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Positive Emotions in Early Algebra Meaning-Making

Abstract: This research reports shifts in facial expressions while working on early algebra tasks observed in three students (grades 1–2) who took part in an early algebra classroom intervention. We found at the beginning of the intervention that the three students showed 'Aha' moments with smiles while they were developing understanding of new concepts. At the conclusion of the intervention, their smiles conveyed confidence when they noticed the tasks were related to previously learned concepts and recalled their meaning. Our analysis of the pre and post interviews revealed not only the ways in which the learners build mathematical meaning, but also how early algebra experiences support students in developing emotional experiences.

Introduction

Although mathematics is viewed as a gateway to technological skills and to higher education, mathematical literacy should be a goal for all students (Schoenfeld, 2002). However, mathematical ability is sometimes viewed as innate and unable to be honed by learners' effort alone (Gutierrez, 2018). As

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a result, some students believe they are not "math people" and give up their effort to learn advanced mathematics. It is not unusual to observe struggling students who are reluctantly participating in mathematical tasks. "When I hear the word math, I get goosebumps," a student explained about his negative feelings toward mathematics (Furner, Duffy, 2002, p. 68). The focus on negativity is reflected in the literature in mathematics education. There are more works of literature on math anxiety than on positive feelings toward mathematics (Ashcraft, 2002; Beilock, Willingham, 2014; Geist, 2015; Gough, 1954; Ramirez, Shaw, Maloney, 2018). Turning the discourse of math anxiety on its head and, instead, paying attention to students' positive emotions in mathematics could lead to an understanding of how to foster positive emotions. This is crucial, since positive emotions can help learners keep working on tasks and accelerate their mastery of them (Frederickson, 2001). Furthermore, mathematical learning that might shape emotions starts at an early age. Along with competency building, shaping positive motivation early on is fundamentally important because this creates long-term attitudes about mathematics (Jansen, Middleton, 2011).

In general, emotional reactions can be captured by features such as language, gestures, and facial expressions. Here, we are interested in facial expressions. The face is the first place to express a person's emotions through the use of various muscles, when students are actively engaging in discussion or tasks (Deleuze, 1986). Since young students have difficulty articulating memories eloquently (Hannula, 2015), interviewing learners after lessons would not capture the whole picture of the emotional responses of young learners. Therefore, this study observes young learners' facial expressions *during* learning.

These ideas of positive emotional experiences in early mathematics and meaning-making of mathematical concepts at an early age guide the research question for this study: *How do students' positive emotional responses during interactions relate to early algebra meaning-making?* This study explores learners' positive emotions, which can be seen as a physiological experience, in early algebra learning. In particular, the study investigates three students' positive emotional shifts and their physiological responses along with their cognitive development in early algebra. Given the challenges students have historically faced with learning algebra (Stigler et al., 1999) and the push in recent decades to begin (early) algebra instruction in kindergarten, it is important that we understand the role of positive emotional experiences in young learners' early algebra instruction. Describing positive emotions and looking for them in facial cues and gestures is important in understanding how these emotional expressions emerge while learners make sense of the early algebra concepts.

Literature Review

Emotions in Mathematics

Understandably, mathematicians and mathematics educators may take for granted that mathematics is enjoyable and easy to learn; however, this is not the case for children who have math anxiety. Many students, in fact, have negative feelings toward mathematics (i.e., math anxiety) (Ashcraft, 2002; Beilock, Willingham, 2014; Geist, 2015; Gough, 1954; Ramirez, Shaw, Maloney, 2018). While this is well documented in the literature, far less attention is paid to the positive emotions in mathematics learning (Black et al., 2019). Recently, some studies have identified positive emotions among 11–14-year-old learners (Hannula, 2015; Holm et al., 2017). However, because emotional experience and mathematics learning – as well as the negative feelings of math anxiety – begin at an early age (Maloney, Beilock, 2012; Ramirez et al., 2016), this paper aims to identify positive emotions in early aged learners in Grades 1 and 2.

Historically, there has been an emotional disparity between a "math person" and a "not a math person" (Palmer, 2009). Traditionally, the world of mathematics is perceived as an elite subject having an arcane image or utility beyond a layperson's knowledge (Schoenfeld, 2002). Katz (1997) explains having a specific image of a certain group of people is not beneficial as it risks creating a barrier between groups of people. In mathematics, those image barriers appear between mathematicians and laypeople. Heller (2015) explained this dichotomy with the expression, "To be or not to be a Math Person" (p. 1). In addition, based on a view of the innate ability of mathematics (Gutierrez, 2018), when some learners feel they are not good at understanding mathematical concepts, they perceive themselves as not a math person. They use this label as an excuse to give up on understanding or participating in mathematics tasks (Kimball, Smith, 2013). To reverse this trend of binary feeling, it is crucial to closely study how young learners' positive emotions develop.

Why Do Positive Emotions Matter in Mathematics?

Trying to closely uncover positive emotions in learning moments is important because positive affect itself triggers persistence in problem solving (Fredrickson, 2001), a key mathematical practice identified by the *Common Core State Standards* (NGA Center, CCSSO, 2010). Even further, positive emotional conditions accelerate mastery of the task compared to all other conditions (Fredrickson, 1998). Bearing this in mind, this study focuses on the learner's positive emotional responses in the context of algebra meaning making. In terms of ways of knowing, we are interested in *subjective knowing*, which refers to the case when an individual considers his or her knowledge "as primarily a result of... affective reactions to information and ideas" (Boaler, Greeno, 2000, p. 174). These affective reactions might be the genesis of building a mathematics identity, so understanding affective reactions to early algebra and at the early formation of children's algebraic identity is important.

In mathematics education, Muis et al. (2015) found that learners experience positive emotions more than negative emotions when they see mathematics as an important way of thinking. This suggests two different ways of thinking about mathematics that leads to two different cycles: positive emotions towards mathematics as an important way of thinking versus negative emotions towards mathematics as not an important subject. Without experiencing feelings of confidence toward mathematics, working on mathematical tasks can be a challenging and painful moment, and those learners who experience pain, again and again, do not feel a sense of belonging in mathematics classrooms (Gholson, Martin, 2019).

To counter negative feelings toward mathematics, Maloney and Beilock (2012) suggest that teaching strong math basic skills in numerical or spatial processing could reduce the negative feelings among early learners. Moreover, a recent study shows that early intervention in mathematics can support competency building (Jordan et al., 2009), and even further, teaching early algebra from kindergarten can be a way of supporting students' readiness to enter an advanced level of mathematics (Blanton et al., 2019). An effective early algebra intervention provides the chance to build a positive emotional experience because emotion is naturally tied to cognition (Hannula, 2015).

The Importance of Early Algebra Experiences

Implementing an effective early algebra intervention offers fundamental benefits to learners before they experience formal algebra in secondary education (Blanton et al., 2019). It not only builds competency but could also build a positive emotional experience given the relationship between experiences of developing competency and human emotion (Hannula, 2006). Nonetheless, there has been little attention to affective features in early aged learners (Batchelor et al., 2019), and none with regards to early algebraic thinking. This paper aims to address these gaps in the literature.

Theoretical Framework

Emotion and Cognition

In mathematics learning, traditional research has dealt with emotion and cognition separately, however, Efklides (2006) argues that emotion and cognition co-exist. Historically, there was a prevalent way of thinking referred to as Cartesian Dualism in education (Shahjahan, 2015), which referred to a way of thinking about 'mind, body, and spirit' as three separate factors in the process of learning. However, mind and body dualism cannot fully explain what is occurring in the situation of a learner (Shahjahan, 2015). Rather, it is important to see mathematics learning holistically to include various factors, such as emotions, language, or gestures, that are not separable from learning. Bringing perspectives of emotion researchers, Hannula (2015) summarizes emotions that have a relationship with the personal goals of human coping and adaptation. In mathematics classrooms, mathematical ideas, social features, and bodily experiences should be integrated in a learning environment (Núñez et al., 1999). A recent study by Roth and Walshaw (2019) confirms this by revisiting Vygotsky's (2012) assertion that affect and intellect are inseparable in psychology because they occur simultaneously in the life of the whole person. This provides an important rationale for the study reported here, which examines emotion in early algebra learning. To address the gap between emotion and cognition, by closely looking at positive emotional responses in early elementary students, this study aims to show how students exhibit their positive emotions along with the learning processes.

Meaning-making is an integral part of learning. Dewey (1933) explained understanding is grasping the meaning of objects and events. Pitvorec (2016) asserted that the pursuit of learning new mathematical concepts becomes a meaning-making activity. This paper deals with three concepts of algebra learning – mathematical equivalence, representations of an unknown quantity and functional thinking – that are closely tied to meaning-making activities. In this study, learners' meaning-making is observed along with their positive emotional responses. In addition, students' meanings are traced through interviews.

Moreover, meaning and its accompanying emotional responses may vary based on the situation such as learning a concept versus recalling the meaning in a different time. According to Moutsios-Rentzos and Kalozoumi-Paizi (2017), emotions can be stimulated by two different triggers. The first type is *externally referenced emotion* (Moutsios-Rentzos, Kalozoumi-Paizi, 2017) that is triggered by *actuality* (Skemp, 1979), or the physical reality surrounding a given situation. For example, a student sees difficult questions in a mathematics exam, and emotional responses are triggered by the physically given problem sets. The second is *internally referenced emotion* (Moutsios-Rentzos, Kalozoumi-Paizi, 2017) that is caused by one's *inner reality* (Skemp, 1979) or an individual awareness that might be different from how other people think. For example, a student shows smiley faces when he recalled his own memory of his successful problem-solving experiences in the past. Since this research includes pre- and post-interviews, Moutsios-Rentzos and Kalozoumi-Paizi's (2017) two emotional triggers are beneficial lenses to look at students' emotional responses within early algebra meaning-making across time.

Emotion and Physiology

Historically, the category of affect has been the subject of much research and discussion. Emotions are closely discussed with various terms: affect, attitude, identity, disposition, and so on. To have a better picture of young learners' emotional responses in algebra tasks, we should understand the extent of affect and emotion. In the late 20th century, McLeod (1992) categorized affect in three areas: beliefs, attitudes, and emotions. More specifically, Ekman (1992) divides emotions into six universal categories by facial expressions based on various cultural observations: happiness, sadness, fear, anger, surprise, and disgust. In addition to Ekman's approach, Sperling (2012) provides a physiological coding manual of emotions (see Appendix A) based on naturalistic observation.

The emotional expressions on the face are part of body movements and may happen in a short time period. Black et al. (2019) suggest that emotional expressions might occur at the moment of the feeling. In addition, Jansen and Middleton (2011) suggested having engagement in a moment allows the learners to realize that mathematics can be interesting and useful, and something they can do. Recently, physiological expressions of emotion are gaining more attention in educational research (Bellocchi et al., 2014; Liaw et al., 2021; Tan et al., 2021). Youdell et al. (2018) discuss physiological expressions in the classroom, but their focus is negative and stressful physiologically expressed emotions in learning. Not only avoiding negative feelings like math anxiety but also highlighting positive emotions could be a beneficial approach to teach mathematics (Batchelor et al., 2019; Boaler, 2008; Sarfaty et al., 2013).

This study takes a physiological frame of emotions (Sperling, 2012) to comprehend positive emotional responses with meaning-making. Among six emotional categories from Sperling's coding manual (2012), this study included two coding sections that are relevant to the aim of this study: happiness and surprise, especially in the moment of 'Aha' experiences. In Sperling's (2012) categories of emotion, happiness is the only positive emotion. However, Sperling's other category, surprise, also can be viewed as having a positive meaning because it includes insightful surprising expressions. Thus, this study included Sperling's two emotional categories of happiness and surprise to look at positive emotional responses in meaning-making. Also, because of the aim of the study, negative emotions are not considered: sadness, fear, anger, and disgust.

Methods of Analysis

The data in this study come from a larger cross-sectional study of grades K – 2 early algebra intervention. Eighteen early algebra lessons were taught at each grade level as part of the intervention. The study included 30 participants from two kindergarten, two grade 1, and two grade 2 classrooms. The classrooms, which are from two schools in two different states, included a diverse population of students. Interviews were collected three times during the academic year of 2018-2019: Pre-interviews in Oct 2018, mid-interviews in Jan 2019, and post-interviews in May 2019. All three cases of this study were selected from one school site based on its diversity in terms of student ethnicity (64% students of color), socioeconomic status (63% of students qualifying for free or reduced-price lunch), and language status (27% English Language Learners).

Among 30 interviewees, this study focused on two first-grade students, and one second-grade student based on occurrences of positive emotions in pre- and post-interviews on common interview questions. The purpose of the interview was to study the development of students' algebraic reasoning in the context of the intervention. The interviews were conducted by a member of the research team.

This paper deals with three new concepts of algebra learning: mathematical equivalence by using a number balance, representations of an unknown quantity, and understanding relationships between numbers in functional thinking. The first case is about a student who is introduced to the meaning of mathematical equivalence by using a number balance. The second case is about a student who is guided to use variables in an expression given a box that represents the unknown quantity. The third case is about a student who is introduced to writing a table relating the number of ducks to the number of duck feet to develop functional thinking.

Three selected students' video clips that included positive emotional responses were transcribed line-by-line. A member of the research team analyzed data based on the emotional observation coding manual in Table 1. Also, when there was a specific category of emotion in the discourse, the researcher added double parentheses to show the gestures and emotional responses. For example, when a student put a chip on the scale, the researcher put notes in the double parentheses: ((putting a chip on 6)) to describle the gesture.

Emotions	Observable Evidence
Happiness	Smiling, cheers, claps, dances, jumps, laughs, sings, eager, lip corners pulled up, cheeks raised, crinkling around eyes, giggling, affection, smirking, warm emotional tone, terms of endearment, physical touching that is intentional and not accidental.
Surprise	Raised eyebrows, dropped jaw and open mouth, widened eyes, intake of air/gasp, surprised emotional tone, an 'Aha' moment as in the person realizes or gets something that previously was not understood.

Table 1. Emotional Observation Coding Manual (Sperling, 2012, See Appendix A).

In addition to physiological coding, the study investigates a connection between emotion and mathematical meaning-making with interviews. To have a better understanding of how mathematical meaning exists or lasts with emotional responses from learners, interviews were conducted with each participant before and after the classroom intervention.

Findings from Analysis of Three Cases

Positive emotion is a factor in promoting competency (Fredrickson, 1998). All three cases showed that positive emotions co-existed with cognition, and these cases displayed a particular pattern: surprise, happiness, then the learners conveyed an understanding of a new early algebra concept regarding mathematical equality, unknown quantity, or functional thinking. Since this research only analyzed three learners, there could be additional types of emotional shifts associated with learning. Yet, these results contribute to our understanding of emotions in mathematics learning:

a) The work focuses on early elementary school children, and
b) The work focuses on positive emotional responses rather than negative ones.

In reporting the results, the following pseudonyms are used throughout: Eric for the first case, Amy for the second case, and Susie for the last case.

Case 1: Eric

Pre-Interview

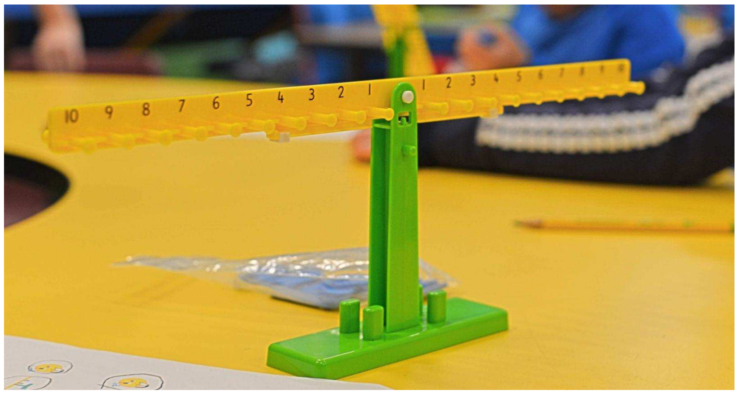

Figure 1. A Number Balance

The first task required students to determine the value of the blank in the number sentence $4 + 2 = \_ + 4$ with the expectation that students would determine the value of 2. Students could use a number balance (see Figure 1) to check whether or not the equation was balanced. This task was intended to support students' understanding regarding equation forms, namely, that equations can be written in different ways (e.g., $a + b = c$, $a + b = c + d$). The task was also intended to help students to develop a relational understanding of the equal sign. In this case, Eric, a first-grade student, insisted on putting 6 in the blank initially (demonstrating an operational view of the equal sign), but while using a number balance, he changed his mind to put a chip on 2.

I: Okay, good. One on 4, and one on 2, perfect.
S: Put this on ((and 4 on the other side of a number balance))
I: Yes. What would we do for the next?
S: We need to put it on this side ((pointing at the other side of the number line scale))
I: Yes, we do. What would we try a number balance to do?
S: To balance?
I: Yes. Very good!
S: ((putting a chip on 6, the number line scale is tilted and he seems confused))
I: Why did you put it on 6?
S: I was just guessing. Because if I put it this side ((pointing to 1, 2, 3)), it won't be enough to make it balanced.
Maybe, I could put a chip on 1, 2, 3.

I: Okay, let's try it.
S: Maybe 2? ((putting chips on 2)) Oh, it does! Oh! Because it's the same. ((Looking chips on both sides with a big smile))
I: What do you mean it is the same?
S: Because, here ((on the left side of the number line scale)) this is 4 and 2, and this is 2 and this is 4 ((on the right side of the number line scale-smiling)).
I: It absolutely does. This is 4 plus 2, and this says 2 plus 4 ((pointing each side of the equation)). So, does this still matter? Is this still the same as adding them together?
S: Yes.
I: How do you know from a number balance?
S: Because these are the same number, for example, if this is five and that is five. It is still balanced.
I: It is still balanced. Really good!

In testing his initial prediction that 6 goes in the blank, he observed that the number balance was tilted, and he seemed to be confused since the result was contradictory to his initial conjecture. He then decided to put a chip on number 2 (Line 15) and noticed that both sides of the number balance had chips in identical locations (i.e., locations 4 and 2 on the left, and 2 and 4 on the right). Upon recognition of the same numbers on both sides, he showed a big smile. Thus, in his work on this task, he displayed a pattern of surprise, happiness, and meaning-making.

Post-Interview

Given the same task of $4 + 2 = \_ + 4$ and use of the number balance to model the equation, the following excerpt shows Eric's post-interview work.

I: How about this one?
S: $4 + 2 = \_ + 4$ ((Looking at the question)) Oh!! ((surprise)) ((Fill number 2 in the blank)) two.
I: Great! How did you get the 2?
S: Because, because, because, it is kind of like this ((pointing above task, $6 = 6$)) but you just do a plus and then a 4 and you can just add two.
I: OK. Did you have to actually figure out how much 4 plus 2 was equal to?
S: Yeah!
I: How did you do it?
S: I had to know, because um, because um because right there might be three or something.
I: But you knew 2 put in there?
S: It is kind of like the same thing.
I: What is the same thing? Show me.
S: It is kind of like the same thing ((pointing at both sides of the equation)) but it is switched around.

I: You mean the numbers ((pointing at both sides of the equation)) are the same, but switched around?
S: Yes. ((Nods. Head up and down))
I: Okay, they are. That's a good thing to notice. That actually helps us make a little easier.
I: Let me ask this using that idea. Let's pick some big numbers here. What about. Can I use 123? Have you seen that number before? That's a big number, isn't it? 123 plus, can I say 29? equals blank plus 123? ((write down $123 + 29 = \_\_ + 123$)) What could we have in the blank?
S: ((Write down 29, Cheeks raised: Smile-Happiness)) Twenty-nine.
I: 29? Good. See how you answered that question, you did not have to be able to add them all together. That's good. Because you noticed something really important that they are the same numbers switched around. ((pointing at the both sides of the equation)). Great!

With the same task from the pre-interview, Eric correctly identified the number 2 without needing to use the number balance. He smiled, but this time his smile appeared to reflect confidence in knowing immediately the correct number rather than the surprise his smile reflected in the pre-interview – the 'Aha' moment seemed to be associated with learning. Another smile of confidence was captured for the next task, $123 + 29 = \_\_ + 123$ (Line 26), as Eric answered 29 without hesitation and again seemed to smile confidently. In short, a difference in the nature of the smiles between the pre- and post-interviews may be that the first smile came from an epistemological moment of Aha, but the later one came from confirmation of the learned knowledge.

Case 2: Amy

Pre-Interview

The second task centered on representations of an unknown quantity. Students were shown a closed box and were told it contained candies, but we didn't know how many. The interviewer introduced the idea of using a letter (H) to symbolically represent this unknown number of candies. "When we do not know the exact number, in mathematics we can use a letter instead of a specific number." Then, the interviewer asked a follow-up question (line 10) about the unknown quantity in a box of paperclips. The interviewer placed three cubes representing three additional paperclips on the top of the box and asked the student how the new quantity might be represented using the letter. The student then wrote $T+3$. The following excerpt captures the exchange between the interviewer and the student:

I: When we do not know the exact number, in mathematics we can use a letter instead of a specific number.
S: ((head down and asks)) What? ((surprise)) That's not a number, it's a letter. ((smile))
I: "H." That's how many pieces are inside the box, ((pointing at 2 cubes above a box)) but I have two more on top. So, I know that all together have the number of pieces inside ((point the letter "H")), plus two more of the outside. That's what I would call it. Have you ever seen anything like that?
S: Uh, uh. ((smile-Happiness))
I: If your friend has 't' clips in his box, and we give him three more in addition to that, how many does he have altogether? Please write down for me?
S: ((grab a pencil and write $T + 3$))
I: Tell me what you wrote. '$T + 3$' Look at you. That is so amazing!
S: T plus 3, J? equals?
I: ((open her two hands widely)) Equals J? We can say equals J. Can you write that?
S: ((Write down $T + 3 = J$))

In Line 3, Amy exhibited a surprise (based on the high tone of her voice) when the unexpected and surprising new idea that a letter could be used to represent an unknown number, and then smiled in happiness (Line 4). After the emotional response faded, the interviewer asked a follow-up question and Amy responded by writing the equation $T + 3$ successfully (Line 12), a response that suggests she is accepting the use of a variable to represent an unknown value.

Post Interview

Given the same task, the following exchange took place.

I: My friend, Fran, has a box of candy. It is Skittles. But, here is what she did. She ate some of them. We don't know how many are in there. And, she taped it up. How can we represent the number of Skittles in the box? We don't know how many there are. What would we do in math to represent that?
S: It sounds like a lot because ((She shakes the box of Skittles))
I: It does. But, she says, "No counting." So, actually, we can't find it. What would we do it?
S: It could be. ((she brings cubes)) Like a hundred?
I: It could be. But, we don't know.
S: It could be like a hundred and two.
I: It could be. But, do we know it for sure?
S: ((Shake her heads left and right))
I: Oh, we don't, you are right. Have you ever used a letter in your math classes? When you don't know how many there are?
S: ((Nods her head))

I: Let's call it something. Can you do something like that? Do you have any ideas?
S: Hmm. "B"?
I: B? Okay. What does B stand for?
S: Skittles.
I: Does B stand for the actual skittles or the number of Skittles in the box?
S: The number.
I: Good! So, you are going to let B stand for the number of skittles in the box. Very good. Here is what is going to happen. I don't have them, but she told her mom gave her three more. How would you talk about the total number of Skittles she now has?
S: ((Write down $B + 3$)) B plus 3 equals B.
I: Okay. B plus 3 equals B? ((pointing to the first B)) That is the number of skittles. Where does the plus 3 come from?
S: ((pointing 3 cubes on the top of the box with her pencil three times))
I: Three on the top? Awesome! Then, why did you say it is equal to B?
S: Because that's the same number. Because if it is not if we just erase it, and ((erase the second B)) put C. But, then we could think of something else.
I: So, if we put C there, what would you think the C stands for?
S: Nothing?
I: If we put there B, to begin with, if that is the number of Skittles, and we are gonna increase it by three, is it gonna be equal to the B ((pointing the first B in the equation)) the amount here?
S: ((shake her heads left to right))
I: Might it be different?
S: But, because, if we knew how many were in here, and we are just adding three.
I: So, could that amount over here be the same as B?
S: ((Nods))
I: Can you call that B, or not?
S: We can't call that B. ((shake her head left and right)) Because it is not the same. It is the same thing, but we are adding ((put three cubes on the top of the box)) to it.
I: Gotcha. Awesome. Okay, so you call it something different, C.

During the post-interview, it seemed that Amy had difficulty recalling how to represent an unknown quantity, but with some support from the interviewer, she changed her mind to accept that $B + 3$ was not the same as B (line 46). In the next task, Amy used K to represent the number of paper clips. Interestingly, Amy tried to add two Ks when she was asked to represent the same number of paper clips: $K + K = KK$. However, here, there was no smile of confidence that might have been associated with a previously learned concept, thus her work did not elicit such an expressive emotion.

I: Here is my next question. My friend Mark has a box of paper clips. Same idea. Taped it up and covered it up. How could we talk about the number of paper clips that Mark has in the box if we don't know the number?
S: We could represent it with a letter.
I: We could represent a letter. What letter would we like to use?
S: K?
I: K? Perfect! Is there any reason to choose K?
S: I like that letter.
I: That's good for me. So, if K is the number of paper clips in here. I have to tell one more thing. His friend John says, “Guess what I have a box of paper clips also". He says, “I don't know how many are in here, but I do know I have the same number of paper clips that John has". So, how we could talk about his number of paper clips?
S: But, wait. If he knows that he has the same amount, he has to know how many are in here?
I: I don't know. Maybe, they weighted, or something else that they actually didn't count them? He didn't tell me, even if he knows the actual amount.
S: Did they open it? Somehow?
I: I don't know if he did that.
S: It says a hundred ((pointing to the box of Skittles))
I: It does say a hundred ((on the box)). But, I think he used some already like friends ate some of the Skittles. If he has the same amount that Mark has, how could we talk about his number of clips in this box?
S: Then, we could say, $K + K$ equals KK.
I: What does this K stand for?
S: Number of paper clips. How many in it?
I: So, this K stands for the number of paper clips here. What does the K stand for? The number of paper clips?
I: So, how many paper clips do they each have?
S: I don't know.
I: Great! We don't know. But, they are having the same amount. What are you calling this amount you don't know?
S: KK.
I: You are. Oh, you are saying KK. Is KK um, the amount in each box? Or, this is the total amount?
S: In each box, when you are adding together.
I: When you are adding together? Is that why you are putting a plus sign there? So, you've got the K amount here, plus, K amount over here. So, altogether, you call that another amount of KK?
S: ((Nods)) because we don't know how many are here, and we don't know how many here, so we call it a letter. So, I am adding ((put her hands together)) it. So, I am adding those letters together. Just imagine we open his box and open his box, and then it can be KK.
I: Okay, is it okay if we just use a single letter instead of two letters? Can we use just another letter? Can we do that?

S: ((Nods))
I: Like $K + K$ equals T? Can we do that?
S: ((Nods))
I: Okay, I was just curious. Great!

During the second task in the post-interview, it seemed clear that Amy had accepted the use of letters to represent an unknown, but she did not vividly express a smile, or any positive emotions associated with already understanding the concept. In the task in the preceding exchange, she represented the addition of two identical amounts as $K + K$, however, she insisted on representing the sum as KK. When the interviewer asked whether she could use a single letter instead of two letters, she accepted using a letter, T.

Case 3: Susie

Pre-Interview

The task in this third case involved thinking about the relationship between the number of ducks in a pond and the number of duck feet in the pond, and students constructed a table to show the number of ducks and the number of duck feet. The interviewer then asked Susie about any patterns she noticed in the table. As the excerpt below illustrates, Susie noticed the relationship of doubling, and gave several specific examples ($1 + 1 = 2$; $2 + 2 = 4$; and $3 + 3 = 6$).

I: Do you see any patterns or any relationships between the number of ducks and the number of duck feet?
S: One, two, three... Oh! ((Happiness & Surprise: open her mouth widely and showing a big smile))
I: What do you see?
S: It is like $1 + 1 = 2$, $2 + 2 = 4$, $3 + 3 = 6$.
I: How did you see that? $1 + 1 = 2$, I want to write it down because it looks really important to me. ((write down the equation on a worksheet))
S: ((Keep write down-Smiling)) $3 + 3 = 6$, $4 + 4 = 8$.
I: What would be the next equation?
S: 6 plus 6 totals ((by making circles with her right hand)) Wow! ((Happiness with her heads up and down))
I: Yes.
S: Because the duck feet will be 12 because 6 plus 6 equals twelve. ((with big hand and arm gesture))

Susie's exclamation "Oh!" (Line 3) suggests an emotion of surprise and happiness associated with finding a pattern in the table, as well as with writing the number sentences step-by-step (Line 9) while smiling. Finally, she added one more number sentence that was not based on the values in the table, suggesting that she understood the functional relationship observed in the table.

Post-Interview

The following excerpt also focuses on the duck pond task.

I: What is happening?
S: Dou, Dou, they are Doubling. ((Smiles))
I: I think you noticed something really important. What is the relationship between 1 and 2, 2 and 4, 3 and 6, 4 and 8?
S: They are doubling?
I: Can you write an equation that shows what happened here?
S: ((write)) $1+1=2$, $2+2=4$, $3+3=6$.
I: Okay, nice, I like how you are doing that. These are all great equations. What if I saw a thousand ducks in the pond? How could we figure out the number of duck feet?
S: Oh, easy-peasy. ((by making circles with her left hand)) If there are a thousand, we just double it, two thousand.
I: So, please write that for me.
S: ((write $1000 + 1000 = 2000$))

Susie's happy voice of "Dou, Dou, they are Doubling" (Line 2) was likely based on her recalling the pattern in the table that she had observed previously. When the interviewer asked Susie how many duck feet there would be with a thousand ducks (Line 10), she said it was easy-peasy (Line 11), and that you just double it and said the answer is two. As she explained her thinking, she showed positive emotions, much like Eric's smile in the previous case – a smile that suggests confidence.

Discussion

Unexpected Concepts, Finding Meaning, and an 'Aha Smile'

When students have never learned a concept or an idea before, they display enjoyment during the unexpected moment of coming to understand the concept or idea, and in the cases presented here, that enjoyment was displayed in an 'Aha smile'. In case 1, after Eric found that his initial thought that 6 should go in the blank $(4+2=\_+4)$, he glanced at the balance scale, realized that the numbers were the same distance from the center (understanding why 2 went in the blank), and showed a big smile. In case 2, Amy initially thought that a letter cannot be used to represent a number, but after saying "It is not a number, it is a letter" she smiled. She then proceeded to use the letter in writing an expression for the task ($T + 3$). Finally, in case 3, Susie created a T-chart, noticed a pattern in the chart, and stated the relationship involved doubling numbers. In her post-interview, she wrote the number sentence, $1000 + 1000 = 2000$, to represent the number of duck feet given 1000 ducks even though she did not know how to represent the relationship prior to the interview. In seeing the pattern in the table and figuring out the relationship in the pattern, she said while smiling "Dou, dou, doubling!"

As Muis et al. (2015) suggested, students might experience more positive emotions than negative emotions when they see that mathematics opens a meaningful way of thinking. In these three cases, we can see that as the students were beginning to make sense of the algebraic concepts being introduced, their emerging understandings were accompanied with Aha smiles. By using the balance scale, Eric showed his happiness as he began to view the equal sign relationally. Amy showed an Aha smile connected to developing her understanding about the use of a variable as a substitute for an unknown number. Finally, when Susie identified the relationship underlying the pattern she noticed in her T-chart, she exclaimed "Oh!". We argue that these three instances are reflective of Muis at al.'s (2015) conjecture about the relationship between positive emotions and meaning making in mathematics.

Aha Smile versus Confident Smile

The category of Surprise includes the 'Aha' moment as noted by Sperling (2012). In terms of learning, the Aha moment was directly connected to making a smile. The smile itself, however, was also displayed in a different way over the course of the interviews. In addition to the 'Aha' smile, there were smiles that seemed to be more associated with feelings of confidence (i.e., confident smiles). Thus, our results suggest two qualitatively distinct patterns of smiles: *Aha smiles* and *Confident smiles*.

When Eric contrasted the unbalanced and balanced number line scales, he noticed the pattern of having same numbers on both sides of a number balance, and the number balance triggered the *externally referenced emotion* (Moutsios-Rentzos, Kalozoumi-Paizi, 2017). We name this triggered emotion as an *Aha smile*. The Aha smile seems to occur as a learner engages with learning a new mathematical concept and comes to make mathematical meaning of the concept, the smile of 'Aha' reflects the happiness of learning the new concept. This is the insightful moment of building mathematical meaning (see the left column of Figure 2).

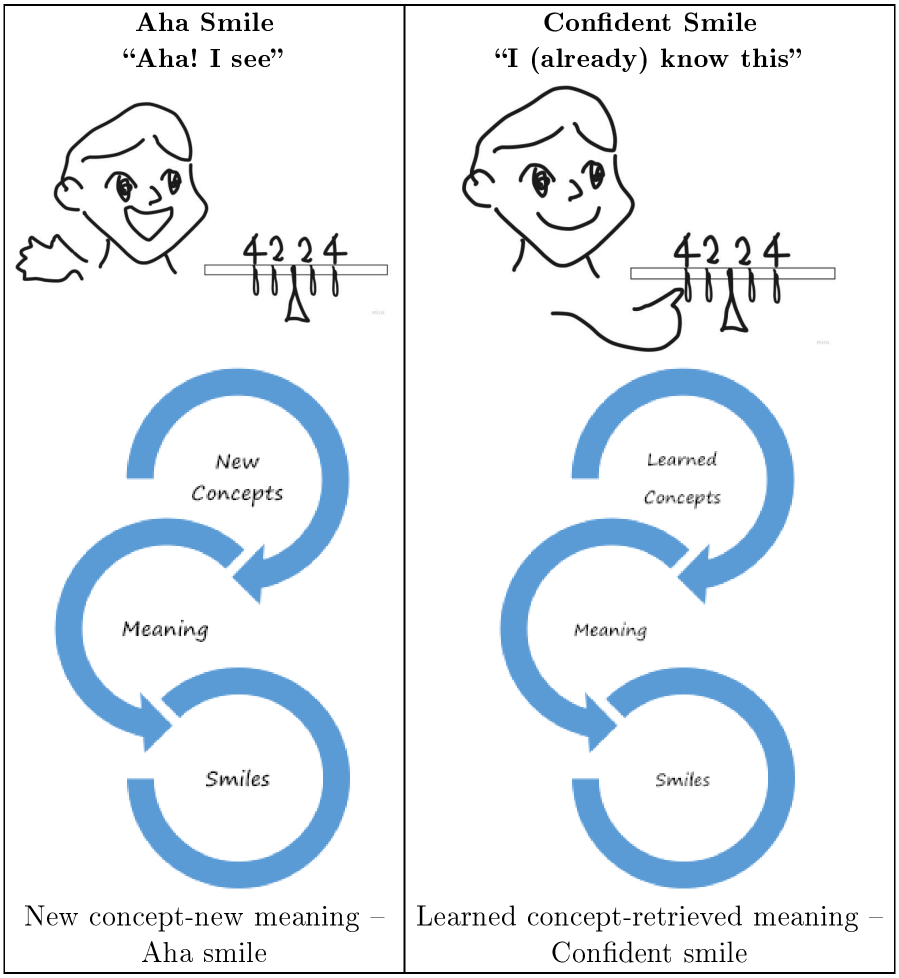

Figure 2. [New concept-new meaning-Aha smile] versus [Learned concept-retrieved meaning-Confident smile]

Eric showed a different smile pattern in the later interview. In this case, he drew on his previous learning about the concept in question to respond to the interviewer's question, and he showed a smile – an *internally referenced emotion (*Moutsios-Rentzos, Kalozoumi-Paizi, 2017). We refer to this internally referenced smile as a *confident smile*, a smile that is reflective of a moment in mathematical meaning (see the right column in Figure 2).

In case 1, an Aha smile is seen in the meaning-making moment, and the student showed a smile that appeared confident in the later interview. In Eric's interview, an Aha moment happened as he worked with a number balance, but later in the interview, there was no more Aha moment since he already learned those algebraic concepts. Instead of showing an Aha smile, the student showed a confident smile in the post-interview since he could say the relationship of numbers which are just switched around in an equation $4 + 2 = 2 + 4$.

In case 2, Amy showed an Aha smile in the meaning-making phase of the pre-interview, then her emotional response looked more neutral in the later interview. In a pre-interview, Amy, while saying "What?" several times, seemed to think that using a letter instead of a number was not acceptable for her. But she immediately showed a big smile, then accepted to use a letter for the unknown quantity. In a later interview, the student still preferred to use a real number, but when she was asked by the interviewer, she used the letter K without any visible emotional responses. In additional tasks, when there were two of the same numbers, she explained $K + K = T$. Then she explained that T is the total of two same of numbers. Amy's performance showed her understanding of the use of letters for the expression of unknown quantities, and the emotional responses were not as salient as before.

In the last case, which is similar to case 1, Susie also showed an Aha smile in the meaning-making stage, and she showed a smile that appears confident in the later interview. Susie said "oh!" and tried to introduce how she thought by writing several equations $1+1 = 2$, $2+2 = 4$, $3+3 = 6$, $4+4 = 8\ldots$ From the later interview, she recalled her memory and said "Dou, dou, doubling" and showed a smile that appeared confident. When she was asked about 100 ducks in the additional questions, she answered that there could be 200 duck feet. Also, when she was asked what if 100 duck feet are there, she brilliantly said, 50 ducks were there. By understanding the relationship, she was able to apply thinking back and forth freely with a confident smile.

Algebra Meaning-Making along with the Positive Emotions

Three students learned three different concepts in algebra: equality in an equation, using a variable for the unknown quantity, and understanding the relationship in functional thinking.

From the first case, by looking at the balance scale, Eric found the numbers were switched around but the same numbers on each side of the equation and a number balance, given an equation of $4 + 2 = \_ + 4$. Although Eric initially wanted to fill the blank with the number 6, which showed operational thinking, by seeing the scale balance, he at least implicitly understood the Commutative Property of Addition and the role it played in the equation. In later interviews, Eric did not manipulate a number balance and just filled in the blank while smiling confidently as if to say, "Umm, I know this." In Amy's case, the positive emotions were captured only in the meaning-making around learning new concepts; there was no later vivid emotional response. In the last case, Susie showed positive emotions in interviews with an understanding of the pattern from the T-chart. In the post-interview, she clearly mentioned the pattern of doubling in the chart. Finally, she generalized the pattern with a bigger number, $1000 + 1000 = 2000$.

Taken together, from all three cases, the presence of positive emotions coexists along with algebraic meaning-making: mathematical equivalence in an equation, using a variable to represent an unknown value, and understanding the relationship in functional thinking.

Limitations

The limitation of this study is that it only focused on several scenes of positive emotions, not seeing how positive emotions build a mathematical identity in the future. The next step of the study might highlight the relationship between positive emotional experiences and mathematical identity.

Another limitation of this study is that only three cases of learning were included. However, this is still beneficial to introduce early learners' accumulating positive emotions along with their mathematics learning. Since this is based on interview observations, further research might investigate how teachers could support positive emotions in a classroom.

Conclusion

These are examples of building positive emotions in young children that may lead to building long-term attitudes (Di Martino, Zan, 2011) in the future. All three concepts were new to the learners and by making meanings of challenging tasks the students expressed positive emotions. These building positive emotions are very important to their future identity formation. From previous researchers, early algebra is a context for developing mathematics competency. Along with that, in this study, early algebra is a context for developing Aha moments related to positive experiences.

In this study, we asked *How do students' positive emotional responses during interactions relate to early algebra meaning-making?* We found that the emotional responses of young learners have almost simultaneously occurring surprise, happiness, and cognitive acceptance. Through the interviews, positive emotions came from two different reasons: First, positive emotions happened when 'Aha' experiences occurred that allowed students to see learning mathematics as a meaningful endeavor (Muis et al., 2015). In addition to the Aha moment of initial learning, these emotional responses are obvious descriptors of successful retention of understanding in post-interviews. In later interviews, the meaning of smiling shifted from understanding to confidence in case 1 and case 3. Among all three cases, learners were good at employing the learned algebraic concepts at the post-interview. This study reconfirms the assertion from Frederickson (2001) that positive emotions keep triggering the motivation to learn. As Shahjahan (2015) asserted, the three cases confirm that it is crucial to understand interrelated emotions from facial expressions and cognitive progress together at the same time to overcome the illusion of separateness between mind–body–spirit–the illusion of Cartesian dualism. Based on these cases, this study urges math educators to guide learners to have more joyful surprises, Aha smiles, and confident smiles in early mathematics learning.

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APPENDIX A

Coding Manual (Sperling, 2012)

Although Sperling provided 6 categories of emotion described below, this paper focused on 2 categories: Happiness and Surprise.

Happiness

- Participant is smiling; eyes/face appear bright or eager
- Cheers–participant waves arms or body in a cheering or self-congratulating motion
- Claps hands–participant claps his/her hands as though to applaud for something
- Dances–participant moves his/her body in a rhythmic way.

  Jumps–participant jumps off of the floor, as though in excitement or happiness
- Laughs–The sound of laughter must be audible
- Sings–participant sings a part of a real or made-up song.
- Eager–participant indicates eagerness or anticipation (Hubbard, 2001)
- Lip corners pulled up
- Cheeks raised
- Crinkling around eyes
- Giggling (Cole et al., 2003)
- Affection
- Smirking
- Warm emotional tone
- Terms of endearment (e.g., "Honey" or "Sweetheart")
- Physical touching that is intentional and not accidental (e.g., a child sitting on a parent's lap or leaning on a parent while reading a story together).

Sadness

- Participant's mouth and/or eyes are turned down or droopy
- Cries–participant has tears in his/her eyes.
- Hides face–participant covers or shields his/her face as though to hide expression from other

- Sighs–participant emits air forcefully from his/her nose or mouth as an indication of disappointment or frustration.
- Slumps–participant slumps shoulders while standing or slumps entire body in seat while sitting.
- Signs to indicate slumping are: sudden drop in the height of the shoulders, shoulders coming in toward each other, and tilt of the head to one side.
- Slumps can be coded while sitting or standing
- Slumps can be only one shoulder
- Slumps can happen when hands are on the table
- Code if he/she puts chin on table
- Facial expressions such as tearfulness, sad frowns, or pained expressions, or looking as if the child is crying or about to cry. Body gestures observed *in conjunction with* other expressions of sadness: slumped shoulders, downcast head or eyes, wringing hands, wiping tears, or putting one's head in one's hands (SCIFF, Lindahl Malik, 2001).
- Pouting
- Sad emotional tone

Fear

- Hand to mouth–participant puts his/her hand to his/her mouth in a gesture that indicates being worried or anxious , not to scratch nose or cover laughter (Hubbard, 2001).
- Eyebrow raising but must be in conjunction with eye widening and lip stretching (sideways) (Wiggers, 2005).
- Brows lowered without any cue of a specific emotion
- Strained voice without harshness
- Frequent eye moving (e.g., rapid glancing) (Cole et al., 2003)

Anger

- Participant's eyebrows are furrowed or pointed inward, and the participant's mouth may be "set" in a hard line. Eyes may narrow. This code may also include an angry lip pout.
- Aggresses–participant engages in behavior that indicates the intention of inflicting physical harm upon another. Examples include hitting, kicking, or throwing objects at the person, or grabbing things from others
- Frustration–examples include swinging fist, punching fist into hand, hitting self in head, screaming "Ahhhh!" and pretending to cry, or grunting.

- Stomps feet–participant pounds one or both feet against the floor, and the sound of his/her feet hitting the floor is audible.
- This does not include someone performing a cheer/dance that involves stomping
- Teases–examples include pointing at someone and laughing
- Tension, and irritation (Lindahl, Malik, 2001)
- Lips pressed or tightened
- Teeth clenched (Cole et al., 2003)
- Angry emotional tone

Surprise

- Raised eyebrows
- Dropped jaw and open mouth (e.g., in awe)
- Widened eyes (Wiggers, 2005).
- Intake of air/gasp (Cole et al., 2003)
- Surprised emotional tone
- An "Aha" moment as in the person realizes or gets something that previously was not understood

Disgust

- Nose wrinkle
- Gaping mouth and tongue extrusion (e.g., responding to a bad taste)
- Raised upper lip
- Cocked eyebrow and raised head (Rozin, Lowery, Ebert, 1994)
- Voice as quality of expelling sound (e.g., the sound made when saying "yuck") (Cole et al., 2003)
- Shaking head and pulling one's face away from object that causes disgust

Pozytywne emocje we wczesnym tworzeniu znaczeń algebraicznych

S t r e s z c z e n i e

W niniejszym artykule przedstawiono badania dotyczące zmian w emocjach przejawiających się w wyrazie twarzy podczas pracy nad zadaniami dotyczącymi wczesnej algebry, zaobserwowanych u trzech uczniów (z klas 1-2). Uczniowie ci brali udział w zajęciach w klasie. Na początku badania odkryto,

iż trzej uczniowie okazywali chwile ‘Aha’ z uśmiechem, podczas gdy rozwijali rozumienie nowych pojęć. Pod koniec badania ich uśmiechy wyrażały inne emocje – pewność siebie, gdy zauważyli, że zadania były związane z wcześniej poznanymi pojęciami i przypomnieli sobie ich znaczenie. Analiza wywiadów przed i po badaniach ujawniła nie tylko sposoby budowania znaczeń matematycznych przez uczniów, ale również to, w jaki sposób doświadczenia z wczesną algebrą wspierają uczniów w rozwijaniu doświadczeń emocjonalnych.

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