Characterizing Connected and Automated Vehicle Platooning Vulnerability under Periodic Perturbation*

Pengcheng Wang, Xinkai Wu and Xiaozheng He

Abstract—**The performance of connected and automated vehicle (CAV) platoons, aimed at improving traffic efficiency and safety, depends on vehicle dynamics and communication reliability. However, CAVs are vulnerable to perturbations in vehicular communication. Such endogenous vulnerability can induce oscillatory dynamics to CAVs, leading to the failure of platooning. Differing from previous work on CAV platoon stability, this research exploits CAV platooning vulnerability under periodic perturbation by formulating the oscillatory dynamics as vibrations in a mechanical system. Akin to other mechanical systems, a CAV platoon has its inherent oscillation frequency, exhibiting unique characteristics in a perturbed travel environment. To this end, this paper proposes an approach to characterize the CAV platooning vulnerability using the mechanical vibration theory. The employed theory reveals that CAV platooning vulnerability mainly associates with its resonance frequency, through which a small periodic perturbation can amplify the platoon oscillation. The analytical formulation and simulation results show that preventing periodic perturbations from a platoon's resonance frequency is crucial to enhance the CAV platooning reliability and suppress large amplitude oscillations, helping to secure the expected benefits of CAV platoons.**

I. INTRODUCTION

Connected and automated vehicle (CAV) technology paves new pathways for improving mobility, safety, sustainability leveraging the advancements in sensing, computation, automation, and communication [1, 2]. Overall, CAVs extend the capability of manual vehicles by allowing vehicles to share information through connections to improve road safety [3], decreasing congestion [4], and reducing energy use [5]. Wireless communication is one of the major enablers for developing connectivity among CAVs, which supports vehicle-to-vehicle and vehicle-to-infrastructure communications to enable collaborative travel among CAVs. Supported by applications such as cooperative adaptive cruise control, CAVs can travel collaboratively with their preceding vehicles to form a platoon maintaining a short headway. Such a platoon-based travel mode conveys additional benefits for CAV systems. It is expected that CAV platoons will be common on the roadways in the future. Hence, platoon stability is crucial to warrant the performance of future CAV traffic flow, where small disturbances could be amplified and lead to stop-and-go traffic waves, even traffic paralysis or collisions [6].

Platoon stability is an important attribute to assess the performance of CAV control strategies. Over the past few years, vast studies have analyzed the stability of CAV platoon under diverse platoon control protocols. For example, Li *et al.* [7] aimed to design a distributed integral sliding-mode control strategy for CAV platoon and analyze its stability by using the Lyapunov technique. Besselink and Johansson [8] presented a delay-based spacing policy for the control of vehicle platoons and analyzed its string stability. Zhou *et al.* [9] developed a car-following control strategy of CAVs to stabilize a mixed vehicular platoon consisting of CAVs and human-driven vehicles and derived a string stability criterion for a mixed vehicular platoon. Moreover, other studies have investigated various platoon stability analysis approaches. For example, Feng *et al.* [10] categorized the commonly-used methods for vehicle platoon into three types, *i.e.*, *z*-domain, *s*-domain, and Time-domain analysis methods, and discussed the relations between ambiguous definitions, analysis methods, and their derived properties.

Unlike those studies focusing on platoon stability, this paper introduces an innovative metric to characterize the platoon *instability*, which is inspired by mechanical vibration in a spring-mass system. As mentioned in [11], each object in the real world is vibrating at a specific frequency. Vibration is everywhere, thereby causing instable systems. To uncover how a CAV platoon responds to ubiquitous perturbations, this research aims to exploit CAV platoon inherent vulnerability from the standpoint of traffic instability by formulating oscillatory dynamics of CAV platoon as vibrations in a mechanical system. To this end, this paper proposes an approach to formulate the vulnerability, *i.e.,* fluctuation characteristics, of CAV platoons under periodic perturbation based on the mechanical vibration theory. The proposed approach helps to analyze platoon instability under periodic perturbations. The employed mechanical vibration theory reveals that a CAV platoon's vulnerability mainly associates with its resonance frequency, through which a small perturbation can amplify the platoon oscillation.

The main contributions of this study are summarized as follows. (i) This paper proposes an approach to characterize the vulnerability of CAV platoons under periodic perturbation

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based on the mechanical vibration theory. (ii) The inherent oscillatory frequency of a CAV platoon is formulated as a function of vehicle control parameter, which is a proxy of the stiffness coefficient of an object in mechanical vibration theory. (iii) The analytical characterization of inherent vulnerability, supported by simulation results, shows that a CAV platoon is vulnerable to periodic perturbations with a frequency same as its inherent frequency, also known as resonance. The analysis of CAV platoon vulnerability conveys the insight that preventing perturbations from the resonance frequency is crucial to enhance the CAV platooning reliability and suppress large amplitude oscillations.

This study is organized as follows. Section II presents a generic car-following model that captures the vehicular dynamical behaviors. Then the generic car-following model is reformulated into the form of a mechanical vibration equation with damper, based on the linear stability theory and mechanical vibration theory. Section III carries out a simulation study to investigate the validity of the proposed metric for CAV platooning vulnerability. Section IV concludes our findings and suggests future research directions.

II. System Model

A. Modeling of the platooned vehicles

To make our proposed approach potentially compatible with most vehicle platoon models, this paper adopts a generic car-following formulation to describe the vehicle's longitudinal dynamics. The continuous-time car-following models have a generalized form as follows [12-14]:

$$\begin{aligned} \dot{x}_n(t) &= v_n(t), \\ \dot{v}_n(t) &= f\left(s_n(t), v_n(t), \Delta v_n(t)\right), \end{aligned} \tag{1}$$

where x_n represents the position of vehicle n, v_n represents the speed of vehicle n, and f represents an acceleration stimuli function, which is determined by vehicle speed v_n, inter-vehicle spacing $s_n = x_{n-1} - x_n$, and relative velocity $\Delta v_n = v_{n-1} - v_n$ between the adjacent vehicles in the platoon. In the typical car-following scheme, each vehicle follows its preceding vehicle in a single lane without overtaking, as shown in Fig. 1.

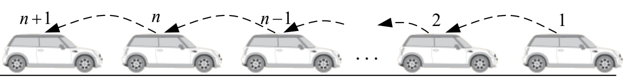


Figure 1 Illustration of the platooned vehicles on a single lane

The generic car-following model presented by Eq. (1) captures the following main characteristics: (i) reaction with current vehicle state and (ii) reaction only to the direct predecessor. In fact, Eq. (1) can be further extended to cover more complicated platooning dynamics models, *e.g.*, with reactions to the platoon leader and other predecessors in the platoon through time-varying communication topologies [15, 16]. To some extent, such models can represent the capabilities of drivers or CAV control systems regarding the input stimuli v_n, s_n and Δv_n. Most car-following models can reproduce traffic waves or other disturbance shapes of traffic flow. The generic formulation is more comprehensive than the various variants in the literature [17, 18], and the derived results can be transferred to any car-following model with a well-defined acceleration stimuli function.

The generic model Eq. (1) can well characterize the car-following behaviors for CAVs since vehicles equipped with the vehicle-to-everything (V2X) communication capability intend to "duplicate" its directly preceding vehicle behavior based on the predecessor-following (PF) communication topology [14]. However, when a periodic perturbation is imposed on the vehicle platoon, the vehicle platoon becomes unstable. If the periodic perturbation is released with different frequencies, the fluctuation pattern of the traffic flow changes. Indeed, periodic perturbation is a ubiquitous event or process in nature, such as sound, noise, and vibration, occurring on varied spatial and temporal scales, which can immediately influence system stability [19, 20]. Therefore, to capture abnormal platoon vehicle behaviors, especially to uncover the relationship between the fluctuation amplitude/frequency of platoon vehicles and external perturbations, the next section introduces a constructive approach to address this issue.

B. Problem formulation

This section considers the scenario where a stable vehicle platoon is exerted by a perturbation [14]. Define v^{e} as the equilibrium velocity and s^{e} denotes the equilibrium velocity. Denote μ and y as the deviation of the velocity and space gap of vehicle n from the equilibrium, respectively, *i.e.*,

$$\begin{aligned} v_n &= v^{\mathrm{e}} + \mu, \\ s_n &= s^{\mathrm{e}} + y. \end{aligned} \tag{2}$$

At the equilibrium state where vehicles have identical space gap and velocity, there exists a function $v^{\mathrm{e}} = V(s^{\mathrm{e}})$ such that $f(s^{\mathrm{e}}, V(s^{\mathrm{e}}), 0) = 0$ for all $s^{\mathrm{e}} > 0$ based on Eq. (1). The perturbation, forcing the changes in space gap and velocity of equilibrium traffic flow, will cause fluctuating behavior of platoon vehicles [21]. This forced fluctuation is similar to mechanical vibration. For example, for a spring-mass system in the field of mechanical vibration, if one exerts an external force on an object and stretches or contracts the spring away from its equilibrium, the spring will vibrate back and forth around its equilibrium location and then gradually stop at the equilibrium location with the help of damper force once removing the external force.

Considering the similarity between the acceleration behavior in platoon vehicles and mechanical vibration, this study develops a formula to facilitate the analysis of platoon instability, as presented in Theorem 1.

Theorem 1. *The oscillatory dynamics of platooning vehicles follows the second-order homogeneous ordinary differential equation:*

$$\frac{d^2 y}{dt^2} + 2\omega_0 \xi \frac{dy}{dt} + \omega_0^2 y = 0, \tag{3}$$

where parameter $\xi = -\frac{1}{2}\left(f_n^v - f_n^{\Delta v}\right) \Big/ \sqrt{\left|f_n^s\right|}$ and $\omega_0^2 = f_n^s$ with f_n^v, f_n^s, and $f_n^{\Delta v}$ defined as

$$f_n^v=\left.\frac{\partial f}{\partial v_n}\right|_{(v^e,s^e)}, \quad f_n^s=\left.\frac{\partial f}{\partial s_n}\right|_{(v^e,s^e)}, \quad f_n^{\Delta v}=\left.\frac{\partial f}{\partial \Delta v_n}\right|_{(v^e,s^e)}.$$

Proof. When a change in the gap and/or velocity occurs, the uniform traffic flow becomes unstable. By applying the linear stability analysis theory of the traffic flow [22], substituting Eq. (2) into Eq. (1) yields the following equation:

$$\frac{\mathrm{d}\mu}{\mathrm{d}t}=f\left(s^e+y,v^e+\mu,-\mu\right). \tag{4}$$

Based on the differential equation $\frac{\mathrm{d}x_n}{dt}=v_n$ and Eq.(2), we have

$$\frac{\mathrm{d}y}{dt}=-\mu, \quad \frac{\mathrm{d}\mu}{\mathrm{d}t}=f\left(s^e+y,v^e+\mu,-\mu\right). \tag{5}$$

The first-order Taylor expansion of Eq. (5) leads to

$$\frac{\mathrm{d}\mu}{\mathrm{d}t}=f_n^v\mu+f_n^s y-f_n^{\Delta v}\mu+\text{nonlinear terms}, \tag{6}$$

Ignoring the nonlinear terms in Eq. (6), the linear part can be written as a single equation for the inter-vehicle spacing deviation:

$$\frac{\mathrm{d}^2 y}{dt^2}-\left(f_n^v-f_n^{\Delta v}\right)\frac{\mathrm{d}y}{dt}+f_n^s y=0. \tag{7}$$

Defining $-\left(f_n^v-f_n^{\Delta v}\right)=2\omega_0\xi$ and $f_n^s=\omega_0^2$, Eq. (7) can be rewritten as

$$\frac{d^2y}{dt^2}+2\omega_0\xi\frac{dy}{dt}+\omega_0^2 y=0,$$

The proof of Theorem 1 is completed. ■

Note that the form of Eq. (3) is akin to the mechanical vibration system, which is also the general form of the differential equation for the displacement of a particle in a linear system with viscous damping [23]. Here, ω_0 represents an inherent oscillation frequency, at which the system oscillates independently of any external excitation.

Parameter ξ in Eq. (3) is a measurement index of system performance, which is used to describe the damping capacity of a system against vibration. In particular, $\xi<1$ indicates an underdamped system while $\xi\geq 1$ indicates an overdamped system. In the context of vehicle platooning, $\xi<1$ represents the scenario that the perturbed platoon will be oscillating back and forth around the equilibrium state and $\xi\geq 1$ represents that the perturbed platoon can converge to its equilibrium state and will not oscillate. Therefore, a platoon control protocol that ensures $\xi\geq 1$ can enable the platoon to resist external perturbation. Such resistance is helpful in enhancing platoon performance reliability in a complex travel environment.

Compared to overdamped systems, this paper focuses on investigating the underdamped condition due to the concern that a perturbed vehicle platoon oscillating back and force may induce more safety risks and traffic efficiency reduction [24]. Furthermore, as shown by [25, 26] in the field of mechanical vibration, the largest vibration amplitude occurs at resonance when the resonance frequency coincides with the inherent oscillation frequency of the underdamped system. The resonance effect in CAV platoon will be verified in Section III.

C. *Vehicle control model*

This section specifies a car-following model, *i.e.*, Helly's model, to be the vehicle control model of CAV platooning due to its simplicity and wide application in car-following behaviors [27-29]. Furthermore, using Helly's model, the acceleration of a vehicle maintains a linear relationship with the deviation from the desired space gap and the velocity difference between two successive vehicles. Also, this model has been found to present a good fit to observed data [28] and has been used to design the Adaptive Cruise Control algorithm [31] that is suitable to describe the vehicle platooning behaviors. The model is expressed as

$$a_n(t)=\lambda_v\Delta v_n(t)+\lambda_x\left(\Delta x_n(t)-D(t)\right) \quad \text{with} \quad D(t)=D_{\min}+l_{n-1}+\tau v_n(t) \tag{8}$$

where λ_v represents the sensitivity to relative velocity, λ_x represents the sensitivity to distance difference between two successive vehicles, $D_n(t)$ is a desired following distance, $D_{\min}$ is the minimum distance allowed as a safety gap, l_n is the length of vehicle n, and τ represents reaction time. Note that distance D_n increases with an increased value of v_n.

When a periodic perturbation is exerted on an underdamped vehicle platoon, the vehicle platoon starts oscillating around its equilibrium state. Applying Eqs. (1)-(3) to the Helly's model, the platoon fluctuation can be characterized by the following equations:

$$f_n^v=-\lambda_x\tau,\ \ f_n^s=\lambda_x,\ f_n^{\Delta v}=\lambda_v, \tag{9}$$

and

$$\omega_0=\sqrt{\lambda_x},\quad \xi=\frac{1}{2}\left(\sqrt{\lambda_x}\tau+\lambda_v/\sqrt{\lambda_x}\right). \tag{10}$$

Eq. (10) establishes the analytical relationships between platoon's inherent oscillation characteristics and vehicle control parameters, as graphically illustrated in Fig. 2.

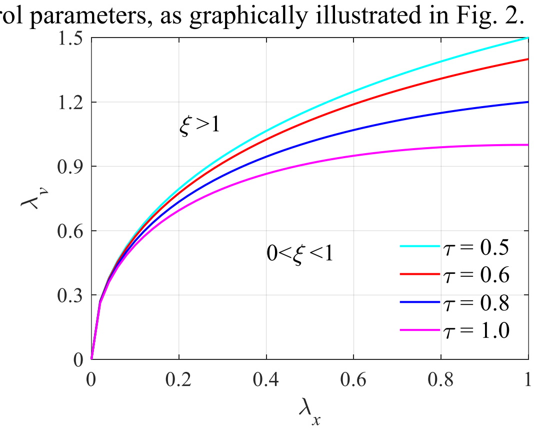


Figure 2 Changes of ξ with λ_v and λ_x

Fig. 2 shows the change of ξ with different reaction time τ and sensitivity parameters λ_v and λ_x. This figure illustrates that increasing the sensitivity parameter of λ_x and/or decreasing the sensitivity parameter of λ_v can weaken the fluctuation feature of the platoon. The area above the curves denotes the overdamped region, the area below the curves the underdamped region.

Moreover, the reaction time affects the performance reliability of the vehicle platoon. In general, the smaller the reaction time, the better the platoon efficiency because the shorter reaction time allows the following vehicle to response the preceding vehicle's operation timely, enabling a shorter headway or a higher traffic throughput. However, based on Eq. (8), the shorter reaction time yields a smaller space gap that could lead to traffic oscillation or even rear-end collisions under perturbation. As illustrated by Fig. 2, the shorter reaction time expands the underdamped region that increases the likelihood of oscillatory dynamics in the platoon. Therefore, Eq. (10) provides a quantitative metric for investigating the tradeoff between platoon efficiency and reliability.

At this point, we have derived analytical formulas for vehicle platoon fluctuation characteristics using mechanical vibration theory and presented that the proposed approach can be used to measure the inherent vulnerability of vehicle platoon. The next section will demonstrate how to apply the approach to analyze or suppress the impact of periodic perturbation.

III. SIMULATION

To prepare a simulation testbed for our study, we adopt the vehicle dynamics with Helly's model and the communication topology with PF scheme, which capture both continuous vehicle dynamics and discrete impact of inter-vehicle communication. The platoon consists of 10 CAVs, and the first leading vehicle remains its initial velocity during the entire simulation. At the initial stage, each vehicle's speed is 15m/s, and each vehicle's space gap is set as 10m. The total simulation duration is 80s. The parameters of the platoon are selected from existing studies [27, 32], as shown in Table 1. Unless otherwise specified, these parameters would not change.

Table 1 Parameters used in simulation

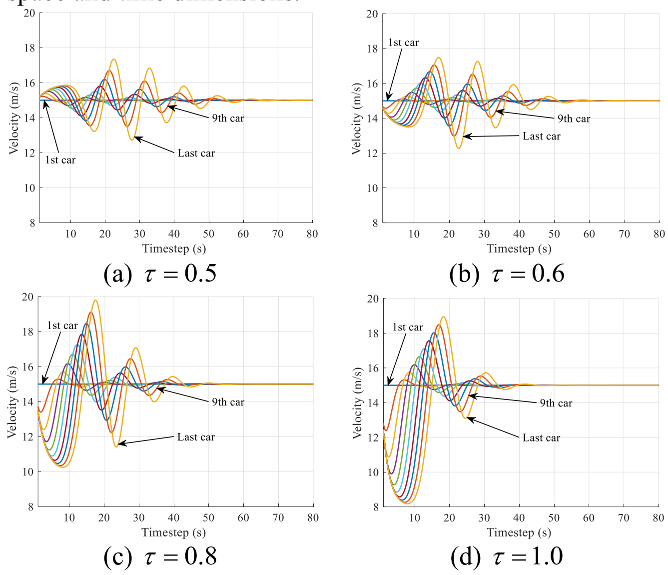
Symbol	Value	Description
v_0	120 km/h	Desired free-flow speed
l	5 m	Vehicle length
s_0	2 m	Minimum safety distance
a	3 m/s^2	Maximum acceleration
b	4 m/s^2	Maximum deceleration
Δt	0.1 s	Sampling time

Next, to verify the theoretical results presented in Section II, we construct two scenarios on the CAV platoon: no perturbation and periodic perturbation. Parameters $\lambda_x = 0.5$ and $\lambda_v = 0.3$ in both scenarios.

Scenario I: no perturbation

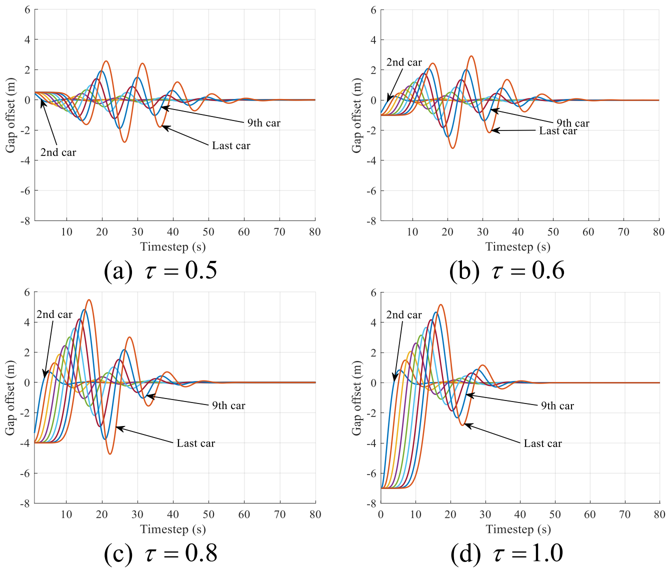
The first case is used to illustrate the performance of the simulated platoon with different reaction time in an environment without perturbation. Let $\tau = 0.5, 0.6, 0.8, 1.0$. Based on Eq. (10), we can derive the value of parameter $\xi = 0.3889, 0.4243, 0.4950, 0.5657$, respectively, and all ω_0 are equal to 0.7071. Hence, $\xi < 1$ representing the simulated platoon is underdamped.

The performance of the platoon is depicted in Figure 3 and Figure 4, where Figure 3 shows the change of each vehicle's velocity given different reaction time, and Fig. 4 shows the change of space gap offset given different reaction time. As shown in these two figures, with increasing reaction time, the fluctuation amplitude of the vehicle platoon turns larger, while the fluctuation ends earlier. In detail, the end time of fluctuation for $\tau = 0.5, 0.6, 0.8, 1.0$ are identified as 72.0s, 63.7s, 56.2s, and 47.4s, respectively. The longer reaction time has a larger stable space gap. The stable space gap is obtained as 9.5m, 11m, 14m, and 17m for $\tau = 0.5, 0.6, 0.8, 1.0$ which leads to the larger fluctuation amplitude. Although the shorter reaction time may cause a smaller space gap, which is a direct reflection of the traffic efficiency, it generates a longer fluctuation time, which agrees with the theoretical results and illustrates the tradeoff effect of increased reaction time in space and time dimensions.



(a) $\tau = 0.5$ (b) $\tau = 0.6$ (c) $\tau = 0.8$ (d) $\tau = 1.0$

Figure 3 Plots of velocity with different reaction time



(a) $\tau = 0.5$ (b) $\tau = 0.6$ (c) $\tau = 0.8$ (d) $\tau = 1.0$

Figure 4 Plots of Gap offset with different reaction time

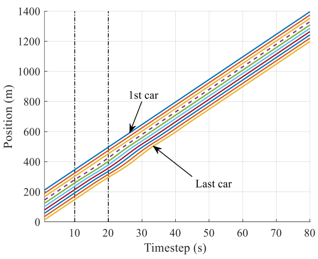
Scenario II: Periodic perturbation

In this scenario, a periodic perturbation with specified frequency and amplitude is exerted on the CAV platoon. Such a periodic perturbation may be from the control system's noise [33] or intentional malicious interference [14, 32]. Under extreme conditions, a small perturbation can amplify vehicle platoon oscillation. Hence, we attempt to verify whether the resonance frequency could lead to serious oscillation. Let $\tau = 1.0$, then, as Scenario I shows, $\omega_0 = 0.7071$ and $\xi = 0.5657 < 1$, representing an underdamped platoon.

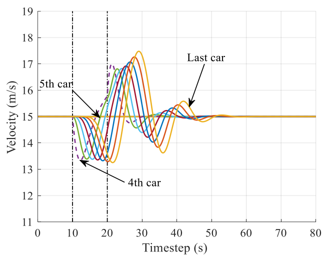
Without loss of generality, we assume the space gap error of the 4th vehicle is disturbed in a sinusoidal fashion, *i.e.*

$$\begin{aligned} s_n(t) &= s^e + y_n(t) \\ y_n(t) &= A\sin(\omega t) \end{aligned} \tag{11}$$

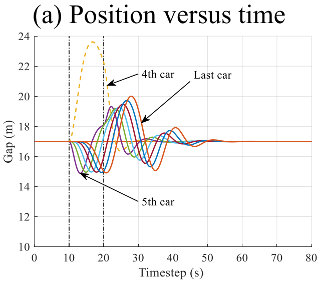
Figs. 5-7 show the impact of a perturbation with low frequency, resonance frequency, and high frequency on the CAV platoon, *i.e.* $\omega = 0.2$, $\omega = \omega_0 = 0.7071$, and ω=1.2, respectively. The disturbance appears in the period of 10-20s. Comparing Figs. 5-7, the disturbance with the resonance frequency in Fig. 6 causes the largest oscillation. In particular, the perturbation amplitudes of velocity, space gap, and acceleration in Fig. 6 are much larger than those in Fig. 5 or Fig. 7. The simulation results demonstrate that the disturbance with the resonance frequency on the vehicle platoon can have serious consequences. Besides, compared to Figs. 3d and 4d with the same reaction time, the periodic perturbation could cause severe oscillation behavior in the platoon.



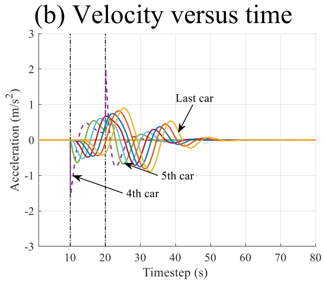
(a) Position versus time



(b) Velocity versus time

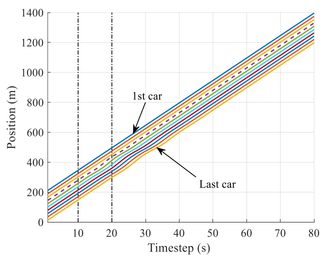


(c) Gap versus time

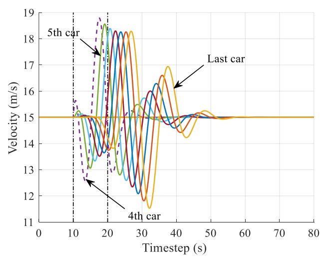


(d) Acceleration versus time

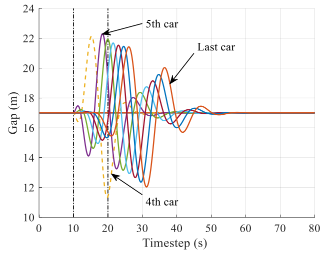
Figure 5 Plots of position, velocity, gap, and acceleration with time under $\omega = 0.2$



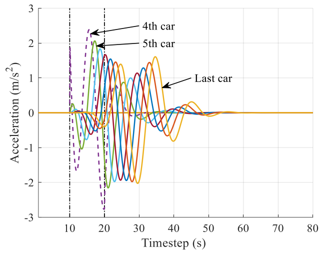
(a) Position versus time



(b) Velocity versus time

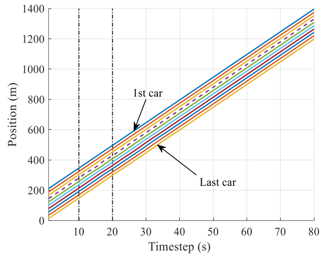


(c) Gap versus time

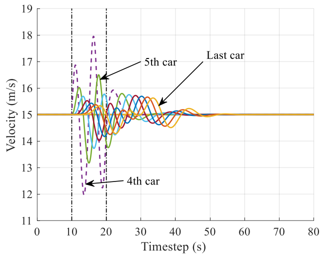


(d) Acceleration versus time

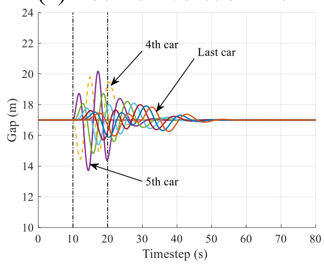
Figure 6 Plots of position, velocity, gap, and acceleration with time under $\omega = \omega_0 = 0.7071$



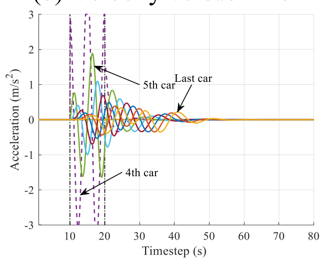
(a) Position versus time



(b) Velocity versus time



(c) Gap versus time



(d) Acceleration versus time

Figure 7 Plots of position, velocity, gap, and acceleration with time under $\omega = 1.2$

The simulation results show that continuous perturbations would lead to platoon oscillations and stop-and-go waves. Among periodic perturbations, the one with resonance frequency could cause the most severe oscillation. The simulation results not only present an application of the proposed approach to quantify CAV platoon vulnerability but also demonstrate that resonance frequency is a dangerous perturbation pattern that could cause dramatic consequences.

IV. CONCLUSIONS

The goal of this study focuses on exploring the vulnerability of the CAV platoon. The generic car-following model and Helly's model have been reformulated as a mathematical representation of the mechanical vibration based on the similarity between mechanical vibrations and platoon oscillations. The proposed approach can be used to characterize the vulnerability of CAV platoon and analyze platoon oscillation behavior. The theoretical results reveal that a CAV platoon's vulnerability mainly associates with its resonance frequency, through which a small perturbation can amplify its effect to increase the platoon oscillation amplitude. The paper demonstrates that resonance frequency could lead to serious oscillations among various periodic perturbation frequencies through simulation. As a result, preventing the perturbation frequency from the resonance frequency is crucial to enhance the CAV platooning reliability and suppress large amplitude oscillations.

The proposed approach for characterizing CAV platooning vulnerability can be further extended in several directions. First, this study only applies the proposed approach to a linear car-following model to develop closed-form formulas of inherent oscillation frequency ω_0 and damping parameter ξ. It will be interesting to investigate the oscillation frequency and damping parameter for nonlinear car-following models, such as the Intelligent Driver Model. Second, the theoretical derivation for CAV platooning vulnerability ignores the communication attributes, such as communication topology and delay, which affect the platoon stability as proved in the literature. These attributes certainly affect CAV platooning vulnerability and are expected to be integrated into formulas as an extension of Eq. (10). Third, this paper analyzes the CAV platooning vulnerability under a periodic perturbation formulated as a continuous sinusoidal function. Perturbations in the real world may present much complicate forms such as discrete pulse, random, or aperiodic perturbation. It will provide more insights by employing the proposed approach to measure the CAV platooning vulnerability under complicated travel environments that will help CAV platoons resist perturbations in the real world.

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