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Gravitomagnetic Dipole Moment of Gravitational Unit Cells

Thomas P. Weldon, Christopher G. Daniel Jr., and Kathryn L. Smith

Department of Electrical and Computer Engineering
University of North Carolina at Charlotte
Charlotte, NC, USA
tpweldon@uncc.edu

Abstract – **It is proposed that gravitational meta-atom unit cells with gravitomagnetic moments could exhibit gravitomagnetic permeability, analogous to the magnetic permeability of materials comprised of atoms with magnetic moments. Recently, a gravitoelectromagnetic (GEM) framework was proposed to explore the possibility of a Veselago-inspired approach to gravitational metamaterials. The prospect of gravitational metamaterials motivates the consideration of candidate gravitational unit cells or gravitational meta-atoms. Although mass serves as a monopole source of a gravitoelectric field similar to positive charge, negative mass would be needed to create a gravitational analog of an electric dipole. However, moving mass is analogous to electric current, and can lead to a gravitomagnetic dipole moment analogous to magnetic dipole moments of magnetic materials and atoms. In this paper, GEM field approximations to general relativity are used to find the gravitomagnetic dipole moment of different rotating systems, ranging in scale from meters to astronomical size.**

I. Introduction

Gravitoelectromagnetic (GEM) field equations have a long history, dating from Heaviside's 1893 proposition of a gravitomagnetic field, to more modern formulations based on weak-field solutions of the Einstein field equations [1–4]. The various formulations of GEM equations are similar to Maxwell's equations of electromagnetism. More recently, gravitationally-small radiators have been proposed as the basis for gravitational unit cells in [5, 6]. Of particular interest in the present work are the gravitomagnetic fields generated by rotating masses and the associated gravitomagnetic dipole moments, where gravitomagnetic dipole meta-atom unit cells may behave analogously to atoms in conventional magnetic materials [7].

In this paper, we consider astronomical objects and small objects that exhibit a gravitomagentic dipole moment that may form the basis for a gravitational unit cell. It is proposed that gravitational meta-atoms with gravitomagnetic moments could exhibit gravitomagnetic permeability, analogous to the magnetic permeability of materials comprised of atoms with magnetic moments. Such unit cells can be at astronomical scales while remaining gravitationally small, since observed gravitational waves are typically well below 1 kHz and orbital phenomina can have hundred-year periods.

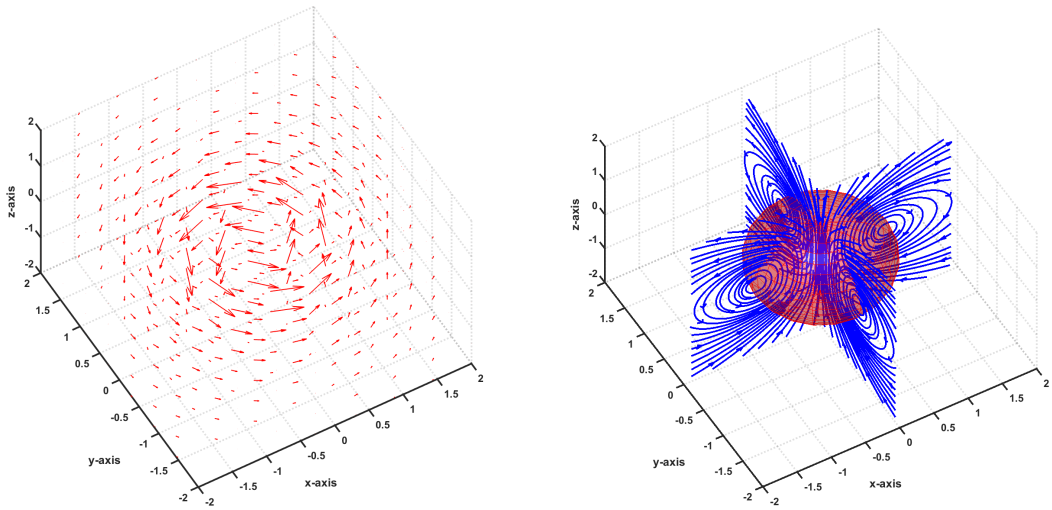
II. GEM Equations

More recently, gravitoelectromagnetic (GEM) field approximations to general relativity have been used in the study of gravitational phenomena [3–5]. In GEM, gravitational analogs of the electric field and magnetic fields result in the following GEM equations [4, 5].

$$\nabla\cdot\mathbf{E}=\nabla\cdot\left(-\nabla V-\frac{\partial\mathbf{A}}{\partial t}\right)=\frac{\rho_e}{\varepsilon}\quad\Rightarrow\quad\nabla\cdot\mathbf{E}_g=\nabla\cdot\left(-\nabla\Phi_g-\frac{1}{c}\frac{\partial\mathbf{A}_g/2}{\partial t}\right)=4\pi G\rho_m=\frac{\rho_m}{\varepsilon_g}\tag{1}$$

$$\nabla\cdot\mathbf{B}=0,\ \text{where}\ \mathbf{B}=\nabla\times\mathbf{A}\quad\Rightarrow\quad\nabla\cdot\mathbf{B}_g/2=0,\ \text{where}\ \mathbf{B}_g=\nabla\times\mathbf{A}_g\tag{2}$$

$$\nabla\times\mathbf{B}=\mu\mathbf{J}_e+\varepsilon\mu\frac{\partial\mathbf{E}}{\partial t}\quad\Rightarrow\quad\nabla\times\mathbf{B}_g/2=\frac{4\pi G}{c}\mathbf{J}_g+\frac{1}{c}\frac{\partial\mathbf{E}_g}{\partial t}=\mu_g\mathbf{J}_g+\frac{1}{c}\frac{\partial\mathbf{E}_g}{\partial t};\quad\mathbf{J}_g=\rho_m v\tag{3}$$



(a) (b)

Fig. 1: Gravitomagnetic field for a one-meter sphere with density 1000 kg/m^3 and spin of 100 rad/s. (a) Computed gravitomagnetic vector potential $\mathbf{A}_g$. (b) Computed gravitomagnetic $\mathbf{B}_g$ in blue for the rotating sphere in red.

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \quad\Rightarrow\quad \nabla\times\mathbf{E}_g = -\frac{1}{c}\frac{\partial\mathbf{B}_g/2}{\partial t} \tag{4}$$

$$\nabla^2\mathbf{E} = \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} \quad\Rightarrow\quad \nabla^2\mathbf{E}_g = \frac{1}{c^2}\frac{\partial^2\mathbf{E}_g}{\partial t^2} \tag{5}$$

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v}\times\mathbf{B} \quad\Rightarrow\quad \mathbf{F}_g = -m\,\mathbf{E}_g - \frac{2m}{c}\,\mathbf{v}\times\mathbf{B}_g\ , \tag{6}$$

where by comparison the left-hand equations are for electromagnetic fields and the right-hand equations are for gravitational fields (where we retain the "$\mathbf{B}_g/2$" terms in the equations above, following the form of equations in [4]). Thus the GEM field equations are quite similar to Maxwell's equations with charge in coulombs replaced by mass in kilograms, and extra factors of $1/c$ appearing in Mashoon's GEM equations. Above, $\mathbf{E}_g$ is in N·kg^{-1}, $\mathbf{B}_g$ is in N·kg^{-1}, Φ_g is in N·m·kg^{-1}, $\mathbf{A}_g$ is in N·m·kg^{-1}, ρ_m is mass density in kg·m^{-3}, v is velocity in m/s, $\epsilon_g = 1/(4\pi G)$ kg^2·N^{-1}·m^{-2}, $\mu_g = 4\pi G/c$ N·m·s·kg^{-2}, and $\epsilon_g\mu_g = 1/c$ where c is the speed of light in vacuum. In (6), the electromagnetic force $\mathbf{F}$ is determined by charge q, and GEM force $\mathbf{F}_g$ is determined by mass m. The less familiar gravitomagnetic force term $\frac{2m}{c}\mathbf{v}\times\mathbf{B}_g$ in (6) is observed in measurements of Lense-Thirring precession of satellites [8]. The A-fields created by static magnetic and gravitomagnetic dipoles are

$$\mathbf{A} = \frac{\mu_\circ}{4\pi}\frac{\mathbf{m}\times\mathbf{R}}{R^3} \quad\Rightarrow\quad \mathbf{A}_g = \frac{G}{c}\frac{\mathbf{m}_g\times\mathbf{R}}{R^3} = \frac{G}{c}\frac{\mathbf{L}_g\times\mathbf{R}}{R^3} = \frac{G}{c}\int_V \frac{2\mathbf{J}_g}{|\mathbf{R}-\mathbf{r}|}d^3\mathbf{r}\ , \tag{7}$$

where $\mathbf{m}$ is magnetic dipole moment in C·m^2·s^{-1}, $\mathbf{m}_g$ is gravitomagnetic dipole moment in kg·m^2·s^{-1}, and $\mathbf{L}_g = \omega\mathbf{I}_g$ is the angular momentum in kg·m^2·s^{-1} of a body with moment of inertia $\mathbf{I}_g$ rotating at ω rad/s. As an example, Fig. 1 shows the gravitomagnetic A$_g$-field and B$_g$-field for a one-meter radius sphere (in red) with density 1000 kg/m^3 and a spin of 100 rad/s. This clearly demonstrates the analogous gravitatomagnetic fields to electromagnetics with the gravitomagnetic A$_g$-field rotating around the origin and the gravitomagnetic B$_g$-field resembling a magnetic dipole with the B$_g$-field going from its north pole to its south pole.

It is proposed that just as materials comprised of atoms with magnetic moments can exhibit magnetic permeability, gravitational meta-atoms with gravitomagnetic moments could exhibit gravitatomagnetic permeability. Since current gravitational waves typically have astrophysical sources below 1 kHz, it is of interest to consider gravitational unit-cell candidates at various scales. Table 1 shows computed gravitomagnetic dipole moment for a range of meta-atom scales from 1 m to the solar system, along with the periods, mass, and angular momentum of the gravitational meta-atoms [9, 10]. The gravitomagnetic dipole moment in Table 1 was calculated from the angular momentum $\mathbf{m}_g = \mathbf{L}_g$, in accordance with (7), using values of $\mathbf{L}_g$ from [9, 10].

Table 1: Comparison of Gravitomagnetic Dipole Moments†

	Sun Spin	Earth Spin	Earth Orbit	Jupiter Orbit	Solar System	1 m Radius Water Sphere
Period (s)	2.2×10^{6}	8.62×10^{4}	3.17×10^{7}	3.75×10^{8}	7.25×10^{15}	π/50
Mass (kg)	1.99×10^{30}	5.97×10^{24}	5.97×10^{24}	1.90×10^{27}	2×10^{30}	4189
Gravitomagnetic Dipole Moment $\mathbf{m}_g$ (kg*m^2/s)	1.92×10^{41}	5.8×10^{33}	2.67×10^{40}	1.94×10^{43}	3.32×10^{43}	1.64×10^{5}

†Data in table is derived from [9, 10].

III. Summary

Gravitational systems exhibit gravitomagnetic dipole moments, just as the atoms comprising conventional materials exhibit magnetic dipole moments. So, at astrophysical scales and the low frequencies of gravitational phenomena, it is proposed that large-scale astronomical objects may serve as meta-atoms with gravitomagnetic dipole moments. In this article, gravitomagnetic dipole moments were presented for gravitational meta-atom systems ranging in size from 1 m to the solar system. The extremely low frequencies of gravitational sources (such as hundred-year orbits) would suggest that even large-scale gravitational systems may serve as meta-atoms at various frequencies. It is even possible that gravitomagnetic dipole moments of the solar systems in a galaxy are in alignment with each other (similar to a ferromagnetic domain), under conservation of momentum theories for galactic formation [11].

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