A Belief Propagation-based Quantum Joint-Detection Receiver for Superadditive Optical Communications

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Abstract: We design a quantum joint-detection receiver for binary-phase-shift-keyed optical communications using belief propagation with quantum messages. For an exemplary tree code, the receiver attains the block-Helstrom limit in discriminating the codewords and achieves superadditive capacity.

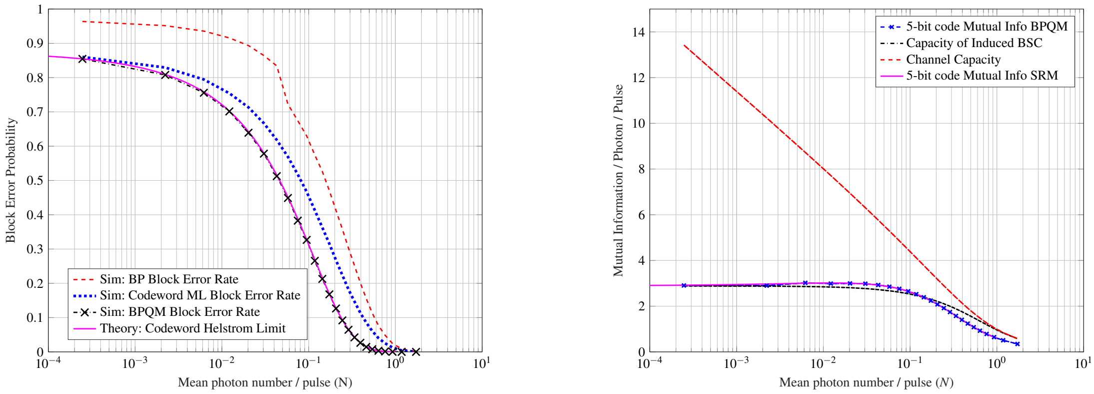
1. Quantum-enhanced Classical Optical Communications

For space-based laser communications, when the mean photon number per received optical pulse is much smaller than one, there is a large gap between communications capacity achievable with a receiver that performs individual pulse-by-pulse detection, and the quantum-optimal joint-detection receiver that acts collectively on long codeword-blocks of modulated pulses; an effect often termed *superadditive capacity* [1]. The simplest scenario where a large superadditive capacity is known is laser communications based on binary-phase-shift-keying (BPSK) modulation for sending classical data over a pure-loss channel of transmissivity $\eta \in (0,1]$. The transmitter modulates each transmitted optical pulse in one of two coherent states $|\pm\alpha\rangle$, $\alpha \in \mathbb{R}$, with mean photon number per pulse $|\alpha|^2 = N_S$. Each received optical pulse is in one of the two coherent states $|\pm\beta\rangle$, where $\beta = \sqrt{\eta}\alpha$ and mean photon number $N = \eta N_S$. These two states are non-orthogonal with an inner product $\langle\beta|-\beta\rangle = e^{-2N} \equiv \sigma$. The coherent states $|\pm\beta\rangle = \sum_{n=0}^{\infty} e^{-|\beta|^2/2} \frac{(\pm\beta)^n}{\sqrt{n!}} |n\rangle$ live in an infinite-dimensional Hilbert space spanned by the complete orthonormal number basis $\{|n\rangle, n \in \mathbb{N}\}$. Yet, each pulse is always one of $|\pm\beta\rangle$, and the subspace spanned by $|\pm\beta\rangle$ can be embedded in a two-dimensional (qubit) Hilbert space via the inner-product-preserving map $|\pm\beta\rangle \longmapsto |\pm\theta\rangle := \cos\frac{\theta}{2}|0\rangle \pm \sin\frac{\theta}{2}|1\rangle$, with $\sigma = \cos\theta$.

When the received BPSK modulated pulses are detected one at a time, the best possible detection error probability is given by the Helstrom bound [2] on the minimum average error probability of discriminating the alphabet states $|\pm\beta\rangle$, which is $p := \frac{1}{2}[1 - \sqrt{1-\sigma^2}]$. A structured optical receiver that achieves this performance was invented by Dolinar in 1973 [3]. This receiver induces a binary symmetric channel (BSC) between the quantum states $|\pm\beta\rangle$ and the receiver's guess "$\pm\beta$", with the fundamentally minimum crossover probability p, thereby enabling the communicating parties to achieve a reliable communication rate given by $C_1 = 1 - h_2(p)$ bits per pulse, the Shannon capacity of the BSC. On the other hand, if a quantum joint-detection receiver that collectively measures over a long codeword-block of n pulses is used, the maximum attainable capacity is given by the Holevo capacity [4,5], $C_\infty = S(\frac{1}{2}|\beta\rangle\langle\beta| + \frac{1}{2}|-\beta\rangle\langle-\beta|) = h_2([1+\sigma]/2)$ bits per pulse, where $S(\cdot)$ denotes the von Neumann entropy. In the limit of low received mean photon number per pulse, i.e., $N \to 0$, or equivalently $\sigma \to 1$, the ratio $C_\infty/C_1 \to \infty$. This regime of operation is especially important for long-haul free-space terrestrial and deep-space laser communications. This large capacity gain can be reaped, e.g., by deploying a Holevo-capacity-achieving code such as a classical-quantum polar code [6] and a receiver that performs block-Helstrom measurement to discriminate the codewords at the minimum probability of error permissible by quantum mechanics, so that the probability of decoding error goes to 0 as the blocklength $n \to \infty$. Given pure quantum state codewords of a linear code, the optimal block-Helstrom measurement is given by the so-called square root measurement (SRM) [7]. However, it is in general hard to translate the mathematical description of this measurement into a physical receiver design.

2. A Joint-detection Receiver based on Belief-Propagation with Quantum Messages

In classical communication theory, the "belief propagation (BP)" algorithm is widely used to efficiently decode codewords of binary linear codes transmitted over classical channels such as the BSC. It works by passing "local



(a) (b)

Fig. 1: (a) The overall block error rate of BPQM along with those of quantum optimal block-Helstrom limit, pulse-by-pulse Helstrom measurement followed by classical optimal block-Maximum a posteriori decoding, and pulse-by-pulse Helstrom measurement followed by classical BP. (b) Mutual information per photon per pulse achieved by BPQM and the SRM on the 5-bit code example, along with the Holevo capacity of the channel and the BSC capacity induced by pulse-by-pulse Helstrom measurements at the channel output, all plotted against the mean photon number per pulse (N). BPQM and SRM on this code produce identical results. The curves indicate that BPQM/SRM provides mutual information gains for certain regimes of N, thereby demonstrating superadditive capacity with an explicit code and decoder.

beliefs" between the nodes of the factor graph corresponding to the parity check matrix of the code. Renes [8] recently proposed a quantum generalization of the classical BP algorithm that we call "BP with quantum messages (BPQM)". Renes' algorithm is well-defined on a tree factor graph and when the received quantum states are pure states. It works by passing "quantum beliefs" (messages encoded in qubits) along with classical messages (bits) between nodes of the code's factor graph and combining them using unitary operations at the nodes. Using the BPQM algorithm, we derive an explicit construction of the quantum circuit of a joint-detection receiver for pure-loss BPSK communications [9]. We analyze the receiver rigorously and show that it achieves the quantum limit of minimum average error probability in discriminating the 8 codewords of a length-5 binary linear code with a tree factor graph, whose parity-check matrix is given by $H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$. Moreover, the receiver attains superadditive capacity with the above code in a regime of N where the mutual-information-per-photon-per-pulse, also known as photon information efficiency (PIE), beats the largest PIE obtained from pulse-by-pulse Helstrom measurement. These results are elucidated in Fig. 1. The receiver can be readily translated into a low-depth quantum circuit composed of single and two-qubit gates, which can be realized using a photonic quantum processor capable of executing "cat-basis" quantum logic.

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