

Impact of Distributed Energy Resource Integration on Real-time Energy Market Oscillation

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Abstract—With the growing penetration of small-scale distributed energy resources (DER), DERs are allowed to participate in the independent system operator (ISO) energy market in the form of aggregation. However, the current approximation model for aggregated DERs, in terms of distribution factors (DF), cannot reflect their actual contributions to power flows of transmission lines, thus leading to market clearing solutions that are suboptimal or even infeasible to the physical system. In this paper, we use real-time security-constrained economic dispatch (RT-SCED) problem as an example to show that DF based DER aggregation approach could lead to oscillations in consecutive time intervals, and analyze the reasons behind this. A novel network equivalence based DER aggregation approach is put forward to mitigate the oscillation and ensure the physical feasibility of RT-SCED solutions. Numerical simulations illustrate the oscillation of DF based approach, and validate effectiveness of the proposed approach.

Index Terms—Distributed energy resources, aggregation bidding, distribution factor, real-time economic dispatch, oscillation.

I. INTRODUCTION

Due to the sophistication of power systems and the large number of market participants, independent system operators (ISO) often adopt certain strategic modeling simplifications in the market clearing process. One example of such strategies adopted by the Midcontinent Independent System Operator (MISO) is to allow load zone commercial pricing (CP) node to represent an aggregation of assets. Currently, the aggregation is in general not allowed for dispatchable resources. Most loads are not dispatchable in real time and load zone aggregation works well in representing the demand side.

With a growing penetration of distributed energy resources (DER), the power industry has been exploring effective approaches to facilitate the participation of DERs in the bulk power market, including the recent FERC Order 2222 [1]. One potential approach is to aggregate DERs spread in different locations of a distribution network as a CP node in the ISO market clearing. To this end, distribution factors (DF) [2], proportions of individual DERs to the total power of the CP node, are used to evaluate their contributions on power flows of individual transmission lines in the ISO market clearing applications, such as the 5-minute based real-time security-constrained economic dispatch (RT-SCED) problem.

However, exact DFs of individual DERs on a CP node remain unknown, until the physical power system operation status is observed following the ISO market clearing instructions. To this end, historical information can be used to provide approximated DFs. Evidently, using approximated DFs would introduce inevitable errors to power flows of individual transmission lines, leading to market clearing solutions that are suboptimal or even infeasible to the actual physical system. In observing that more recent information of system operation status could provide more accurate DF approximation, state estimation (SE) results have also been used to update DFs for the rolling RT-SCED calculations. However, this feedback strategy could induce severe uneconomic phenomenon, while the locational marginal prices (LMP) as well as dispatches of generators and DERs could keep oscillating between the suboptimal and infeasible regions.

Oscillation under the context of RT-SCED has been concerned for a long time, especially in terms of the interactive oscillation between the ISO market clearing and the price responsive assets. Specifically, during the interactive process, the market clearing targets on identifying the optimal market operation solutions and calculating LMPs according to the most recent information from price responsive assets; while the price responsive assets focus on arbitrage based on LMPs and send updated electricity generation/ consumption strategies back to the ISO market [3]. Reference [4] illustrates such an interactive procedure via the cobweb plot, and points out that the RT-SCED may not converge with respect to the current market clearing design. To improve convergence of the dynamic process, consensus-based designs are conducted via distributed coordination [5] and multi-agent system [6] approaches.

Although similar oscillation phenomena are observed, the dispatch and LMP oscillations caused by inaccurate DFs of DERs are, in principle, different from the ones discussed in [3]-[6], and their solutions cannot be simply applied. In fact, the most direct way to correct the DF approximation errors is to explicitly calculate RT-SCED according to element pricing (EP) nodes [7]. However, this might not be practically viable because ISOs do not directly collect configuration details, such as DERs and distribution network data, to exhaustively model the distribution systems. Moreover, explicitly modeling a large number of EP nodes equipped with DERs in the ISO models

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could introduce significant computational burdens.

This paper demonstrates the oscillation phenomena in the ISO RT-SCED calculation process with DER aggregation models, and discusses an innovative approach to resolve the oscillation issue. Specifically, the proposed aggregation model can project the available transmission capabilities to a CP node via a network equivalent transformation. The projected constraints ensure that the cleared quantity of a CP node always meets the transmission requirements in various system operation states, promoting the RT-SCED solution falling in the feasible region and close to the optimal value. Therefore, the RT-SCED will not oscillate between infeasible and suboptimal solutions. A 4-node system and an IEEE 30-bus system are simulated to investigate oscillation issue of the current DF based practice and validate efficacy of the proposed design.

II. OSCILLATION OF THE MARKET CLEARING WITH DERS

Figs. 1 and 2 are used to illustrate the market clearing oscillation issue studied in this paper, where DERS are integrated into the transmission system at multiple locations. Under the EP node aggregation model, as shown in Fig. 1, the DER integration is simulated at EP nodes 2 and 3, and their contributions on transmission line 2-3 can be accurately evaluated. In comparison, if the CP node aggregation on DERS is applied, as shown in Fig. 2, inaccurate DFs of individual DERS could lead to over or under estimations on their contributions to the power flow of line 2-3, leading to uneconomic or infeasible operations.

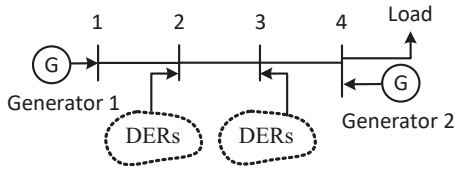


Fig. 1. EP node aggregation

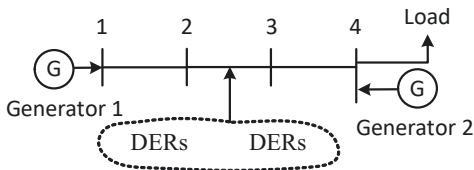


Fig. 2. CP node aggregation

For the sake of discussion, time index is not explicitly presented in the following formulations, while the proposed model can be naturally extended to multiple time intervals. Notations \mathcal{J} , \mathcal{L} , \mathcal{G} , \mathcal{D} , \mathcal{A} , and \mathcal{F} denote sets of nodes, lines, generators, DERS, aggregators, and loads in a transmission system, which are respectively indexed by i , l , g , d , a , and f .

A. ED Problem with CP Node Based DER Aggregation

According to the CP node model of the MISO as shown in Fig. 2, DERS of a single market participant connected at multiple EP nodes can be aggregated through 2 steps: (i) the aggregated bidding curve is created by combining bidding curves of individual DERS in an ascending order; (ii) distribution factor $DF_{i,a}$ of DER at node i in DER aggregator a is pre-fixed

according to its capacity $P_{D,i}^{UB}$ as in (1), or updated according to the most recent SE results $\hat{P}_{D,i}$ as in (2).

$$DF_{i,a} = P_{D,i}^{UB} / \sum_{i \in \mathcal{J}_a} P_{D,i}^{UB}; \quad i \in \mathcal{J}_a \quad (1)$$

$$DF_{i,a} = \hat{P}_{D,i} / \sum_{i \in \mathcal{J}_a} \hat{P}_{D,i}; \quad i \in \mathcal{J}_a \quad (2)$$

To this end, P_l , power flow of transmission line l , can be calculated as in (3), where $SF_{l,i}$ denotes the shift factor of bus i to line l and $\sum_{i \in \mathcal{J}_a} (SF_{l,i} \cdot DF_{i,a})$ calculates the approximated sensitivity of DER aggregator a to line l . $P_{G,i}$ and $P_{L,i}$ are power dispatches of generator and load at node i , and P_a is power dispatch of DER aggregator a . Power flow is further limited by its transmission capacity P_l^{UB} , given as in (4).

$$P_l = \sum_{i \in \mathcal{J}} SF_{l,i} \cdot (P_{G,i} - P_{L,i}) + \sum_{a \in \mathcal{A}} [\sum_{i \in \mathcal{J}_a} (SF_{l,i} \cdot DF_{i,a}) \cdot P_a]; \quad l \in \mathcal{L} \quad (3)$$

$$-P_l^{UB} \leq P_l \leq P_l^{UB}; \quad l \in \mathcal{L} \quad (4)$$

Therefore, the ISO market can treat each DER aggregator as one CP node, which reduces the system decision variables from $|\mathcal{G}| + |\mathcal{D}|$ to $|\mathcal{G}| + |\mathcal{A}|$, where $|\cdot|$ measures cardinality of a set. Considering one aggregator usually contains dozens of DERS, it will significantly reduce the number of variables and non-zeros in constraint (3). However, it is noted that parameter $DF_{i,a}$ based on (1) or (2) could deviate from the actual responses.

With the above CP node based DER aggregation model, the conventional ED problem for the energy market is formulated as in (5)-(9). The objective (5) is to minimize the total cost with bidding curves of all participants, where $c_{G,i}$ and c_a are bidding prices of generators at node i and aggregator a . Constraint (6) defines the power balance requirement. Constraints (7) and (8) represent dispatch limits of generators and DER aggregators, described by lower bounds P^{LB} and upper bounds P^{UB} .

$$\min C = \sum_{i \in \mathcal{J}} c_{G,i} \cdot P_{G,i} + \sum_{a \in \mathcal{A}} c_a \cdot P_a; \quad (5)$$

$$\sum_{i \in \mathcal{J}} (P_{G,i} - P_{L,i}) + \sum_{a \in \mathcal{A}} P_a = 0; \quad (6)$$

$$P_{G,i}^{LB} \leq P_{G,i} \leq P_{G,i}^{UB}; \quad i \in \mathcal{J} \quad (7)$$

$$P_a^{LB} \leq P_a \leq P_a^{UB}; \quad a \in \mathcal{A} \quad (8)$$

$$\text{Constraints (3) and (4);} \quad (9)$$

B. SE based RT-SCED with Oscillations

Different from the conventional ED model (5)-(9), MISO builds the incremental dispatch based RT-SCED model which leverages the most recent SE results to mitigate network modeling inaccuracy and make a good utilization of network capacities. Specifically, with the feedback of SE results $\hat{P}_{G,i}$, \hat{P}_a , $\hat{P}_{L,i}$, and \hat{P}_l , constraint (3) is revised as (10).

$$P_l = \hat{P}_l + \sum_{i \in \mathcal{J}} SF_{l,i} \cdot [(P_{G,i} - P_{L,i}) - (\hat{P}_{G,i} - \hat{P}_{L,i})] + \sum_{a \in \mathcal{A}} [\sum_{i \in \mathcal{J}_a} (SF_{l,i} \cdot DF_{i,a}) \cdot (P_a - \hat{P}_a)]; \quad l \in \mathcal{L} \quad (10)$$

With constraint (10), MISO executes the rolling based 5-minute RT-SCED calculation as follows: (i) RT-SCED for time interval t is executed; (ii) the system actual operation at time interval t occurs, following the RT-SCED instruction while making necessary adjustment as needed (i.e., when such instructions would lead to infeasibility of the physical system); (iii) SE results of time interval t are used to update $DF_{i,a}$; (iv) RT-SCED for the next time interval is executed.

However, owing to the inaccuracy of $DF_{i,a}$, contributions of

DERs to transmission lines could be over or under estimated, consequently RT-SCED results could be suboptimal or even infeasible to the physical system. Moreover, the SE feedback design in the incremental dispatch could lead to oscillation of LMPs and dispatch results along the consecutive RT-SCED intervals, as shown in Fig. 3. The details oscillation phenomena will be presented and analyzed in the case study section.

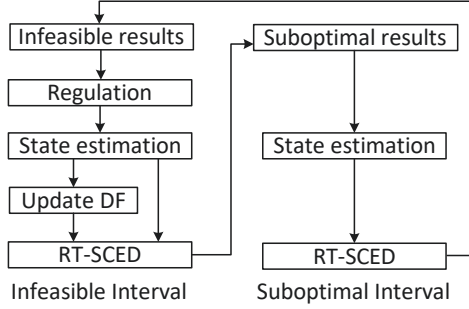


Fig. 3. Oscillation procedures of the RTED rolling process

III. THE PROPOSED DER AGGREGATION APPROACH

To avoid the inaccuracy of DF estimations introduced in the RT-SCED, we propose a network equivalent transformation approach, together with a projection method, to accurately formulate DER integration in the RT-SCED model.

A. Network Equivalent Transformation

The basic idea of the network equivalent transformation is shown in Fig. 4, which substitutes the original transmission line l by two pseudo lines l'' and l' with the same reactance as the original line. This equivalent transformation converts the DER aggregator to be single-node connected to the transmission system, so that its contribution to power flows of transmission lines can be accurately evaluated without the use of DF.

Specifically, the total power flow to the original transmission line is represented as $P_{l''} + P_{l'}$, as shown in (11), where $P_{l'}$ describes the contribution of the aggregated DERs to power flow of the original transmission line, and $P_{l''}$ represents the contribution of all other assets to power flow of the original transmission line. It is noteworthy that this equivalence holds irrelevant to flow directions of $P_{l''}$ and $P_{l'}$.

$$-P_l^{UB} \leq P_{l''} + P_{l'} \leq P_l^{UB}; \quad l, l', l'' \in \mathcal{L} \quad (11)$$

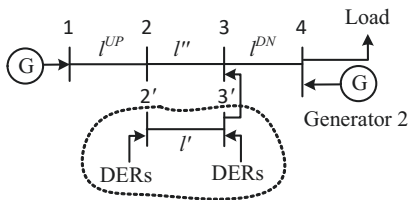


Fig. 4. Network equivalent transformation

However, with the above network equivalent transformation, the aggregated bidding curve cannot be constructed as the simple combination of individual DERs' bidding curves. The reason is that the aggregated bidding curve is also affected by the available transmission capacity, i.e., the distance of $P_{l''}$ to its boundary. With this, the bidding curve can be synthesized via an optimization problem of aggregator (12)-

(16). Constraint (14) calculates power flow of line l' . Constraint (15) is the total bidding power of the aggregator. Constraint (16) limits the outputs of individual DERs.

$$\min C_a(P_a, P_{l''}) = \sum_{i \in \mathcal{a}} c_{D,i} \cdot P_{D,i}; \quad (12)$$

$$-P_l^{UB} - P_{l''} \leq P_{l'} \leq P_l^{UB} - P_{l''}; \quad l, l', l'' \in \mathcal{L} \quad (13)$$

$$P_{l'} = \sum_{i \in \mathcal{a}} S F_{l,i} \cdot P_{D,i}; \quad l, l' \in \mathcal{L} \quad (14)$$

$$P_a = \sum_{i \in \mathcal{a}} P_{D,i}; \quad (15)$$

$$P_{D,i}^{LB} \leq P_{D,i} \leq P_{D,i}^{UB}; \quad i \in \mathcal{J}_a \quad (16)$$

The steps described above are tabulated as Algorithm 1.

Algorithm 1

1. For each group of DERs to be aggregated and the associated critical lines, build the equivalent network with extra pseudo lines, as shown in Fig. 4.
2. Execute the optimization model (12)-(16) multiple times, one for each pair of the aggregated dispatch and pricing values.
3. Calculate the aggregated power flows through pseudo lines for each dispatch level in step 2.
4. Generate the aggregated bidding curve for each group of DERs via dispatch, power flow, and pricing results from steps 2 and 3.

Similar to the CP node aggregation, the proposed approach generates one aggregated bidding curve for each DER aggregator. This will greatly reduce the computational complexity as compared to directly modeling all DERs in the RT-SCED model. Moreover, unlike the CP node aggregation which approximates power flows via DF, the proposed aggregation accurately calculates power flow contributions of individual DERs within the aggregation to the original transmission lines via (12)-(16). Thus, the DF estimation is not needed, and the oscillation can be effectively mitigated.

B. Bidding Curve Linearization

Instead of solving the optimization problem (12)-(16) for innumerable pairs of P_a and $P_{l''}$, we propose to construct the aggregated bidding curve via the linearization approach on the 2-D map of P_a and $P_{l''}$.

First, the feasible region of the DER aggregator on the 2-D map of P_a and $P_{l''}$ is calculated, as the shaded area shown in Fig. 5. This can be done by projecting all the constraints involved in the DER aggregator on the 2-D map of P_a and $P_{l''}$. That is, each equality constraint is processed by Gaussian elimination [8], and other constraints are projected by Fourier-Motzkin elimination [9].

Then, a rectangular area containing the entire feasible region is identified. We set the number of equidistant segments as 2^M for each axis via an integer parameter M , which produces $(2^M + 1)^2$ vertices in the entire rectangular area. Based on each set of the 3-neighbored vertices, a triangulated area, in a form of Union Jack structure, is established as shown in Fig. 5, where M is set as 3. Therefore, the bidding price of any point $(P_a, P_{l''})$ in the feasible region can be linearly approximated via the weighted average of its three neighbor vertices.

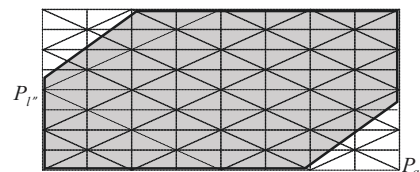


Fig. 5. linearly approximation according to Union Jack structure

IV. CASE STUDY

In this section, the 4-node system shown in Section III is first used to elaborate the oscillation procedure caused by inaccurate DFs, as well as efficacy of the proposed network equivalence aggregation approach in mitigating such oscillations. The proposed approach is further tested on an IEEE 30-bus system with multiple DER aggregators.

A. 4-node System Study

Bidding parameters of the 4-node system in Fig. 1 is listed in Table I. With the load of 15MW at bus 4, we simulate the rolling RT-SCED process based on current MISO practice, considering potential congestion of line l_{23} with the capacity of 10MW.

TABLE I. ORIGINAL BIDDING INFORMATION OF EP NODES

Asset	G,1	G,4	D,2	D,3
P^{UB} (MW)	8	10	4	2
c (\$/MWh)	6	16	12	8

• The current DER Aggregation Strategies

$DF_{2,a}$ and $DF_{3,a}$ are fixed as 66.7% and 33.3% according to their capacities (i.e., 4MW and 2MW). The rolling RT-SCED solution enters an oscillation process, as shown in Table II.

Numerically, $P_{G,1}$ with the lowest cost is dispatched at its maximum capacity of 8MW. The aggregated bidding prices of $P_{D,2}$ and $P_{D,3}$ are 8 and 12\$/MWh, lower than that of $P_{G,4}$, so the aggregator would be dispatched next. At this point, the remaining transmission line capacity from node 2 to node 3 is 2MW. According to the DF parameters of the aggregator, power flow constraints (3)-(4) allow the aggregator to generate 3MW (i.e., 66.7%*3MW=2MW). The remaining 4MW load is balanced by $P_{G,4}$. However, when the aggregator receives the 3MW dispatch instruction, it will schedule 1MW from $P_{D,2}$ and 2MW from $P_{D,3}$ with the minimum cost, which results 9MW actual power flowing through line l_{23} . Thus, the fixed DFs in this RT-SCED slot over-estimates the contributions of DERs on line 2-3, leading system operated in a suboptimal status. Indeed, $P_{D,2}$, with a lower bidding price, is capable to supply more power but replaced by the higher pricing generator $P_{G,4}$.

With the SE results indicating 1MW spare capacity on line 2-3, ISO in the next time slot can promptly adjust the economic dispatch according to incremental power constraint (10). To this end, the unused 1MW capacity on the line enables the DER aggregator to provide 1.5 MW extra power, because DFs approximate only 66.7% of the incremental power from P_a will be added to l_{23} . However, as shown in Table II, the increased dispatch of 1.5MW is all added on the $P_{D,2}$ because $P_{D,3}$ has been binding at the upper bound already. To this end, the DF approximation causes infeasible dispatch results, as power flow of l_{23} would be 10.5MW if the dispatch is strictly enforced, violating the capacity limit.

In the third slot of RT-SCED, based on the SE feedback information, the aggregator will reduce 0.75MW generation to relieve the approximated 0.5MW line power. After that, the dispatch falls into the suboptimal region again, because 0.25MW spare capacity remains on the transmission line.

Hereby, RT-SCED is oscillating between suboptimal and infeasible solutions in the consecutive time slots, with oscillated dispatches of $P_{G,4}$ and $P_{D,2}$ as shown in Table II. On the other hand, $P_{D,2}$ acts as the marginal asset and LMP of the aggregator remains 12\$/MWh.

TABLE II. RT-SCED WITH FIXED DISTRIBUTION FACTORS

Time Interval	1	2	3	4	5	6
$P_{G,1}$ (MW)	8.00	8.00	8.00	8.00	8.00	8.00
$P_{G,4}$ (MW)	4.00	2.50	3.25	2.88	3.06	2.97
$P_{D,2}$ (MW)	1.00	2.50	1.75	2.13	1.94	2.03
$P_{D,3}$ (MW)	2.00	2.00	2.00	2.00	2.00	2.00
$DF_{2,a}$	66.7%	66.7%	66.7%	66.7%	66.7%	66.7%
LMP_a (\$/MWh)	12	12	12	12	12	12

Other potential DF update strategies have also been tested, including using: (i) ratios of unscheduled capacities of DERs; (ii) ratios of DERs state estimation amount; and (iii) the weighted average of DFs in the past several time intervals. However, none of these methods can fully eliminate the oscillation.

• The Proposed DER Aggregation Strategy

Our proposed method, by adopting the network equivalent transformation to formulate certain critical transmission constraints and building the aggregated bidding curves, could effectively avoid the usage of approximated DFs and resolve the oscillation issue. First, all constraints are projected onto the 2-D map of $P_{l_{23}}''$ and P_a ; After Gaussian and Fourier Motzkin elimination processes, the feasible region can be obtained as shown in Fig. 6.

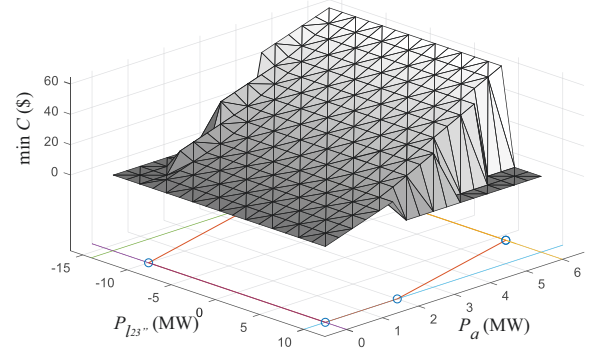


Fig. 6. Projected feasible region and bidding curve of aggregation

Fig. 6 shows that when $P_a = 0$, $P_{l_{23}}''$ can vary from -10MW to 10MW, shown as the purple boundary lines of the feasible region. In addition, the blue boundary lines of the feasible region show that when $P_{l_{23}}'' = 10$ MW (i.e., the original line is binding), P_a can only provide power up to 2MW from $P_{D,3}$. As the spare transmission capacity emerges, P_a can provide more power from $P_{D,2}$ flowing through the critical line. Fig. 6 also shows that $P_{l_{23}}''$ can be smaller than -10MW, if $P_{l_{23}}''$ provides enough power flow in the positive direction to keep $P_{l_{23}}$ within the boundary. The power flow in the positive direction is supplied by the more expensive $P_{D,2}$, making the bidding price higher than the non-congestion situation with the same amount

of P_a . The local optimization method guarantees a feasible power flow solution of market clearing, comparing to the inaccurate power flow calculation with the DF estimation.

Economic performance against different M values are compared in Table IV. Linearization will inevitably lose accuracy to certain extent, but when M is larger than 4, the economic difference between the linearized model and global optimal solution (i.e., the optimal solution obtained by the EP node model) is within 1%. In addition, because all constraints indicated in Fig. 6 are included in the RT-SCED, all solutions remain feasible to the physical system.

TABLE III. AGGREGATION PERFORMANCE WITH DIFFERENT M

	$P_{G,1}/\text{MW}$	$P_{G,4}/\text{MW}$	$P_{D,2}/\text{MW}$	$P_{D,3}/\text{MW}$	Total cost/\$
$M=3$	7.49	3.01	2.50	2	139.08
$M=4$	8.00	3.25	1.75	2	137.00
$M=5$	8.00	3.16	1.84	2	136.63
Global Optimal	8.00	3.00	2.00	2	136.00

B. IEEE 30-bus System Study

We further use the IEEE 30-bus system to demonstrate the oscillation phenomena induced by the DF based DER integration approach, as well as efficacy of the proposed approach. Multiple DERs are added at different locations of the system, as shown in Fig. 7, with 5MW capacity of each location. Grouping these DERs into 3 aggregators and conducting the RT-SCED simulation as described in Section II, the oscillation occurs at nodes 22, 26, and 27.

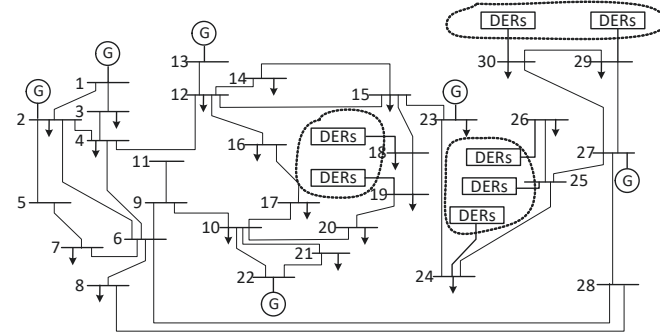


Fig. 7. Modified IEEE 30-node system

Fig. 8 compares solutions from different aggregation approaches, benchmarked with the global optimal solution. It is noted that the global optimal solution is obtained by explicitly formulating individual DERs in RT-SCED, which will not be achievable for practical systems at high DER penetration levels. Fig. 8 shows that the DF based aggregation approaches will cause oscillations at $P_{D,26}$. Specifically, using the fixed DF of 33.3% via (1), RT-SCED results of $P_{D,26}$ persistently oscillate between a suboptimal solution 0.69MW and an infeasible solution 1.9MW. Improvements via the updated DF through equation (2) or the weighted average DF of the past slots (i.e., adaptive DF) could converge the dispatch solution after several RT-SCED runs. In comparison, the proposed approach dispatches $P_{D,26}$ at 0.625MW with $M=4$. It benefits the DER aggregator to constantly and steadily operate within the feasible region. Although the system is operated in a suboptimal point, the profit loss is negligible because only a limited amount of

cheaper DER supplies is substituted by other generators of similar costs. In this case, the total cost of the system is \$331.05, which is merely 0.02% larger than the global optimal solution \$330.98.

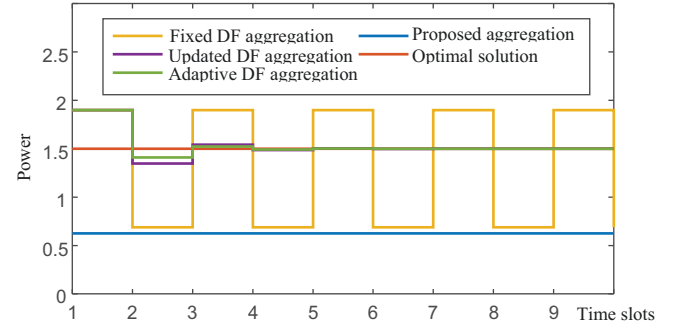


Fig. 8. Comparison on dispatch results for $P_{D,26}$ via different approaches

V. CONCLUSION

This paper analyzes the oscillation phenomenon across multiple time intervals when DER aggregations are simulated in the RT-SCED model via the DF approach. To this end, we propose a network equivalence based aggregation approach to mitigate the oscillation and ensure the feasibility of RT-SCED solutions. Numerical case studies show that the proposed approach can: (i) effectively mitigate oscillation by restricting solutions in the feasible region; and (ii) derive dispatch results that are close to the global optimal solution.

Dispatch and LMP oscillations could cause significant economic losses and even insecure operation. In the future, we will explore other RT-SCED oscillation issues in practical operations, with respect to aggregated bidding curves, system disturbance, price response, etc.

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