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Sampled-data control of a class of uncertain nonlinear systems based on direct method ☆

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ABSTRACT

This paper employs the direct method to design sampled-data state/output feedback controller for a class of uncertain nonlinear systems under a general linear growth condition. Both state feedback controller and observer are constructed directly in the discrete-time domain for the difference-integral systems equivalent to the continuous-time systems. To dominate the influence of the uncertain nonlinearities, a co-design process is used to determine the sampling period and scaling gain together. Compared to the emulation method, the direct co-design method can handle a broader class of nonlinear systems and has more flexibility in choosing the sampling period that is less restrictive for digital implementation.

1. Introduction

Recently, the problem of designing sampled-data controllers for nonlinear continuous-time systems has received a lot of attention as more and more controllers are being implemented using digital computers [1]. Many design methods have been developed and reported in classical textbooks or some of tutorial papers [1–4].

In the linear case, design of discrete-time controllers, either in state feedback form or output feedback form, can be perfectly solved as there is a discrete-time system equivalent to the sampled continuous-time system. However, in the nonlinear case, it is impossible to find a discrete-time system that is completely equivalent to the sampled nonlinear continuous-time system. Therefore, the emulation method has been used to derive regional or global stabilizers for nonlinear systems in [5] where the possible interval of sampling period is very small. Similar conditions are also assumed in [6] where fast enough sampling is required to guarantee global asymptotic stability. Designing controllers in the discrete-time domain based on an approximated system has also been conducted in [7] and lately extended in [8] to system modeled by differential inclusions plus controller in terms of difference inclusions. However, only local or semi-global stabilization was obtained due to the inevitable approximating error between the approximated linear discrete-time system and the original nonlinear continuous-time system.

Another challenge caused by the presence of nonlinearities is that the well-known separation principle in output feedback controller design does not work for nonlinear system [9], and a hybrid version of the separation principle has been recently proposed in [10] for globally sampled-data stabilization of nonlinear systems. The situation is even worse when the nonlinearities in controlled system are not known. Although global output feedback stabilization has been proved to be solvable in [11] using technique of input saturation, the nonlinear systems considered in [11] are restricted to those having stable-free dynamics. For a lower-triangular nonlinear system satisfying the linear growth condition, globally exponentially stabilization has also been successfully solved in [12] where the observer was first designed without considering nonlinearities and then a scaling gain was employed to dominate the nonlinearities. The work [13] used an emulation method and introduced a linear domination approach to achieve global output feedback stabilization using sampled-data control for the same system under the same condition. The approach in [13] has a two-step design process: (i) a scaling gain is firstly injected into the continuous-time controller to dominate the influence of the nonlinearities in the continuous-time domain, and (ii) the sampling period for the discretized controller is tuned to make sure that the difference between the continuous-time controller and discretized controller is sufficiently small. The drawbacks of this method are that the nonlinearities are independent of the control input and only very small sampling periods are allowed in the end. Even in the linear case, the emulation method

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still requires that the sampling period should be very small due to the presence of approximation error. The independent design between control gain and sampling period can also be observed in recent results [14] on semi-global asymptotic stabilization of non-affine systems using sampled-data output feedback.

Selection of appropriate sampling periods is a fundamental problem in designing digital controller [1] and many results have been achieved from different aspects, such as controllability and observability of discrete-time system in [15], maximally allowable sampling/transmission interval in [16], system performance in [17,18], or even optimization problem in [19]. In [20], comparisons under different sampling periods and different design methods have been studied for linear discrete-time stochastic system which shows that common control strategies, frequently applied in the control literature, often give poor results especially for shorter sampling periods. Similarly, high sampling rates are often seen when the sampled-data output feedback controller has been constructed based on high-gain observers in [13,21,22].

In this paper, to handle more general nonlinearities and for more flexibility in selecting the sampling period, we introduce a new method where the scaling gain and sampling period will be co-designed. Both sampled-data state feedback controller and output feedback controller are constructed directly in the discrete-time domain. Then the sampled-data control is implemented to a difference-integral system which is equivalent to the sampled continuous-time nonlinear system. Depending on the nonlinearities, an appropriate sampling period can be selected to render the closed-loop system globally asymptotically stable. Compared to the emulation method, the direct co-design method enables us to use a less restrictive sampling period. Especially for the nominal linear system, the sampling period can be arbitrarily large which is impossible in the emulation case. In addition, by utilizing the discrete-time controller to dominate the nonlinearities in the difference-integral system, our direct method is able to handle more general nonlinearities dependent of the control input.

2. Problem statement and preliminaries

Considering the following nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x, u), \\ \dot{x}_2 = x_3 + f_2(x, u), \\ \quad \vdots \\ \dot{x}_n = u + f_n(x, u), \\ \ y = x_1 \end{cases} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^{\intercal}$ is the system state, $u(t) \in R$ is the control input, $y \in R$ is the system output and $f_i(x, u)$, $i = 1, 2, \dots, n$ are unknown functions. We are interested in designing sampled-data controllers using state feedback and output feedback to globally stabilize the nonlinear system (1).

To solve the aforementioned problems, in this paper we assume the following condition for the nonlinearities.

Assumption 2.1. For $i = 1, 2, \dots, n$, there exists a constant c_i such that

$$\begin{aligned} &\left|f_i(z_1,\ Nz_2,\ \cdots, N^{n-1}z_n,\ N^n v)\right| \\ &\le c_i N^{i-1}(|z_1| + \cdots + |z_n| + |v|), \quad \forall N \ge 1. \end{aligned} \tag{2}$$

Remark 1. The growth condition (2) is a more general form of the linear growth condition, specifically,

$$|f_i(x, u)| \le c_i(|x_1| + \cdots + |u|), \quad i = 1, \dots, n. \tag{3}$$

which has been a commonly used condition for global output feedback stabilization of (1) via continuous-time control [12] and sampled-data control [13]. In fact, this linear growth condition (3) is somehow necessary for global sampled-data stabilization of a nonlinear system. For example, for an unknown constant $\theta \in [0.5, 1]$, system $\dot{x} = u + \theta x^2$ can be globally stabilized by a continuous-time controller $u(x) = -x - x^3$ satisfying (i) $u(x)x < 0$ and (ii) $\lim_{x\to-\infty} u(x) = \infty$. However, there is no any sampled-data state feedback controller with the properties (i)-(ii) to globally stabilize the above system.

3. Sampled-data state feedback stabilization

In this section, a sampled-data state feedback controller will be designed directly in the discrete-time domain.

First, through the following coordinate transformation

$$z_1 = x_1,\ z_2 = \frac{x_2}{N},\ \cdots, z_n = \frac{x_n}{N^{n-1}},\ v = \frac{u}{N^n} \tag{4}$$

for a constant $N \ge 1$ to be determined later, system (1) can be written as

$$\begin{cases} \dot{z}_1 = Nz_2 + \bar{f}_1(z, v), \\ \dot{z}_2 = Nz_3 + \bar{f}_2(z, v), \\ \quad \vdots \\ \dot{z}_n = Nv + \bar{f}_n(z, v), \\ \ y = z_1, \end{cases} \tag{5}$$

where $\bar{f}_i(z, v) = \frac{f_i(x,u)}{N^{i-1}}$, $i = 1, \dots, n$. By Assumption 2.1, it is clear that

$$\left|\bar{f}_i(z, v)\right| \le c_i(|z_1| + \cdots + |z_n| + |v|). \tag{6}$$

Defining

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^{\intercal},$$

system (5) can be written as the following compact form

$$\begin{cases} \dot{z}(t) = NAz(t) + NBv(t) + \bar{f}(z(t), v(t)) \\ y(t) = Cz(t). \end{cases} \tag{7}$$

For a given constant $T_0 > 0$, denote $T := T_0/N$ in the followings sections. The nominal part of system (7) is

$$\dot{z}(t) = NAz(t) + NBv(t). \tag{8}$$

The system (8) under a sampler and a sampled-data controller with a zero-order-hold, i.e., $v(t) = v(kT), t \in [kT, (k+1)T)$, is equivalent to the following discrete-time system

$$z((k+1)T) = \Phi z(kT) + \Gamma v(kT) \tag{9}$$

where $\Phi = e^{NAT} = e^{AT_0}$ and $\Gamma = \int_0^T e^{NAs}NBds = \int_0^{T_0} e^{As}Bds$, or ,

$$\Phi = \begin{bmatrix} 1 & T_0 & \frac{T_0^2}{2!} & \cdots & \frac{T_0^{n-1}}{(n-1)!} \\ 0 & 1 & T_0 & \ddots & \vdots \\ & & \ddots & \ddots & \frac{T_0^2}{2!} \\ 0 & 0 & \cdots & 1 & T_0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}, \Gamma = \begin{bmatrix} \frac{T_0^n}{n!} \\ \frac{T_0^{n-1}}{(n-1)!} \\ \vdots \\ \frac{T_0^2}{2!} \\ T_0 \end{bmatrix}. \tag{10}$$

It is obvious that (Φ, Γ) is a controllable pair for any constant $T_0 > 0$. Therefore, there exists appropriate $K = (k_1, \dots, k_n)$ such that under the following sampled-data state feedback controller

$$v = -k_1 z_1(kT) - k_2 z_2(kT) - \cdots - k_n z_n(kT), \tag{11}$$

the following closed-loop system

$$z((k+1)T) = \Omega z(kT), \tag{12}$$

where Ω is defined as

$$\begin{bmatrix} 1-\frac{k_1T_0^n}{n!} & T_0-\frac{k_2T_0^n}{n!} & \frac{T_0^2}{2!}-\frac{k_3T_0^n}{n!} & \cdots & \frac{T_0^{n-1}}{(n-1)!}-\frac{k_nT_0^n}{n!} \\ \frac{-k_1T_0^{n-1}}{(n-1)!} & 1-\frac{-k_2T_0^{n-1}}{(n-1)!} & T_0-\frac{-k_3T_0^{n-1}}{(n-1)!} & \ddots & \vdots \\ & & \ddots & \ddots & \\ \frac{-k_1T_0^2}{(2)!} & \frac{-k_2T_0^2}{(2)!} & \cdots & 1-\frac{k_{n-1}T_0^2}{(2)!} & T_0-\frac{k_nT_0^2}{(2)!} \\ -k_1T_0 & -k_2T_0 & \cdots & -k_{n-1}T_0 & 1-k_nT_0 \end{bmatrix}$$

is globally asymptotically stable.

Lemma 1. *Under the sampled-data controller (11), the continuous-time system (7) is equivalent to the following difference-integral system at $t = kT,\ k = 0, 1, 2, \ldots$,*

$$z((k+1)T) = \Phi z(kT) + \Gamma v(kT) + \frac{1}{N}\Psi(kT) \tag{13}$$

where $\Psi(kT) = \int_0^{T_0} e^{A\tau}\bar{f}\left(z((k+1)T - \frac{\tau}{N}), v(kT)\right) d\tau$ and $T_0 = NT$.

Proof. The solution of system (7) is described by

$$z(t) = e^{NAt}z(t_0) + \int_{t_0}^{t} e^{NA(t-s)}\left[NBv(s) + \bar{f}\,(z(s), v(s))\right] ds.$$

Letting $t = (k+1)T$ and $t_0 = kT$, we can obtain the following equation with the sampled-data controller (11)

$$z((k+1)T) = e^{NAT}z(kT) + \int_{kT}^{(k+1)T} e^{NA((k+1)T-s)}NBv(kT)ds + \int_{kT}^{(k+1)T} e^{NA((k+1)T-s)}\bar{f}\,(z(s), v(kT))\, ds.$$

Denoting $\tau = N((k+1)T - s)$, the above equation can be rewritten as

$$z((k+1)T) = \underbrace{e^{AT_0}}_{\Phi} z(kT) + \underbrace{\int_0^{NT} e^{A\tau}Bd\tau}_{\Gamma}\, v(kT) + \frac{1}{N}\int_0^{NT} e^{A\tau}\bar{f}\left(z((k+1)T - \frac{\tau}{N}), v(kT)\right) d\tau$$

which is the difference-integral system (13) corresponding to (7). □

Theorem 1. *Under Assumption 2.1, there exist a scaling gain $N > 1$ and a sampling period T such that the sampled-data state feedback controller (11) globally asymptotically stabilizes system (1), where $N > 2\left(2\|\Omega\| + \rho_0\right)\|P\|\,\rho_0$ with $\rho_0 = \rho_1\left[(1+T_0\gamma_2)\,e^{T_0\gamma_1} + \|K\|\right]\int_0^{T_0}\left\|e^{A\tau}\right\| d\tau$, $\gamma_1 = \|A\| + \sqrt{c_1^2 + \cdots + c_n^2}$, $\gamma_2 = \|BK\| + \sqrt{c_1^2 + \cdots + c_n^2}\|K\|$ and $T_0 = NT$ is selected such that Ω is Schur-stable.*

Proof. First, integrating both sides of (7) on $[kT, t]$ yields

$$z(t) - z(kT) = \int_{kT}^{t}(NAz(s) + \bar{f}(z, v(s)) + NBv(s))ds.$$

Substituting (11) and (6) into the above equation yields

$$\begin{aligned} &\|z(t)\| \\ &\leq \|z(kT)\| + \int_{kT}^{t}\left[N\|A\|\|z(s)\| + \|\bar{f}(z, v)\| + N\|BK\|\,\|z(kT)\|\right] ds \\ &\leq \|z(kT)\| + \int_{kT}^{t} N\gamma_1\,\|z(s)\|\, ds + NT\gamma_2\,\|z(kT)\| \end{aligned}$$

where $\gamma_1 = \|A\| + \sqrt{c_1^2 + \cdots + c_n^2}$ and $\gamma_2 = \|BK\| + \sqrt{c_1^2 + \cdots + c_n^2}\|K\|$. Thus on the kth sampling interval $[kT, t]$, we have

$$\|z(t)\| \leq (1 + T_0\gamma_2)\,\|z(kT)\| + \int_{kT}^{t} N\gamma_1\,\|z(s)\|\, ds.$$

Applying Gronwall inequality, one gets

$$\|z(t)\| \leq (1 + T_0\gamma_2)\,e^{T_0\gamma_1}\cdot\|z(kT)\| \tag{14}$$

for $t \in [kT, (k+1)T)$. Then with the help of (6), $\Psi(kT)$ defined in Lemma 1 can be estimated as

$$\begin{aligned} \|\Psi(kT)\| &\leq \int_0^{T_0}\left\|e^{A\tau}\right\|\left\|\bar{f}\left(z((k+1)T - \frac{\tau}{N}), v(kT)\right)\right\| d\tau \\ &\leq \int_0^{T_0}\left\|e^{A\tau}\right\|\rho_1\left(\left\|z((k+1)T - \frac{\tau}{N})\right\| + |v(kT)|\right) d\tau \end{aligned} \tag{15}$$

for a constant $\rho_1 = \sqrt{c_1^2 + \cdots + c_n^2}$. After substituting $v(kT)$ using (11) and replacing $z((k+1)T - \frac{\tau}{N})$ with the upper bound obtained in (14), (15) can be further estimated as

$$\|\Psi(kT)\| \leq \underbrace{\rho_1\left[(1+T_0\gamma_2)\,e^{T_0\gamma_1} + \|K\|\right]\int_0^{T_0}\left\|e^{A\tau}\right\| d\tau}_{\rho_0}\,\|z(kT)\|\,.$$

In what follows, we will show that the sampled-data state feedback controller (11) can globally asymptotically stabilize the uncertain nonlinear system (1) using the linear domination approach (LDA). Since the system (9) without perturbations is globally asymptotically stable under the sampled-data state feedback controller (11), there exists a positive definite $P = P^{\top}$ such that $\Omega^{\top}P\Omega - P = -I$.

Construct $V(z(k)) = z^{\top}(k)Pz(k)$ for the closed-loop system (13), then we have

$$\begin{aligned} &V(z((k+1)T)) - V(z(kT)) \\ &= \left[\Omega z(kT) + \frac{\Psi(kT)}{N}\right]^{\top} P\left[\Omega z(kT) + \frac{\Psi(kT)}{N}\right] - z^{\top}(kT)Pz(kT) \\ &= -\|z(kT)\|^2 + 2z^{\top}(kT)\Omega^{\top}P\frac{\Psi(kT)}{N} + \frac{1}{N^2}\Psi^{\top}(kT)P\Psi(kT) \\ &\leq -\|z(kT)\|^2 + \frac{2\rho_0}{N}\|\Omega\|\,\|P\|\,\|z(kT)\|^2 + \frac{\rho_0^2\,\|P\|}{N^2}\|z(kT)\|^2 \\ &\leq -\|z(kT)\|^2 + \frac{(2\|\Omega\| + \rho_0)\,\|P\|\,\rho_0}{N}\|z(kT)\|^2. \end{aligned}$$

Thus if we select a large enough N such that $\frac{(2\|\Omega\| + \rho_0)\|P\|\rho_0}{N} \leq 1/2$, we will have

$$V(z((k+1)T)) - V(z(kT)) \leq -\frac{1}{2}\|z(kT)\|^2 \tag{16}$$

which implies that the closed-loop system (13) is globally asymptotically stable i.e. $z(kT) \longrightarrow 0\ (k \longrightarrow \infty)$. Then $z(t) \to 0\ (t \to \infty)$ is ensured based on (14). Thus with the help of coordinate transformation (4), $x(t) \longrightarrow 0\ (t \longrightarrow \infty)$ can be guaranteed, and system (1) is globally stabilized by the sampled-data state feedback controller (11). □

Remark 2. The controller (11) is directly designed for the discrete equivalent system (9) of the nominal system (8) and then is applied to the difference-integral system (13). This method is different to the emulation method (also called indirect method) where the controller is firstly designed as a continuous-time one for (8) and then is discretized as a sampled-data controller. One of the disadvantages of the emulation method is that the sampling period has to be very small to maintain stability even for the linear case, due to its approximation nature. However, the direct method will not have this restriction. In the linear case when $c_i = 0, \forall i$, it is clear that $\rho_0 = 0$ since $\rho_1 = 0$. By (16), it is possible to choose any $N > 0$ which means $T = T_0$ for any given T_0. Even in the presence of nonlinearities, it is still possible to choose a smaller N (consequently a larger sampling period T) to achieve global asymptotic stability if the growth rate is small.

4. Sampled-data output feedback stabilization

To start with, the following discrete-time state observer for system (7) will be firstly constructed

$$\hat{z}((k+1)T) = \Phi\hat{z}(kT) + \Gamma v(kT) + H(y(kT) - C\hat{z}(kT)),$$
$$\hat{z} = [\hat{z}_1, \hat{z}_2, \dots, \hat{z}_n]^\intercal,\ C = [1, 0, \dots, 0] \quad (17)$$

where $\Phi = e^{AT_0}$, $\Gamma = \int_0^{T_0} e^{As}Bds$ and H is designed such that the eigenvalues of $\Phi - HC$ are inside of the unit circle.

Since $z_2, \dots, z_n$ are not measurable, we design

$$v(kT) = -K\hat{z}(kT), \quad (18)$$

with the same $K = [k_1, k_2, \dots, k_n]$ used in (11) to ensure that all eigenvalues of $\Phi - \Gamma K$ lie inside of the unit circle. Substituting (18) into (17) yields

$$\hat{z}((k+1)T) = (\Phi - \Gamma K - HC)\hat{z}(kT) + Hy(kT) \quad (19)$$

Putting the difference-integral system (13) together with the sampled-data output feedback controller (18)–(19) leads to the following closed-loop system in $[t_k, t_{k+1})$

$$\begin{bmatrix} z((k+1)T) \\ \hat{z}((k+1)T) \end{bmatrix} = \Xi \begin{bmatrix} z(kT) \\ \hat{z}(kT) \end{bmatrix} + \begin{bmatrix} \frac{1}{N}\Psi(kT) \\ 0 \end{bmatrix}, \quad (20)$$

$$\Xi = \begin{pmatrix} \Phi & -\Gamma K \\ HC & \Phi - \Gamma K - HC \end{pmatrix}. \quad (21)$$

With the following similar transformation of matrix

$$\begin{pmatrix} I & -I \\ 0 & I \end{pmatrix} \Xi \begin{pmatrix} I & I \\ 0 & I \end{pmatrix} = \begin{pmatrix} \Phi - HC & 0 \\ HC & \Phi - \Gamma K \end{pmatrix},$$

we can conclude that the eigenvalues of Ξ are composed of eigenvalues of $\Phi - HC$ and eigenvalues of $\Phi - \Gamma K$. With the previously selected K and H, all eigenvalues of system matrix Ξ lie inside of the unit circle. Hence, there is a positive definite matrix $\bar{P} = \bar{P}^\intercal \in R^{2n\times 2n} > 0$ such that $\Xi^\intercal \bar{P}\Xi - \bar{P} = -I$.

Theorem 2. *Under Assumption 2.1, there exist appropriate scaling gain N and sampling period T such that the uncertain system (1) is globally asymptotically stable under the sampled-data output feedback controller (18)–(19), where $N > 4\hat{\rho}_0\|\Xi^\intercal\bar{P}\| + 2\hat{\rho}_0^2\|\bar{P}_1\|$ with $\hat{\rho}_0 = \rho_1\left(T_0\gamma_2 e^{T_0\gamma_1} + \|K\| + e^{T_0\gamma_1}\right)\cdot\int_0^{T_0}\|e^{A\tau}\|\,d\tau$ and $T_0 = NT$ is selected such that Ξ is Schur-stable.*

Proof. Integrating both sides of (7) on $[kT, t]$ with the output-feedback controller (18) will yield

$$\|z(t)\| \le \|z(kT)\| + \int_{kT}^{t}\left[N\|A\|\|z(s)\| + \|\bar{f}(z,v)\| + N\|BK\|\,\|\hat{z}(kT)\|\right]ds.$$

Similar to the proof in Theorem 1, the upper bound of $\|z(t)\|$ on the kth sampling interval is

$$\|z(t)\| \le \|z(kT)\| + NT\gamma_2\left\|\hat{z}(kT)\right\| + \int_{kT}^{t} N\gamma_1\|z(s)\|\,ds. \quad (22)$$

Applying Gronwall inequality for $t \in [kT, (k+1)T)$, we have

$$\|z(t)\| \le \left(\|z(kT)\| + NT\gamma_2\left\|\hat{z}(kT)\right\|\right)e^{T_0\gamma_1}. \quad (23)$$

Similar to (15), with the help of (18) and (23), we have

$$\begin{aligned}&\|\Psi(kT)\| \\ &\le \int_0^{T_0}\left\|e^{A\tau}\right\|\rho_1\left(\left\|z((k+1)T - \frac{\tau}{N})\right\| + \|K\|\left\|\hat{z}(kT)\right\|\right)d\tau \\ &\le \rho_1\left[\left(T_0\gamma_2 e^{T_0\gamma_1} + \|K\|\right)\left\|\hat{z}(kT)\right\| + e^{T_0\gamma_1}\|z(kT)\|\right] \\ &\quad\times\int_0^{T_0}\left\|e^{A\tau}\right\|d\tau \le \hat{\rho}_0\|Z(kT)\| \end{aligned} \quad (24)$$

where $\hat{\rho}_0 = \rho_1\left(T_0\gamma_2 e^{T_0\gamma_1} + \|K\| + e^{T_0\gamma_1}\right)\cdot\int_0^{T_0}\left\|e^{A\tau}\right\|d\tau$.

For the stability analysis of (20), select

$$\bar{V}(Z(kT)) = Z^\intercal(kT)\bar{P}Z(kT), \quad Z(kT) = \begin{bmatrix} z(kT) \\ \hat{z}(kT) \end{bmatrix}$$

as the Lyapunov function. Then we have

$$\begin{aligned}&\bar{V}\left(Z((k+1)T)\right) - \bar{V}(Z(kT)) \\ &= \left[\Xi Z(kT) + \frac{1}{N}\begin{bmatrix}\Psi(kT) \\ 0\end{bmatrix}\right]^\intercal \bar{P}\left[\Xi Z(kT) + \frac{1}{N}\begin{bmatrix}\Psi(kT) \\ 0\end{bmatrix}\right] \\ &\quad - Z^\intercal(kT)\bar{P}Z(kT) \\ &= -\|Z(kT)\|^2 + 2Z^\intercal(kT)\Xi^\intercal\bar{P}\begin{bmatrix}\frac{\Psi(kT)}{L} \\ 0\end{bmatrix} + \frac{\Psi^\intercal(kT)}{N}\bar{P}_1\frac{\Psi(kT)}{N} \\ &\le -\|Z(kT)\|^2 + 2\|Z(kT)\|\left\|\Xi^\intercal\bar{P}\right\|\frac{\|\Psi(kT)\|}{N} + \frac{\left\|\bar{P}_1\right\|}{N^2}\|\Psi(kT)\|^2\end{aligned}$$

where $\bar{P}_1$ is the first $n \times n$ block of $\bar{P}$.

Based on the upper bound of perturbations obtained in (24), the above difference of Lyapunov function $\bar{V}(Z(k))$ can be further estimated as

$$\begin{aligned}&\bar{V}\left(Z((k+1)T)\right) - \bar{V}(Z(kT)) \\ &= -\|Z(kT)\|^2 + \frac{2\hat{\rho}_0\|\Xi^\intercal\bar{P}\| + \hat{\rho}_0^2\left\|\bar{P}_1\right\|}{N}\|Z(kT)\|^2.\end{aligned} \quad (25)$$

By selecting a large enough N satisfying $\frac{2\hat{\rho}_0\|\Xi^\intercal\bar{P}\| + \hat{\rho}_0^2\|\bar{P}_1\|}{N} \le 1/2$, (25) becomes

$$\bar{V}\left(Z((k+1)T)\right) - \bar{V}(Z(kT)) \le -\frac{1}{2}\|Z(kT)\|^2.$$

Thus, asymptotic stability of the closed-loop system (20) can be guaranteed i.e. $z(kT) \longrightarrow 0$ and $\hat{z}(kT) \longrightarrow z(kT)$, as $k \longrightarrow \infty$. Then $z(t) \to 0\,(t \to \infty)$ is ensured based on (23). Thus with the help of coordinate transformation (4), $x(t) \longrightarrow 0\,(t \longrightarrow \infty)$ can also be guaranteed, and system (1) is globally stabilized by the sampled-data output feedback controller (18)–(19). □

Remark 3. Selections of N in the proof of Theorems 1 and 2 are based on the decrease of Lyapunov function $V(z(k))$ or $\bar{V}(Z(k))$ on each sampling interval where $\frac{1}{2}$ has been selected for simplicity of proof. Any other positive number that is smaller than 1 can be used to guarantee decreasing of the Lyapunov function. The larger of N is selected, the faster decrease of Lyapunov function can be obtained.

Remark 4. For any given control gain of $K = [k_1, k_2, \dots, k_n]$ that can continuously stabilizes the nominal linear system in

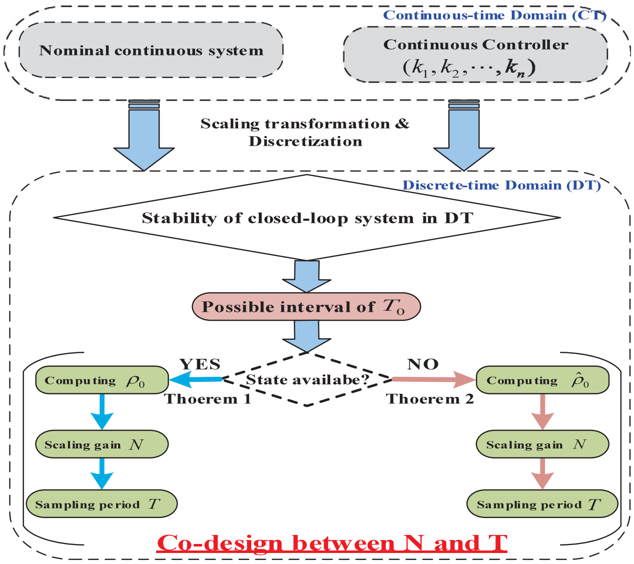

Fig. 1. Interplay among T_0, ρ_0, N and T.

continuous-time, the process for deriving the co-designed sampled-data controller for the nonlinear system (1) has been summarized in Fig. 1 where the interplay among different parameters are explicitly shown.

Example 1. Consider the following system

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u + a\sin(x_1^2 u^2), \\ \; y = x_1, \end{cases} \tag{26}$$

where a is a constant. It is obviously that the nonlinearity $f_i(\cdot)$ in (26) satisfies Assumption 2.1 with $c_1 = 0$ and $c_2 = a$ since

$$|a\sin(x_1^2 u^2)| \le a\sqrt{|x_1 u|} = a\sqrt{|z_1 N^2 v|} \le aN(|z_1| + |v|).$$

Hence, by Theorem 2, we can construct the following sampled-data output feedback controller

$$\begin{bmatrix} \hat{z}_1((k+1)T) \\ \hat{z}_2((k+1)T) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{z}_1(kT) \\ \hat{z}_2(kT) \end{bmatrix} + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} v(kT) + H(z_1(k) - \hat{z}_1(k)) \tag{27}$$

$$v(t) = -k_1\hat{z}_1(kT) - k_2\hat{z}_2(kT), \ \forall t \in [kT, (k+1)T), \tag{28}$$

with $K = [0.9702 \quad 1.4849]$ and $H = [1.7 \quad 0.72]^{\mathsf{T}}$.

Under the above selection of control gain K, the system matrix Ω defined in (12) is

$$\begin{bmatrix} 1 - \frac{0.9702}{2}T_0^2 & T_0 - \frac{1.4849}{2}T_0^2 \\ -0.9702T_0 & 1 - 1.4849T_0 \end{bmatrix}$$

where the possible interval of $T_0 = NT \in (0, 1.3)$ can be numerically obtained such that all eigenvalues of the above Ω lie inside of the unit circle. For simplicity, we have selected $T_0 = NT = 1$ in the following studies.

Firstly, we consider the sampled-data stabilization of system (26) without nonlinearity using the co-designed controller (27)–(28). Simulation results under large sampling period ($T = 1$) have been shown in Figs. 2 and 3, and the sampling period can be arbitrary large due to the compensation of scaling gain in this case.

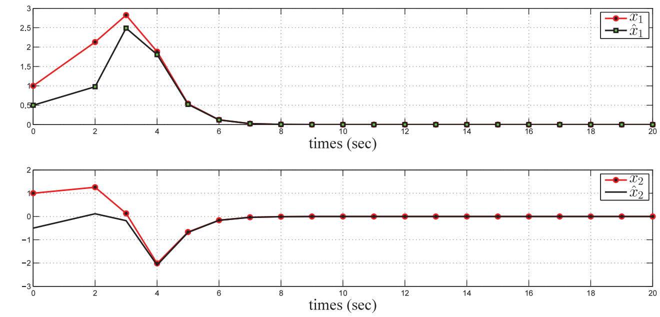

Fig. 2. States and estimated states of (26) without nonlinearities ($x(0) = [1, 1]^{\mathsf{T}}, z(0) = [0.5, -0.5]^{\mathsf{T}}$).

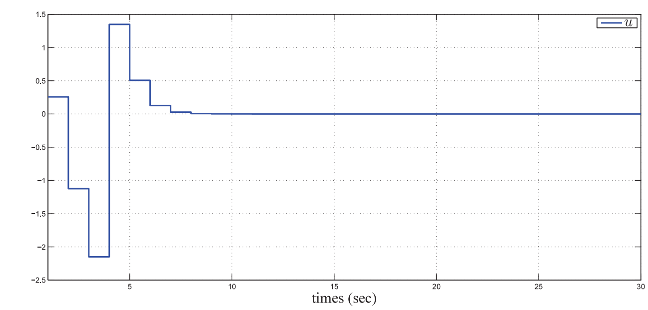

Fig. 3. Control input of (26) without nonlinearities ($x(0) = [1, 1]^{\mathsf{T}}, z(0) = [0.5, -0.5]^{\mathsf{T}}$).

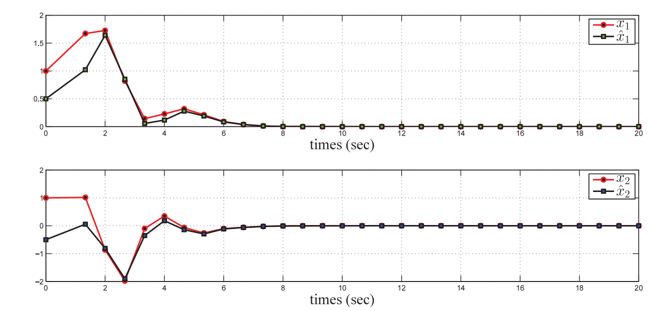

Fig. 4. States and estimated states of (26) ($x(0) = [1, 1]^{\mathsf{T}}, z(0) = [0.5, -0.5]^{\mathsf{T}}$).

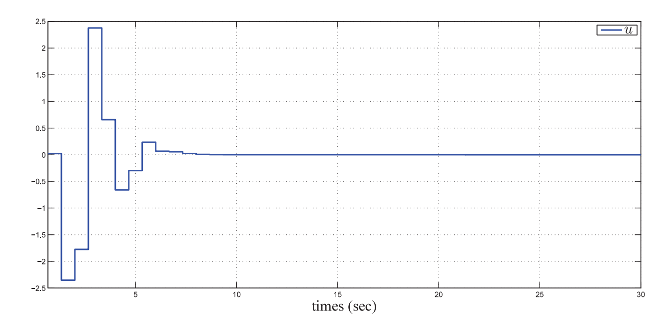

Fig. 5. Control input of (26) ($x(0) = [1, 1]^{\mathsf{T}}, z(0) = [0.5, -0.5]^{\mathsf{T}}$).

Secondly, global stabilization of system (26) with $a = 1$ is further considered using the co-designed controller (27)–(28). Due to the presence of perturbation, $N = 1.5$ has been selected to dominate the effects of perturbations, and then the allowable sampling period is $0 < T \le 0.86 \approx \frac{1.3}{1.5}$. Simulation results shown in Figs. 4 and 5 with $N = 1.5, T = 0.67$ have shown the effectiveness of our co-designed controller (27)–(28).

Remark 5. The emulation method for sampled-data output control in [13] first designs a continuous-time output feedback controller and then discretizes it as a sampled-data controller. The emulation method in [13] works well by contracting the difference between the discretized and continuous-time controllers, but it has two drawbacks: (i) the nonlinearities are *independent of the control input*, and (ii) the approximation error is small only for a *very small sampling period*. Within the framework of direct design in discrete-time domain, these two drawbacks can be avoided. As seen in (26), $f_2(\cdot) = a\sin(x_1^2 u^2)$ not only contains u but also is not Lipschitz in u, which now can be handled by the direct method. In addition, less restrictive sampling periods can be selected, which is an important implementation issue in computer-controlled systems.

Example 2. For comparisons with the emulated controllers proposed in [13], global state feedback stabilization and global output feedback stabilization of the following system are considered in this example

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3 + \sin(x_1)\ln(1+|x_2|), \\ \dot{x}_3 = u + \sin(x_2)\ln(1+|x_3|), \\ y = x_1. \end{cases} \tag{29}$$

Comparisons between the emulated controller of [13] and the co-designed controllers (11) have been conducted with the same control gain $K = \begin{bmatrix} 1 & 2 & 2 \end{bmatrix}$ where $T_{max} = 0.4$ if the emulated controller of [13] is used. Similar to the previous example, the system matrix Ω defined in (12) becomes

$$\begin{bmatrix} 1-\frac{1}{6}T_0^3 & T_0 - \frac{1}{3}T_0^3 & \frac{T_0^2}{2} - \frac{1}{3}T_0^3 \\ -\frac{1}{2}T_0^2 & 1-T_0^2 & T_0 - T_0^2 \\ -T_0 & -2T_0 & 1-2T_0 \end{bmatrix}$$

from which the allowable selection of T_0 lies in $(0, 1.03)$ can be numerically obtained. For simplicity, $LT = 1$ has been used in the following simulation studies. Based on the fact that $|\sin(z_1)\ln(1+|x_2|)| < (|z_1| + |z_2|)$, Assumption 2.1 is satisfied with $c_i = 1$. Then we select $N = 1.1$ to dominate the nonlinearities, and $T_{max} = 0.9$ can be obtained due to the compensation between the scaling gain N and the sampling period T. Simulation results presented in Figs. 6 and 7 with $T = 0.8, N = 1.25$ and $x(0) = [3, 2, -3]^\top$ have shown that the allowable sampling period has been enlarged if our co-designed controller (11) is used.

Sampled-data output stabilization of system (29) using (18)–(19) is further considered in the following. With results obtained in Theorem 2, the allowable selection of NT lies in $(0, 0.65)$ for the output stabilization of system (29) under the co-designed controller (18)–(19). Under the selection of control gain $K = \begin{bmatrix} 1 & 2 & 2 \end{bmatrix}$ and observer gain $H = \begin{bmatrix} 2.4 & 3.3 & 1.8 \end{bmatrix}^\top$, the maximum sampling period for the emulated controller of [13] is $T_{max} = 0.28$ while $T_{max} = 0.45$ for the co-designed controller (18)–(19). Simulation results obtained in Figs. 8 and 9 with $z(0) = [-1, -5, -5]^\top$ have illustrated the effectiveness of Theorem 2 under $T = 0.3$ which is larger than the maximum sampling period obtained by [13] using emulation method.

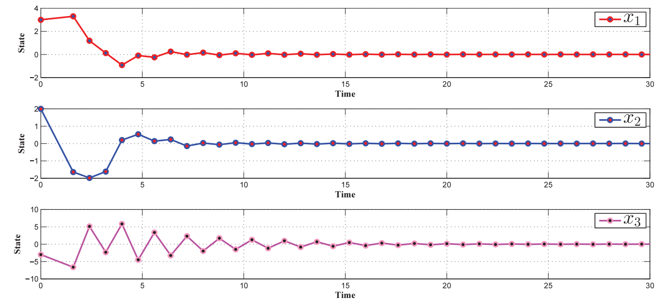

Fig. 6. States of (29) under state-feedback controller (11).

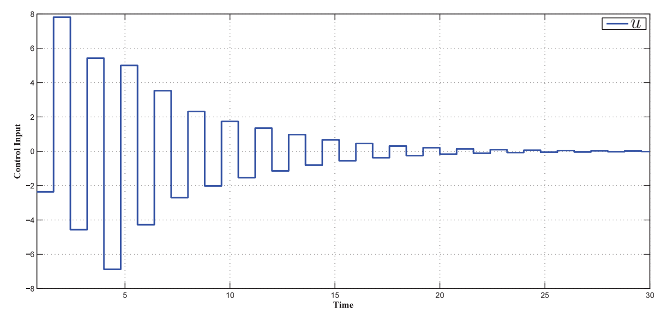

Fig. 7. Control input of (29) under state-feedback controller (11).

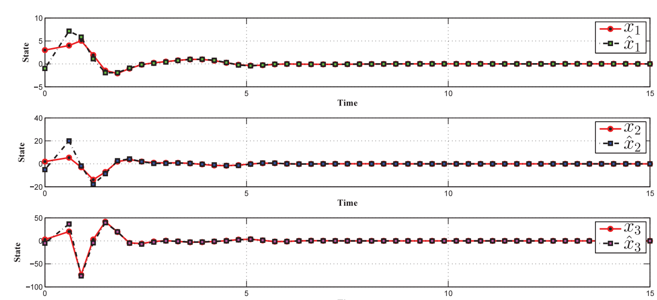

Fig. 8. States and estimated states of (29) under controller (18)–(19).

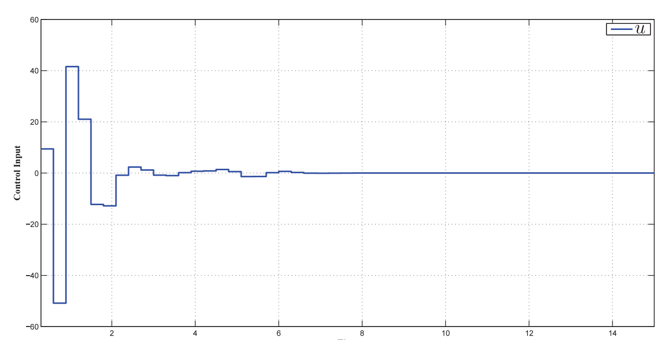

Fig. 9. Control input of (29) under controller (18)–(19).

5. Conclusion

In this paper, global stabilization of a class of uncertain nonlinear systems has been considered using sampled-data control. Both sampled-data state feedback controller and sampled-data output feedback controller have been constructed directly in the discrete-time domain without resorting to the method of emulation. The direct method has enlarged the class of nonlinear systems which are globally stabilized by sampled-data control. Moreover, as the sampling period and scaling gain have been co-designed in the construction process, the sampling period can be selected in a less conservative way for relaxed hardware requirements to implement our sampled-data controllers.

With the flexibility provided by the co-designed sampled-data controller, results of this paper are much preferred for control problems with sparse sampling where event-triggered and self-triggered strategies maybe not applicable due to hardware or physical restrictions. Thus an interesting extension will involve the possibility of finding a minimal dwell-time to prevent the Zeno phenomenon through combining the co-designed time-regularizing controller with those event-triggered strategies. Because work of this paper are mainly based on Assumption 2.1 where the nonlinearities are vanishing at the equilibrium, the validation of obtained results for system with much stronger nonlinearities will also be considered in the future.

CRediT authorship contribution statement

Kecai Cao: Conceptualization, Methodology, Data curation, Writing – original draft, Software, Visualization, Writing – review & editing. **Chunjiang Qian:** Supervision, Validation, Data curation, Writing – review & editing. **Juping Gu:** Revising, Software, Visualization, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] K.J. Åström, B. Wittenmark, Computer-Controlled Systems: Theory and Design, Prentice Hall, NJ, 1997.
[2] T. Chen, B. Francis, Optimal Sampled-Data Control Systems, Springer, 1995.
[3] D. Nešić, A.R. Teel, Sampled-data control of nonlinear systems: An overview of recent results, in: Perspectives in Robust Control, Springer, 2001, pp. 221–239.
[4] D. Nesic, R. Postoyan, Nonlinear sampled-data systems, Springer London, 2014, pp. 1–7,
[5] D.H. Owens, Y. Zheng, S.A. Billings, Fast sampling and stability of nonlinear sampled-data systems: Part 1. existence theorems, IMA J. Math. Control Inform. 7 (1) (1990) 1–11.
[6] F.H. Clarke, Y.S. Ledyaev, E.D. Sontag, A.I. Subbotin, Asymptotic controllability implies feedback stabilization, IEEE Trans. Automat. Control 42 (10) (1997) 1394–1407.
[7] D. Nešić, A.R. Teel, P. Kokotović, Sufficient conditions for stabilization of sampled-data nonlinear systems via discrete-time approximations, Systems Control Lett. 38 (4) (1999) 259–270.
[8] D. Nešić, A.R. Teel, A framework for stabilization of nonlinear sampled-data systems based on their approximate discrete-time models, IEEE Trans. Automat. Control 49 (7) (2004) 1103–1122.
[9] F. Mazenc, L. Praly, W.P. Dayawansa, Global stabilization by output feedback: examples and counterexamples, Systems Control Lett. 23 (2) (1994) 119–125.
[10] W. Lin, When is a nonlinear system semiglobally asymptotically stabilizable by digital feedback? IEEE Trans. Automat. Control 65 (11) (2020) 4584–4599.
[11] W. Lin, Input saturation and global stabilization of nonlinear systems via state and output feedback, IEEE Trans. Automat. Control 40 (4) (1995) 776–782.
[12] C. Qian, W. Lin, Output feedback control of a class of nonlinear systems: A nonseparation principle paradigm, IEEE Trans. Automat. Control 47 (10) (2002) 1710–1715.
[13] C. Qian, H. Du, Global output feedback stabilization of a class of nonlinear systems via linear sampled-data control, IEEE Trans. Automat. Control 57 (11) (2012) 2934–2939.
[14] W. Lin, J. Sun, New results and examples in semiglobal asymptotic stabilization of nonaffine systems by sampled-data output feedback, Systems Control Lett. 148 (11) (2021) 104850.
[15] E.D. Sontag, Mathematical Control Theory: Deterministic Finite Dimensional Systems, Springer-Verlag, 1998.
[16] W.P.M.H. Heemels, A.R. Teel, N. van de Wouw, D. Nesic, Networked control systems with communication constraints tradeoffs between transmission intervals, delays and performance, IEEE Trans. Automat. Control 55 (8) (2010) 1781–1796.
[17] Y. Baba, Choice of sampling period for minimum-time computer control system, Trans. Soc. Instrum. Control Eng. 16 (4) (1980) 491–497.
[18] J. Ackermann, Sampled-Data Control Systems: Analysis and Synthesis, Robust System Design, Springer-Verlag, 1985.
[19] X. Peng, B. Lennartson, C.-M. Fransson, On the choice of sampling period and robust pole placement, IFAC Proc. Vol. 35 (1) (2002) 97–102.
[20] B. Lennartson, On the choice of controller and sampling period for linear stochastic control, Automatica 26 (3) (1990) 573–578.
[21] A.M. Dabroom, H.K. Khalil, Output feedback sampled-data control of nonlinear systems using high-gain observers, IEEE Trans. Automat. Control 46 (11) (2001) 1712–1725.
[22] H.K. Khalil, Performance recovery under output feedback sampled-data stabilization of a class of nonlinear systems, IEEE Trans. Automat. Control 49 (12) (2004) 2173–2184.