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Droop-Free Distributed Control of DC Microgrids with Voltage Profile Guarantees and Relaxed Current Sharing

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***Abstract*—In this paper we propose a droop-free distributed secondary control with admissible voltage profile guarantees for DC microgrids. The proposed distributed control includes an average voltage regulator, a voltage variance regulator, and a relaxed current sharing regulator. The voltage regulator ensures global average voltage of the distributed generators (DGs) to be the rated voltage and the voltage variance regulator regulates the global variance of the DG voltage magnitudes to a predetermined reference. In order to achieve the objectives of voltage regulation, the current sharing from one of the DGs which may be owned by the microgrid community is relaxed. The global dynamic model of the DC microgrid with the proposed control is derived. Besides, steady-state analysis is performed to show that all objectives can be achieved. Finally, the effectiveness of the proposed control is validated through simulations on a 4-DG DC microgrid.**

***Index Terms*—Current sharing, DC microgrid, distributed control, droop-free, global dynamic model, variance, voltage profile.**

I. Introduction

Due to their inherent DC nature, the increased penetration of renewable energy resources and energy storages expedites the development of DC microgrids [1]–[3]. Compared to their AC counterparts, DC microgrids eliminate the requirement of redundant DC-AC conversion stages and thus improve efficiency, reliability, and scalability [4], [5]. Furthermore, DC microgrids can either coexist with the existing AC systems or operate in an independent fashion, and are free from the traditional challenges such as synchronization, frequency regulation, and reactive power sharing issues [4].

Hierarchical control strategy is conventionally adopted to achieve maximum utilization from DC microgrids which resembles the control hierarchy of the traditional legacy grid [6]–[8]. The primary control is usually realized through a droop-based approach which is responsible for voltage stabilization and current sharing [7], [9]. The voltage deviations caused by primary control is compensated by secondary control whereas tertiary control ensures economic operation [7], [8].

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For the primary control in DC microgrids, droop control is usually implemented in a decentralized way for the voltage regulation and current sharing [10]. However, the primary droop control may lead to steady-state voltage deviations due to line impedance mismatch [10] and can have poor dynamic performance in the presence of nonlinear loads [10], [11].

The secondary control of DC microgrids is usually implemented through a communication network and could have centralized [12] or distributed structure [10]. A major limitation of the centralized control is that it is prone to single point of failure [1], [13], [14]. Also, a high-bandwidth, point-to-point communication is required between the central controller and local distributed generator (DG) control units, increasing communication and computational cost [6]. By contrast, distributed control utilizes a sparse communication network and can improve resiliency, economic efficiency, and scalability [15]–[17]. Distributed control of DC microgrids is usually realized on top of the droop control that allows proportional current sharing, which is essential for preventing circulating currents and over-stressing of the DGs [3], [10]. However, when droop control is implemented a trade-off is required between voltage regulation and current sharing because high gain may lead to proper current sharing but at the cost of poor voltage regulation [3].

To solve the voltage regulation problem in DC microgrids, some recent literature proposes distributed control methods with average voltage regulation for which the average voltage of the DG output voltages is regulated to a reference value [1], [4], [10]. However, this type of control may not be able to guarantee that the DG output voltage profile is admissible, especially under heavy loading conditions [18].

To achieve proper voltage regulation and ensure appropriate current sharing among the DGs, in this paper we propose a droop-free distributed secondary control with an average voltage regulator, a voltage variance regulator, and a current regulator. The average voltage regulator ensures the global average voltage to be regulated to the rated voltage of the microgrid and the voltage variance regulator regulates the global voltage variance to a proper voltage variance reference. These two voltage regulation objectives are achieved by a relaxed current sharing regulator for which a predetermined special DG does not participate in the current sharing.

Although droop-free distributed control with voltage profile guarantees is presented for AC microgrids [18], the control approach remains unexplored for DC microgrids. For DC microgrids, a trade-off is required between voltage regulation and current sharing whereas in AC microgrids the reactive power from one DG is relaxed to achieve admissible voltage profiles. Furthermore, the dynamic model of the microgrid system under the distributed control is not provided in [18]. The main contributions of this paper are summarized below.

1) A droop-free distributed secondary control with an average voltage regulator, a voltage variance regulator, and a current regulator is proposed to achieve voltage profile guarantees and relaxed current sharing among the DGs.
2) The dynamic model of the DC microgrid system under the proposed control is derived based on the linearization of the nonlinear voltage variance observer and voltage variance regulator.
3) Based on the global dynamic model, steady-state analysis of the DC microgrid under the proposed control is performed to show that the objectives of average voltage, voltage variance, and current sharing regulations can be achieved in steady-state.

The remainder of this paper is organized as follows. In Section II, the proposed droop-free distributed control for DC microgrids is discussed. Then the global dynamic model of the DC microgrid with the proposed control is developed in Section III. Steady-state analysis of the proposed control is performed in Section IV to show convergence of the regulators in steady state. Detailed simulation results in Matlab/Simulink is presented in Section V to validate the effectiveness of the proposed control. Finally conclusions are drawn in Section VI.

II. Proposed Droop-Free Distributed Control for DC Microgrids

In DC microgrid, the DG output voltages need to be maintained within a specified range. In case of heavy loading conditions, the output voltages will drop which will lead to increased current and overheating of the devices. In some extreme scenarios this may eventually cause voltage collapse and complete shutdown of the system.

The layout of the proposed droop-free distributed control for node i of the DC microgrid is illustrated in Fig. 1. The distributed controller deploys an average voltage regulator, a voltage variance regulator, and a current regulator to select appropriate control actions for the lower level controller. The controller utilizes a sparse communication network to exchange information among the neighboring source nodes.

A directed graph (digraph) $\mathcal{G}$ is used to model the communication network where nodes represent agents and edges represent communication links between nodes. $\mathcal{G}$ can be represented by a time-invariant and scalar adjacency matrix $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ where N is the number of DC dispatchable sources. The Laplacian matrix is defined as $\mathbf{L} = \mathbf{D}^{\text{in}} - \mathbf{A}$ where $\mathbf{D}^{\text{in}} = \text{diag}\{d^{\text{in}}\}$ is the in-degree matrix with $d^{\text{in}} = \sum_{j\in\mathcal{N}_i} a_{ij}$ and $\mathcal{N}_i$ as the set of neighbors of node i [10]. It is assumed that the Laplacian matrix is balanced and $\mathcal{G}$ has at least a spanning tree and minimum redundancy.

A. Average Voltage Regulator

The average voltage regulator ensures global average voltage of the microgrid to be the rated voltage. To achieve this objective, the regulator implements a distributed average voltage estimator at each DG location. The estimator at DG i updates its information about the average voltage $\overline{v}_i$ utilizing the local voltage measurement v_i and the neighbors' estimated average voltage $\overline{v}_j$ as follows [10]:

$$\overline{v}_i(t) = v_i(t) + \int_0^t \sum_{j\in\mathcal{N}_i} a_{ij}\Big(\overline{v}_j(\tau) - \overline{v}_i(\tau)\Big)d\tau. \quad (1)$$

The estimated average voltage $\overline{v}_i$ is then compared with the microgrid rated voltage v_{rated}. The difference of the comparison is then fed to a PI controller $G_i(s)$ to generate a voltage correction term Δv_i^1 which ensures the objective of the global average voltage regulation across the microgrid.

Differentiating (1) for $i = 1, \dots, N$, the global average voltage observer dynamics can be obtained as:

$$\dot{\overline{\mathbf{v}}} = \dot{\mathbf{v}} - \mathbf{L}\overline{\mathbf{v}}, \quad (2)$$

where $\mathbf{v} = [v_1, v_2, \cdots, v_N]^T$ and $\overline{\mathbf{v}} = [\overline{v}_1, \overline{v}_2, \cdots, \overline{v}_N]^T$ are the voltage measurement and average voltage estimation vector, respectively. In frequency domain (2) becomes [10]:

$$\overline{\mathbf{V}} = s(s\boldsymbol{I}_N + \mathbf{L})^{-1}\mathbf{V} \triangleq \mathbf{G}_{\text{av}}\mathbf{V}, \quad (3)$$

where $\overline{\mathbf{V}}$ and $\mathbf{V}$ are, respectively, the Laplace transform of $\overline{\mathbf{v}}$ and $\mathbf{v}$, $\boldsymbol{I}_N \in \mathbb{R}^{N\times N}$ is an identity matrix, and $\mathbf{G}_{\text{av}}$ is the distributed average voltage observer transfer-function matrix.

B. Voltage Variance Regulator

The average voltage regulator ensures global average voltage regulation but cannot always guarantee that the local DG voltage deviations are within $\pm 5\%$ of the rated voltage, especially under heavy loading conditions. The voltage variance regulator is utilized to guarantee an admissible voltage profile based on a voltage variance estimator. To update the local voltage variance estimation σ_i^2, the estimator at DG i utilizes information about the local voltage measurement v_i, distributed estimated average voltage $\overline{v}_i$, and the voltage variance estimation σ_j^2 of the neighboring DGs [18]:

$$\sigma_i^2(t) = \big(v_i(t) - \overline{v}_i(t)\big)^2 + \int_0^t \sum_{j\in\mathcal{N}_i} a_{ij}\Big(\sigma_j^2(\tau) - \sigma_i^2(\tau)\Big)d\tau. \quad (4)$$

It has been proven in [18] that the estimate in (4) can converge to the true global voltage variance if the communication graph has a spanning tree and the associated Laplacian matrix is balanced. The estimated voltage variance σ_i^2 is compared with the voltage variance reference σ^{2*} and the error term is then multiplied with $\big(v_i - \overline{v}_i\big)$ before being sent to the second PI controller $K_i(s)$. The term $(v_i - \overline{v}_i)$ guides the controller

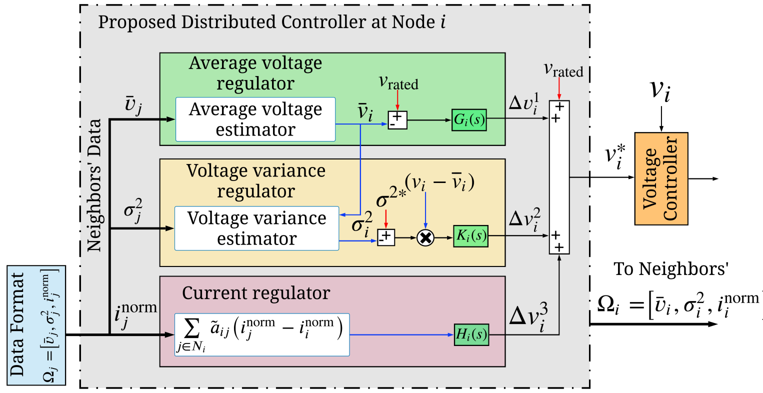

Fig. 1. Schematic of the proposed droop-free distributed secondary control for node i of the DC microgrid.

in selecting proper direction of control. The PI controller generates voltage correction term Δv_i^2 to regulate the voltage variance to the reference value.

The nonlinear term $f(v_i, \overline{v}_i) = \big(v_i(t) - \overline{v}_i(t)\big)^2$ in (4) can be approximated by first-order Taylor series expansion at $v_{i_{\text{init}}}$ and v_{rated}, where $v_{i_{\text{init}}}$ is the actual output voltage of DG i and v_{rated} is the rated voltage of the microgrid:

$$f(v_i, \overline{v}_i) = \Delta_{v_i}^2 + 2\Delta_{v_i}(v_i - v_{i_{\text{init}}}) - 2\Delta_{v_i}(\overline{v}_i - v_{\text{rated}}),$$

where $\Delta_{v_i} = v_{i_{\text{init}}} - v_{\text{rated}}$. Then (4) can be modified to be:

$$\begin{aligned}\sigma_i^2(t) = \Delta_{v_i}^2 + 2\Delta_{v_i}(v_i - v_{i_{\text{init}}}) - 2\Delta_{v_i}(\overline{v}_i - v_{\text{rated}}) \\ + \int_0^t \sum_{j\in\mathcal{N}_i} a_{ij}\big(\sigma_j^2(\tau) - \sigma_i^2(\tau)\big)d\tau. \quad (5)\end{aligned}$$

By differentiating (5) we have

$$\dot{\sigma}_i^2 = 2\Delta_{v_i}(\dot{v}_i - \dot{\overline{v}}_i) + \sum_{j\in\mathcal{N}_i} a_{ij}\sigma_j^2 - d_i^{\text{in}}\sigma_i^2. \quad (6)$$

Let $\boldsymbol{\sigma}^2 = [\sigma_1^2, \sigma_2^2, \cdots, \sigma_N^2]^\top$ and its Laplace transform as $\mathbf{\Sigma}$. Since $\overline{v}_i(0) = v_i(0)$, $\sigma_i^2(0) = 0$. Then in frequency domain (6) becomes:

$$s\mathbf{\Sigma}^2 = 2s\mathbf{\Delta}_v(\mathbf{V} - \overline{\mathbf{V}}) - \mathbf{L}\mathbf{\Sigma}^2, \quad (7)$$

where $\mathbf{\Delta}_v = \text{diag}\{\Delta_{v_i}\}$.

From (7) and (3) we have:

$$\begin{aligned}\mathbf{\Sigma}^2 &= 2s(s\boldsymbol{I}_N + \mathbf{L})^{-1}\mathbf{\Delta}(\boldsymbol{I}_N - \mathbf{G}_{\text{av}})\mathbf{V} \\ &\triangleq \mathbf{G}_{\text{var}}\mathbf{V}, \quad (8)\end{aligned}$$

where $\mathbf{G}_{\text{var}}$ is the distributed voltage variance observer transfer-function matrix.

C. Current Regulator

A current regulator is employed to ensure proportional current sharing among DGs and prevent circulating currents and over-stressing of the DG sources. However, bounded voltage regulation and proportional current sharing are contradictory and a trade-off between them is required. For this purpose, we have considered one of the DGs as a special one which does not participate in current sharing but involves in average voltage and voltage variance regulations.

A DC microgrid can accommodates different types of DGs (e.g. diesel generators, solar PVs, wind turbines, fuel-cells), energy storage (e.g. batteries, super-capacitors, flywheels), and loads. A microgrid usually employs community owned energy storage with sufficient capacity to ensure power balance and reduce generation costs. Such energy storage can be considered as the special DG in our proposed control to help improve voltage regulation with relaxed current sharing.

Specifically, the current regulator at DG i updates the current mismatch mi_i as:

$$mi_i = \sum_{j\in\mathcal{N}_i} \tilde{a}_{ij}\left(i_j^{\text{norm}} - i_i^{\text{norm}}\right), \quad (9)$$

where $i_j^{\text{norm}} = i_j / i_j^{\text{rated}}$ is the normalized current of DG j, $\tilde{\mathbf{A}} = [\tilde{a}_{ij}] \in \mathbb{R}^{N\times N}$ is obtained by setting the row and column corresponding to the special DG of the matrix $\mathbf{A}$ to be zero. The error term mi_i is then sent to the PI controller $H_i(s)$ to update the third voltage correction term Δv_i^3 and thereby ensure relaxed current sharing among the DGs.

The three correction terms from the average voltage regulator, voltage variance regulator, and current regulator are added to v_{rated} to update the voltage set-point of the ith DG as:

$$v_i^* = v_{\text{rated}} + \Delta v_i^1 + \Delta v_i^2 + \Delta v_i^3. \quad (10)$$

PI controller is usually used as zero-level control which compares the reference voltage from the secondary control with the DG output voltage and produces an output signal to appropriately track the reference voltage [19]. The output of the PI controller is then sent to a PWM comparator which generates the switching signals for the DC-DC converters.

III. Development of Global Model

Let $\mathbf{i} = [i_1, i_2, \cdots, i_N]^\top$ be the supplied current vector and its Laplace transform be $\mathbf{I}$. In the proposed DC microgrid control, v_{rated} and σ^{2*} are the global inputs whereas the outputs are $\mathbf{V}$ and $\mathbf{I}$. Therefore, the global dynamic model is developed to formulate the transfer functions from the input v_{rated} and σ^{2*} to the outputs $\mathbf{V}$ and $\mathbf{I}$.

In the proposed control in Fig. 1, we have three voltage correction terms Δv_i^1, Δv_i^2, and Δv_i^3 for each DG. The voltage variance correction term Δv_i^2 at DG i can be represented as:

$$\begin{aligned}\Delta v_i^2 &= K_i\big(v_i - \overline{v}_i\big)\big(\sigma^{2*} - \sigma_i^2\big)\\ &= K_i\big(\sigma^{2*} v_i - v_i\sigma_i^2 - \sigma^{2*}\overline{v}_i + \overline{v}_i\sigma_i^2\big). \end{aligned} \tag{11}$$

By first-order Taylor series expansion, $v_i\sigma_i^2$ can be approximated as:

$$v_i\sigma_i^2 = \sigma^{2*}v_i + v_{i_{\rm init}}\sigma_i^2 - v_{i_{\rm init}}\sigma^{2*}, \tag{12}$$

where $v_{i_{\rm init}}$ and σ^{2*} represent the operating points of the Taylor series approximation. Similarly, $\overline{v}_i\sigma_i^2$ can be approximated around the operating point $v_{\rm rated}$ and σ^{2*} as:

$$\overline{v}_i\sigma_i^2 = \sigma^{2*}\overline{v}_i + v_{\rm rated}\sigma_i^2 - v_{\rm rated}\sigma^{2*}. \tag{13}$$

Substituting (12)–(13) into (11), Δv_i^2 becomes:

$$\Delta v_i^2 = K_i\big(v_{i_{\rm init}}\sigma^{2*} - v_{\rm rated}\sigma^{2*} + v_{\rm rated}\sigma_i^2 - v_{i_{\rm init}}\sigma_i^2\big).$$

Let the three voltage correction terms be $\Delta \mathbf{v}^1 = [\Delta v_1^1, \Delta v_2^1, \cdots, \Delta v_N^1]^\top$, $\Delta \mathbf{v}^2 = [\Delta v_1^2, \Delta v_2^2, \cdots, \Delta v_N^2]^\top$, and $\Delta \mathbf{v}^3 = [\Delta v_1^3, \Delta v_2^3, \cdots, \Delta v_N^3]^\top$. In frequency domain they can be represented by:

$$\Delta\mathbf{V}^1 = \mathbf{G}\Big(\frac{v_{\rm rated}}{s}\mathbf{1} - \overline{\mathbf{V}}\Big) \tag{14}$$

$$\Delta\mathbf{V}^2 = \mathbf{K}\Big(\frac{\sigma^{2*}}{s}\big(\mathbf{V}_{\rm init}\mathbf{1} - v_{\rm rated}\mathbf{1}\big) + \big(v_{\rm rated}\boldsymbol{I}_N - \mathbf{V}_{\rm init}\big)\boldsymbol{\Sigma}^2\Big) \tag{15}$$

$$\Delta\mathbf{V}^3 = -\mathbf{H}\tilde{\mathbf{L}}\mathbf{i}_{\rm rated}^{-1}\mathbf{I}, \tag{16}$$

where $\Delta\mathbf{V}^1$, $\Delta\mathbf{V}^2$, and $\Delta\mathbf{V}^3$ are the Laplace transforms of $\Delta\mathbf{v}^1$, $\Delta\mathbf{v}^2$, and $\Delta\mathbf{v}^3$, $\mathbf{1} \in \mathbb{R}^{N\times 1}$ is a column vector with all ones, $\mathbf{V}_{\rm init} = {\rm diag}\{v_{i_{\rm init}}\}$, and $\mathbf{i}_{\rm rated} = {\rm diag}\{i_i^{\rm rated}\}$. The controller matrices for the average voltage regulator, voltage variance regulator, and current regulator are $\mathbf{G} = {\rm diag}\{G_i\}$, $\mathbf{K} = {\rm diag}\{K_i\}$, and $\mathbf{H} = {\rm diag}\{H_i\}$, respectively.

Let the local voltage reference vector be $\mathbf{v}^* = [v_1^*, v_2^*, \cdots, v_N^*]^\top$ and its Laplace transform be $\mathbf{V}^*$. The proposed controller sets the local voltage reference point as:

$$\mathbf{V}^* = \frac{v_{\rm rated}}{s}\mathbf{1} + \Delta\mathbf{V}^1 + \Delta\mathbf{V}^2 + \Delta\mathbf{V}^3. \tag{17}$$

Substituting (14)–(16) into (17) and applying (3) and (8), we have:

$$\begin{aligned}\mathbf{V}^* = \frac{v_{\rm rated}}{s}(\mathbf{G} + \boldsymbol{I}_N)\mathbf{1} + \frac{\sigma^{2*}}{s}\mathbf{K}(\mathbf{V}_{\rm init}\mathbf{1} - v_{\rm rated}\mathbf{1})\\ + \mathbf{R}\mathbf{V} - \mathbf{H}\tilde{\mathbf{L}}\mathbf{i}_{\rm rated}^{-1}\mathbf{I},\end{aligned} \tag{18}$$

where $\mathbf{R} = -\mathbf{G}\mathbf{G}_{\rm av} + \mathbf{K}(v_{\rm rated}\boldsymbol{I}_N - \mathbf{V}_{\rm init})\mathbf{G}_{\rm var}$.

Using the admittance matrix $\mathbf{Y}_{\rm bus}$, the injected current and the DG output voltage can be related as $\mathbf{I} = \mathbf{Y}_{\rm bus}\mathbf{V}$. Furthermore, the input-output relationship of the DG converters can be represented as $\mathbf{V} = \mathbf{G}_{\rm c}\mathbf{V}^*$ [10] where $\mathbf{G}_{\rm c} = {\rm diag}\{G_i^{\rm c}\}$ is the transfer-function matrix of the DC-DC converters. Therefore, (18) can be rewritten as follows:

$$\begin{aligned}\mathbf{V} = \Big[\mathbf{G}_{\rm c}^{-1} - \mathbf{R} + \mathbf{H}\tilde{\mathbf{L}}\mathbf{i}_{\rm rated}^{-1}\mathbf{Y}_{\rm bus}\Big]^{-1}\Big[\\ \frac{v_{\rm rated}}{s}(\mathbf{G} + \boldsymbol{I}_N)\mathbf{1} + \frac{\sigma^{2*}}{s}\mathbf{K}(\mathbf{V}_{\rm init}\mathbf{1} - v_{\rm rated}\mathbf{1})\Big]\end{aligned} \tag{19}$$

$$\begin{aligned}\mathbf{I} = \Big[\mathbf{G}_{\rm c}^{-1}\mathbf{Y}_{\rm bus}^{-1} - \mathbf{R}\mathbf{Y}_{\rm bus}^{-1} + \mathbf{H}\tilde{\mathbf{L}}\mathbf{i}_{\rm rated}^{-1}\Big]^{-1}\Big[\\ \frac{v_{\rm rated}}{s}(\mathbf{G} + \boldsymbol{I}_N)\mathbf{1} + + \frac{\sigma^{2*}}{s}\mathbf{K}(\mathbf{V}_{\rm init}\mathbf{1} - v_{\rm rated}\mathbf{1})\Big].\end{aligned} \tag{20}$$

Eqs. (19)–(20) represent the global dynamic model of the DC microgrid under the proposed control.

IV. Steady-State Analysis

The steady-state analysis can be performed to check whether the proposed controller can achieve the operational requirements. Applying final value theorem to the global dynamic model in (19), the steady-state solution of the DC bus voltage can be obtained as:

$$\begin{aligned}\mathbf{v}^{\rm ss} &= \lim_{t\to\infty}\mathbf{v}(t) = \lim_{s\to 0} s\mathbf{V}(s)\\ &= \lim_{s\to 0}\Big[s\mathbf{G}_{\rm c}^{-1} - s\mathbf{R} + s\mathbf{H}\tilde{\mathbf{L}}\mathbf{i}_{\rm rated}^{-1}\mathbf{Y}_{\rm bus}\Big]^{-1}\Big[\\ &\quad s v_{\rm rated}(\mathbf{G} + \boldsymbol{I}_N)\mathbf{1} + s\sigma^{2*}\mathbf{K}(\mathbf{V}_{\rm init}\mathbf{1} - v_{\rm rated}\mathbf{1})\Big].\end{aligned} \tag{21}$$

Since the DC gain of the closed loop converters can be equal to one [10], we have

$$\lim_{s\to 0}\mathbf{G}_c^{-1} = \mathbf{I}_N.$$

For the PI controllers there are

$$\lim_{s\to 0} s\mathbf{G} = \mathbf{G}_{\rm I},\ \ \lim_{s\to 0} s\mathbf{K} = \mathbf{K}_{\rm I},\ \ \lim_{s\to 0} s\mathbf{H} = \mathbf{H}_{\rm I},$$

where $\mathbf{G}_{\rm I}$, $\mathbf{K}_{\rm I}$, and $\mathbf{H}_{\rm I}$ respectively denotes the integral gains of the PI controllers $\mathbf{G}$, $\mathbf{K}$, and $\mathbf{H}$. For the voltage estimator we have $\lim_{s\to 0}\mathbf{G}_{\rm av} = \mathbf{M}$ where $\mathbf{M}$ is the averaging matrix whose elements are equal to $1/N$ with N as the total number of DGs. Also $\lim_{s\to 0} s\boldsymbol{I}_N = \mathbf{0}$. Therefore, (21) can be written as follows:

$$\begin{aligned}&\big[\mathbf{G}_{\rm I}\mathbf{M} + \mathbf{K}_{\rm I}(\mathbf{V}_{\rm init} - v_{\rm rated}\boldsymbol{I}_N)\mathbf{G}_{\rm var} + \mathbf{H}_{\rm I}\tilde{\mathbf{L}}\mathbf{i}_{\rm rated}^{-1}\mathbf{Y}_{\rm dc}\big]\mathbf{v}^{\rm ss}\\ &= v_{\rm rated}\mathbf{G}_{\rm I}\mathbf{1} + \sigma^{2*}\mathbf{K}_{\rm I}(\mathbf{V}_{\rm init}\mathbf{1} - v_{\rm rated}\mathbf{1}),\end{aligned} \tag{22}$$

where $\mathbf{Y}_{\rm dc} = \mathbf{Y}(0)$ represents the DC admittance matrix. Substituting $\mathbf{U}_1 = \mathbf{H}_{\rm I}^{-1}\mathbf{G}_{\rm I}$ and $\mathbf{U}_2 = \mathbf{H}_{\rm I}^{-1}\mathbf{K}_{\rm I}$ in (22) the following equation can be obtained:

$$\begin{aligned}&\big[\mathbf{U}_1\mathbf{M} + \mathbf{U}_2(\mathbf{V}_{\rm init} - v_{\rm rated}\boldsymbol{I}_N)\mathbf{G}_{\rm var} + \tilde{\mathbf{L}}\mathbf{i}_{\rm rated}^{-1}\mathbf{Y}_{\rm dc}\big]\mathbf{v}^{\rm ss}\\ &= v_{\rm rated}\mathbf{U}_1\mathbf{1} + \sigma^{2*}\mathbf{U}_2(\mathbf{V}_{\rm init}\mathbf{1} - v_{\rm rated}\mathbf{1}).\end{aligned} \tag{23}$$

In steady state, the elements of $\overline{\mathbf{v}}$ converge to the true average voltage [10], i.e. $\overline{\mathbf{v}}^{\rm ss} = \mathbf{M}\mathbf{v}^{\rm ss} = \langle\mathbf{v}^{\rm ss}\rangle\mathbf{1}$, where $\mathbf{M} \in \mathbb{R}^{N\times N}$ is an averaging matrix [18]. $\mathbf{x}^{\rm ss}$ is the vector of the steady-state value of $\mathbf{x}$ and $\langle\mathbf{x}\rangle$ is the average value of the elements in $\mathbf{x}$. Pre-multiplying (23) by $\mathbf{M}\mathbf{U}_2^{-1}$, we have

$$\begin{aligned}&(v_{\rm rated} - \langle\mathbf{v}^{\rm ss}\rangle)\mathbf{M}\mathbf{U}_2^{-1}\mathbf{U}_1\mathbf{1} - \mathbf{M}\mathbf{U}_2^{-1}\tilde{\mathbf{L}}\mathbf{i}_{\rm rated}^{-1}\mathbf{i}^{\rm ss} =\\ &\mathbf{M}(\mathbf{V}_{\rm init} - v_{\rm rated}\boldsymbol{I}_N)\mathbf{G}_{\rm var}\mathbf{v}^{\rm ss} - \sigma^{2*}\mathbf{M}(\mathbf{V}_{\rm init}\mathbf{1} - v_{\rm rated}\mathbf{1}).\end{aligned} \tag{24}$$

Based on the proof of convergence of (4) in [18], there is $\mathbf{G}_{\rm var}\mathbf{v}^{\rm ss} = \sigma^{2,\rm ss}\mathbf{1}$. Assume the average value of the initial DG output voltages is controlled to be the rated voltage before the

TABLE I
PARAMETERS OF THE TEST SYSTEM AND CONTROL

	Parameters		Value
	Symbol	Quantity	
DGs	i_1^{rated} – i_3^{rated}	DG1 – DG3 rated current	12 A
	i_4^{rated}	DG4 rated current	24 A
Buck Converters	L	Inductance	4 mH
	C	Capacitance	5 mF
	Fs	Switching Frequency	60 kHz
Lines	R_{12}, R_{34}	Line Resistance 1–2, 3–4	0.5 Ω
	R_{25}, R_{35}	Line Resistance 2–5, 3–5	1 Ω
	L_{12}, L_{34}	Line Inductance 1–2, 3–4	5 μH
	L_{25}, L_{35}	Line Resistance 2–5, 3–5	10 μH
Proposed Control	$G_{\text{P}}, G_{\text{I}}$	Voltage control P, I term	0.012, 20
	$K_{\text{P}}, K_{\text{I}}$	Variance control P, I term	0.02, 10
	$H_{\text{P}}, H_{\text{I}}$	Current Control P, I term	5, 350

disturbance injection, there is $\mathbf{M}(\mathbf{V}_{\text{init}}\mathbf{1} - v_{\text{rated}}\mathbf{1}) = \mathbf{0}$. Then (24) can be written as:

$$(v_{\text{rated}} - \langle \mathbf{v}^{\text{ss}} \rangle)\mathbf{M}\mathbf{U}_2^{-1}\mathbf{U}_1\mathbf{1} - \mathbf{M}\mathbf{U}_2^{-1}\tilde{\mathbf{L}}\mathbf{i}_{\text{rated}}^{-1}\mathbf{i}^{\text{ss}} = \mathbf{0}. \quad (25)$$

For the balanced Laplacian matrix $\tilde{\mathbf{L}}$, there is $\mathbf{1}^\top\tilde{\mathbf{L}} = \mathbf{0}^\top$. Premultiplying (25) by $\mathbf{1}^\top\mathbf{U}_2\mathbf{M}^{-1}$, we have

$$\left(v_{\text{rated}} - \langle \mathbf{v}^{\text{ss}} \rangle\right)\sum_{i=1}^{N} u_{1,ii} = 0, \quad (26)$$

where $u_{1,ii} > 0$ is the ith diagonal element of the diagonal matrix $\mathbf{U}_1$. Therefore, we have $\langle \mathbf{v}^{\text{ss}} \rangle = v_{\text{rated}}$, implying that the steady-state average voltage converges to the rated voltage. Substituting $v_{\text{rated}} - \langle \mathbf{v}^{\text{ss}} \rangle = 0$ into (25), we have

$$\tilde{\mathbf{L}}\mathbf{i}_{\text{rated}}^{-1}\mathbf{i}^{\text{ss}} = \mathbf{0}, \quad (27)$$

which indicates that the controller can share currents among the sources except the special DG in proportion to their ratings.

Now substituting $v_{\text{rated}} = \langle \mathbf{v}^{\text{ss}} \rangle$ and $\tilde{\mathbf{L}}\mathbf{i}_{\text{rated}}^{-1}\mathbf{i}^{\text{ss}} = \mathbf{0}$ into (24) and premultiplying both sides by $\mathbf{M}^{-1}$, we have

$$(\sigma^{2,\text{ss}} - \sigma^{2*})(\mathbf{V}_{\text{init}}\mathbf{1} - v_{\text{rated}}\mathbf{1}) = \mathbf{0}, \quad (28)$$

which requires for each DG $i = 1, \ldots, N$ that

$$(\sigma^{2,\text{ss}} - \sigma^{2*})(v_{i_{\text{init}}}^{\text{ss}} - v_{\text{rated}}) = 0. \quad (29)$$

Note that it is only assumed that the average value of the initial DG output voltages is controlled to be the rated voltage before the disturbance. Controlling the initial DG voltages $v_{i,\text{init}}^{\text{ss}}$ for $\forall i = 1, \ldots, N$ to be the rated voltage is a much stronger condition with zero voltage variance, which cannot hold in most cases [18]. Therefore, (29) will be true only when $\sigma^{2,\text{ss}} = \sigma^{2*}$, which implies that the distributed control can successfully regulate the voltage variance to the reference value σ^{2*}.

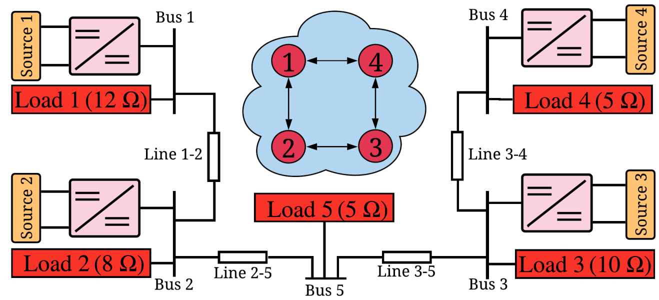

Fig. 2. Schematic diagram of the 4-DG DC microgrid.

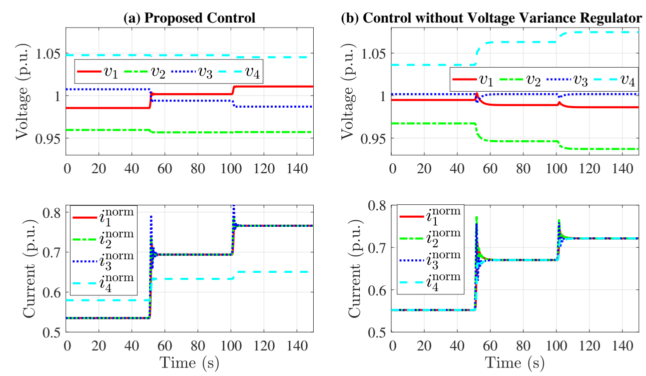

Fig. 3. Comparison between the proposed control and the control without voltage variance regulator. Load changes are applied at Load 5 at 50 s and 100 s.

V. RESULTS

The performance of the proposed control is tested on a 4-DG DC microgrid shown in Fig. 2. The rated voltage of the system is 48 V-DC. The topology of the communication graph is also demonstrated in Fig. 2. We set $a_{ij} = 1$ in $\mathbf{A}$ if there is a communication link between nodes i and j, and $a_{ij} = 0$ otherwise. In this paper, we choose $\sigma^{2*} = 2.4\,\text{V}^2$. DG 4 is selected as the special DG that is relaxed from current sharing. We consider buck converters as the DC-DC converters. The parameters of the buck converter components, the rated current of the DC-DC converters, the microgrid test system line parameters, and the PI control parameters of the voltage and current regulators are listed in Table I.

A. Performance Under Load Change

The proposed control is compared with the droop-free distributed control that only has the average voltage regulator and current sharing regulator in [10]. In Fig. 3, the per unit output voltages of the DGs and the normalized currents are given, for which Load 5 is increased by 5 Ω at 50 s and by 10 Ω at 100 s. It is seen that after the load change the control without voltage variance regulator fails to limit the voltage within ±5% of the rated voltage whereas the proposed control can do so by relaxing the current sharing of DG 4.

Fig. 4 shows the detailed performance of the proposed control under the same load changes as in Fig. 3, including the average voltage estimation, voltage variance estimation,

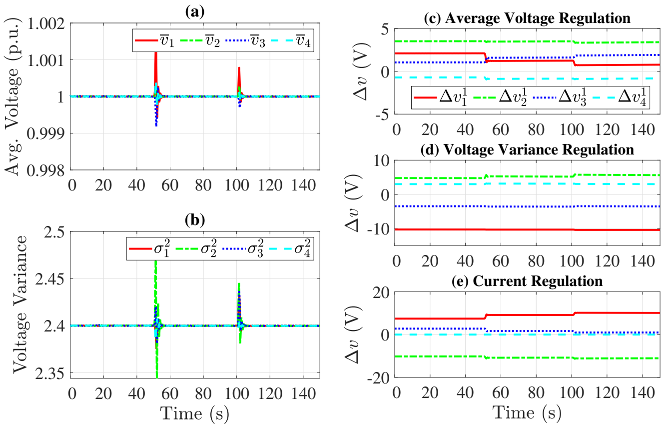

Fig. 4. Performance of the proposed control: (a) average voltage estimation; (b) voltage variance estimation; (c) correction term from average voltage regulator; (d) correction term from voltage variance regulator; and (e) correction term from current regulator.

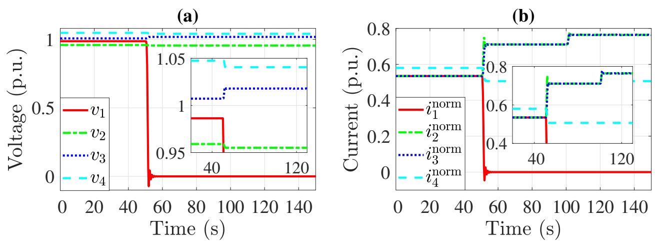

Fig. 5. Performance of the proposed control when Line 12 is disconnected due to a fault at 50 s and Load 5 is increased at 100 s.

and the three correction terms. It is seen that both average voltage estimation and voltage variance estimation can quickly converge to the reference values.

B. Performance Under Disconnection of Lines

Fig. 5 shows the controller performance when Line 1-2 is disconnected due to a fault on the line. Under this condition DG 1 and Load 1 are isolated from the microgrid. At 100 s Load 5 is increased by 30 Ω. It is seen that despite the line fault and the disconnection of DG 1, the remaining three DGs can still regulate the voltage of the microgrid and share the remaining load among them.

VI. Conclusion

In this paper, a droop-free distributed control with an average voltage regulator, a voltage variance regulator, and a relaxed current sharing regulator is proposed for DC microgrids. The global dynamic model of the DC microgrid with the proposed control is derived based on which the steady-state analysis is performed. The effectiveness of the proposed control is validated through simulations on a 4-DG DC microgrid. Under load changes and/or faults the proposed control can always guarantee admissible voltage profiles which is made possible by the added flexibility from the relaxed current sharing regulator. In our future work we will develop a unified control for grid-forming and grid-feeding converters in DC microgrids with average voltage regulation and current sharing. Besides, an optimization based unified distributed control for grid-forming and grid-feeding converters will be developed to achieve an optimal trade-off between voltage regulation and current sharing.

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