and data as a mapping between two normed spaced as illustrated in Fig. 3.

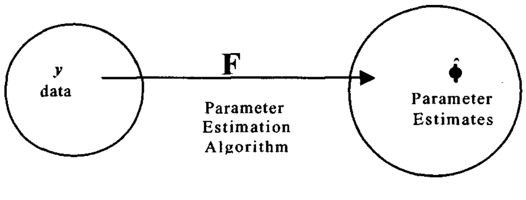

Fig. 3. Parameter Estimation as a Mapping.

In the scalar case, the relative change in $\hat{\phi}$ can be related to the relative change in y as follows

$$\frac{\Delta\hat{\phi}}{\hat{\phi}} = S_y^{\hat{\phi}} \frac{\Delta y}{y} \tag{8}$$

where

$$S_y^{\hat{\phi}} = \frac{y}{\hat{\phi}} \frac{\partial\hat{\phi}}{\partial y}$$

is the (relative) *condition number* of $\hat{\phi}$ [11]. In the control systems literature, (8) is called the *sensitivity function*. A large magnitude of $S_y^{\hat{\phi}}$ will denote an ill-conditioned parameter while a small $S_y^{\hat{\phi}}$ will denote a well-conditioned parameter.

Let $\hat{\phi} = \mathbf{F}(\mathbf{y})$ be the vector function relating data and parameter estimate, then the sensitivity function can be generalized for the vector case as follows

$$CN_i = S_y^{\hat{\phi}_i} = \frac{\|y\|\|\nabla f_i\|}{\|\hat{\phi}\|} \frac{\|\hat{\phi}\|}{|\hat{\phi}_i|} \tag{9}$$

where $\mathbf{f}_i$ and $\hat{\phi}_i$ are the i-th component of $\mathbf{F}$ and $\hat{\phi}$ respectively, and $\|\cdot\|$ is the 2-norm. We can identify two components in (9) which point out to potential sources of ill conditioning in the estimated parameter. The first component is the colinearity factor

$$CF_i = \frac{\|y\|\|\nabla f_i\|}{\|\hat{\phi}\|}$$

which is associated to the conditioning of the Jacobian of the parameter estimation mapping and a scaling factor

$$SF_i = \frac{\|\hat{\phi}\|}{|\hat{\phi}_i|}$$

This later one is associated with the relative scaling of the i-th parameter to the remaining parameters. We consider the colinearity factor to be the most important factor in analyzing parameter conditioning. Since it is a measure of the ill conditioning of the mapping Jacobian while ill conditioning caused by scaling seems more artificial and with little influence on numerical algorithms used for estimation. This is a topic that needs further investigation.

For nonlinear least square estimation, let $\hat{\phi} = \mathbf{F}_{NLS}(\mathbf{y})$, be the parameter estimation map implicitly defined by the optimization problems (4) or (7). The Jacobian of nonlinear least square problem was derived in [12] and is given by

$$\frac{\partial \mathbf{F}_{NLS}}{\partial \mathbf{y}} = (\mathbf{J}'*\mathbf{J})^{-1} * \mathbf{J}' \tag{10}$$

where $\boldsymbol{J}$ is the Jacobian of the error.

V. Simulation Results

A key issue in developing robust speed and parameter estimation algorithms for induction motors is to characterize the information content of measured signals. We are conducting research to characterize the information content of stator currents as a function of input stator voltage and operating conditions. Our simulation study considers an induction motor fed by a voltage source inverter. Here we present preliminary results for the analysis of equation error and output error approaches at nominal speed conditions for PWM, and six-step voltage input. The condition number (9) and the colinearity factor for each parameter are presented in Tables I-IV. For each stator voltage the condition number are computed for the speed and electrical parameters. The results for output error are presented in Tables I to II and the results for equation error are presented in Tables III to IV.

It is observed that for the OE error case we get from little to moderate levels of ill-conditioning while parameters in the EE case have much higher condition numbers signaling significant ill-conditioning problems. In fact, the condition numbers for the EE case are, for most parameters, from one to two orders of magnitude larger. In both cases, we can see that σ, L_s, and T_r are the best-conditioned parameters while R_s and ω_r are the worst. Another observation is that as the harmonic content of the input signal increases, the conditioning of individual parameter estimates improves as expected. This is an expected result due to improved harmonic richness when going from PWM to six-step voltage.

In [13], the subset selection method developed in [12] was applied to the EE approach suggesting to estimate only three parameters and rotor speed from available data and to fix the stator resistance to a prior value. This resulted in significant reduction in sensitivity and improvement in estimation algorithm and sensorless controller performance as shown in [4,13]. Here we repeated the analysis for the OE and EE approach and the results are presented in Tables V and VI for PWM and Tables VII and VIIII for six step.

TABLE I

SPEED AND PARAMETERS SENSITIVITIES FOR THE INDUCTION MACHINE EXCITEDWITH PWM VOLTAGES (OE)

Parameter	Condition Number	Colinearity Factor
σ	12.2748	0.0020
L_s	2.1766	0.0039
T_r	56.8011	0.0171
R_s	36.4057	0.7669
ω_r	2.3033	2.3028

TABLE II

SPEED AND PARAMETERS SENSITIVITIES FOR THE INDUCTION MACHINE EXCITEDWITH SIX STEP VOLTAGES (OE)

Parameter	Condition Number	Colinearity Factor
σ	3.3389	0.00053
L_s	1.8350	0.0033
T_r	55.4492	0.0167
R_s	27.4672	0.5788
ω_r	2.2849	2.2849

TABLE III

SPEED AND PARAMETERS SENSITIVITIES FOR THE INDUCTION MACHINE EXCITEDWITH PWM VOLTAGES (EE)

Parameter	Condition Number	Colinearity Factor
σ	70.8411	0.0113
L_s	66.8193	0.1195
T_r	905.3600	22.8248
R_s	513.2742	10.8088
ω_r	38.0503	38.0297

TABLE IV

SPEED AND PARAMETERS SENSITIVITIES FOR THE INDUCTION MACHINE EXCITEDWITH 6 STEP VOLTAGES (EE)

Parameter	Condition Number	Colinearity Factor
σ	55.7010	0.0089
L_s	55.5100	0.0993
T_r	472.7416	11.9182
R_s	321.4566	6.7694
ω_r	20.0037	19.9928

In these tables, we show the condition numbers for the parameter estimates where one electrical parameter or speed is fixed to a prior value for EE case and repeated for OE. The results are quite interesting, in particular it is important to point out that the resulting condition numbers for the EE reduced order case (one parameter fixed) are of the same order as those for the OE full case (all parameters estimated). In other words, to get the same level of well conditioning as found in the OE case with the EE case, we had to fix one parameter to reduce problem sensitivity.

An interesting issue with the Jacobian is how well the Jacobian for the reduced order problem approaches the stable part of the full Jacobian. In [13] it is shown that the goodness of this approximation is dependent on the existence of a gap between contiguous singular values. In the analysis presented here, we can see the goodness of the approximation by looking at how close are the singular values of the reduced order Jacobian to the first four singular values of the full order Jacobian. It can be argued that these elements are associated with the most identifiable directions in the parameter space. The best approximation for both EE and OE is when the rotor speed is known. Here all four singular values are well approximated. When estimating speed together with the electrical parameters, fixing the stator resistance results in the best approximation. The significant improvement in estimation due to fixing the stator resistance has been pointed out in the literature [4,5,6,13]. Notice that when we estimate speed together with the parameters the best we can do is to approximate the first three singular values.

VI. CONCLUSIONS

This paper introduced a comparison between OE and EE formulations for speed and parameter estimation. Parameter estimates computed using the output error are significantly better conditioned than those computed using the equation error approach. It is also shown that the best-conditioned case is to measure speed and estimate the electrical parameters while when estimating rotor speed with the electrical parameters fixing the stator resistance is the best approach from a sensitivity point of view.

Further research is being done to investigate the conditioning of the speed and parameter estimates as a function of input frequency and to further understand the use of condition number to analyze estimation problems and development of robust estimation algorithms.

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TABLE V

CONDITION NUMBER OF SPEED AND PARAMETER ESTIMATION WITH ONE PARAMETER FIXED (OE CASE PWM)

Fixed Parameter	–	ω_r	R_s	T_r	L_s	σ
Singular Values	3327.56 1153.3 823.7 1.706 0.5659	3327.5 1153.3 823.8 1.70	3327.56 1153.3 823.8 0.56	1245.9 1003.2 18.90 1.70	3276.3 1140.9 4.3 0.56	3296.73 836.9 2.9 0.56
Parameter	Condition Number	Condition Number	Condition Number	Condition Number	Condition Number	Condition Number
σ	12.2748	12.2526	7.1452	12.2559	8.4993	–
L_s	2.1766	2.1630	0.8654	2.1662	–	1.5071
T_r	56.8011	1.7662	56.6428	–	56.5291	56.7139
R_s	36.4057	36.2814	–	36.3042	14.4748	21.1919
ω_r	2.3033	–	2.2959	0.0716	2.2889	2.2997

TABLE VI

CONDITION NUMBER OF SPEED AND PARAMETER ESTIMATION WITH ONE PARAMETER FIXED (EE CASE PWM)

Fixed Parameter	–	ω_r	R_s	L_s	T_r	σ
Singular Values	1.6×10^7 7.3×10^6 7.7×10^5 2.7×10^4 1.5×10^3	1.6×10^7 7.3×10^6 7.7×10^5 2.7×10^4	1.6×10^7 7.3×10^6 7.7×10^5 0.74×10^4	1.6×10^7 7.1×10^5 0.27×10^4 0.23×10^3	1.6×10^7 1.5×10^6 0.31×10^4 2.2×10^4	0.07×10^7 0.15×10^6 2.7×10^4 0.22×10^4
Parameter	Condition Number	Condition Number	Condition Number	Condition Number	Condition Number	Condition Number
σ	70.8411	49.3275	47.9537	23.9560	51.5372	–
L_s	66.8193	45.0176	43.5949	–	47.4941	22.5959
T_r	905.3600	54.6046	209.5636	643.5156	–	658.6541
R_s	513.2742	112.51	–	334.8714	118.8075	347.4447
ω_r	38.0503	–	8.3412	25.6353	2.2949	26.4949

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TABLE VII

CONDITION NUMBER OF SPEED AND PARAMETER ESTIMATION WITH ONE PARAMETER FIXED (OE CASE SIX STEP)

Fixed Parameter	–	ω_r	R_s	σ	T_r	L_s
Singular Values	4638.08 3891.5 1026.6 4.3716 0.7459	4638.1 3891.4 1026.6 4.30	4638.1 3891.5 1026.6 1.10	4603.3 3855.9 6.744 0.95	4528.4 1231.0 24.7 4.20	4026.5 1061.7 5.1 0.85
Parameter	Condition Number	Condition Number	Condition Number	Condition Number	Condition Number	Condition Number
σ	3.3389	2.9411	2.5049	–	2.9508	2.4878
L_s	1.8350	1.4487	0.9264	1.3668	1.4688	–
T_r	55.4492	1.8005	38.9474	49.0288	–	44.4210
R_s	27.4672	19.0994	–	20.6178	19.2938	13.8787
ω_r	2.2849	–	1.5869	2.0113	0.0741	1.8033

TABLE VIII

CONDITION NUMBER OF SPEED AND PARAMETER ESTIMATION WITH ONE PARAMETER FIXED (EE CASE SIX STEP)

Fixed Parameter	–	ω_r	R_s	L_s	T_r	σ
Singular Values	1.1×10^8 7.3×10^6 6.7×10^5 7.6×10^4 2×10^4	1.1×10^8 7.3×10^6 67×10^4 7.2×10^4	1.1×10^8 7.3×10^6 67×10^4 2.4×10^4	1.1×10^8 7.3×10^6 7.6×10^4 2.2×10^4	1.1×10^8 1.4×10^6 7.7×10^4 2.2×10^4	0.1×10^8 7.6×10^6 7.6×10^4 0.3×10^4
Parameter	Condition Number	Condition Number	Condition Number	Condition Number	Condition Number	Condition Number
σ	55.7010	45.6733	45.5242	3.0802	48.6748	–
L_s	55.5100	45.6105	45.4635	–	48.5946	3.0696
T_r	472.7416	56.0147	67.6595	412.7574	–	412.02
R_s	321.4566	36.3992	–	262.7280	46.0326	262.17
ω_r	20.0037	–	2.2800	16.5105	2.3872	16.47

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