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Collaborative Human Decision-Making With Heterogeneous Agents

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***Abstract*— While there has been extensive work on modeling of human decision-making both for individuals and groups from a cognitive psychology point of view, research on this topic from a signal processing and information fusion perspective is relatively recent. In this work, we consider a distributed detection problem consisting of a number of human local decision makers and a fusion center (FC). Signal detection theory is exploited to answer why promoting heterogeneity could improve the performance of collaborative human decision-making. We consider the following two scenarios: 1) the local decision makers are independent and the level of heterogeneity is measured in terms of the variability of human expertise and 2) humans make correlated local decisions due to their perceptual and behavioral similarities and heterogeneity is measured by the amount of correlation. In both cases, we show that the detection performance of the FC can be improved with the increase of heterogeneity. In particular, in the second scenario, we develop a portfolio theory-based framework to select participants from correlated human agents so that heterogeneity is enhanced resulting in improved decision-making performance. Simulations are provided for illustration and performance comparison.**

***Index Terms*— Correlated local decisions, group decision-making, heterogeneity, human decision-making, information fusion, portfolio theory.**

I. Introduction

In different workplaces and organizations, heterogeneity, or variety, is a key factor that guides the recruitment and hiring processes. It is widely acknowledged that heterogeneity in workplace drives innovation and fosters creativity as the team members bring a variety of backgrounds, experiences, and perspectives to the table [1], [2]. Heterogeneity can be promoted by having people from different ethnic groups, genders, cultures, religions, languages, education, viewpoints, and abilities. It is the collaborative effort from people who think differently and act differently that facilitates the solution of complex problems in an efficient manner. In contrast to the literature from psychology and sociology, the objective of this article is to study the impact of heterogeneity of human expertise, cognitive biases, and perceptual abilities on collaborative human decision-making from a signal processing and information fusion perspective.

In numerous applications such as national security, natural disaster forecasting, and healthcare, modeling of autonomous and semiautonomous decision-making systems that involve human agents as participants is becoming an important research area. The problem of distributed detection where humans act as local decision makers has been studied in different contexts. For example, the quantization of priors in hypothesis testing was analyzed to model the fact that humans make categorical observations [3]. Vempaty *et al.* [4] developed a Bayesian hierarchical structure to characterize human behavior of decision fusion at individual level, group level, and population level. The performance of collaborative human decision-making was analyzed when each individual is assumed to make local decisions by comparing the observations to a random threshold [5], [6]. By utilizing crowd wisdom, crowdsourcing has become an efficient paradigm to solve problems that are easy for humans but hard for machines, e.g., handwriting recognition, image labeling, and voice transcription. Different methods were proposed for aggregation of the local decisions by considering the unreliability and uncertainty of the human crowd workers [7], [8]. Moreover, since humans are selfish who request monetary rewards to be motivated to perform the sensing tasks, Li *et al.* [9], Cao *et al.* [10], and Geng *et al.* [11], [12] have incorporated game theory into the design of efficient incentive mechanisms.

Another line of work on human decision-making is the consideration of cognitive biases of human agents in the decision-making process. It is well known in the psychology literature that cognitive biases and uncertainties can be found in the human judging and decision-making processes at individual and group levels [13], [14]. The significance of cognitive psychology has been demonstrated by its ability to outperform machine learning methodologies when predicting people's choice behavior [15]. Among these biases are people's distorted representations of outcomes and probabilities, which are accurately captured by the Nobel prize-winning prospect theory (PT) [13]. There have been a few works that incorporate PT into hypothesis testing to model human

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decision-making. In the Bayesian framework, Nadendla *et al.* [16] exploited PT to analyze the behavior of optimists and pessimists of different types. The optimality of the likelihood ratio test (LRT) was investigated in PT-based hypothesis testing in [17]. In this work, we showed that the LRT may or may not be optimal for behavioral decision makers in terms of Neyman–Pearson and/or Bayesian criterion. Moreover, Geng *et al.* [18] employed a utility-based approach to investigate prospect theoretic human decision-making and decision fusion in multiagent systems, where several decision fusion scenarios that include humans were studied. As the behavior of cognitively biased humans deviates from being rational, Geng *et al.* [19] investigated several strategies to ameliorate the biases and help humans make higher quality decisions. Geng *et al.* [20] analyzed the prospect theoretic behavioral differences of honest workers and spammers in crowdsourcing systems. A weighted majority voting scheme was proposed to assign an optimal weight for every worker to maximize the system performance.

In the context of collaborative human decision-making through crowdsourcing, responses (e.g., binary decisions) of the humans have typically been assumed to be independent of each other for analytical convenience by past work [5]–[7], [20]–[22]. However, in real application scenarios, human decisions may be correlated due to their common characteristics and backgrounds. Moreover, the above literature has overlooked the problem of selecting human agents for the task under consideration, which is an important problem when there are restrictions on the number of humans that can be selected. The few works that do consider the selection of human agents in crowdsourcing simply rely on reputation, where individuals who have been more accurate in the past are chosen to ensure that reliable responses can be obtained [23], [24]. However, this approach considers human decision qualities only at an individual level, without considering the group-level properties that arise from the existence of complex correlation relationships among the behaviors of individuals. The worker selection problem from a pool of correlated decision makers for optimized decision-making performance, to the best of our knowledge, has not been studied in the previous literature. Such a problem is not only complicated by the fact that there is a lack of concrete methods to model the correlations among the workers' decisions in real decision-making scenarios but also by the fact that it is challenging to develop a worker selection mechanism from a pool of heterogeneous decision makers.

In this work, we aim to alleviate this problem by analyzing how heterogeneity could affect the performance of collaborative human decision-making under consideration of complex correlation relationships among the behavior of humans and design the human selection strategy at the population level. The major technical contributions of our work are as follows.

1) *Development of Collaborative Human Decision-Making Framework:* We construct the model where humans are assumed to provide binary decisions to a fusion center (FC), and the FC makes a final decision regarding the phenomenon of interest (PoI). We first assume that the humans make decisions independently of each other and study the impact of human expertise variability, i.e., the variance of the local decisions' qualities, on the decision-making performance at the FC.
2) *PT-Based Approach to Model the Correlations Among Human Agents:* We employ the behavioral economics concept of PT to model the fact that the quality of human decisions is affected by individual cognitive biases. Correlations among the decision-making behaviors of agents are considered to stem from correlations among their observations as well as correlations among their prospect theoretic parameters. We provide an analytical method to derive the mean vector and covariance matrix of the humans' average probabilities of error in binary decision-making.
3) *Selection of Correlated Human Agents for Optimized Decision-Making Performance:* We innovatively exploit concepts from portfolio theory [25] to address the problem of selecting cognitively biased human agents for performing a distributed decision-making task while carefully considering the complex correlation relationships that may exist among their decision-making behaviors. In the optimization problem, we minimize the sum of error probabilities of the selected humans while constraining the variability of the system performance to be below a certain level. We show that an appropriate level of heterogeneity measured by the amount of correlation among the selected humans helps improve the final decision-making performance.

The rest of this article is organized as follows. In Section II, we study the performance of decision-making by groups composed of independent local decision makers. In Section III, we investigate human decision-making under cognitive biases and study how the correlations among local decisions affect the accuracy of the majority rule-based decision fusion. A portfolio theory-based human selection scheme for decision-making is proposed in Section IV. We provide simulation results in Section V to demonstrate the effectiveness of our approach and conclude our work in Section VI.

II. Group Decision-Making With Independent Local Decision Makers

Let us formulate the problem where a group of n human decision makers needs to choose between two options denoted by hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$. In this section, we consider a structure where the humans make binary decisions independently regarding the PoI. The decision of the ith human $g_i \in \{0, 1\}$ is modeled using a binary symmetric channel (BSC) shown in Fig. 1. The parameter α_i represents the ith human's accuracy/expertise in making a decision, i.e., $\Pr(g_i = j|\mathcal{H}_j \text{ is true}) = \alpha_i$ for $j = 0, 1$. We assume that $\alpha_i \geq 0.5$ and the worst case $\alpha_i = 0.5$ simply corresponds to a random guess. As noted, the local decisions g_i for $i = \{1, 2, \ldots, n\}$ are assumed to be independent of each other.

An FC collects the local decisions $G = [g_1\ g_2 \ldots g_n]^T$ and makes the final decision on which hypothesis is true. To decide on one of the two hypotheses based on G, it was shown that

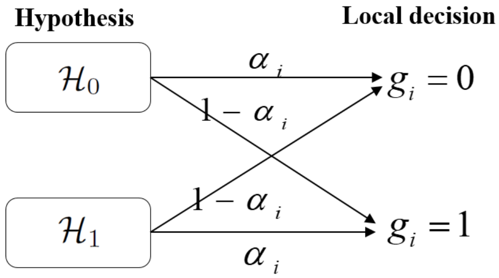

Fig. 1. BSC model with accuracy α_i.

the LRT is optimal as it minimizes the Bayesian risk [26]

$$\frac{\Pr(G|\mathcal{H}_1)}{\Pr(G|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta \tag{1}$$

where η represents the appropriate threshold. The derivation of the exact error probability of the system is intractable as the number of human participants is large. To evaluate system performance and to guide the worker recruiting strategy, we compute and employ the asymptotic detection performance of the system via the Bhattacharyya distance[1] as follows:

$$\lim_{\mathcal{N}\to\infty} \frac{\ln P_e}{\mathcal{N}} \leq -\mathcal{BD}(Pr(G|\mathcal{H}_0), \Pr(G|\mathcal{H}_1)) \tag{2}$$

where P_e is the average error probability and $\mathcal{N}$ is the number of samples of the decision vector G such that the test statistic $G^{\mathcal{N}}$ is an $\mathcal{N} \times n$ matrix. $Pr(G|\mathcal{H}_0)$ and $Pr(G|\mathcal{H}_1)$ are the likelihoods under $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. Intuitively, the probability of error decreases exponentially as the sample size $\mathcal{N}$ increases and $\mathcal{BD}(\Pr(G|\mathcal{H}_0), \Pr(G|\mathcal{H}_1))$ upper bounds the decay rate. It is desired to have a large value of $\mathcal{BD}(\cdot)$ so that the probability of error decreases faster [27], [28].

To characterize the decision-making behavior of the group of human decision makers, we assume that α_i is random and follows a certain probability density function (PDF). The mean and the variance of the random variable, denoted by $\mathbb{E}(\alpha_i) = \mu_\alpha$ and $\mathrm{Var}(\alpha_i) = \sigma_\alpha^2$, are used to represent the average and heterogeneity of the accuracy of the decisions made by the group members, respectively. We may interpret μ_α to be the average level of expertise and σ_α^2 to be the level of heterogeneity of the expertise in the group. A large value of σ_α^2 indicates that the humans in the group have diverse decision-making/cognitive abilities resulting in a large variance in their decision-making accuracy. The objective is to study the impact of μ_α and σ_α^2 on the FC's decision-making performance in terms of the Bhattacharyya distance.

By assuming that the independent local decisions are modeled via BSCs shown in Fig. 1, the probability mass functions (pmfs) of the decision vector G under both hypotheses are given by

$$\Pr(G|\mathcal{H}_0) = \prod_{i=1}^{n} \alpha_i^{1-g_i}(1-\alpha_i)^{g_i}$$
$$\Pr(G|\mathcal{H}_1) = \prod_{i=1}^{n} \alpha_i^{g_i}(1-\alpha_i)^{1-g_i}.$$

[1]Bhattacharyya distance is a special case of Chernoff information when its parameter is equal to 1/2.

The Bhattacharyya distance between the two discrete distributions can be expressed as

$$\begin{aligned}
&\mathcal{BD}(Pr(G|\mathcal{H}_0), Pr(G|\mathcal{H}_1)) \\
&\quad = -\ln \sum_{G\in\mathcal{G}} \sqrt{Pr(G|\mathcal{H}_0)Pr(G|\mathcal{H}_1)} && \text{(3a)}\\
&\quad = -\ln \sum_{G\in\mathcal{G}} \sqrt{\prod_{i=1}^{n} \alpha_i(1-\alpha_i)} && \text{(3b)}\\
&\quad = -\ln \left\{ 2^n \sqrt{\prod_{i=1}^{n} \alpha_i(1-\alpha_i)} \right\} && \text{(3c)}\\
&\quad = \sum_{i=1}^{n} \left\{ -\frac{1}{2}\ln(\alpha_i(1-\alpha_i)) - \ln 2 \right\} && \text{(3d)}
\end{aligned}$$

where $\mathcal{G}$ in (3a) represents all possible combinations of the decision vector G. The term 2^n in (3c) comes from the fact that $\mathcal{G}$ has the cardinality of 2^n.

The result in (3d) shows that because of the contribution of the ith human's decision g_i, the Bhattacharyya distance at the FC is incremented by $\epsilon_i = -1/2\ln(\alpha_i(1-\alpha_i)) - \ln 2$, which is nonnegative for $\alpha_i \geq 0.5$. Note that $\epsilon_i = 0$ when $\alpha_i = 0.5$, indicating that a random guess does not contribute any useful information. Moreover, from (3d), we know that the expected Bhattacharyya distance can be expressed as

$$\begin{aligned}
&\mathbb{E}(\mathcal{BD}(Pr(G|\mathcal{H}_0), Pr(G|\mathcal{H}_1))) \\
&\quad = \mathbb{E}\left[\sum_{i=1}^{n}\left(-\frac{1}{2}\ln(\alpha_i(1-\alpha_i)) - \ln 2\right)\right] \\
&\quad = n\mathbb{E}\left[-\frac{1}{2}\ln(\alpha_i(1-\alpha_i))\right] - n\ln 2. && \text{(4)}
\end{aligned}$$

Next, we derive a lower bound on the expected Bhattacharyya distance at the FC.

Proposition 1: In a group composed of n human decision makers with average level of expertise μ and level of heterogeneity σ_α^2, the lower bound of the expected Bhattacharyya distance at the FC increases as μ_α and σ_α^2 become larger.

Proof: Note that $h(\cdot) = -1/2\ln(\cdot)$ in (4) is a convex function. Applying Jensen's inequality, we have

$$\begin{aligned}
\mathbb{E}\left[-\frac{1}{2}\ln\left(\alpha_i(1-\alpha_i)\right)\right] &\geq -\frac{1}{2}\ln\left(\mathbb{E}\left[\alpha_i(1-\alpha_i)\right]\right)\\
&= -\frac{1}{2}\ln\left(\mathbb{E}(\alpha_i) - \mathbb{E}\left(\alpha_i^2\right)\right)\\
&= -\frac{1}{2}\ln\left(\mu_\alpha - \mu_\alpha^2 - \sigma_\alpha^2\right). \quad (5)
\end{aligned}$$

Since $-1/2\ln(\cdot)$ in (5) is a decreasing function and $\mu_\alpha - \mu_\alpha^2$ is decreasing for $\mu_\alpha \in \{0.5, 1\}$, it is clear that $-(1/2)\ln(\mu_\alpha - \mu_\alpha^2 - \sigma_\alpha^2)$ increases as μ and σ_α^2 become larger. Hence, the term $\mathbb{E}\left[-(1/2)\ln\left(\alpha_i(1-\alpha_i)\right)\right]$ and, consequently, the lower bound on the expected Bhattacharyya distance given in (4) become larger as μ_α and σ_α increase. ■

Using the lower bound of the expected Bhattacharyya distance as the surrogate judging criterion, it can be seen that a group performs better with higher average accuracy μ_α as expected. At the same time, it is interesting to observe that for a fixed value of μ_α, a group that has a larger variance

σ_α^2 achieves better performance. Essentially, a group performs better in collaborative decision-making if the parameters yield a larger value in the mean–variance tradeoff given in (5). Note that the best system performance occurs when decision makers are always correct, i.e., $\alpha_i = 1$ for $i = 1, \ldots, n$. In this case, the lower bound given in (5) becomes $+\infty$.

Remark 1: In collaborative human decision-making, if the average expertise μ_α of the humans is kept the same, the group with higher heterogeneity (quantified in terms of σ_α^2) yields better decision-making performance.

In the last part of this section, we present the criterion of human selection for collaborative decision-making under the assumption of independence. Recall that in the above analysis, α_i for $i \in \{1, \ldots, n\}$ is assumed to be a group-level parameter that follows a PDF with mean μ_α and variance σ_α^2. In realistic environments, another aspect of human decision uncertainty comes from variability that exists at individual level. Individual variability, a prominent feature in human behavior, is observed in perception and decision-making even when the external condition, such as sensory signal and the task environment, is kept the same [29]. This is also known as trial-to-trial variability in psychology experiments, i.e., differences of responses are noticeable when the same experiment is repeated in the same human subject. Hence, we assume that for the ith human, α_i is also a random variable at the individual level. Let μ_{α_i} and $\sigma_{\alpha_i}^2$ denote the mean and variance of the ith human's decision accuracy at the individual level, respectively. Following the results in (5), a preferred candidate that contributes more in collaborative decision-making should have a smaller value of $\mu_{\alpha_i} - \mu_{\alpha_i}^2 - \sigma_{\alpha_i}^2$.

III. Group Decision-Making With Correlated Local Decision Makers

In Section II, we analyzed the performance of collaborative human decision-making by assuming that humans make independent decisions. In real applications, we have to account for humans' cognitive biases and uncertainties in order to estimate their decision accuracy in performing a particular task [13], [18]. The accuracy of decision-making, namely, parameters α_i, should be impacted by the human's behavioral properties. Moreover, there may be subgroups within the group whose members have similar background so that their perceptions of the environment and cognitive biases are close to each other. Therefore, in contrast to the assumption that humans make independent local decisions, the decisions made by them for the same problem are correlated and the amount of correlation depends on the degree of their similarities. In addition, the performance measure in terms of the Bhattacharyya distance (presented in Section II) is effective only when the FC employs the LRT to fuse the individual human decisions. In most applications, instead of using the LRT as the decision rule, the FC often uses the majority voting rule due to its simplicity and efficiency even though it is not necessarily optimal [30]. In the majority voting rule, a decision is made in favor of a hypothesis if it gets more than half of the votes. It is desirable to design appropriate strategies that maximize the system performance under the criterion of majority voting.

In the following, we aim to tackle these challenges step by step, starting with a discussion on how humans make decisions in binary hypothesis testing under cognitive biases.

A. Impact of Behavioral Properties on the Accuracy of Human Decisions

In realistic collaborative human decision-making scenarios, it is often the case that decisions have to be made on a problem that has not been seen before. It is shown in [18] and [31] that one could characterize the inherent behavioral properties and cognitive biases of human in decision-making and use this information to infer the human's decision quality. In this section, we study human decision-making in binary hypothesis testing problems that include the consideration of human cognitive biases. According to psychology studies, one prominent feature of human cognitive biases is their loss attitude characterizing the asymmetric valuation toward gains and losses [13], [31]. In the following, we show the impact of loss aversion on the quality of human decision-making under the framework of hypothesis testing.

The Nobel prize-winning PT is widely employed to model human rationality when humans choose between probabilistic alternatives that involve risks. According to PT, a prominent feature of human cognitive biases is loss aversion that characterizes the asymmetric valuation toward gains and losses. PT suggests that humans are usually loss averse in the sense that loss hurts more than the equivalent amount of gain feels good. Under PT, humans distort the valuation of the actual cost c of an event through a value function

$$v(c) = \begin{cases} \lambda c^\beta & c \geq 0 \\ -|c|^\beta & c < 0 \end{cases} \tag{6}$$

where positive values of cost c corresponds to losses and negative values of c correspond to gains. λ is the loss aversion coefficient and β is the parameter that characterizes the phenomenon of diminishing marginal utility [13]. By varying λ, the value function reflects humans' different loss aversion attitudes. The value function shown in Fig. 2 illustrates that positive costs weigh more than negative costs. According to the experiment conducted on a group of human subjects [31], the behavioral parameters λ and β of each individual can be estimated using a nonlinear regression method. The median values of the parameters λ and β are 2.25 and 0.88, respectively. It has been demonstrated in the literature that the loss aversion effect has a larger impact on human decision-making compared to diminishing marginal utility phenomenon [18], [32]. Hence, in this work, we adopt a similar approach by fixing the value of $\beta = 0.88$ and focusing on analyzing how the loss aversion parameter λ affects the decision-making of humans.

Consider that a human decision maker solves a binary hypothesis testing problem. Let the set of the two hypotheses be denoted by $\mathcal{H} = \{\mathcal{H}_0, \mathcal{H}_1\}$, and the priors are given by $\pi_0 = Pr(\mathcal{H}_0)$ and $\pi_1 = Pr(\mathcal{H}_1)$. The human makes an observation $r \in \Gamma$ regarding the PoI, where Γ represents the observation space. Under $\mathcal{H}_0$ and $\mathcal{H}_1$, the observation r has

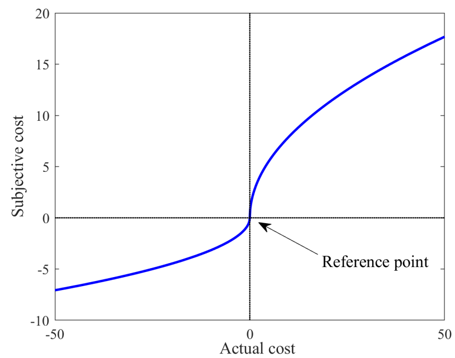

Fig. 2. Value function $\upsilon(c)$.

conditional PDFs

$$\mathcal{H}_0 : r \sim f_0(r), \quad \mathcal{H}_1 : r \sim f_1(r). \tag{7}$$

Let c_{ij} denote the cost of deciding $\mathcal{H}_i$ when $\mathcal{H}_j$ is the true hypothesis. In practical applications, the costs of failing to decide the correct hypothesis, i.e., c_{10} and c_{01}, are positive values, while the costs of successfully choosing the true hypothesis, i.e., c_{00} and c_{11}, are set to 0 or negative values (which can be interpreted gains or something earned).

In solving binary hypothesis testing problems, psychologists have shown that humans first calculate the expected cost of declaring each hypothesis based on some observed evidence and, then, choose the one that has lower expected cost [33], [34]. However, unlike rational decision makers who are able to perceive the expected cost accurately, humans' subjective perception of costs toward a risky event is distorted due to cognitive biases. Given an observation r, we incorporate the value function to calculate the human's subjective perception of costs of declaring $\mathcal{H}_0$ and $\mathcal{H}_1$ [18], [19]

$$\begin{aligned}\mathbf{SC}(\mathcal{H}_0) &= \Pr(\mathcal{H}_0|r)\upsilon(c_{00}) + \Pr(\mathcal{H}_1|r)\upsilon(c_{01})\\ \mathbf{SC}(\mathcal{H}_1) &= \Pr(\mathcal{H}_0|r)\upsilon(c_{10}) + \Pr(\mathcal{H}_1|r)\upsilon(c_{11})\end{aligned} \tag{8}$$

where $\Pr(\mathcal{H}_i|r)$ is the probability that $\mathcal{H}_i$ is true when the observation is r. Following the Bayesian rule, $\Pr(\mathcal{H}_i|r) = (f_i(r)\pi_i/f(r))$ for $i = \{0, 1\}$, where $f(\cdot)$ and $f_i(\cdot)$ denote the appropriate PDFs. The human selects the hypothesis $\mathcal{H}_0$ or $\mathcal{H}_1$ that has a smaller subjective expected cost

$$\mathbf{SC}(\mathcal{H}_0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \mathbf{SC}(\mathcal{H}_1). \tag{9}$$

Substituting the expression of (8) into (9) and simplifying, we have

$$\frac{f_1(r)}{f_0(r)} \underset{H_0}{\overset{H_1}{\gtrless}} \left(\frac{\upsilon_{10} - \upsilon_{00}}{\upsilon_{01} - \upsilon_{11}}\right)\frac{\pi_0}{\pi_1} \triangleq \eta_p. \tag{10}$$

where υ_{ij} represents the distorted valuation of the cost when the value function is applied on c_{ij}. Note that if $\beta = \lambda = 1$, η_p reduces to $\eta = ((\pi_0(c_{10} - c_{00}))/(\pi_1(c_{01} - c_{11})))$, and (10) is the classical LRT studied in the signal processing literature [26].

With the decision rule given in (10), the human decides $\mathcal{H}_1$ if $r \in \mathcal{R}^1 \triangleq \{r \in \Gamma|(f_1(r)/(f_0(r) \geq \eta_p\}))$ where $\mathcal{R}^1$ is known as the critical region and the human decides $\mathcal{H}_0$ if $r \in \mathcal{R}^0 \triangleq \{r \in \Gamma|(f_1(r)/(f_0(r) < \eta_p\}))$ where $\mathcal{R}^0$ is known as the acceptance region. Note that $\mathcal{R}^1$ and $\mathcal{R}^0$ are mutually exclusive and $\mathcal{R}^1 \bigcup \mathcal{R}^0 = \Gamma$. In this case, the human's probability of false alarm (i.e., declaring $\mathcal{H}_1$ when $\mathcal{H}_0$ is true) and probability of miss detection (i.e., declaring $\mathcal{H}_0$ when $\mathcal{H}_1$ is true) in solving the hypothesis testing problem are given by $p_f = \int_{\mathcal{R}^1} f_0(r)dr$ and $p_m = \int_{\mathcal{R}^0} f_1(r)dr$, respectively. It follows that the average probability of error can be expressed as $p_e = \pi_0 \ p_f + \pi_1 \ p_m$.

For humans with different loss aversion attitudes, the variation of λ causes the threshold of the LRT in (10) to be different. As a result, the humans have different decision regions $\mathcal{R}^1$ and $\mathcal{R}^0$, which leads to different decision-making performances in terms of p_e.

B. Correlation Between Local Decision Makers

Humans from the same demographic subgroup often share similar behavioral properties that include emotion state, loss attitudes, and perception of the environment, while the variations of those behavioral properties are significant across different ages, genders, and cultural backgrounds [35]. For example, psychologists have studied the impact of cultural differences on economic decision-making, where they showed that cross-cultural differences such as experiences, individualism, power distance, and masculinity are highly correlated with the level of loss aversion and subjective perceptions [36]. In experiments conducted in two countries (China and Ethiopia), it was shown that the intercountry differences in behavioral patterns are more significant than intracountry differences. We in this work concluded that the intercountry variations in risk attitudes can be ascribed to cultural differences [37].

Inspired by the above evidence, we develop the correlation structure of local decision makers in the following. Consider that there are n humans participating in the collaborative decision-making process regarding the hypothesis testing problem (7). Each human provides a local decision d_i for $i = \{1, \ldots, n\}$ by employing the decision rule (10). Let r^i and λ^i be the random variables that denote the ith human's observation regarding the PoI and his/her loss aversion parameter, respectively. Analogous to the models presented in the quantitative psychology literature [38], [39] that employ a physical measure to quantify the distance between representations of objects on *a priori* grounds, we establish a measure m_{ij} to represent the cognitive profile difference between humans i and j. To model the perceptual and behavioral similarity among the local decision makers, we consider that the correlation coefficient between human observations r_i and r_j follows an exponential decay model [38]:

$$\rho_r^{i,j} = \exp\big(-\phi_r(m_{ij})/l_0\big) \tag{11}$$

and the correlation coefficient between human loss aversion parameters λ_i and λ_j is given by

$$\rho_\lambda^{i,j} = \exp\big(-\phi_\lambda(m_{ij})/l_0\big) \tag{12}$$

where $\phi_r(\cdot)$ and $\phi_\lambda(\cdot)$ are appropriate distance functions that project m_{ij} to the correlation measures applicable to

r and λ, respectively. l_0 is a constant parameter. We hereby assume that the ith human's observation r^i has PDF $f_r^i(r)$. Since all the humans make observations regarding the same PoI (7), we have $f_r^i(r) = f_0(r)$ under $\mathcal{H}_0$ and $f_r^i(r) = f_1(r)$ under $\mathcal{H}_1$ for $i = \{1, \ldots, n\}$, with $\rho_r^{i,j}$ characterizing their correlation structures. Moreover, let the loss aversion parameter λ^i follow PDF $f_\lambda^i(\lambda)$ with $\rho_\lambda^{i,j}$ being the correlation coefficient between λ^i and λ^j.

Here, in contrast to Section II where the local decisions were assumed to be independent, d_i for $i = 1, \ldots, n$ have a dependence structure due to the similarity in human decision makers' behavioral and perceptual properties. To characterize the quality and correlation of local decisions, we define an error indicator random variable δ_i that is equal to 1 if the ith human's local decision d_i is wrong and is equal to 0 if the decision is correct. Let $\boldsymbol{\delta} = [\delta_1, \ldots, \delta_n]$ so that its mean vector $\mu_{\boldsymbol{\delta}} = [\mu_{\delta_1}, \ldots, \mu_{\delta_n}]$ represents the humans' average probabilities of error and the covariance matrix $\Sigma_{\boldsymbol{\delta}}$ shows the dependence structure of δ_i.

Following the analysis and notation in Section II, we further denote the acceptance region and the critical region of the ith human as $\mathcal{R}_i^0$ and $\mathcal{R}_i^1$, respectively. Given a particular hypothesis testing problem, both $\mathcal{R}_i^0$ and $\mathcal{R}_i^1$ are determined by the ith human's loss aversion parameter λ_i. Note that δ_i is equal to 1 if $r_i \in \mathcal{R}_i^0$ under H_1 or $r_i \in \mathcal{R}_i^1$ under H_0. Hence, we have the expected value of δ_i given as

$$\begin{aligned}\mu_{\delta_i} &= \mathbb{E}_{r_i,\lambda_i,H}(\delta_i)\\ &= \int_{\lambda_i}\left\{\pi_0\int_{\mathcal{R}_i^1} f_0^i(r)dr_i + \pi_1\int_{\mathcal{R}_i^0} f_1^i(r)dr_i\right\} f_\beta^i(\lambda)d\lambda_i\end{aligned}$$

where the expectation is taken with respect to r_i, λ_i, and $\mathcal{H}$. Since δ_i takes its value from $\{0, 1\}$, its second moment is $\mathbb{E}(\delta_i^2) = \mu_{\delta_i}$. Hence, the variance of δ_i is given by

$$\text{var}(\delta_i) = \mathbb{E}(\delta_i^2) - \mathbb{E}^2(\delta_i) = \mu_{\delta_i} - \mu_{\delta_i}^2.$$

To evaluate the covariance of δ_i and δ_j, we need to compute the expected value of $\delta_i\delta_j$. Note that $\delta_i\delta_j = 1$ only when both δ_i and δ_j are equal to 1. Hence, we have $\mathbb{E}(\delta_i\delta_j)$ given in (13), as shown at the bottom of the page, where $f_k^{ij}(r_i r_j)$ is the joint PDF of observations r_i and r_j. f_λ^{ij} is the joint PDF of the loss aversion parameters λ_i and λ_j.[2] Hence, the covariance of δ_i and δ_j is given by

$$\text{cov}(\delta_i, \delta_j) = \mathbb{E}(\delta_i\delta_j) - \mu_{\delta_i}\mu_{\delta_j}.$$

At this point, we have been able to compute the values of $\mu_{\boldsymbol{\delta}}$ and $\Sigma_{\boldsymbol{\delta}}$, which will be used to perform human selection in collaborative decision-making in Section IV.

[2]The derivation can be easily extended to incorporate the consideration of diminishing marginal utility parameter β by calculating $\mathbb{E}_{r_i,r_j,\lambda_i,\lambda_j,\beta_i,\beta_j,H}(\delta_i\delta_j)$ instead of $\mathbb{E}_{r_i,r_j,\lambda_i,\lambda_j,H}(\delta_i\delta_j)$ in (13) of the manuscript, where in $\mathbb{E}_{r_i,r_j,\lambda_i,\lambda_j,\beta_i,\beta_j,H}(\delta_i\delta_j)$ we compute the expectation by averaging over $r_i, r_j, \lambda_i, \lambda_j, \beta_i, \beta_j, H$.

IV. Portfolio Theory-Based Human Selection

The objective of this section is to develop a methodology to select a subgroup from a pool of heterogeneous human decision makers to participate in a binary decision-making task. It should be noted that such a human selection problem is not only complicated by the fact that it is difficult to evaluate the performance of decision fusion in realistic multihuman decision-making applications but also by the fact that there exist correlations among the quality of local decisions.

The majority rule is widely adopted as the aggregation rule in collaborative human decision-making due to its simplicity and efficiency. Under the majority rule, the FC collects all the local decisions $D = [d_1 \ldots d_n]$ where $d_i \in \{0, 1\}$ and compares the statistic $\Gamma = \sum_{i=1}^n d_i$ to a threshold $z = \lceil n/2 \rceil$. The FC chooses $\mathcal{H}_1$ if $\Gamma \geq z$ and chooses $\mathcal{H}_0$ otherwise, i.e., whichever hypothesis that has the majority votes is declared to be true.

In past works on majority rule-based collaborative human decision-making or crowdsourcing systems, e.g., [8], [23], [24], and the references therein, it is always the practice to select human agents whose error probabilities are small. Note that this surrogate approach, although intuitive, yields a guaranteed level of system performance when the humans make local decisions independently of each other as we prove in the following.

Proposition 2: In collaborative human decision-making where the humans submit local decisions independently, the majority rule-based decision rule at the FC has lower probability of error when the average error probability of the humans decreases.

Proof: In a group of workers with size $n = 2k+1 (k > 0)$, suppose that each worker provides a binary answer 0 or 1 independently. The average probability of each worker making an error in the local decision is p_o. According to the majority rule, the FC computes the sum of local decisions Γ and makes the final decision by comparing Γ with $z = k + 1$. Without loss of generality, we assume that the true answer is 1. In this case, the FC decides 0 (makes an error) only when the number of 1s submitted by the humans is less or equal to k. Note that Γ follows a binomial distribution with a total of n trials and expected success probability $1 - p_o$. The probability of error can be expressed using the regularized incomplete beta function

$$Pr(x \leq k) = I_{p_o}(k+1, k+1)$$

where $I_{p_o}(a, b) = (B(p_o; a, b)/B(1; a, b))$ with $B(p_o; a, b) = \int_0^{p_o} w^{a-1}(1-w)^{b-1}dw$. Since $B(1, k+1, k+1)$ is a constant given k and $B(p_o, k+1, k+1)$ is an increasing function of p_o, it is clear that the probability of error at the FC decreases as p_o becomes smaller. ■

When there are correlations among the local decisions, the probability that the majority rule makes an error, i.e., less

$$\mathbb{E}_{r_i,r_j,\lambda_i,\lambda_j,H}(\delta_i\delta_j) = \int_{\lambda_i\lambda_j}\left\{\pi_0\int_{\mathcal{R}_i^1\bigcap\mathcal{R}_j^1} f_0^{ij}(r_i r_j)dr_i r_j + \pi_1\int_{\mathcal{R}_i^0\bigcap\mathcal{R}_j^0} f_1^{ij}(r_i r_j)dr_i r_j\right\} f_\lambda^{ij}(\lambda_i\lambda_j)d\lambda_i\lambda_j \tag{13}$$

than z humans submit correct decisions, is expressed as

$$P_e = \sum_{\gamma=0}^{z} \sum_{A \in \mathcal{S}_\gamma} Pr(\boldsymbol{O}_A)\Pr(\boldsymbol{Q}_{A'}) \tag{14}$$

where $\mathcal{S}_\gamma$ is the set that contains all possible combinations of γ humans out of a total of n humans. $\boldsymbol{O}_A$ represents the event that all the humans in subset A make correct decisions and $\boldsymbol{Q}_{A'}$ represents the event that all the humans in the complement set of A make wrong decisions. Quantifying the value of P_e using (14) is difficult because the cardinality of $\mathcal{S}_\gamma$ is $\binom{n}{\gamma}$, which increases quite rapidly as n and γ becomes large, and both $Pr(\boldsymbol{O}_A)$ and $Pr(\boldsymbol{Q}_{A'})$ depend on the joint PDFs of local decisions, which are hard to compute in general applications.

Markowitz's portfolio theory (MPT) [25], [40] is the first to analyze portfolio risk, diversification, and asset allocation in a mathematically consistent framework. In portfolio selection, each asset is an investment instrument that can be bought and sold in the market, e.g., company stock. The return value of each asset is modeled as a random value where the mean value represents the expected value growth of the asset and the variance represents the measure of risk. The expected return of the portfolio is calculated as a weighted sum of the individual assets' returns. The portfolio's risk is a function of the variances of each asset and the correlations of each pair of assets. MPT provides the solution of how to construct a portfolio of multiple assets that the expected return is maximized for a given level of risk.

We aim to solve the human selection problem by mapping it to the portfolio selection problem under the MPT model. There is an analogy between the two problems where we relate the ith human's average probability of making a correct decision to the return of asset i. In such an analogy, $1-\delta_i$ corresponds to the expected return (equivalently, δ_i corresponds to the expected cost) and the covariance matrix $\Sigma_{\boldsymbol{\delta}}$ corresponds to the uncertainty (or risk). Similar to assembling the portfolio of assets under MPT, we select a subgroup of humans that maximizes the sum of their probabilities of making correct decisions (equivalently, minimizing the sum of their error probabilities), while constraining the variability of the system performance is below a certain level. Here, note that minimizing the sum of the humans' error probabilities is consistent with the objective of human selection where the humans make decisions independently of each other.

A. Motivation for Portfolio Theory-Based Human Selection

Because of the correlation among the local decisions, variability (or variance) of the system performance is an important criterion in determining the subgroup of human participants. Selecting humans with a smaller average error probability does not necessarily result in the highest accuracy at the FC. In the following, we provide a toy example to illustrate our concern.

Motivating Toy Example: Consider that there is a pool of six humans and we aim to select 3 of them to participate in an inference task. Whether or not the humans make a mistake in their local decisions are modeled as Bernoulli random variables b_i for $i = 1, \ldots, 6$ with the probabilities of error given by $\boldsymbol{p} = [p_1\ p_2\ p_3\ p_4\ p_5\ p_6] = [0.25\ 0.25\ 0.25\ 0.3\ 0.3\ 0.3]$. We assume the case that the first three decision makers are highly correlated such that they make correct or wrong decisions at the same time. Hence, we have the correlation coefficient of each pair among the first three decision makers equal to 1, i.e., $\rho_{ij} = 1$ if $i, j \in \{1, 2, 3\}$. On the other hand, we assume that each of the last three human agents $i = 4, 5$, and 6 makes the decisions independently of any other decision maker in the pool. As a result, the correlation coefficient $\rho_{ij} = 0$ if i or $j \in \{4, 5, 6\}$ and $i \neq j$. Under this model, the covariance matrix of the random variables b_i, denoted by Σ_b, can be written as

$$\Sigma_b = \begin{bmatrix} 0.1875 & 0.1875 & 0.1875 & 0 & 0 & 0 \\ 0.1875 & 0.1875 & 0.1875 & 0 & 0 & 0 \\ 0.1875 & 0.1875 & 0.1875 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.21 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.21 \end{bmatrix}$$

where the variance of a Bernoulli random variable is obtained by $\mathrm{var}(b_i) = p_i(1-p_i)$ and the covariance is given by $\mathrm{cov}(b_i, b_j) = \rho_{ij}(\mathrm{var}(b_i)\mathrm{var}(b_j))^{1/2}$. To select three out of the six decision makers to perform the inference task without considering their dependence structure, first, we choose those that have low error probabilities. In the above problem, the first three humans $i = 1, 2$, and 3 have the lowest error probabilities so that we choose the human selection vector to be $s_1 = [1\ 1\ 1\ 0\ 0\ 0]$, where 1 represents the selection of the corresponding human and 0 represents no selection. When the local decisions are aggregated via the majority rule, the subgroup selected by s_1 has the probability $p_{e1} = 0.25$ to make a mistake as the participants make wrong decisions at the same time with a probability of 0.25. On the other hand, if we set $s_2 = [0\ 0\ 0\ 1\ 1\ 1]$ and select the last three humans whose error probabilities are larger, the majority rule-based decision rule has the probability of error $p_{e2} = \binom{3}{2}(0.3)^2(1-0.3) + (0.3)^3 = 0.216$, where $\binom{3}{2} = 3$ represents the number of combinations of selecting two humans out of three humans. We find that although the last three humans have higher error probabilities, the selection of humans using s_2 achieves a better system performance compared to the selection using s_1. Since the humans $i = 4, 5$, and 6 act independently of each other, it allows for more freedom in terms of diversification, which reduces the probability that two or more humans make mistakes together.

In the MPT model, an investor can reduce the risk by holding a combination of assets that are not perfectly positively correlated. In collaborative human decision-making, let C_N denote the number of selected humans that make incorrect decisions. When the recruited human decision makers are less correlated with each other, the variance of C_N becomes smaller. To provide an intuition, we continue with our toy example and compute the variances of C_N when employing the selection vector s_1 and s_2

$$\mathrm{var}_{C_N}(s_1) = s_1 \Sigma_b s_1{}' = 1.6875$$
$$\mathrm{var}_{C_N}(s_2) = s_2 \Sigma_b s_2{}' = 0.63$$

where the superscript $'$ represents the transpose of the vector. Compared to s_1, the selection of independent decision makers using s_2 has a smaller value of C_N's variance. When the correlation among humans is low, it is unlikely that they make mistakes at the same time. In such a case, C_N remains small most of the time and the probability that C_N takes a large value is negligible, which makes the variance of C_N small. On the other hand, when the local decisions have a strong correlation, there is a relatively large chance that they make mistakes at the same time, causing the variance of C_N to be large. It was also shown that along with the smaller variance achieved by s_2, the average probability of error p_{e2} is smaller. This motivates the application of MPT in our human selection problem in the sense that a smaller variance of C_N corresponds to diversification among human decision makers (i.e., the local decisions are not highly correlated with each other), which avoids the possibility of concurrent failures so that the system performance can be improved.

B. MPT-Based Human Selection and Optimization Method

Following MPT, the risk-averse investors wish to design portfolios that have the best expected return–risk tradeoff. In our problem, the portfolio set corresponds to the pool of human workers and we wish to select a subgroup to participate in an inference task to ensure the quality of system performance. The expected means and the covariance matrix of the random variables that represent that the ith local decision is incorrect are given by μ_δ and Σ_δ, which have been derived in Section III. We seek to select the subgroup to achieve two objectives, i.e., minimize the sum of expected error probabilities[3] and reduce the variance of C_N.

Let $\boldsymbol{s} = [s_1, \dots, s_i, \dots, s_n]$ denote the human selection vector, where s_i represents whether or not the ith human is selected. In the first formulation, we aim to minimize the sum of the error probabilities of the selected humans while keeping the variance of C_N below a target value σ_t^2

$$\min_{\mathbf{s}} \quad \mu_s = \boldsymbol{s}\mu_\delta' \tag{15a}$$

$$\text{s.t.} \quad \sigma_s^2 = \boldsymbol{s}\Sigma_\delta \boldsymbol{s}' \le \sigma_t^2, \tag{15b}$$

$$\boldsymbol{s}\mathbb{1}' = m \text{ and } s_i \in \{0, 1\} \text{ for } i = 1, \dots, n \tag{15c}$$

where $\mathbb{1}$ represents the all-one vector. In (15c), we constrain that a total number of m humans are selected and each s_i has to be a Boolean variable. In MPT, the problem of maximizing the expected return at a given level of risk has an equivalent dual representation where we minimize the variance of the portfolio subject to a target value of expected return. Hence, the dual formulation of the optimization problem (15a)–(15c) is given by

$$\min_{\mathbf{s}} \quad \sigma_s^2 = \boldsymbol{s}\Sigma_\delta \boldsymbol{s}' \tag{16a}$$

$$\text{s.t.} \quad \mu_s = \boldsymbol{s}\mu_\delta' \le \mu_t, \tag{16b}$$

$$\boldsymbol{s}\mathbb{1}' = m \text{ and } s_i \in \{0, 1\} \text{ for } i = 1, \dots, n \tag{16c}$$

where μ_t in (16b) denotes the threshold that upper bounds the sum of selected humans' error probabilities. The advantage of the second formulation given in (16a)–(16c) compared to the one given in (15a)–(15c) is that it is preferable to constrain the value of μ_t rather than the target variance levels σ_t^2. This is because typically it is hard for the project manager to quantitatively relate the value of σ_t^2 to a specific level of variability.

[3]This objective coincides with the surrogate criterion for human selection when they make local decisions independently, i.e., minimizing the average error probability of the selected crowd workers as shown in Proposition 2.

Algorithm 1 Solving the Optimization Problem (16a)–(16c)

1: **PROCEDURE: Find the human selection vector $\boldsymbol{s}$**

2: Set the iteration count $t = 0$ and the initial weights $w_i^{(t)} = 1, i = 1, \dots, n$.

3: Construct the weight matrix $W^t = \text{diag}([w_1^{(t)} w_2^{(t)} \dots w_n^{(t)}]')$.

4: Solve the minimization problem

$$s^t = \arg\min_{\boldsymbol{s}} \ \boldsymbol{s}\Sigma_\delta \boldsymbol{s}' + \phi \left\| W^t \boldsymbol{s}' \right\|_{\ell_1}$$
$$\text{s.t.} \quad \boldsymbol{s}\mu_\delta' \le \mu_t, \quad \boldsymbol{s}\mathbb{1}' = m,$$
$$0 \le s_i \le 1 \text{ for } i = 1, \dots, n$$

where ϕ is a properly designed parameter that is positive.

5: Update the weights for $i = 1, \dots, n$

$$w_i^{t+1} = \frac{1}{\left|s_i^t\right| + \epsilon}$$

where $\epsilon > 0$ is a parameter to provide stability to the algorithm.

6: Repeat step 3-5 until a specified maximum iteration number $t_{\max}$ is reached.

7: For the largest m entries from the final weight vector, set the corresponding entries in $\boldsymbol{s}$ to be 1. Set the other entries in $\boldsymbol{s}$ to be 0.

In contrast to MPT where the optimization variable is continuous, we have $s_i \in \{0, 1\}$ so that $\boldsymbol{s}$ is in a nonconvex set, making the problem generally impossible to solve as the solution requires an intractable combinatorial search. We employ the reweighted ℓ_1 minimization approach [41] to solve this binary constrained optimization problem by assigning the weight w_i to each element s_i, where the algorithm iteratively alternates between optimizing $\boldsymbol{s}$ and redefining the weights. After a certain number of iterations, $\boldsymbol{s}$ converges to a steady state and the entries that have large weights are set equal to 1, indicating that the corresponding humans will be selected. To provide an example, we show the detailed procedures to solve the problem (16a)–(16c) in Algorithm 1.

In this section, we propose a collaborative human decision-making mechanism while using PT to model the correlations of the workers' decisions and using concepts from portfolio theory for worker selection. If the desired system performance does not achieve a certain level of accuracy, the FC may expand or reselect the worker pool to enhance heterogeneity and improve system performance. The flowchart of the system is presented in Fig. 3.

V. Simulation Experiments

We conduct numerical experiments using MATLAB. First, we evaluate the performance of collaborative decision-making system that consists of independent local decision makers.

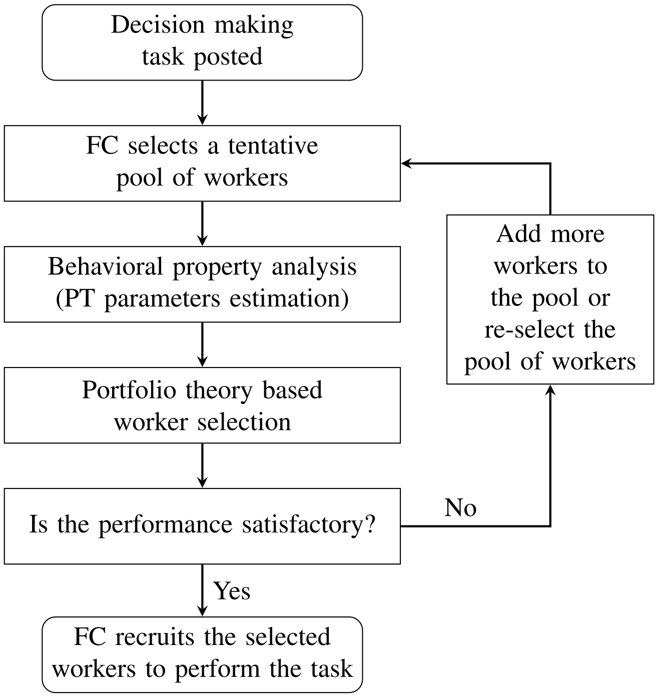

Fig. 3. Flowchart of the PT and portfolio theory-based collaborative human decision-making system.

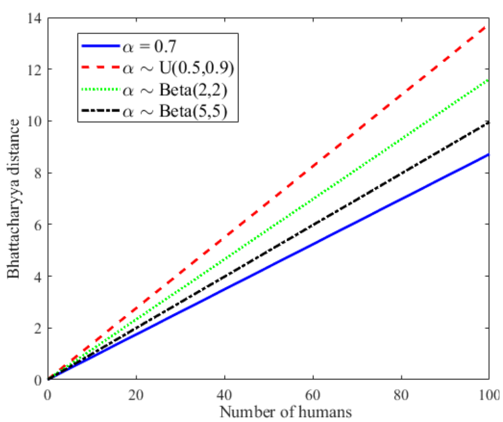

Fig. 4. Bhattacharyya distance as the number of human experts increases for different distributions of α.

As described in Section II, the Bhattacharyya distance is used to measure the decision-making performance. In Fig. 4, we plot the Bhattacharyya distance between the conditional observation distributions under the two hypotheses as the group size n increases from $n = 1$ to $n = 100$. In each of the four curves, human decision-making accuracy α_i follows the following distribution: fixed at 0.7, uniform distribution $U(0.5, 0.9)$ and Beta distribution within the interval $[0.5, 0.9]$ with parameters $(2, 2)$ and $(5, 5)$. The expected means of the four distributions of α are the same $\mu = 0.7$, but they have different variances. The variances of α in the four curves (red, green, yellow, and blue) are 0.013, 0.008, 0.004, and 0, respectively. Results are obtained by averaging over 5000 Monte Carlo simulations. It can be observed that in these four distributions of α, the Bhattacharyya distance increases linearly as more human experts collaborate on the task. As the variance of the distribution from which α is sampled becomes larger, the Bhattacharyya distance has a higher increment rate. This motivates us to select those groups that are composed of humans of diverse backgrounds to achieve better decision-making performance.

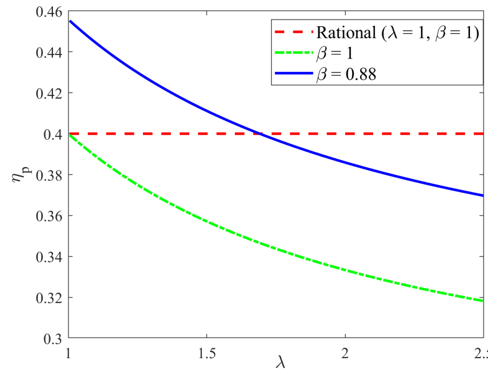

Fig. 5. Threshold of LRT employed by a behavioral decision maker.

Next, we provide some simulation results for the scenario where local decision makers are correlated as described in Section III. For illustration, we consider that a human decision maker solves a hypothesis testing problem, where under $\mathcal{H}_0$ and $\mathcal{H}_1$, the conditional PDFs of the observation r are Gaussian with means $m_0 = 0$ and $m_1 = 3$, respectively, and the same variance $\sigma_s^2 = 3$. We assume that the priors are $\pi_0 = \pi_1 = 0.5$ and the costs are $c_{01} = 80, c_{10} = 20$, and $c_{11} = c_{00} = -20$. In Fig. 5, the threshold η_p of the LRT employed by the human is plotted as a function of the human's loss aversion parameter λ. For comparison, the red line provides the benchmark where the human is rational and the threshold of LRT is a constant, i.e., η. It is observed that when $\beta = 1$, the LRT threshold of the human deviates from η and keeps decreasing as λ increases. That is because as λ becomes larger, the subjective cost of misdetection $c_{01} = 80$ is more significant than the cost of false alarm $c_{10} = 20$. Hence, the threshold of the LRT should be decreasing to avoid the possibility of miss detection. Similarly, when $\beta = 0.88$, the LRT threshold η_p decreases as λ increases. At a particular value of λ^*, however, the blue curve intersects the red line, indicating that the loss aversion parameter λ^* counteracts the diminishing marginal utility effect with $\beta = 0.88$. Hence, a human with certain cognitive bias parameters β and λ could achieve the same decision-making performance as rational decision makers.

We further conduct experiments for the worker selection problem from a 30-human pool where we use assign the label i to each human: $i = 1, 2, \ldots, 30$. For the correlation coefficient equations (11) and (12), we assume that the human i and j have cognitive profile difference given by $m_{ij} = 0.2|i - j|$ for $i, j \in 1, \ldots, 30$. For simplicity, the projection functions $\phi_r(\cdot)$ and $\phi_\lambda(\cdot)$ are assumed to be identity functions and the constant parameter $l_0 = 1$. In this case, we have $\rho_r^{i,j} = \rho_\lambda^{i,j} = \rho^{i,j} = \exp(-m_{ij})$. Moreover, the loss aversion parameter of the ith human λ_i is assumed to follow a Beta distribution $\text{Beta}(a_i, b_i)$ with support $[0\ 3]$ and the parameters $a_i = 2 + i, b_i = 3$. Meanwhile, the ith and jth humans'

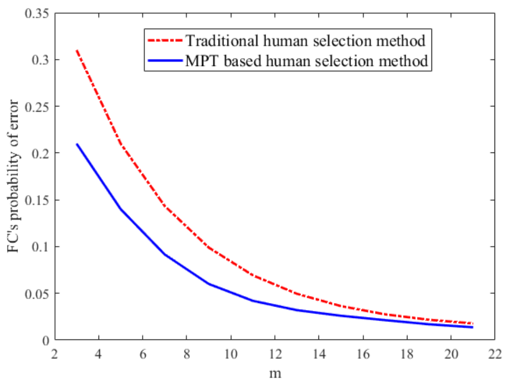

Fig. 6. FC's probability of error as m increases.

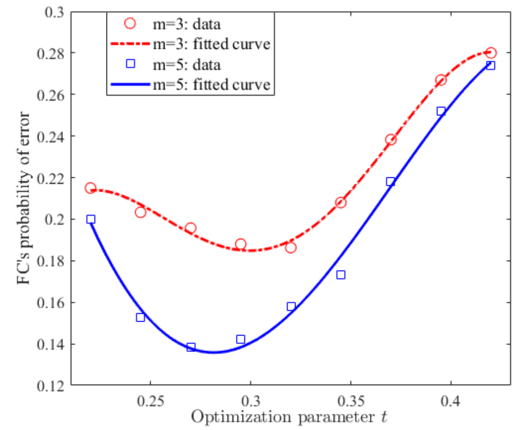

Fig. 7. FC's detection performance as a function of optimization parameter t.

observations r_i, r_j as well as their loss aversion parameters λ_i and λ_j have a correlation structure with correlation coefficient $\rho^{i,j}$. In simulations using MATLAB, for example, we use the *mvnrnd* function to generate correlated Gaussian random variables $r_1, \ldots, r_n$. To generate correlated Beta distributed random variables, we first exploit the *copularnd* function to get a vector of random variables generated from a Gaussian copula with a certain correlation structure and, then, employ the *betainv* function to transform the output of *copularnd* into random numbers that follow the beta distribution.

Without loss of generality, we assume that $\mathcal{H}_1$ is true so that the ith human makes a wrong decision (i.e., $\delta_i = 1$) when he/she submits $d_i = 0$ and makes a correct decision (i.e., $\delta_i = 0$) when he/she submits $d_i = 1$. We obtain the mean vector μ_δ and covariance matrix Σ_δ that characterize the quality and dependence structure of the humans' local decisions. We formulate the MPT-based optimization problem for human selection as given in (16a)-(16c), where we set the target error probabilities $\mu_t = 0.3\,m$, indicating that the selected humans should have their averaged error probability below 0.3. Algorithm 1 is used to solve the optimization problem where m out of 30 humans are selected to participate in the inference problem.

As m takes its value from $\{3, 5, \ldots, 21\}$, we plot the error probability of the majority rule-based decision fusion with respect to m in Fig. 6. The blue curve represents the scenario where humans are selected using our proposed approach based on MPT optimization. The red curve corresponds to the case in which humans with the lowest individual error probabilities are selected without considering their correlation structure. As m increases, the FC's error probability decreases for both of the scenarios and it is observed that our proposed method performs better for every value of m. By minimizing the variance of the number of humans that make mistakes, our algorithm does not favor selecting highly correlated local decision makers. The diversification (or independence) among the selected humans ensures that they are not likely to make mistakes at the same time. Therefore, the system performance improves.

Finally, we vary the optimization parameter μ_t in (16b) and see how it affects the system performance. It should be noted that μ_t controls the tradeoff between the two conflicting objectives: 1) minimizing the average error probability of the selected humans and 2) reducing the variance of C_N. A small value of μ_t gives more emphasis to the first objective and a large value of μ_t gives more emphasis to the second objective. On the one hand, μ_t cannot be too small as it limits the human selection pool to a small range, where heterogeneity might not be promoted. On the other hand, μ_t cannot be too large, and otherwise, we might select humans whose error probabilities are quite large (such as spammers and Byzantines) where the quality of the system performance is not guaranteed. In the previous simulation, we fixed the value of μ_t to be $\mu_t = 0.3\,m$, which might not necessarily be optimal. In Fig. 7, we set $\mu_t = tm$ and let t vary. We plot the FC's probability of error for different values of t for $m \in \{3, 5\}$. We also fit a 4° polynomial curve to the data samples and show that how the system performance changes with respect to t. It is observed that in each case, there is a certain value of t that achieves the best system performance. It should also be noted that the optimal parameter t when $m = 3$ is larger than the optimal value of t when $m = 5$, indicating that when the number of selected humans is small, it is desirable to enhance the emphasis on heterogeneity to improve the performance of group decision-making.

VI. Conclusion

From a distributed detection and information fusion perspective, we showed that promoting heterogeneity enhances the performance of collaborative human decision-making. First, we assumed that the independent local decisions are modeled via the BSC model and the final result is aggregated through the LRT-based decision rule. We showed that when the average level of human accuracy is kept the same, high variability of the human expertise leads to better system performance as it results in a larger lower bound of the Bhattacharyya distance at the FC. Next, we considered the more practical scenario where humans have similarities in their behavioral properties so that they make correlated local decisions. When the FC employs the majority rule-based decision fusion, we proposed an MPT-based human selection scheme so as to promote heterogeneity and reduce the probability of error while decision-making. Our results corroborate the widely recognized benefits and advantages of encouraging heterogeneity by providing justification using signal detection theory.

In future work, the model developed in this work can be applied to guide the recruitment and decision fusion of committee members, human scouts, and so on in various decision-making scenarios that have humans collaboratively making a final decision. The analysis of human cognitive biases and the correlations among their decisions modeled and analyzed in this article can be exploited to study the group role assignment problem in role-based collaboration where different agents are needed to be assigned to different roles [42], [43].

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