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Partner with a Third-Party Delivery Service or Not? A Prediction-and-Decision Tool for Restaurants Facing Takeout Demand Surges During a Pandemic

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Abstract. Amidst the COVID-19 pandemic, restaurants become more reliant on no-contact pick-up or delivery ways for serving customers. As a result, they need to make tactical planning decisions such as whether to partner with online platforms, to form their own delivery team, or both. In this paper, we develop an integrated prediction-decision model to analyze the profit of combining the two approaches and to decide the needed number of drivers under stochastic demand. We first use the susceptible-infected-recovered (SIR) model to forecast future infected cases in a given region and then construct an autoregressive-moving-average (ARMA) regression model to predict food-ordering demand. Using predicted demand samples, we formulate a stochastic integer program to optimize food delivery plans. We conduct numerical studies using COVID-19 data and food-ordering demand data collected from local restaurants in Nuevo Leon, Mexico, from April to October 2020, to show results for helping restaurants build contingency plans under rapid market changes. Our method can be used under unexpected demand surges, various infection/vaccination status, and demand patterns. Our results show that a restaurant can benefit from partnering with third-party delivery platforms when (i) the subscription fee is low, (ii) customers can flexibly decide whether to order from platforms or from restaurants directly, (iii) customers require more efficient delivery, (iv) average delivery distance is long, or (v) demand variance is high.

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Keywords: on-demand grocery or food delivery • demand uncertainty • susceptible-infected-recovered (SIR) model • autoregressive-moving-average (ARMA) • stochastic integer programming

1. Introduction

As of mid-April 2021, almost 1.5years after its first known outbreak in December 2020 in Wuhan China, the SARS-CoV-2 virus (COVID-19) infections continue being transmitted worldwide with the total number of confirmed cases being almost 140 million and nearly 3 million deaths (WHO 2020). In many countries, public gathering prohibition and social distancing are mandatory to reduce the risk of contacts between susceptible and infected groups (Dingel and Neiman 2020, Koren and Pető 2020). These nonpharmaceutical interventions decrease travel frequency and reshape service patterns of traditional restaurant dine-in businesses.

In recent years, rapid growth of online food ordering and grocery delivery using digital platforms such as Doordash, Grubhub, and so on, has been seen in many places because of the blooming of smartphones and proliferation of e-commerce (Hirschberg et al. 2016). This trend became much more significant worldwide in the last year, especially after the pandemic was announced by the World Health Organization (WHO). An increasing number of restaurants decide to only offer curbside pick-up or no-contact delivery, especially in places with high number of infections (Banskota et al. 2020). Yang et al. (2020) show the negative effects of COVID-19 on dine-in businesses in the United States from February to April 2020, and Abay et al. (2020) show the increasing demand for no-contact food delivery service worldwide after March 2020. The resilience of most businesses during and after the pandemic mainly relies on continuous access to customers. Therefore, well-designed and properly deployed online service platforms play essential roles in reliable food delivery (Raj et al. 2020). Restaurants also need to prepare for increasing food delivery demand because of shelter-in-place order from local governments. To this end, they may choose to partner with third-party service platforms by paying subscription fees or partial revenue from their sales (typically 20%–30%) to expand their capacities of delivery while ensuring safe and timely delivery to customers.

To offer on-demand food ordering and delivery services, there are mainly three types of decisions to make: strategic planning, tactical, and operational. For example, strategic decisions include where to locate restaurants, which locations to provide online ordering options, and how to improve users' experience, whereas operational decisions take into account individual order–related activities and involve routes and schedules for delivering food to specific customer locations. As something in between, tactical decisions are made for targeted periods and include whether to partner with a third-party platform and how many drivers to recruit as we consider in this paper. Before COVID-19, a large number of restaurants focused on dine-in service and did not partner with service platforms because of their high subscription fee, and one purpose of this paper is to provide adequate methods for restaurants to make tactical planning decisions under order demand uncertainty because of the stochastic and fluctuating trends of infections in many countries worldwide.

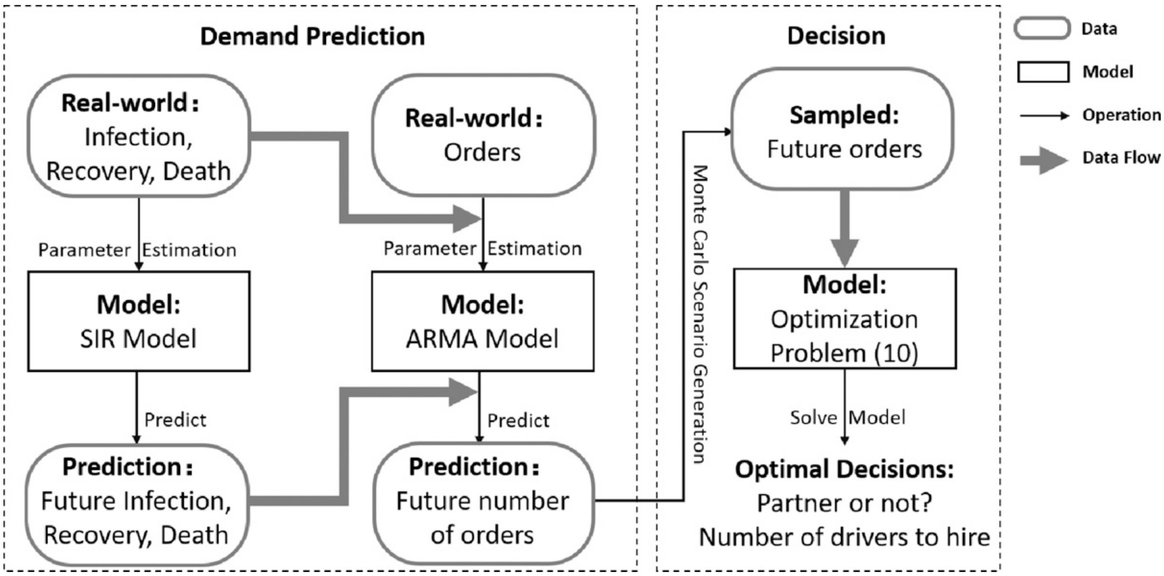
In this paper, we solve for tactical decisions that are the most relevant to restaurant owners who aim to adapt their business operations to daily demand fluctuations during the pandemic. We consider that strategic decisions, such as restaurant locations, are fixed and cannot be easily changed. We also do not involve operational decision variables such as specific routes and schedules for delivering each received takeout order, but instead estimate the overall net profit based on restaurants' partnership strategies. The focus of this paper is to integrate different methods from epidemiology, statistics, and optimization for developing a decision tool for restaurants to evaluate the profit of collaborating or not with third-party service platforms. The main contributions of the paper are threefold. First, we provide restaurants a scientific integrated model for making collaboration and service-expansion decisions under the unprecedented pandemic and future demand uncertainties. The proposed stochastic optimization model can solve this problem under decision-dependent demand uncertainty through a decision-independent modeling approach. Second, the derived theoretical solution properties and computational results provide insights to guide the process of partnering with online food ordering platforms or hiring dedicated drivers under different demand patterns. Third, this study fills the gap of limited work on tactical decision making in the application of shared mobility and food delivery, although a substantial amount of research has been conducted on strategic and operational problems in this area.

We build a model that integrates different methods in epidemiological modeling, statistical learning, and optimization for restaurants to integrate different approaches for attracting online orders and safe delivery. The model consists of two parts: demand prediction and decision. We first use the susceptible-infected-recovered (SIR) model to analyze the disease spread based on historical COVID-19 infection data. This SIR model is used to understand the infection pattern and scale, so that one can estimate potential infected cases during the targeted periods. Then, we construct a linear autoregressive-moving-average regression model (ARMA; Fuller 2009) to predict the amount of takeout food orders in the targeted periods. The ARMA model takes the forecasted infected cases as input. Finally, we formulate a stochastic integer program to make the tactical decisions for maximizing the total expected profit during the targeted periods. The optimization-based decision model uses samples of future demand predicted by the above SIR and ARMA, stochastic revenue brought by the randomness of order sales, and delivery travel cost given possible locations of future orders. We present an overview of the prediction-and-decision model in Figure 1.

The severity of infection and government policies such as mask requirement, travel restriction can have significant impacts on customers' attitudes toward online shopping for food and groceries, and mainly on strategic planning decisions. We refer the interested readers to the following work related to strategic planning for online food shopping. Ingham et al. (2015) review a dozen years of prolific and versatile empirical research on factors that have influence on consumers' incentives of using online shopping. Yeo et al. (2017) study consumer experiences toward online food delivery services, and Suhartanto et al. (2019) analyze the effects of food delivery service on customer loyalty. Based on the existing diverse literature, one can conclude that the main factors for choosing online services include convenience, age (Chopra and Rajan 2016), usefulness (Littler and Melanthiou 2006, Kimes 2011, Saarijärvi et al. 2014), values and pleasure from shopping (Alavi et al. 2016), previous experiences of using Internet (Rezaei et al. 2018), and website design and information security (Zulkarnain et al. 2015). Zhao and Bacao (2020) derive the key factors that determine customer continuously using food delivery platforms during the COVID-19 pandemic period.

In contrast, operational decisions for grocery and food delivery are typically modeled using the framework of the vehicle routing problem with time windows (VRPTM; Bräysy and Gendreau 2005, Pisinger and Ropke 2007). Hsu et al. (2007) extend the general VRPTM by considering the randomness in food delivery processes and formulate a model to obtain optimal delivery routes, loads, fleet dispatching, and departure times. Gera et al. (2018) take the drivers'

Figure 1. An Overview of the Prediction-and-Decision Model



availability into account when formulating food delivery models to satisfy customers' demand. Zulvia et al. (2020) propose a VRPTW model for delivering perishable food and take into account multiple objectives, such as the operational cost, deterioration cost, carbon emissions, and customer satisfaction.

Regarding tactical decisions considered in this paper, the demand prediction model we develop is closely related to the following three papers that also use statistical models to predict future sales and demand of restaurants: Liu et al. (2001) apply data mining techniques, including autoregressive integrated moving average (ARIMA) model, a series of automatic modeling procedures, and outlier detection; Meneghini et al. (2018) first fit an exponential smoothing model and then incorporate qualitative contextual factors through expert elicitation; and Huber et al. (2017) apply a clustered hierarchical approach based on ARIMA to predict the demand of several stores in different organization levels. The main difference between our work and these three papers is that they analyze and predict stationary demand including both dine-in and takeout orders in normal time, whereas in our setting, we consider time-varying takeout-order demand dependent on time-varying disease infections. Therefore, we use an ARMA model and take the daily infected cases as our independent variables rather than the ARIMA model. We refer the interested readers to a survey paper by Lasek et al. (2016) about food demand forecast using different techniques based on variables such as time, weather conditions, economic factors, random situations, and so on. During the COVID-19 pandemic, it is crucial for restaurant owners and managers to understand the new demand patterns of takeout food orders and make tactical decisions to guarantee their profitability accordingly.

The rest of this paper is organized as follows. In Sections 2 and 3, we provide problem description and develop the mathematical prediction-and-decision model. Specifically, Section 2 focuses on demand prediction, and Section 3 provides details of the stochastic integer program for optimal partnership decision making. In Section 4, we numerically validate our approach using instances generated based on real data from April to October 2020 of COVID-19 infections and the related food-ordering demand at local restaurants in Nuevo Leon, Mexico. In Section 5, we conclude the paper and describe future research directions.

2. Demand Prediction

Restaurants are facing unprecedented challenges to survive in this pandemic. As online food delivery demand surges, restaurants must seek help from third-party food delivery service platforms even the latter may charge substantially high fees. In this paper, we consider the partnership and having a fleet of drivers dispatched by individual restaurants as tactical decisions that are made and kept unchanged for certain time periods, for example, weeks or months. We refer to the time periods as future targeted periods and predict food demand during the targeted periods as follows. In Section 2.1, we describe an SIR model to analyze the disease spread and predict trends of infections. In Section 2.2, we construct an ARMA model using the weekdays, the number of daily infected cases, and the number of takeout orders during previous days as inputs to predict takeout food orders during the targeted periods.

2.1. SIR Model for Infection Forecast

We use the traditional SIR model to predict the future spread of an infectious disease (Bagal et al. 2020, Biswas et al. 2020, Dhanwant and Ramanathan 2020). This epidemiological compartmental SIR model separates the total population in a given region into three subgroups for each period t: The susceptible subgroup S_t of populations, the infectious subgroup I_t of populations who have been infected and are capable of infecting susceptible individuals, and the recovered subgroup R_t of populations who have been infected and have either recovered from the disease or died. The total population, denoted by N, is a constant, such that $N = S_t + I_t + R_t,\ \ \forall t \geq 0$. There are two important parameters in the SIR model: β represents the contact rate, that is, the probability of transmitting disease between a susceptible and an infectious individual; and γ represents the recovery rate, which commonly takes the value as $1/D$ where D is the average duration of the infection. The actual values of β and γ can be different when considering different periods during different stages after an outbreak.

During period $t + 1$, we expect to see new infected cases as $\beta S_t I_t/N$, where I_t/N can be viewed as the expected number of contacts with infectious people for each susceptible person. Similarly, we can expect that γI_t individuals will recover. Therefore, an SIR model consists of the following three differential equations of dynamic population subgroups for each period t as

$$\text{(Susceptible Equation)} \qquad \frac{dS_t}{dt} = -\frac{\beta S_t I_t}{N}, \tag{1a}$$

$$\text{(Infectious Equation)} \qquad \frac{dI_t}{dt} = \frac{\beta S_t I_t}{N} - \gamma I_t, \tag{1b}$$

$$\text{(Recovered Equation)} \qquad \frac{dR_t}{dt} = \gamma I_t. \tag{1c}$$

After analyzing the historical infection time series, one can estimate the values of β and γ, denoted as $\hat{\beta}$ and $\hat{\gamma}$, and further construct the correspondent SIR model of the service region. The SIR model assumes fixed transmission rate β and recovery rate γ over all time periods, and thus the decision makers should use the most recent infectious data to fit the SIR model for reliable infection forecast. In the remaining of the paper, we denote the actual size of the subgroups in the past period t as $\{S_t, I_t, R_t\}$ and the predicted size of the subgroups for future period t' as $\{\breve{S}_{t'}, \breve{I}_{t'}, \breve{R}_{t'}\}$. We use one day as the length of one period and thus refer to period t and day t alternatively.

2.2. ARMA Model for Takeout-Order Demand Prediction

Next, we predict the takeout-order demand of a given restaurant for the targeted periods. We construct a multivariate linear ARMA model that is widely used in the analysis of time series (Whittle 1951, Brillinger 2001, Fuller 2009). The dependent variable of this ARMA model is O_t, representing the number of takeout orders received during period t. We include three types of independent variables. First, consider the effects of the weekdays on potential customers' behavior of ordering food online. Thus, we use a six-dimensional dummy vector w_t to represent the categorical weekdays of period t, where the vector with kth position being one represents the kth weekday from Monday to Saturday and the all-zero vector representing Sunday. We denote the correspondent coefficients as $\beta_w \in \mathbb{R}^6$. We also incorporate the relationship between takeout demand and disease outbreak and thus include the daily new infected cases as data input. Because of the latency between the time when people prefer to order online and when they obtain current infection status, we use the daily infected cases in day $t-1$ to predict the demand in day t. For the daily new infected cases i_t, the infected population I_t, and the recovered population R_t in day t, we have

$$R_t = R_{t-1} + r_t, \tag{2a}$$

$$I_t = I_{t-1} + i_t - r_t, \tag{2b}$$

where r_t denotes the newly added population from the infected population I_t to the recovered population R_t in day t. By analyzing the relationship between the daily infected cases in previous J_1 days and the number of takeout orders, we use $\log i_{t-j},\ j = 1, 2, \ldots, J_1$ as independent variables and denote the correspondent coefficients as $\beta_{i_j},\ j = 1, 2, \ldots, J_1$. Furthermore, the number of daily takeout orders forms a time series, and this is also the reason that we use the ARMA model to capture the endogenous relationship among dependent variables, O_t. We assume that the orders in day t depend linearly on orders in previous J_2 days and thus consider autoregressive (AR) variables $O_{t-j},\ j = 1, 2, \ldots, J_2$ and denote the correspondent coefficients as $\alpha_j,\ j = 1, 2, \ldots, J_2$. We assume that the stochastic terms for the orders of each day are correlated and thus consider moving-average (MA) variables, that is, the noise of the current and previous J_3 days, ϵ_{t-j}, $j = 0, 1, 2, \ldots, J_3$, and denote the correspondent coefficients for previous noise as $\phi_j,\ j = 1, 2, \ldots, J_3$.

Finally, we present the ARMA model as follows:

$$O_t = \beta_0 + \beta_w^T w_t + \sum_{j=1}^{J_1} \beta_{i_j} \cdot \log i_{t-j} + \sum_{j=1}^{J_2} \alpha_j O_{t-j} + \sum_{j=1}^{J_3} \phi_j \epsilon_{t-j} + \epsilon_t, \tag{3}$$

where $\epsilon_{t-j:i=0,1,\ldots,J_3} \sim \mathcal{N}(0, \sigma)$ denote random noises. Consider the prediction function for the number of takeout orders $\hat{O}_t$ such that

$$\hat{O}_t = \beta_0 + \beta_w^T w_t + \sum_{j=1}^{J_1} \beta_{i_j} \cdot \log i_{t-j} + \sum_{j=1}^{J_2} \alpha_j O_{t-j} + \sum_{j=1}^{J_3} \phi_j \epsilon_{t-j}. \tag{4}$$

We estimate the coefficients $(\beta_0, \beta_w, \beta_d, \alpha, \phi)$ from the observed number of historical orders O_t and the correspondent independent variables (w_t, i_t) during periods $t, t \in T_{\text{his}}$, where we use T_{his} to denote the days of historical order data. The estimated values of coefficients are reflecting the feature importance of independent variables, such as infection, weekdays, and past sales. Specifically, parameters are estimated by a least-square loss function:

$$\hat{\beta}_0, \hat{\beta}_w, \hat{\beta}_d, \hat{\alpha}, \hat{\phi} = \underset{\beta_0, \beta_w, \beta_d, \alpha, \phi}{\arg\min} \sum_{t \in T_{\text{his}}} (O_t - \hat{O}_t)^2. \tag{5}$$

For a future target day t' that we need to predict the number of takeout orders, we may not know its previous days' infected cases. Therefore, we use the forecasted infected populations in previous days before day t' to estimate $i_{t'-j}, j = 1, 2, \ldots, J_1$. That is, for each future time period t':

$$\breve{O}_{t'} = \hat{\beta}_0 + \hat{\beta}_w^T w_{t'} + \sum_{j=1}^{J_1} \hat{\beta}_{i_j} \cdot \log \breve{i}_{t'-j} + \sum_{j=1}^{J_2} \hat{\alpha}_j \breve{O}_{t'-j} + \sum_{j=1}^{J_3} \hat{\phi}_j \breve{\epsilon}_{t'-j}, \tag{6}$$

where $\breve{i}_{t'-j} = \breve{S}_{t'-j} - \breve{S}_{t'-j-1} + \breve{R}_{t'-j} - \breve{R}_{t'-j-1}$ is the forecasted daily infected cases for day $t' - j$ with $(\breve{S}, \breve{R})$ from the SIR model (see Equation (2)); $\breve{O}_{t'-j}$ is the predicted order for day $t' - j$ (if we can access the actual order data of day $t' - j$, then use the actual demand); and $\breve{\epsilon}_{t'-j}$ is predicted noise for day $t' - j$ (similarly, if day $t' - j$ is before the day we make decisions, then use the actual noise). There could exist redundant AR and MA variables depending on the historical data.

3. Decision Model Formulation and Solution Properties

We present the stochastic optimization model in Section 3.1. We discuss the problem settings and how our model can be used to decide delivery radius optimally for restaurants (if they choose to partner with a platform) without explicitly modeling decision-dependent uncertainty in Section 3.2. We derive three solution properties and summarize the managerial insights under uncertain markets in Section 3.3. These properties are also verified in the computational experiments in Section 4.

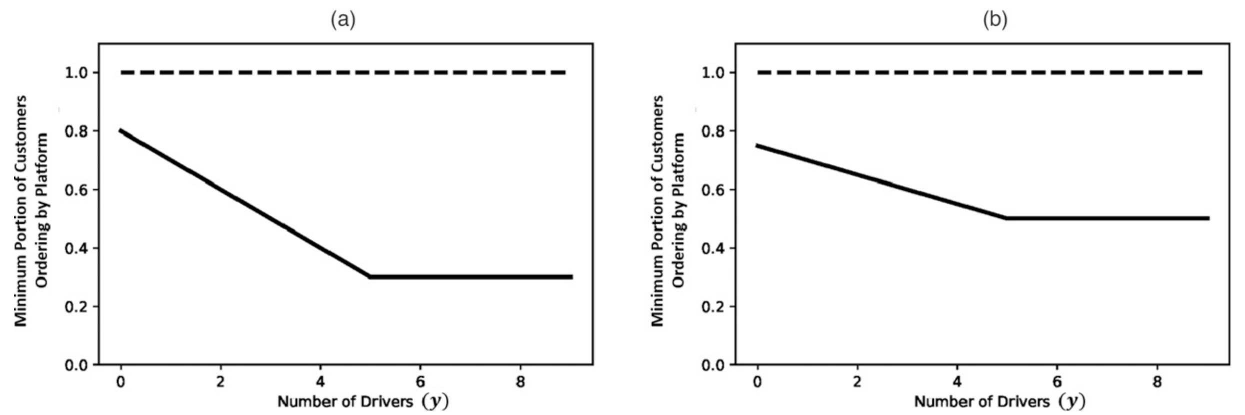
3.1. Optimizing Partnership Decisions via Stochastic Integer Programming

We formulate a stochastic integer program to solve for the tactical decisions. Consider the targeted periods $\mathcal{T}$. For every hired driver, the restaurant needs to pay a fixed base salary c_v per day. The driver will also receive an additional stipend, proportional to the travel time and distance of each delivery, at unit cost rates c_τ and c_d, respectively. Every hired driver can serve at most Δ number of orders each day. (This value can be relaxed if we allow service delays and thus the number of orders queued in the system could be more than the current capacity.) Let $\tilde{\mathcal{O}}_t$ denote the uncertain future takeout orders in day $|\tilde{\mathcal{O}}_t| = \breve{O}_t$, where $\breve{O}_t$ is predicted in Section 2. For each order $o \in \tilde{\mathcal{O}}_t, t \in \mathcal{T}$, we denote the unknown random delivery distance, delivery time, and sales profit as $\tilde{d}_o$, $\tilde{\tau}_o$, and $\tilde{p}_o$, respectively. For notation convenience, we use $\tilde{\mathcal{O}}$ to denote $\tilde{\mathcal{O}}_{t, t\in\mathcal{T}}$ and the correspondent uncertain parameters $\{\tilde{d}_o, \tilde{\tau}_o, \tilde{p}_o\}$ of any future order o.

3.1.1. Partnering with Third-Party Food Delivery Service Platforms. If the restaurant partners with a food delivery service platform, then some potential customers will choose to use the service platform, such as a mobile app, to make takeout-food orders. For orders received from the platform, the restaurant needs to pay δ portion of the sales profit to the platform and the platform will take over the delivery process by assigning a driver to deliver the food to the customer. We consider a threshold Y for the number of drivers hired by the restaurant. If the restaurant hires more than Y number of drivers, at least θ portion of the customers will order food through the platform rather than directly contacting the restaurant. If the number of hired drivers is smaller than Y, we assume that more customers will order food through the service platform because of the lack of service capacity of the own delivery team for the restaurant. In particular, we assume that another Γ portion of customers will use the service platform once there is one reduction of hired drivers from Y. Therefore, we also have that $\Gamma Y \le 1 - \delta$ to guarantee that at most 100% of potential customers will use the service platform. Therefore, if the restaurant hires y number of drivers and has in total O number of takeout orders, the number of orders received from the service platform is at least $\theta O + \Gamma O (Y - y)^+$, where $(\cdot)^+ = \max\{0, \cdot\}$. Figure 2 shows two examples of the minimum portion of orders received from the service platform versus the number of hired drivers.

Based on the previous assumption on the number of orders received from the third-party delivery platform, we formulate a two-stage stochastic integer programming model. The first-stage decisions include a binary variable x, where $x = 1$ denotes partnering with third-party platforms and $x = 0$ otherwise, and a nonnegative integer variable y denoting the number of drivers that will be hired during the targeted periods $\mathcal{T}$. The second-stage decisions are binary variables u_o

Figure 2. Two Examples of Portion of Orders Received from Delivery Platform



Notes. Solid line, the minimum percentage of orders received from app and the number of hired drivers; dashed line, the 100% reference line. (a) $Y = 5$, $\theta = 0.3$, $\Gamma = 0.1$. (b) $Y = 5$, $\theta = 0.5$, $\Gamma = 0.05$.

and v_o, $\forall o \in \tilde{\mathcal{O}}$. For order $o \in \tilde{\mathcal{O}}$, $u_o = 1$ represents that this order o is delivered by the drivers hired by the restaurant, and $v_o = 1$ means that the order is delivered by the service platform. Let parameter M represents the maximum number of daily takeout orders during targeted periods, that is, $M = \max_{t\in\mathcal{T}} |\tilde{\mathcal{O}}_t| = \max_{t\in\mathcal{T}} \breve{O}_t$. Then, we can formulate a stochastic integer program to maximize the total expected profit as

$$\max_{x,y} - c_v\,|\mathcal{T}|\,y + \mathbb{E}_{\tilde{\mathcal{O}}}[Q(x,y,\tilde{\mathcal{O}})], \tag{7a}$$
$$\text{s.t. } x \in \{0,1\}, \tag{7b}$$
$$y \geq 0 \,\text{integer}, \tag{7c}$$

where the function $Q(x,y,\tilde{\mathcal{O}})$ represents the profit of the restaurant under the first-stage decisions (x, y) and future orders $\tilde{\mathcal{O}}$:

$$Q(x,y,\tilde{\mathcal{O}}) = \max_{u,v} \sum_{o\in\tilde{\mathcal{O}}} (\tilde{p}_o - c_d\tilde{d}_o - c_\tau\tilde{\tau}_o)u_o + (1-\delta)\tilde{p}_o v_o, \tag{8a}$$
$$\text{s.t. } u_o + v_o \leq 1 \quad \forall o \in \tilde{\mathcal{O}}, \tag{8b}$$
$$\sum_{o\in\tilde{\mathcal{O}}_t} u_o \leq \Delta y \quad \forall t \in \mathcal{T}, \tag{8c}$$
$$\sum_{o\in\tilde{\mathcal{O}}_t} v_o \leq Mx \quad \forall t \in \mathcal{T}, \tag{8d}$$
$$\sum_{o\in\tilde{\mathcal{O}}_t} v_o \geq \theta\breve{O}_t + \Gamma\breve{O}_t(Y-y) - M(1-x) \quad \forall t \in \mathcal{T}, \tag{8e}$$
$$\sum_{o\in\tilde{\mathcal{O}}_t} v_o \geq \theta\breve{O}_t x \quad \forall t \in \mathcal{T}, \tag{8f}$$
$$u_o, v_o \in \{0,1\} \quad \forall o \in \tilde{\mathcal{O}}. \tag{8g}$$

The second-stage objective function (8a) computes the total profit from serving orders by their own hired drivers or by the third-party delivery platform. Constraints (8b) restrict that each order can be delivered by at most one approach, that is, by the restaurant, or by the service platform, or neither one. Constraints (8c) are delivery capacity constraints of hired drivers for each day. (One can also define a penalty term $P_t \geq 0$ for each period t, revise Constraints (8c) as $\sum_{o\in\tilde{\mathcal{O}}_t} u_o \leq \Delta y + P_t$ for all $t \in \mathcal{T}$, and subtract $\sum_{t\in\mathcal{T}} P_t$ in the objective function to penalize the number of deliveries that exceeds drivers' capacities in the objective function to allow service delay.) Constraints (8d), (8e), and (8f) together restrict the number of orders delivered by the service platform. Constraints (8d) require that orders can be delivered by the service platform only if the restaurant decides to partner with a third-party platform. Constraints (8e) and (8f) restrict the minimum number of orders delivered by the service platform if the restaurant decides to partner with a service platform, based on the aforementioned assumption. Constraints (8e) and (8f) allow more orders to be delivered by the service platform, which is consistent with real service context. For each order received by the restaurant, if the delivery cost by hired drivers is higher than the partial sale profit charged by the service platform, the restaurant can make an identical order through the service platform to reduce the cost and thus to improve the total profit.

Proposition 1. *Constraints* (8e) *and* (8f) *are the linear formulation of*

$$\sum_{o\in\tilde{\mathcal{O}}_t} v_o \geq x(\theta\breve{O}_t + \Gamma\breve{O}_t(Y-y)^+) \quad \forall t \in \mathcal{T}. \tag{9}$$

Proof. Without loss of generality, in the following proof we consider Inequality (9) and Constraints (8e) and (8f) of a specific time period $t \in \mathcal{T}$. Inequality (9) means, if the restaurant partners with a third-party service platform, that is, $x = 1$, then the number of orders that will be delivered by the service platform is at least $\theta\breve{O}_t + \Gamma\breve{O}_t(Y-y)^+$. Otherwise, there is no restriction on the minimum number of orders delivered by the service platform; that is, we have $\sum_{o\in\tilde{\mathcal{O}}_t} v_o \geq 0$.

Consider Constraints (8e) and (8f). If $x = 1$, then we have $\sum_{o\in\tilde{\mathcal{O}}_t} v_o \geq \theta\breve{O}_t + \Gamma\breve{O}_t(Y - y)$ and $\sum_{o\in\tilde{\mathcal{O}}_t} v_o \geq \theta\breve{O}_t$, which is equivalent to $\sum_{o\in\tilde{\mathcal{O}}_t} v_o \geq \max\{\theta\breve{O}_t + \Gamma\breve{O}_t(Y - y), \theta\breve{O}_t\} = \theta\breve{O}_t + \Gamma\breve{O}_t(Y - y)^+$. If $x = 0$, by the definition of M, we have the right-hand side of (8e) is less than or equal to zero and the right-hand side of (8f) is zero. Based on the previous analysis, we have $\sum_{o\in\tilde{\mathcal{O}}_t} v_o \geq 0$. □

We approximate the optimization formulation (7) through a finite number of realizations of uncertain parameters $(\tilde{p}, \tilde{d}, \tilde{\tau})$, as in the sample average approximation (SAA) approach (Kleywegt et al. 2002). Thus, we consider a finite sample set Ω of the realizations for the random variables $(\tilde{p}, \tilde{d}, \tilde{\tau})$, also denoted for short as the order set $\tilde{\mathcal{O}}$. All scenarios $\omega \in \Omega$ are generated following the Monte Carlo sampling scheme and thus they are independent with each other. Denote the realization of random parameters for scenario $\omega \in \Omega$ as $\tilde{\mathcal{O}}_\omega$. Then, we can use the following optimization problem to approximate formulation (7):

$$\max_{x,y} -c_v\,|\mathcal{T}|\,y + \sum_{\omega\in\Omega} \frac{1}{|\Omega|}[Q(x, y, \tilde{\mathcal{O}}_\omega)], \tag{10a}$$

$$\text{s.t.}\;\; x \in \{0, 1\}, \tag{10b}$$

$$y \geq 0 \,\text{integer}, \tag{10c}$$

where the function $Q(\cdot)$ is the same as that defined in (8).

3.1.2. Random Order-Scenario Generation by Monte Carlo Sampling. We generate the scenario set Ω for future orders. For all scenarios, we generate a fixed number of orders $\breve{O}_t$ in the future day $t \in \mathcal{T}$, i.e., $|\tilde{\mathcal{O}}_t| = \breve{O}_t$, where $\breve{O}_t$ is predicted by ARMA model from Equation (6). The orders $\mathcal{O}_{\omega,t}$ in each scenario $\omega \in \Omega$ for day t, $t \in \mathcal{T}$ is generated through the following procedures. First, we independently simulate $\breve{O}_t$ number of demand locations within a radius r from the restaurant's location following certain distributions through random coordinates generation. These simulated locations are delivery locations for orders $o \in \mathcal{O}_{\omega,t}$. The radius r and the probability that an order shows in a certain area depend on the location and the service pattern of the given restaurant, and they can be either estimated from the historical orders or given by the restaurant operators. Next, we use Google Maps application programming interface (API) to estimate the travel time τ_o and the distance d_o between the generated delivery locations and the restaurant in the given road network, for $o \in \mathcal{O}_{\omega,t}$. Finally, we independently generate random sale profit $p_o \sim \mathcal{P}$ for $o \in \mathcal{O}_{\omega,t}$, where the profit distribution $\mathcal{P}$ can also be estimated from historical order data.

3.2. Discussions

Remark 1. The modules of this integrated tool, such as the prediction and the optimization models, can be modified according to specific practical scenarios, depending on the forms of historical sales data and specific operational restrictions of a restaurant. For example, if a restaurant can access a high-dimensional context variable as prediction input and there exists a large volume of historical data to use, it can use a neural network as the prediction module instead of the ARMA used here.

Remark 2. The optimization model (10) can be used during normal business time where future demand orders can be predicted through other statistical learning approaches without the consideration of infections. In Section 4.2, we present numerical results where the demand orders follow different patterns.

3.2.1. Order Delivery Options. The formulation (8) is an optimistic model with respect to order delivery assignments between restaurants and the platform. In each scenario ω, optimal solutions of the decision variables v_o, $\forall o \in \tilde{\mathcal{O}}_\omega$, which indicate whether each order is delivered by the service platform (if $v_o = 1$ for order o) or by the restaurant (if $v_o = 0$), are obtained to maximize the total profit, through solving Model (8). That is, Model (8) can estimate the best outcome for a restaurant if it decides to partner with a third-party service platform because it also performs an optimal order assignment between the platform and restaurants' own delivery teams. One can flexibly extend this model to accommodate specific requirements on delivery options related to each order. For example, if some orders are submitted directly and will be delivered by the platform, one can set $v_o = 1$ for the corresponding orders and add them as constraints in Model (8).

3.2.2. Modeling Advantage. Another benefit of using Model (8) is to decide the optimal delivery radius without specifically modeling decision-dependent uncertain demand and its probability distributions. According to the partnering process between a restaurant and a third-party delivery platform, the delivery radius shown to users is a decision that needs to be made by the restaurant, which will affect the number of orders received through the platform (i.e., the demand). Thus, if we specify the delivery radius as a decision variable in our formulation, we will have decision-dependent uncertain demand parameter and the model cannot be directly solved via state-of-the-art optimization methods or solvers. To avoid specifically modeling the relationship between delivery range and the resultant demand (which is also ambiguously known), in Model (8), we set the delivery radius as large as possible to cover all potential demand orders. After obtaining the optimal solutions, one can then infer a reasonable choice of the preferred delivery radius, for example, the distance to the farthest location of all the orders that are assigned to the platform to deliver. The specific structure of the delivery radius and the corresponding decision-dependent order uncertainty allow us to (i) first omit the radius decision and (ii) then recover the optimal radius decision based on the optimal solutions and uncertainty realizations. In this way, we avoid modeling the specific function between delivery radius and the demand, as well as the resulting nonlinearity and computational difficulty of the problem.

3.3. Model and Solution Properties

In this section, we derive three solution properties and summarize the related managerial insights.

Proposition 2. *Consider two possible values $\delta_1 < \delta_2$. Let $Z(\delta)$ denote the optimal value of Model (8) with input data δ, and specifically, let $Z_p(\delta)$ denote the optimal value conditional on $x = 1$, that is, if partnering with a third-party platform, and $Z_n(\delta)$ denote the optimal value conditional on $x = 0$, that is, if not partnering with a third-party platform. Therefore, $Z(\delta) = \max\{Z_p(\delta), Z_n(\delta)\}$. Then, we have the following:*

1. Always $Z(\delta_1) \geq Z(\delta_2)$;
2. *If $Z(\delta_1) = Z_n(\delta_1)$, then $Z(\delta_2) = Z_n(\delta_2)$;*
3. *If $Z(\delta_2) = Z_p(\delta_2)$, then $Z(\delta_1) = Z_p(\delta_1)$.*

Proof. We denote Model (8) with δ_1 as P_1 and that with δ_2 as P_2. Consider that one optimal decision of P_2 for achieving $Z(\delta_2)$ is x_2^*, then we can conclude that (i) x_2^* is also feasible to P_1 because the feasible regions of P_1 and P_2 are exactly the same; and (ii) $P_1(x_2^*) \geq P_2(x_2^*)$ because $\delta_1 < \delta_2$ and the parameter δ only shows in the objective function. The optimal value of P_1 is at least the objective value of any feasible solution, and thus we have

$$Z(\delta_1) \geq P_1(x_2^*) \geq P_2(x_2^*) = Z(\delta_2),$$

which completes the proof of the first statement.

If $Z(\delta_1) = Z_n(\delta_1)$, we have $Z_n(\delta_1) \geq Z_p(\delta_1)$. Parameter δ only affects the model when the participating decision x equal to one, and thus we also have $Z_n(\delta_1) = Z_n(\delta_2)$. Therefore, we have

$$Z_n(\delta_2) = Z_n(\delta_1) \geq Z_p(\delta_1) \geq Z_p(\delta_2),$$

where the last inequality is because $\delta_1 < \delta_2$ and the parameter δ only shows in the objective function. We derive that $Z_n(\delta_2) \geq Z_p(\delta_2)$, and thus we have $Z(\delta_2) = Z_n(\delta_2)$, which completes the proof of the second statement.

If $Z(\delta_2) = Z_p(\delta_2)$, we have $Z_p(\delta_2) \geq Z_n(\delta_2)$. Similarly, we always have $Z_n(\delta_1) = Z_n(\delta_2)$. Therefore, we have

$$Z_p(\delta_1) \geq Z_p(\delta_2) \geq Z_n(\delta_2) \geq Z_n(\delta_1),$$

where the first inequality is because $\delta_1 < \delta_2$ and the parameter δ only shows in the objective function. We derive that $Z_p(\delta_1) \geq Z_n(\delta_1)$, and thus we have $Z(\delta_1) = Z_p(\delta_2)$, which completes the proof of the third statement. □

Proposition 3. *Consider two possible values $\theta_1 < \theta_2$. Let $Z(\theta)$ denote the optimal value of Model (8) with input data θ, and specifically, let $Z_p(\theta)$ denote the optimal value conditional on $x = 1$, that is, if partnering with a third-party platform, and $Z_n(\theta)$ denote the optimal value conditional on $x = 0$, that is, if not partnering with a third-party platform. Therefore, $Z(\theta) = max\{Z_p(\theta), Z_n(\theta)\}$. Then, we have*

1. Always $Z(\theta_1) \geq Z(\theta_2)$;
2. *If $Z(\theta_1) = Z_n(\theta_1)$, then $Z(\theta_2) = Z_n(\theta_2)$;*
3. *If $Z(\theta_2) = Z_p(\theta_2)$, then $Z(\theta_1) = Z_p(\theta_1)$.*

Proof. The proof here is similar to that of Proposition 2. We define P_1 and P_2 regarding θ_1 and θ_2. The parameter θ only shows in Constraints (8e) and (8f). Consider that one optimal decision of P_2 for achieving $Z(\theta_2)$ is x_2^*, then we can conclude that (i) x_2^* is also feasible to P_1 because the feasible region of P_1 is larger than that of P_2, given $\theta_1 < \theta_2$; and (ii) $P_1(x_2^*) = P_2(x_2^*)$ because the objective functions are the same. The optimal value of P_1 is at least the objective value of any feasible solution, and thus we have

$$Z(\theta_1) \geq P_1(x_2^*) = P_2(x_2^*) = Z(\theta_2),$$

which completes the proof of the first statement.

If $Z(\theta_1) = Z_n(\theta_1)$, we have $Z_n(\theta_1) \geq Z_p(\theta_1)$. Parameter θ only affects the model when the participating decision x equal to one, and thus we also have $Z_n(\theta_1) = Z_n(\theta_2)$. Denote the optimal decision to P_2 conditional on participating with the delivery platform as x_{2p}^*, then by $\theta_1 < \theta_2$, we have x_{2p}^* is also feasible to P_1 and $P_1(x_{2p}^*) = P_2(x_{2p}^*) = Z_p(\theta_2)$. Therefore, similarly to the proof of Proposition 2, we have

$$Z_n(\theta_2) = Z_n(\theta_1) \geq Z_p(\theta_1) \geq P_1(x_{2p}^*) = P_2(x_{2p}^*) = Z_p(\theta_2),$$

where the inequality $Z_p(\theta_1) \geq P_1(x_{2p}^*)$ is because the former is the optimal objective value conditional on $x = 1$ and the latter is the objective value of a feasible solution conditional on $x = 1$. We derive that $Z_n(\theta_2) \geq Z_p(\theta_2)$, and thus we have $Z(\theta_2) = Z_n(\theta_2)$, which completes the proof of the second statement.

If $Z(\theta_2) = Z_p(\theta_2)$, we have $Z_p(\theta_2) \geq Z_n(\theta_2)$. Similarly, we always have $Z_n(\theta_1) = Z_n(\theta_2)$. Therefore, we have

$$Z_p(\theta_1) \geq Z_p(\theta_2) \geq Z_n(\theta_2) = Z_n(\theta_1),$$

where the first inequality is proved in the previous paragraph in the proof of the second statement. We derive that $Z_p(\theta_1) \geq Z_n(\theta_1)$, and thus we have $Z(\theta_1) = Z_p(\theta_1)$, which completes the proof of the third statement. □

Proposition 4. *Consider two possible values $\Gamma_1 < \Gamma_2$. Let $Z(\Gamma)$ denote the optimal value of Model (8) with input data Γ, and specifically, let $Z_p(\Gamma)$ denote the optimal value conditional on $x = 1$, that is, if partnering with a third-party platform, and $Z_n(\Gamma)$ denote the optimal value conditional on $x = 0$, that is, if not partnering with a third-party platform. Therefore, $Z(\Gamma) = max\{Z_p(\Gamma), Z_n(\Gamma)\}$. Then, we have*

1. *Always $Z(\Gamma_1) \geq Z(\Gamma_2)$;*
2. *If $Z(\Gamma_1) = Z_n(\Gamma_1)$, then $Z(\Gamma_2) = Z_n(\Gamma_2)$;*
3. *If $Z(\Gamma_2) = Z_p(\Gamma_2)$, then $Z(\Gamma_1) = Z_p(\Gamma_1)$.*

Proof. The proof here follows the same analysis steps as those of Propositions 2 and 3. We define P_1 and P_2 regarding Γ_1 and Γ_2. The same as the input θ as discussed in Proposition 3, the parameter Γ also only shows in Constraints (8e).

Consider that one optimal decision of P_2 for achieving $Z(\Gamma_2)$ is x_2^*, then we can conclude that (i) x_2^* is also feasible to P_1 because the feasible region of P_1 is larger than that of P_2, given $\Gamma_1 < \Gamma_2$; and (ii) $P_2(x_2^*) = P_1(x_2^*)$ because the objective functions are the same. The optimal value of P_1 is at least the objective value of any feasible solution, and thus we have

$$Z(\Gamma_1) \geq P_1(x_2^*) = P_2(x_2^*) = Z(\Gamma_2),$$

which completes the proof of the first statement.

If $Z(\Gamma_1) = Z_n(\Gamma_1)$, we have $Z_n(\Gamma_1) \geq Z_p(\Gamma_1)$. Parameter Γ only affects the model when the participating decision x equal to 1, and thus we also have $Z_n(\Gamma_1) = Z_n(\Gamma_2)$. Denote the optimal decision to P_2 conditional on participating with the delivery platform as x_{2p}^*, then by $\Gamma_1 < \Gamma_2$, we have x_{2p}^* is also feasible to P_1 and $P_1(x_{2p}^*) = P_2(x_{2p}^*) = Z_p(\Gamma_2)$. Therefore, we have

$$Z_n(\Gamma_2) = Z_n(\Gamma_1) \geq Z_p(\Gamma_1) \geq P_1(x_{2p}^*) = P_2(x_{2p}^*) = Z_p(\Gamma_2),$$

where the inequality $Z_p(\Gamma_1) \geq P_1(x_{2p}^*)$ is because the former is the optimal objective value conditional on $x=1$ and the later is the objective value of a feasible solution conditional on $x = 1$. We derive that $Z_n(\Gamma_2) \geq Z_p(\Gamma_2)$, and thus we have $Z(\Gamma_2) = Z_n(\Gamma_2)$, which completes the proof of the second statement.

If $Z(\Gamma_2) = Z_p(\Gamma_2)$, we have $Z_p(\Gamma_2) \geq Z_n(\Gamma_2)$. Similarly, we always have $Z_n(\Gamma_1) = Z_n(\Gamma_2)$. Therefore, we have

$$Z_p(\Gamma_1) \geq Z_p(\Gamma_2) \geq Z_n(\Gamma_2) = Z_n(\Gamma_1),$$

where the first inequality is proved in the previous paragraph in the proof of the second statement. We derive that $Z_p(\Gamma_1) \geq Z_n(\Gamma_1)$, and thus we have $Z(\Gamma_1) = Z_p(\Gamma_1)$, which completes the proof of the third statement. □

Propositions 2–4 indicate that a restaurant can benefit from partnering with a third-party platform, when (i) the proportion of the sales that needs to be paid to the platform drops (i.e., δ decreases), (ii) the minimum amount of orders placed through the platform decreases (i.e., θ decreases), or (iii) the percentage of customers, who will only choose the platform rather than placing orders directly with the restaurant, decreases (i.e., Γ decreases), respectively. The results of the three propositions are also reflected in the computational results later in Section 4.1.3.

4. Numerical Studies

We conduct two sets of numerical experiments. In Section 4.1, we evaluate our model, consisting of demand prediction and tactical decision, with COVID-19 infection data and historical order data of a restaurant.

The numerical results are consistent with the solution properties derived in Section 3.3.

Section 4.2 focuses on testing the optimization formulation developed in Section 3 for different demand patterns.

From the numerical results in Sections 4.1 and 4.2, we conclude that the tactical decisions are mainly affected by partnering policies, demand response, and demand patterns. Specifically, when (i) the proportional revenue charged by the platform is low, (ii) customers can flexibly decide whether to order from platforms or restaurants directly, (iii) customers require more efficient delivery, (iv) average delivery distance of all the orders is long, and (v) the demand variance is high, restaurants can gain more benefits from partnering with third-party delivery platforms.

4.1. Results of a Real-World Example During the COVID-19 Pandemic

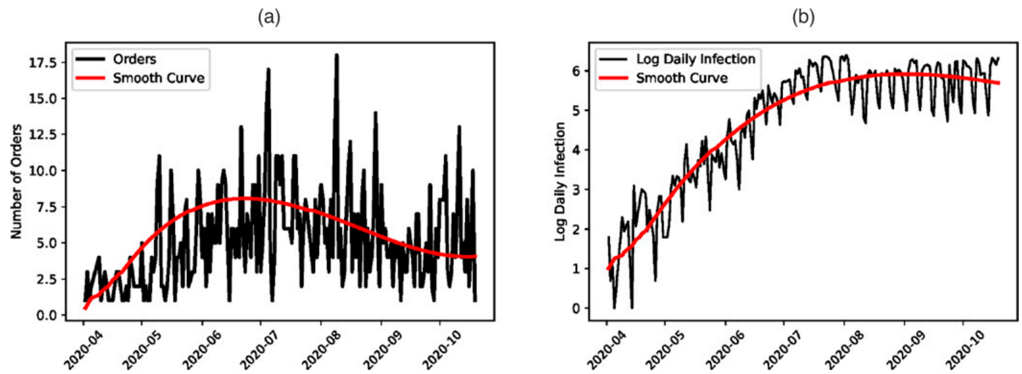
We consider a real-world restaurant located in Nuevo Leon, a state in the northeast of Mexico. The historical daily infection data and the number of takeout orders of the restaurant from April 2 to October 19, 2020, are presented in Figure 3.

4.1.1. Results of SIR Model and Infection Prediction.

We follow the approaches in Bagal et al. (2020) to estimate the two parameters of the SIR model, β and γ, for Nuevo Leon, Mexico. We use COVID-19 data from COVID-19 Mexico (2020), an interactive website maintained by the Mexican government. We first convert the daily infected, recovery, and death data into the population of subgroups I_t and R_t for each day t. We estimate the parameters and compute the susceptible population S_t by plugging in the total population of Nuevo Leon as $n = 5{,}000{,}000$.

We first construct an SIR model with data from April 2 to October 19, 2020. The estimation results are shown in Figure 4. Figure 4(a) shows the real-world

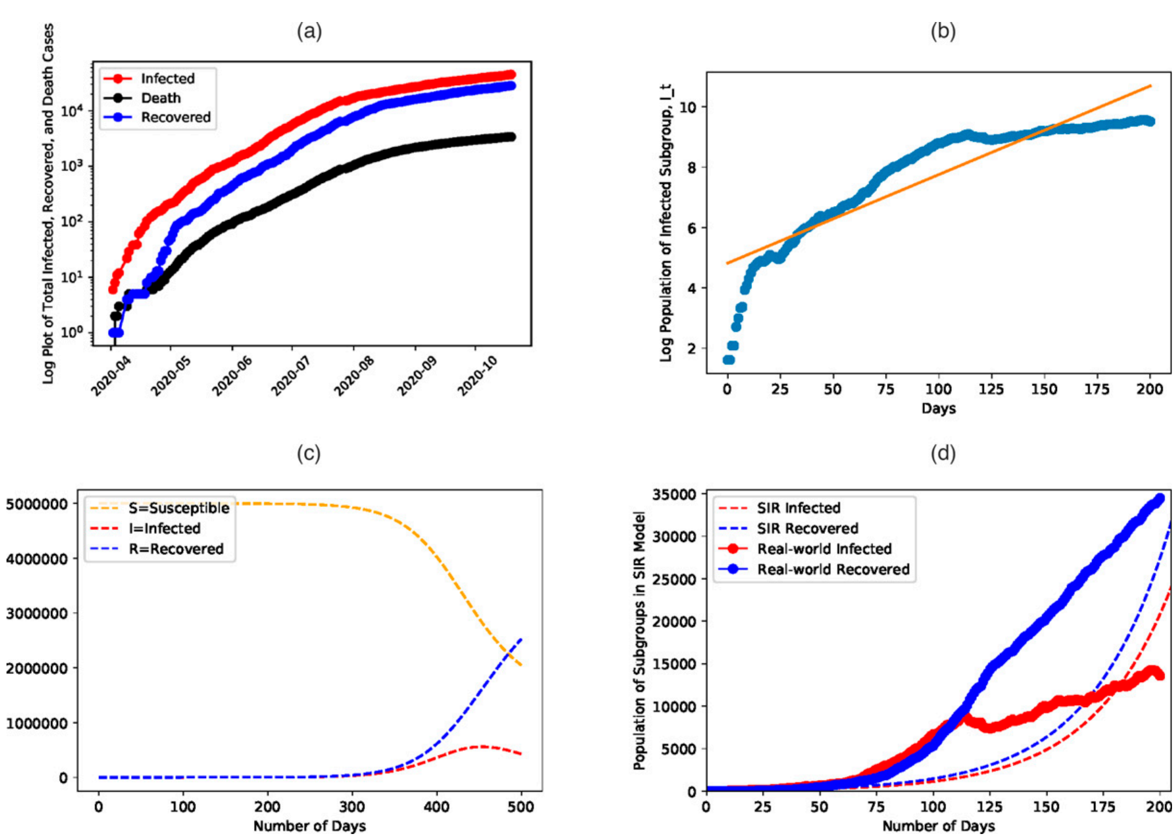
Figure 3. (Color online) Historical Data from April 2 to October 19, 2020



Notes. (a) Number of takeout orders of a local restaurant in Nuevo Leon, Mexico. (b) Log plot of daily new infected cases of Nuevo Leon, Mexico.

COVID-19 data we collected from COVID-19 Mexico (2020), including cumulative infected cases, death tolls, and recovery cases. Figure 4(b) shows the log-plot of I_t versus t, where we compute I_t by the difference between the cumulative infected cases minus the cumulative recovered cases (including cases who have been infected and recovered later or died). We estimate the value of $\hat{\beta} - \hat{\gamma}$ by the slope of the fitted line in the log-plot as 0.0293 and further estimate that $\hat{\beta} = 0.0675$ and $\hat{\gamma} = 0.0382$. Figure 4(c) shows the predicted population of three subgroups, that is, $(\breve{S}_t, \breve{I}_t, \breve{R}_t)$, for 500 days by SIR starting from $I_0 = 10$. Figure 4(d) compares the real-world populations of three subgroups (solid) and the predicted populations of three subgroups by the SIR model (dashed) from April 2 to October 19, 2020.

Figure 4. (Color online) Results for SIR Model with Data from April 2 to October 19, 2020



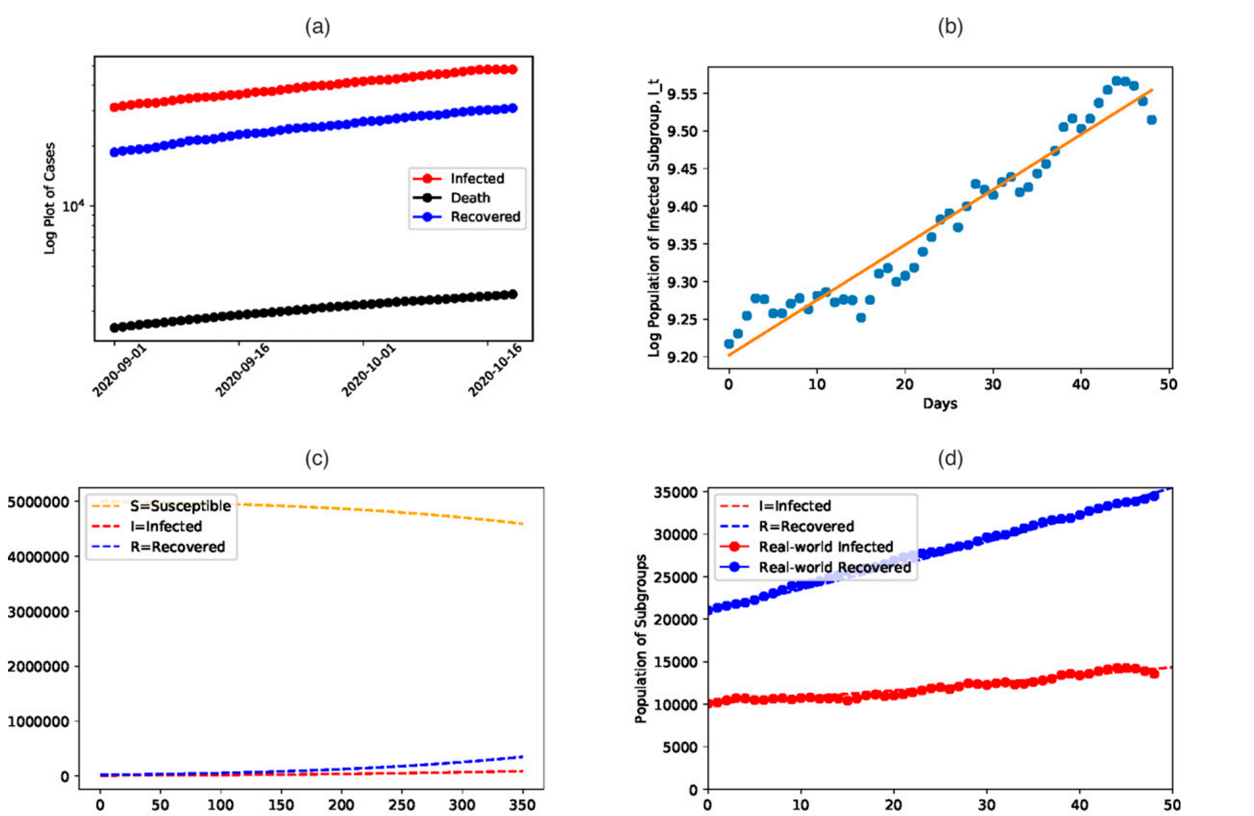
Notes. (a) Real-world infected, recovery, and death cases. (b) Estimation from the slope of the fitted line (orange): $\hat{\beta} - \hat{\gamma} = 0.0293$. (c) SIR prediction of subgroup populations for 500 days. (d) Real-world subgroup populations versus predicted subgroup populations by SIR.

From the comparison results in Figure 4(d), we can observe that the real-world infected subgroup grows faster than that of the SIR model in the first 150 days and then slower than that of the SIR model in the rest 50 days. The reason is that the SIR model assumes fixed transmission rate β and recovery rate γ over all time periods while in the real-world, both rates change because of the reshaping of human activities, policies issued by governments, improvement of disease treatment, and so on. Therefore, we construct another SIR model with COVID-19 data in shorter time periods, where we assume the transmission and the recovery rates are fixed during this period and in future days, so that we can predict future infected cases through this SIR model. We assume that both transmission and recovery rates keep stable from September 1, 2020, to November 3, 2020. We use COVID-19 data from September 1 to October 19, 2020, to estimate the parameters for the SIR model and later predict the daily infected cases for October 20 to November 3, 2020, that is, two weeks in the future. The estimation results are shown in Figure 5. The contents of Figure 5, (a) to (d), are similar to those of Figure 4, (a) to (d). The estimated transition rate is $\hat{\beta} = 0.0313$, and the estimated recovery rate is $\hat{\gamma} = 0.0240$. From the estimated values of β and γ in these two SIR models, we notice that the spread of COVID-19 disease in Nuevo Leon in September 2020 is slower than that in April 2020. This can also be directly observed by comparing Figures 4(c) and 5(c), where in the latter, the infected subgroup grows much slower. Figure 4(c) predicts 500 days starting on April 2, and Figure 5(c) predicts 350 days starting on September 1, and thus these two figures end almost on the same day. In Figure 5(d), we conclude that this new SIR model fits the historical data well, and we believe it can also well predict the infected and recovery cases for the two weeks in the future.

4.1.2. Results of ARMA Model and Order Prediction.

We analyze the historical order data of the given restaurant from April 2, 2020, to October 19, 2020. As stated in Section 2.2, we tested different combinations of independent variables, including log daily infected cases in previous days, the historical orders in previous days, and random errors in previous days. We also verify that the model including log daily infected cases performs significantly better than that directly using daily infected cases. We finally build the following

Figure 5. (Color online) Results for SIR Model with Data from September 1 to October 19, 2020



Notes. (a) Real-world infected, recovery, and death cases. (b) Estimation from the slope of the fitted line (orange): $\hat{\beta} - \hat{\gamma} = 0.0073$. (c) SIR prediction of subgroup populations for 350 days. (d) Real-world subgroup populations versus predicted subgroup populations by SIR.

model according to the significance of independent variables:

$$O_t = \beta_0 + \sum_{i=0,1,2,3,4} \beta_{w_i} w_i + \beta_{i_2} \cdot \log i_{t-2} + \alpha_1 O_{t-1} + \epsilon_t. \tag{11}$$

All the independent variables in the ARMA Model (11) are significant, and the p values associating with all coefficients are less than 0.05, which means that all the coefficients are not zero with at least 95% confidence. The R^2 of Model (11) is 0.471, and the adjusted R^2 is 0.447.

The p value of coefficient β_{w_5} of Model (3) is 0.7, and thus we cannot conclude that w_5 can affect the number of orders. This result indicates that there are no significant changes in takeout orders on Saturday and Sunday. This observation is consistent with people's intuition that Saturday and Sunday are weekends, and thus human activities are similar on these two weekdays. Therefore, Model (11) does not differentiate between Saturday and Sunday when predicting the orders. We also compare the results of using the daily infected cases during the order date versus using the daily infected cases of previous dates. We find that the daily infected cases of previous dates have a more significant effect on the number of takeout orders, which validates our assumption on the latency between the time of notifying the infected cases and the time of making online takeout food orders. Moreover, we find that for this specific order data set, the ARMA model has the best performance when only including the number of takeout orders of yesterday and the log value of the number of daily infected cases of the day before yesterday.

4.1.3. Results of Tactical Decision. We make tactical decisions for the future two weeks, that is, from October 20, 2020, to November 3, 2020. We forecast the daily infected cases by the SIR model and use the forecasted results to predict the number of orders by Model (11). The results are presented in Table 1.

We sample 10 scenarios for the locations of future orders within a radius $r = 10$ km. We use Google Maps distance matrix API to export the travel distance and time between the restaurant and sampled order locations. The order sales profit is generated following a truncated normal distribution with the mean and variance computed from the historical order data. We set the daily salary paid to each driver as \$50. The restaurant also pays hired drivers \$4 for every 1 km and \$15 for every hour as a stipend during the delivery process. The restaurant needs to pay 20%–30% of the sales profit to the third-party delivery platform for the orders delivered by the platform. We further assume different values for model parameter, θ and δ, which affect the percentage of orders received by the third-party service once deciding to partner. The results of solving Optimization Program (10) are shown in Table 2. The columns Partner decision and Number of drivers show the tactical decisions for the restaurant. The column Delivered by restaurant shows the average percentage (for all scenarios) of orders delivered by the restaurant, computed through the optimal decisions u^* as $(\sum_{\omega\in\Omega}\sum_{o\in\mathcal{O}_\omega} u^*_{o,\omega})(|\Omega|\sum_{t\in\mathcal{T}}\breve{O}_t)$. Similarly, the column Delivered by third-party service shows the average percentage of orders delivered by the service platform, computed through the optimal decisions v^* as $(\sum_{\omega\in\Omega}\sum_{o\in\mathcal{O}_\omega} v^*_{o,\omega})/(|\Omega|\sum_{t\in\mathcal{T}}\breve{O}_t)$. The last column computes the average percentage of orders being served by either the restaurant or the delivery service platform.

4.1.4. Delivery Capability of Driver Δ. Comparing the solutions under different assumptions on the number of orders that a driver can deliver, we notice that when the capability Δ is larger, then the restaurant tends to not partner with a third-party delivery service platform. This result is consistent with our intuition. When the capability is larger, the number of required drivers is smaller (it can also be observed

Table 1. Predicted Daily Infection and Orders for Future Two Weeks

Date	Predicted daily infection	Predicted demand
10/20/20	446	7
10/21/20	450	5
10/22/20	454	7
10/23/20	455	6
10/24/20	459	9
10/25/20	463	9
10/26/20	465	4
10/27/20	469	6
10/28/20	471	5
10/29/20	475	7
10/30/20	479	6
10/31/20	482	9
11/01/20	485	9
11/02/20	489	4

Table 2. Optimal Decisions Under Different Settings When Partnering with Third-Party Services

Index	Δ	δ	θ	Obj	Partner decision	Number of drivers	Delivered by restaurant (%)	Delivered by third-party service (%)	Served orders (%)
1	2	0.1	0.2	248,323.0	Yes	1	22.04	77.96	100.00
2	2	0.1	0.4	248,219.4	Yes	1	21.29	78.71	100.00
3	2	0.1	0.6	247,245.5	Yes	2	18.49	81.51	100.00
4	2	0.3	0.2	233,624.1	Yes	4	70.75	29.25	100.00
5	2	0.3	0.4	231,943.9	No	5	93.66	0.00	93.66
6	2	0.3	0.6	231,943.9	No	5	93.66	0.00	93.66
7	2	0.5	0.2	231,943.9	No	5	93.66	0.00	93.66
8	2	0.5	0.4	231,943.9	No	5	93.66	0.00	93.66
9	2	0.5	0.6	231,943.9	No	5	93.66	0.00	93.66
10	6	0.1	0.2	248,618.3	Yes	1	26.99	73.01	100.00
11	6	0.1	0.4	248,315.7	Yes	1	23.01	76.99	100.00
12	6	0.1	0.6	247,245.5	Yes	2	18.49	81.51	100.00
13	6	0.3	0.2	234,324.1	Yes	3	70.75	29.25	100.00
14	6	0.3	0.4	234,043.9	No	2	93.66	0.00	93.66
15	6	0.3	0.6	234,043.9	No	2	93.66	0.00	93.66
16	6	0.5	0.2	234,043.9	No	2	93.66	0.00	93.66
17	6	0.5	0.4	234,043.9	No	2	93.66	0.00	93.66
18	6	0.5	0.6	234,043.9	No	2	93.66	0.00	93.66
19	10	0.1	0.2	248,618.3	Yes	1	26.99	73.01	100.00
20	10	0.1	0.4	248,315.7	Yes	1	23.01	76.99	100.00
21	10	0.1	0.6	247,245.5	Yes	2	18.49	81.51	100.00
22	10	0.3	0.2	234,743.9	No	1	93.66	0.00	93.66
23	10	0.3	0.4	234,743.9	No	1	93.66	0.00	93.66
24	10	0.3	0.6	234,743.9	No	1	93.66	0.00	93.66
25	10	0.5	0.2	234,743.9	No	1	93.66	0.00	93.66
26	10	0.5	0.4	234,743.9	No	1	93.66	0.00	93.66
27	10	0.5	0.6	234,743.9	No	1	93.66	0.00	93.66

from the table). Therefore, the cost of operating their delivery service by the restaurant is smaller.

4.1.5. Percentage of Sales Profit Paid to the Third-Party Delivery Service Platform δ.

The results are consistent with Proposition 2. If the delivery service fee charged by the third-party platform is relatively lower, then the restaurant tends to partner with the service platform to reduce the cost and thus improve the profit. We also observe that under the current parameter settings, the restaurant always hires drivers while deciding to partner with the third-party service platform. The reason is that for orders whose locations are close to the restaurant, the cost of delivering by hired drivers is less than the partial sales profit paid to the third-party service. Therefore, the restaurant prefers to deliver these orders by themselves to improve the total profit. When the restaurant decides to partner with a third-party delivery service platform, all the orders can be served. When the restaurant does not partner with a third-party platform, it may reject some orders to maximize the total profit.

4.1.6. Percentage of Customers Using the Third-Party Service θ.

The decision to partner with a third-party service platform will also affect the percentage of orders that will be received from the service platform or from the restaurant itself. Comparing settings 4, 5, and 6 (also by settings 13, 14, and 15), we conclude that when the minimum percentage of orders received from the service platform is larger, the restaurant may decide not to partner. From the results of Settings 13 and 14 (or by Settings 21 and 22), we can observe that in some cases, the restaurant decides to hire one more driver when partnering with a delivery service platform. This contradicts our intuition, while we typically assume the restaurant will hire fewer drivers when partnering with third-party platforms because the total number of orders that need to be delivered by hired drivers is smaller. The reason is the assumption that the minimum number of orders received by the service platform increases when the hired drivers are fewer than a threshold value of Y. Therefore, in some cases, the restaurant even hires more drivers, that is, pays more for operating its own delivery team, to decrease the minimum number of orders delivered by the platform to increase the total profit. Therefore, we can infer that, although the idleness of drivers increases, the total profit rises also. The results are consistent with Proposition 3.

4.2. Results of Different Order Patterns

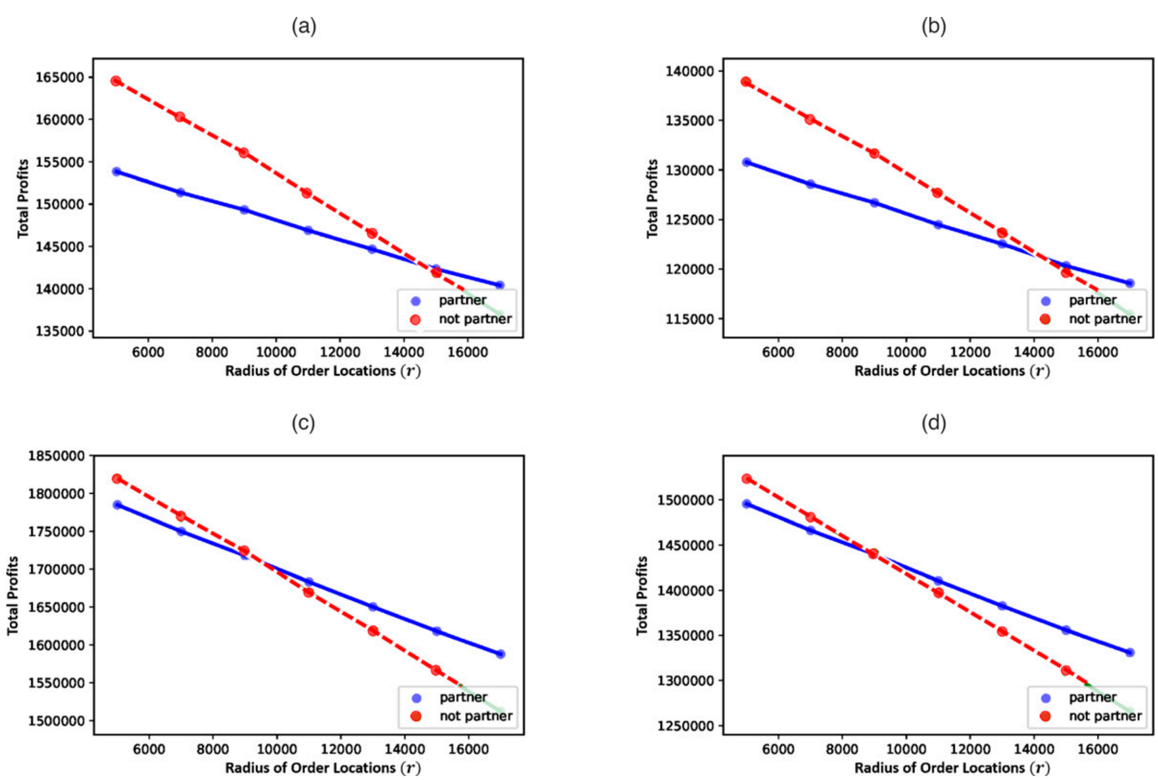
We consider the tactical decisions under different order patterns. We focus on time-stationary demand, where the patterns are not changing over time, and specifically, we take three features of order pattern into consideration. The first feature is the average number of orders received each day. For large restaurants, they receive more delivery orders compared with restaurants on a relatively small scale. The second feature is the standard deviation of the number of orders received each day. For restaurants that serve a relatively fixed group of customers, the orders they received each day is more stable compared with the restaurants that do not have a fixed group of customers. The third feature is the service radius for the delivery orders. For restaurants in downtown or other population-gathering areas, most of the orders are from customers nearby, and thus the service radius is small. For restaurants in the suburb, the orders can from customers living in different areas of the city, and therefore the service radius is large.

We present the results of several instances, where we vary the value of one feature and keep the other two features fixed. For all instances, we consider the tactical decisions for future 14 days, that is, $\mathcal{T} = [14]$. The number of orders $|\tilde{\mathcal{O}}_t|$ of day t, $\forall t \in \mathcal{T}$ are sampled from truncated independent and identically distributed (i.i.d.) normal distributions with mean $\bar{O}$ and standard deviation (STD) σ_O. Then, the order realizations in each scenario $\omega \in \Omega$ are generated following the same Monte Carlo sampling approach under radius r. To fully show the changes in profits when the restaurant partners with the third-party service platform or not and to show the transition between the decision for partnering or not partnering, we present the optimal profit of the restaurant when it partners or does not with the third-party service platform separately. Therefore, the optimal objective profit of Problem (10) is the maximum value of these two types of profit.

4.2.1. Decisions Under Different Service Radius. We consider two levels of the average order numbers, $\bar{O} \in \{5, 50\}$ and two levels of the STD of order numbers, $\sigma_O = \{0.1, 0.6\} \times \bar{O}$. We vary the radius r in $\{3 + 2 \times i : i = 0, 1, 2, \ldots, 6\}$ km. We present the results in Figure 6.

Generally, when the service radius is becoming larger, the optimal decision for the restaurant changes from not partnering to partnering by observing the blue line becomes above the red line. When the average number of orders per day is larger, the restaurant benefits more from partnering with a third-party platform by observing the intersection points are at smaller radii in Figure 6, (c) and (d) compared with Figure 6, (a) and (b). When the variance of the number of orders per day is larger, that is, larger STD, the restaurant benefits more from partnering with a third-party platform, by observing the intersection points are at

Figure 6. (Color online) Optimal Profits of the Restaurant When the Service Radius Changes from 3 to 15 Km



Notes. Blue, profits when restaurant partners with a third-party service platform; red, profits when restaurant does not partner with a third-party service platform. (a) $\bar{O} = 5$, $\sigma_O = 0.1\bar{O}$. (b) $\bar{O} = 5$, $\sigma_O = 0.6\bar{O}$. (c) $\bar{O} = 50$, $\sigma_O = 0.1\bar{O}$. (d) $\bar{O} = 50$, $\sigma_O = 0.6\bar{O}$.

smaller radii in Figure 6, (b) and (d) compared with Figure 6, (a) and (c) (slightly smaller in these instances). Our model will always choose the decision that achieves the higher value. Therefore, if a restaurant makes a different partnering decision, then the difference between these two curves will reflect the revenue enhancement that will be brought by our model (the same in Figures 7 and 8).

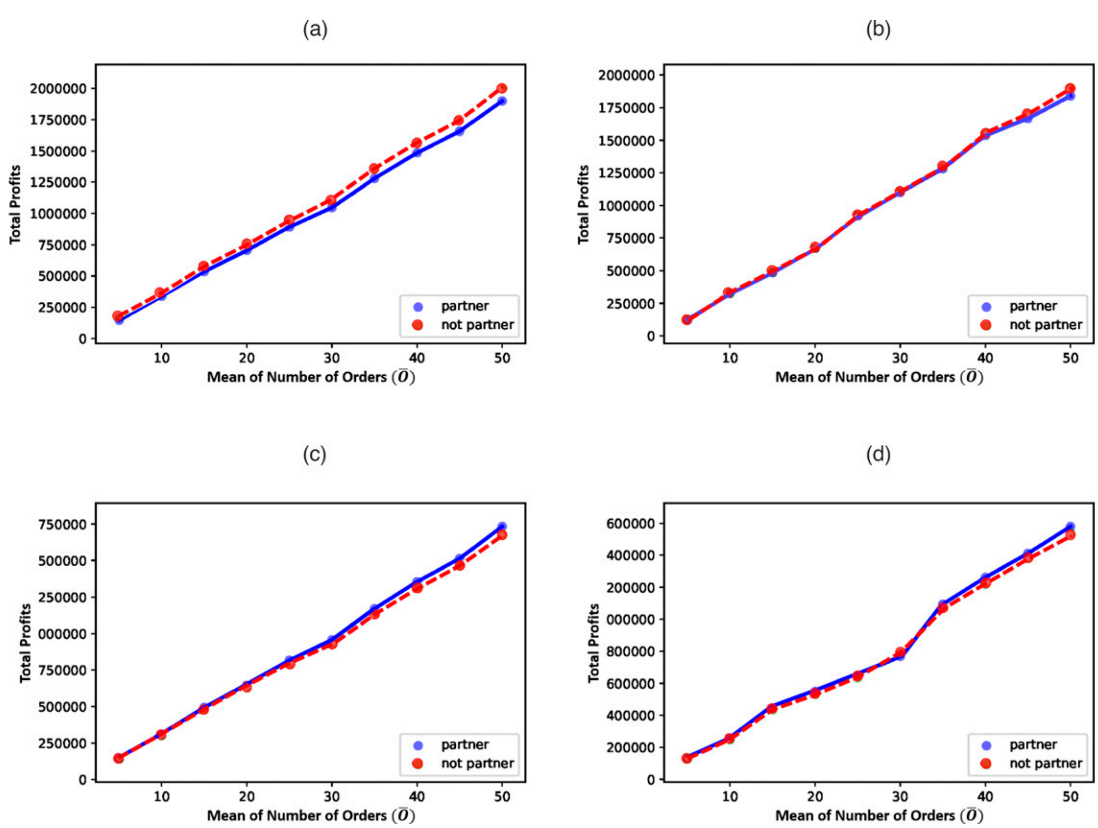
4.2.2. Decisions Under Different Average Number of Orders. We consider two levels of radius, $r \in \{3, 15\}$ km and two levels of the STD of order numbers, $\sigma_O = \{0.1, 0.6\} \times \bar{O}$. We vary the average order numbers $\bar{O} \in \{5 + 5 \times i : i = 0, 1, 2, \ldots, 9\}$. We present the results in Figure 7.

When the average number of orders is becoming larger, the difference between partnering or not partnering is also becoming larger, which requires the restaurant to pay more attention to tactical decisions. We draw the same conclusion as in Section 4.2.1, when the service radius is larger, the restaurant tends to partner with the delivery service platform by observing that the blue line is above the red line in Figure 7, (c) and (d), and vice versa in Figure 7, (a) and (b). Comparing Figure 7, (a) and (b) (or Figure 7, (c) and (d)), we observe that the difference between partnering or not partnering is smaller when the variance of daily orders is large. Because under both decisions, partnering or not, the restaurant hires drivers and thus needs to pay a fixed salary to drivers, which may lead to idleness and cost because of the unstable demand.

4.2.3. Decisions Under Different Standard Deviation of Number of Order. We consider two levels of radius, $r \in \{3, 15\}$ km and two levels of the average number of daily orders $\bar{O} = \{5, 50\}$. We vary the STD of the number of daily orders in the range $\sigma_O = \{0.1 \times i : i = 1, 2, \ldots, 6\} \times \bar{O}$. We present the results in Figure 8.

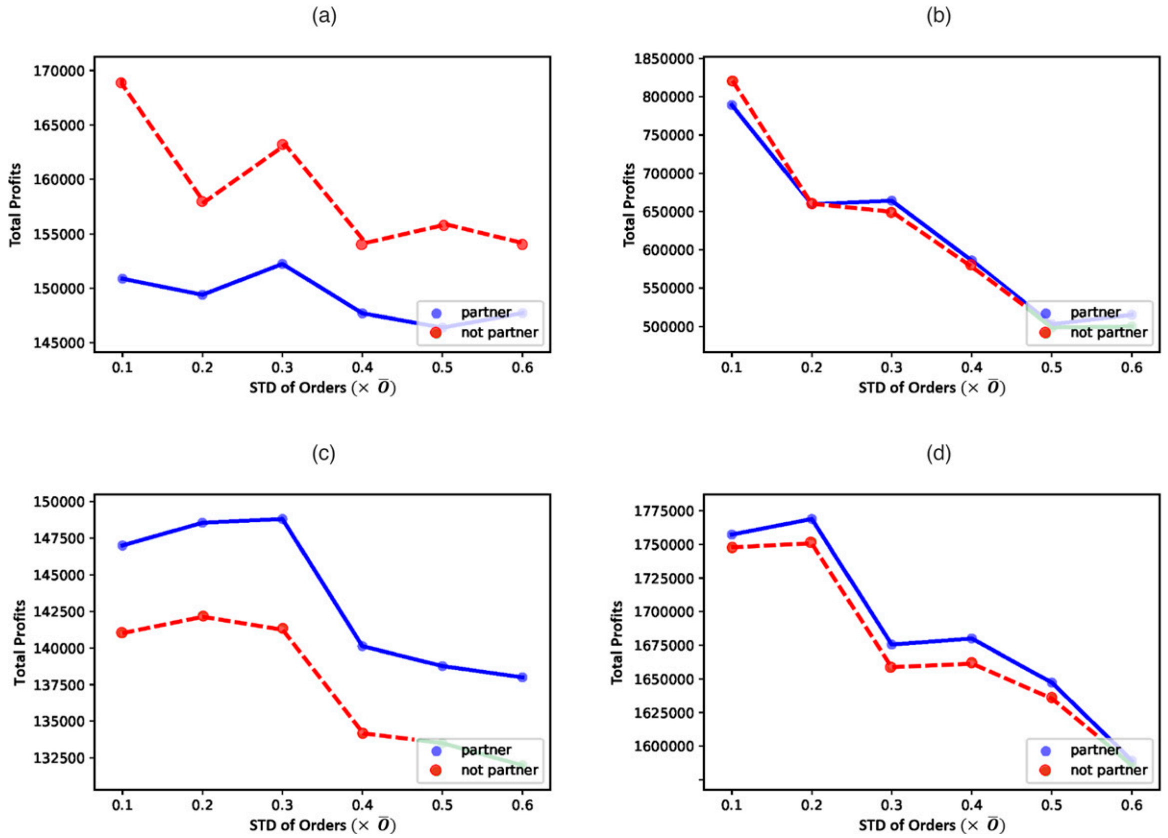
We conclude that when the variance of the number of orders is larger, the total profit of both partnering decisions, partnering or not, is decreasing. Similar to Sections 4.2.1 and 4.2.2, we can conclude that when the service radius is smaller, the restaurant tends to not partner with a third-party platform because the cost for operating an own delivery team is relatively low. However, in Figure 8(b), we can observe that when the variance is becoming larger and larger, the

Figure 7. (Color online) Result 2: Optimal Profits of the Restaurant When the Average Number of Daily Orders Changes from 5 to 50



Notes. Blue, profits when restaurant partners with a third-party service platform; red, profits when restaurant does not partner with a third-party service platform. (a) $r = 3$, $\sigma_O = 0.1\bar{O}$. (b) $r = 3$, $\sigma_O = 0.6\bar{O}$. (c) $r = 15$, $\sigma_O = 0.1\bar{O}$. (d) $r = 15$, $\sigma_O = 0.6\bar{O}$.

Figure 8. (Color online) Result 3: Optimal Profits of the Restaurant When the STD of Number of Daily Orders Changes from 0.1 to 0.6 Times the Average Number of Orders



Notes. Blue, profits when restaurant partners with a third-party service platform; red, profits when restaurant does not partner with a third-party service platform. (a) $r = 3, \bar{O} = 5$. (b) $r = 3, \bar{O} = 50$. (c) $r = 15, \bar{O} = 5$. (d) $r = 15, \bar{O} = 50$.

restaurant will switch to partner with a third-party platform to counter the risk from the unstable demand and decrease the cost.

5. Conclusion

In this paper, we developed a prediction-and-decision model to help restaurants to make tactical decisions, including whether to partner with third-party delivery services and the number of drivers to be hired during or post a pandemic under stochastic demand surge. Our approach implemented demand prediction and decision making sequentially. We used an SIR model in epidemiology to analyze the disease spread and further constructed an ARMA model to predict the number of future orders, which took the forecasted infected cases as inputs. Then, we formulated a stochastic integer program based on the predicted amount of orders and sampled sales profit and locations of future orders. We conducted numerical experiments using real-world COVID-19 infection data, restaurant food ordering data, and simulated various demand patterns to derive computational results.

Results along the following future research directions can provide more support for restaurants' tactical decision making. First, one can explore and involve more features that have impacts on order demand to improve the prediction power of the regression model. Second, it is interesting to compare results obtained by using optimization frameworks based on different risk measures of the undesirable outcomes under stochastic demand surges.

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References

Abay KA, Tafere K, Woldemichael A (2020) Winners and losers from COVID-19: Global evidence from Google search. Policy Research Working Paper No. 9268, World Bank, Washington, DC. https://openknowledge.worldbank.org/handle/10986/33852 License: CC BY 3.0 IGO.

Alavi SA, Rezaei S, Valaei N, Wan Ismail WK (2016) Examining shopping mall consumer decision-making styles, satisfaction and purchase intention. *Internat. Rev. Retail Distribution Consumer Res.* 26(3):272–303.

Bagal DK, Rath A, Barua A, Patnaik D (2020) Estimating the parameters of susceptible-infected-recovered model of COVID-19 cases in India during lockdown periods. *Chaos Solitons Fractals* 140: 110154.

Banskota S, Healy M, Goldberg EM (2020) 15 smartphone apps for older adults to use while in isolation during the COVID-19 pandemic. *Western J. Emerging Medicine* 21(3):514.

Biswas K, Khaleque A, Sen P (2020) COVID-19 spread: Reproduction of data and prediction using a SIR model on Euclidean network. Preprint, submitted March 16, https://arxiv.org/abs/2003.07063.

Bräysy O, Gendreau M (2005) Vehicle routing problem with time windows, part I: Route construction and local search algorithms. *Transportation Sci.* 39(1):104–118.

Brillinger DR (2001) *Time Series: Data Analysis and Theory* (SIAM, Philadelphia).

Chopra S, Rajan P (2016) Modeling intermediary satisfaction with mandatory adoption of e-government technologies for food distribution. *Inform. Tech. Internat. Development* 12(1):15.

COVID-19 Mexico (2020) Covid-19 Mexico. Accessed July 10, 2021, https://datos.covid-19.conacyt.mx/#DownZCSV.

Dhanwant JN, Ramanathan V (2020) Forecasting COVID-19 growth in India using susceptible-infected-recovered (SIR) model. Preprint, submitted April 1, https://arxiv.org/abs/2004.00696.

Dingel JI, Neiman B (2020) How many jobs can be done at home? Technical report, National Bureau of Economic Research, Cambridge, MA.

Fuller WA (2009) *Introduction to Statistical Time Series*, vol. 428 (John Wiley & Sons, Hoboken, NJ).

Gera M, Nawander N, Tharwani N, Bhatia P (2018) Operations research in food delivery. *Internat. J. Adv. Res. Development.* 3(10): 73–78.

Hirschberg C, Rajko A, Schumacher T, Wrulich M (2016) The changing market for food delivery. Technical Report, McKinsey & Company. Accessed July 10, 2021, https://www.mckinsey.com/industries/technology-media-and-telecommunications/our-insights/the-changing-market-for-food-delivery.

Hsu CI, Hung SF, Li HC (2007) Vehicle routing problem with time-windows for perishable food delivery. *J. Food Engrg.* 80(2): 465–475.

Huber J, Gossmann A, Stuckenschmidt H (2017) Cluster-based hierarchical demand forecasting for perishable goods. *Expert Systems Appl.* 76:140–151.

Ingham J, Cadieux J, Berrada AM (2015) E-shopping acceptance: A qualitative and meta-analytic review. *Inform. Management* 52(1): 44–60.

Kimes SE (2011) The current state of online food ordering in the US restaurant industry. *Cornell Hospitality Report* 11(17):6–18.

Kleywegt AJ, Shapiro A, Homem-de Mello T (2002) The sample average approximation method for stochastic discrete optimization. *SIAM J. Optim.* 12(2):479–502.

Koren M, Pető R (2020) Business disruptions from social distancing. Preprint, submitted March 31, https://arxiv.org/abs/2003.13983.

Lasek A, Cercone N, Saunders J (2016) *Restaurant Sales and Customer Demand Forecasting: Literature Survey and Categorization of Methods. Smart City 360 Degree* (Springer, Berlin).

Littler D, Melanthiou D (2006) Consumer perceptions of risk and uncertainty and the implications for behaviour toward innovative retail services: The case of Internet banking. *J. Retailing Consumer Services* 13(6):431–443.

Liu LM, Bhattacharyya S, Sclove SL, Chen R, Lattyak WJ (2001) Data mining on time series: An illustration using fast-food restaurant franchise data. *Comput. Statist. Data Anal.* 37(4):455–476.

Meneghini M, Anzanello M, Kahmann A, Tortorella G (2018) Quantitative demand forecasting adjustment based on qualitative factors: Case study at a fast food restaurant. *Sistemas Gestão* 13(1):68–80.

Pisinger D, Ropke S (2007) A general heuristic for vehicle routing problems. *Comput. Oper. Res.* 34(8):2403–2435.

Raj M, Sundararajan A, You C (2020) COVID-19 and digital resilience: Evidence from Uber Eats. Preprint, submitted June 12, https://arxiv.org/abs/2006.07204.

Rezaei S, Shahijan MK, Valaei N, Rahimi R, Ismail WKW (2018) Experienced international business traveller's behaviour in Iran: A partial least squares path modelling analysis. *Tourism Hospitality Res.* 18(2):163–190.

Saarijärvi H, Mitronen L, Yrjölä M (2014) From selling to supporting–leveraging mobile services in the context of food retailing. *J. Retailing Consumer Services* 21(1):26–36.

Suhartanto D, Helmi Ali M, Tan KH, Sjahroeddin F, Kusdibyo L (2019) Loyalty toward online food delivery service: the role of e-service quality and food quality. *J. Foodservice Bus. Res.* 22(1):81–97.

Whittle P (1951) *Hypothesis Testing in Time Series Analysis*, vol. 4 (Almqvist & Wiksells).

WHO (2020) WHO coronavirus disease (COVID-19) dashboard. Accessed July 10, 2021, https://covid19.who.int/.

Yang Y, Liu H, Chen X (2020) COVID-19 and restaurant demand: Early effects of the pandemic and stay-at-home orders. *Internat. J. Contemporary Hospitality Management* 32(12):3809–3834.

Yeo VCS, Goh SK, Rezaei S (2017) Consumer experiences, attitude and behavioral intention toward online food delivery (OFD) services. *J. Retailing Consumer Services* 35:150–162.

Zhao Y, Bacao F (2020) What factors determining customer continuingly using food delivery apps during 2019 novel coronavirus pandemic period? *Internat. J. Hospitality Management* 91:102683.

Zulkarnain K, Yusof I, Ahasanul H, Selim A (2015) Key success factors of online food ordering services: An empirical study. *Malaysian Inst. Management* 50(2):19–36.

Zulvia FE, Kuo R, Nugroho DY (2020) A many-objective gradient evolution algorithm for solving a green vehicle routing problem with time windows and time dependency for perishable products. *J. Cleaner Production* 242:118428.