

An Overview on Passive Magnetic Bearings

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Abstract—Magnetic bearings are an area of interest for high speed applications, such as flywheel energy storage systems, to remove friction losses. Stable levitation cannot be achieved through a static passive magnetic bearing system, as indicated by Earnshaw's Theorem. Solutions using active magnetic bearings have been presented which achieve stable levitation, but induce losses in the current used to actively stabilize one or more degrees of freedom. To overcome these losses while retaining stable operation a fully passive architecture can be created utilizing superconducting magnetic bearings. This paper will review the key passive magnetic and superconducting technologies, their design and optimizations methods for different topologies, and existing work done to combine both into a functional stable system.

Index Terms—Passive Magnetic Bearing, Permanent Magnetic Bearing, Superconducting Bearing.

I. INTRODUCTION

Bearings are essential components for machines that require linear or rotational movement. In recent years, passive magnetic bearings have been extensively studied by engineers and scientists as an alternative to traditional mechanical bearings. Passive magnetic bearings utilize magnetic interaction to provide suspension or levitation forces for components such as a rotor or shaft. [1] The use of passive magnetic bearings eliminates physical contact between mechanical components, hence no friction losses and wear and tear.

Earnshaw's theorem [2] states that the sum of stiffness in three dimensions must equal 0. The result of this is that stability can only be achieved in at most 2 directions using a fully passive permanent magnet solution. Several methods of getting around this seeming limitation are to use an additional mechanical constraint such as mechanical bearings [3], active control via electric coils [4], superconducting or diamagnetic materials [5], or halbach stabilization coils [6].

In [7] the loss comparison between mechanical bearings, active control, and passive stabilization with a combination of passive magnetic bearings and superconducting bearing is presented with significant savings being achievable. This paper will give an overview of passive PMBs, SMBs, hybrids comprised of both, and full system level design utilizing a combination of PMBs and SMBs with the goal of providing design options for a completely passively stabilized system.

II. PRINCIPLES AND TOPOLOGIES

Passive magnetic bearings are generally divided into three main categories: 1) permanent magnetic bearings (PMB), 2)

superconducting magnetic bearings (SMB), and 3) hybrid magnetic bearings utilizing both PMBs and SMBs. These categories can be further subdivided based on the geometry of the bearings as shown in Fig. 1. Additionally, machine system designs may involve combinations of PMB and SMB which will be further discussed in the review.

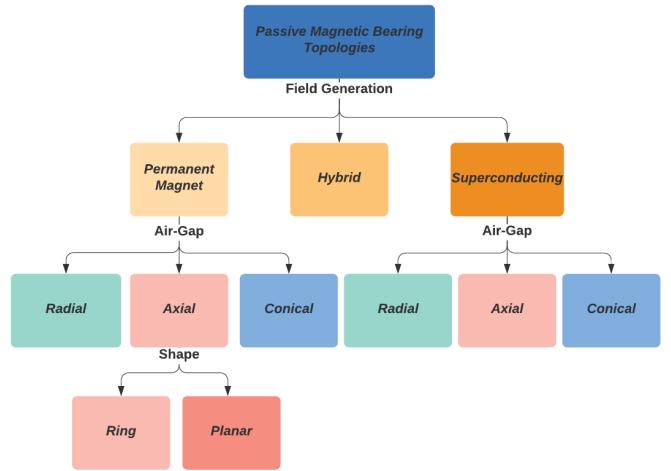


Fig. 1. Overview of passive magnetic bearing topologies as they are organized in this paper.

A. Permanent Magnetic Bearing (PMB)

Permanent magnetic bearings utilize their geometry, magnet placement, and magnetization direction to generate forces as a function of displacement. This is typically given as the achievable stiffness. The design and analysis considerations for PMBs are presented in this section.

Ring shaped magnetic bearings can be classified by airgap direction [8], magnetization direction [8]–[15], or thrust direction [10]. Here the air gap will be used as it is typically constrained by the application. Many works optimizing a given air gap structure will evaluate numerous magnetization direction schemes [8]–[10], [16]. In general, cylindrical shapes are extensions of ring shapes by either increasing the height parameter [17], or stacking rings in the axial direction possibly with consideration for magnetization by individual ring [16].

Common metrics used to analyze these systems include the obvious force or stiffness in a given direction, but also include more application specific metrics such as stiffness per magnetic volume [4], [18]–[20], ratio of axial and radial stiffness [21], ratio of force per current [22], or aspect ratio [23].

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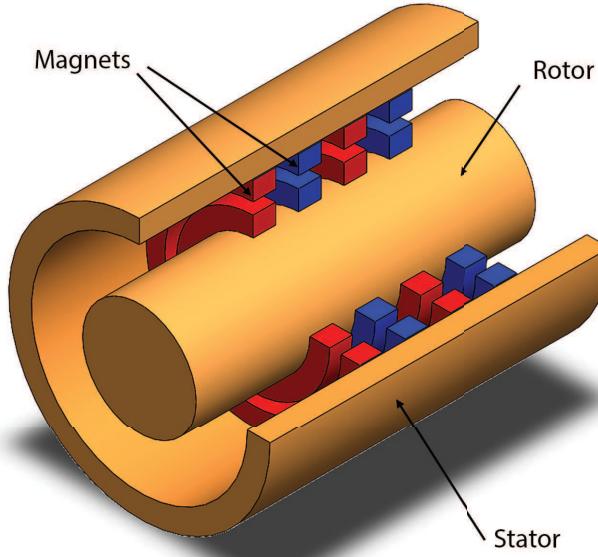


Fig. 2. Ring bearing with radial air-gap, featuring multiple ring segments.

1) Radial Ring PMB

Radially oriented bearings feature two ring or cylindrical structures nested inside one another with a radial air-gap as shown in Fig. 2. Numerous magnetization schemes are employed with several common ones shown in Fig. 3. Much effort has been expended to analytically estimate the force and stiffness of these structures under different magnetization schemes [8], [11]–[13], [15], [24]–[26]. Design and optimization focus on the magnetization employed [11]–[13], [15], [16], [18], [19], [24]–[27], the height of the magnets [12], [13], [16], the number of stacks [16], and the presence of iron either in the form of back iron or inset between the magnets [20].

Methods focus on Finite Element Analysis (FEA) solutions [18], [20], [24], [25], [28], [29] with or without the backing of analytical design [6], [11]–[13], [15], [18]–[20], [24], [25], [27]. Methods typically focus on optimizing the force [6], [12], [13], [15], [16], [18]–[20], [24], [25], [27], [28] or stiffness in a particular direction [6], [12], [13], [15], [18]–[20], [24], [25], [28] with several sources seeking to minimize cost and magnetic material [18], [19], [29]. Optimization results tend to favor halbach arrays with back iron [15], [20] to increase force and stiffness.

2) Axial Ring PMB

Axial air gaps are extensively documented [6], [8]–[10], [16]–[19], [21], [26], [30], [31]. They feature an axial air-gap between interacting rings as shown in Fig. 4. Typically these bearings are located near the axial ends of the system, often times they are designed to counteract gravity [32].

Design of these bearings focuses on the optimization of the height and width of the magnets [16], [21], [31], the magnetization direction and scheme [6], [8]–[10], [16], [21], [30], the number of layers [16], [21], and the presence of back iron [21]. This process utilizes a combination of analytical [21] as well as

FEA [16], [18], [21], [30] consideration to optimize the force [16], [18], [21] and stiffness [18], [19], [21] in the direction of interest (for axial bearings this is typically the axial direction), and to minimize magnetic volume [16]. Experimental results are presented in [18] where special optimal magnetization patterns are applied to ring magnets to maximize axial force and stiffness. Additional results are shown in [31] where an axial magnetic bearing was paired with a mechanical bearing to reduce the amount and non-linearity of the bearing friction during rotational operation.

3) Planar PMB

The planar design is closely related to ring bearings which have an axial air-gap and are used to provide a force along the axial direction, often to counteract gravitational loads as shown in Fig. 5. In [32] the design process is demonstrated analytically but primarily using FEA. Key design parameters include the air-gap, magnet height, direction of magnetization, number of air-gaps, and arrays utilizing bidirectional magnetization or halbach arrays as shown in Fig. 6 [32].

Maximal lifting force is achieved in the case where bidirectional magnetization is applied, using a small number of magnet segments and multiple air gaps, and with a high magnet height. With only a single air gap the halbach array is shown to have higher force with smaller magnet height i.e. larger force density. However, when using a halbach array consideration must be given to the magnet remanence to prevent demagnetization [32].

4) Conical PMB

The conical design takes the linear slide bearing and wraps it about a rotational axis to achieve a conic shape as shown in Fig. 7. [33] This design when placed axially along a rotor, paired with a mirrored conic bearing on the opposite end may appear to be stable. However, Earnshaw's theorem still applies and instability in one of the directions will result. In [33] this design is paired with a mechanical bearing to provide full system stability while reducing the amount of force and thereby friction applied to the mechanical bearing.

This design gives further options to the design process over ring shapes as in addition to the direction of magnetization, there is also the angle of the two conic sections to consider. This design method can offer some advantages over ring geometries in shaping the direction of the force to give priority to either radial or axial force generation. However, this comes at the expense of additional complexity in the geometry and potentially manufacturing cost if conic magnetic sections cannot be easily obtained [3], [33].

B. Superconducting Magnetic Bearing (SMB)

The operating principle of SMB relies on two superconducting phenomena: the Meissner effect and flux pinning. When a type-II superconductor (such as YBaCuO (YBCO) crystals) reaches its superconducting state under critical temperature, the Meissner effect causes the superconductor to expel any external magnetic field and generates an opposing magnetic field, resulting in a repulsive force. Meanwhile, the flux pinning phenomenon causes some of the external magnetic field lines to be trapped inside the superconductor resulting

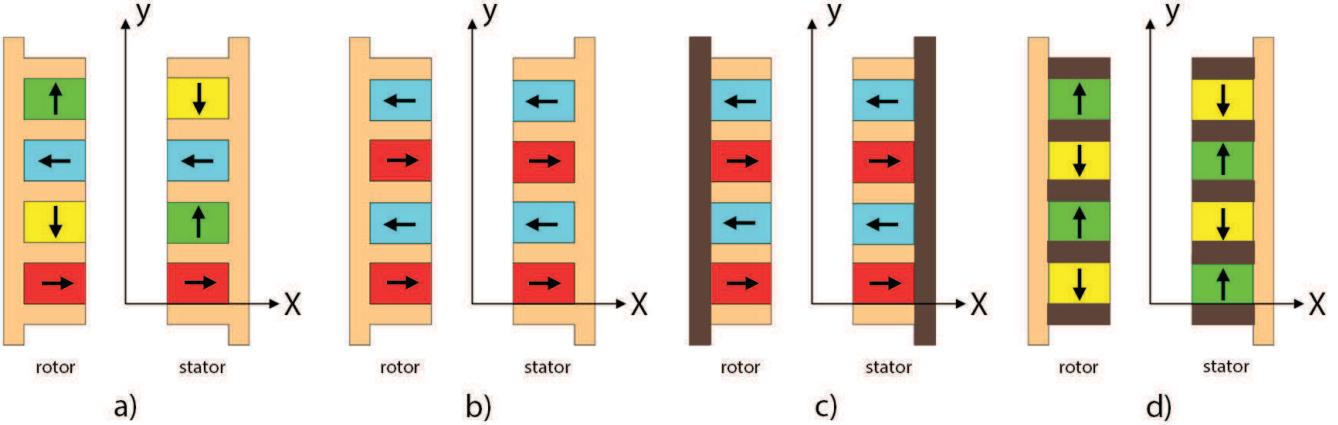


Fig. 3. A sample of common magnetization schemes with design options of iron placement (c) and (d) shown in brown to demonstrate configurations from [20].

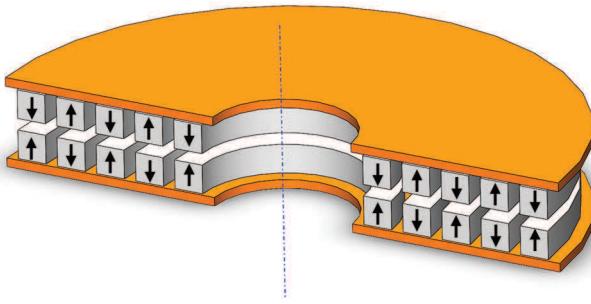


Fig. 4. Ring magnet structure with an axial air-gap.

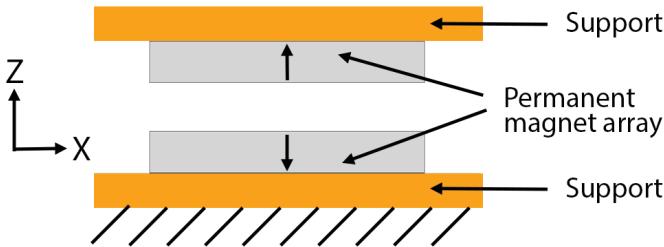


Fig. 5. General planar design with air gap definition.

in a resistive/attractive force between the superconductor and permanent magnet. As a result, the combination of these two phenomena creates a stable magnetic levitation for objects exerting magnetic fields. Further studies on these two superconducting phenomena can be found in [1], [34].

The main modeling methods of force between permanent magnets and superconductors include the critical-state model method and differential Maxwell equations method as studied in [35]. Other numerical and FEM modeling method for superconductors are also investigated in [36]–[43].

The geometry of SMB varies in each design. However, they can generally be summarized into two main categories: axial and radial geometry. SMB may also contain a tertiary geometry, conical, which will be addressed at a later point in this paper.

1) Axial SMB

Commonly used superconductors and permanent magnets for constructing an axial SMB are YBCO bulk and NdFeB permanent magnets. The bearing shown in Fig. 8 is a simple form of an axial SMB. The axially magnetized permanent magnet is attached to a rotor while the superconductor is positioned underneath the permanent magnet to provide levitation. During its operation, the flux pinning and Meissner effect allow the rotor to spin freely around the permanent magnet's field symmetry axis. These two phenomena create stabilizing axial and lateral supporting forces which result in complete rotor levitation without any physical contact with the superconductor.

While axial SMB is typically positioned directly underneath the rotor to provide axial support, the axial SMB can also be positioned above the rotor to provide additional support to the rotor. For example, bearing system designs where axial SMB are fixed at the two ends of the shaft are studied in [5], [44], [45].

Numerous studies have been conducted to optimize the stiffness and load capability of an axial SMB. A study conducted by [46] found that the use of a Halbach array configuration resulted in a 50% increase in levitation force. Other studies, such as [1], [47], [48], suggest using a multipole permanent magnet configuration to improve the load capability and stiffness of an axial SMB. Another method for optimization is to decrease the zero-field cooling position of the superconductor inside the SMB, as suggested in [34], [47]. Other methods, such as adding PM rings to the SMB stator, are suggested in [49]. FEA optimization and mathematical studies can also be found in [50], [51].

Two major advantages of using axial SMB is the minimization of mechanical losses and high axial load capability, which are ideal for applications such as flywheel energy storage systems (FESS). An example where an axial SMB design is used in the FESS is the Boeing flywheel project. The Boeing flywheel project used iron rings in between permanent magnets to increase the flux in one direction which allows for a higher gradient against the SMB and generates a higher

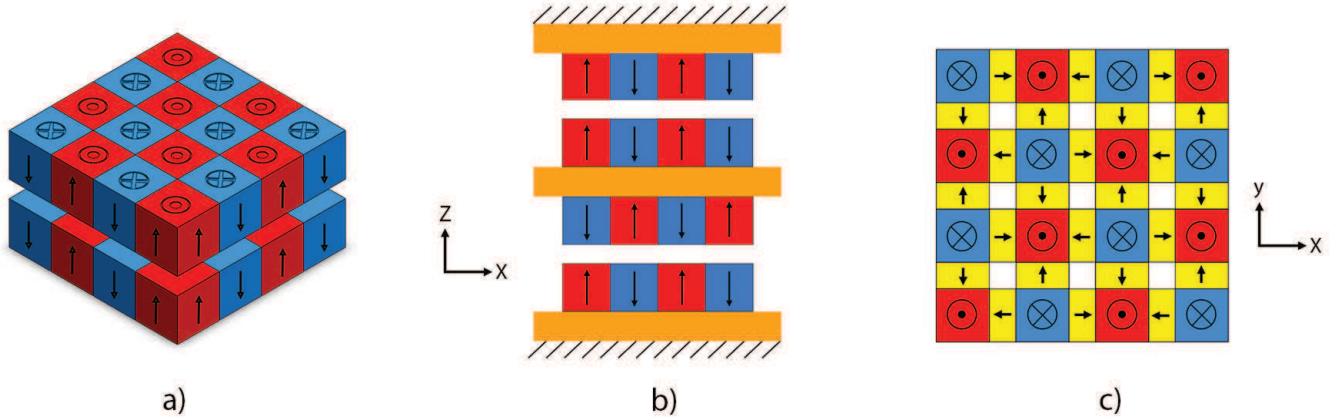


Fig. 6. Planar design parameters considering a) bidirectional magnetization, b) multiple airgaps, c) utilization of a 2D halbach array as shown in [32].

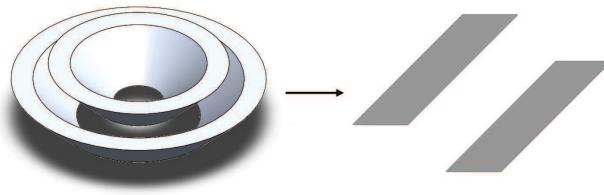


Fig. 7. Operating principle showing a conic bearing as a wrapped version of angled slide/linear bearings.

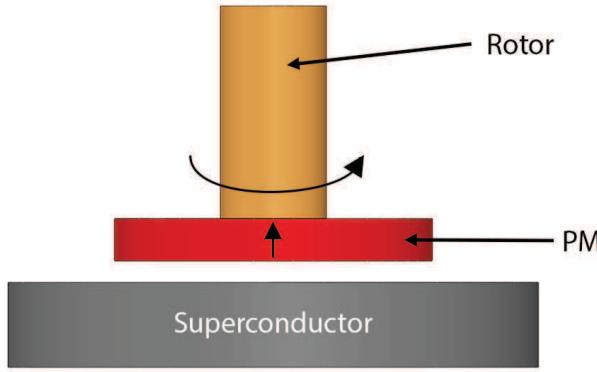


Fig. 8. A simple axial superconducting magnetic bearing, consists of a superconducting stator levitating a permanent magnetic rotor as described in [1]

levitation force [52], [53]. Other FESS designs where axial SMB are utilized can be found in [54], [55]. One disadvantage of using an axial SMB is the effect of the surface area of superconducting material on the size of the design. In order to increase the axial load capacity, the surface area of superconductors in an axial SMB must increase, resulting in an increase in the size of the system [56]–[58]. Another disadvantage of axial SMB is its low stiffness in lateral/radial directions when compared to a radial SMB [57], [59], [60].

2) Radial SMB

The operating principle of an axial and radial SMB are similar as they both rely on the stabilizing and levitation force generated by the superconductor. However, the geometry of

these two types of SMBs varies as radial SMBs are typically ring or cylindrical in shape. A common radial SMB design is shown schematically in Fig. 9.

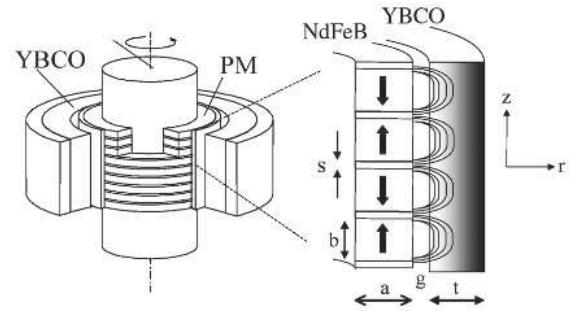


Fig. 9. A simple radial SMB with inner rotor design, with axially magnetized permanent magnetic rings in alternating configuration describing configurations from [61].

Notably, depending on the application, radial type SMBs can also be configured to have an outer rotor design [62]. Prominent studies that utilize radial SMBs in FESS design can be found in [57], [58], [61], [63], [64].

One of the major challenges of using radial SMB is the tendency of force creep during operation. Force creep is a natural or perturbed drift of the position of a superconductor levitated in a static magnetic field [34]. To combat this issue, studies in [62], [65], [66] have suggested the use of preloading and supercooling methods to drastically increase the axial levitation force and stiffness. In addition, methods of extending the radial SMB in axial direction have also been found to be effective [59], [62]. Furthermore, methods such as performing field cooling to improve radial stiffness are studied in [60], [67], [68]. Other studies such as [67], [69] have suggested the implementation of iron shims in between permanent magnet layers to increase the overall flux gradient and optimize stiffness. Lastly, analytical models and FEA geometric parameter optimization methods can be found in [59], [70]–[72]. Specific method of using Genetic Algorithms to optimize cost and volume is also studied in [73].

As stated earlier, the main advantage of radial SMB is its compactness and ability to provide higher lateral/radial stiffness in comparison to that of an axial SMB [58], [59]. However, the axial stiffness of radial type SMB is typically lower than that of an axial SMB [60]. Moreover, effective radial SMB also requires the use of arc-shaped permanent magnets and YBCO bulks, which increase the complexity and overall costs [74].

3) Conical SMB

An alternative geometry is a conically shaped SMB. In recent years, a group of researchers in New Zealand have investigated the possibility of using a conically shaped SMB as an alternative to the traditional planar and radial geometry [75]. A schematic diagram of the conical SMB design is shown in Fig. 10.

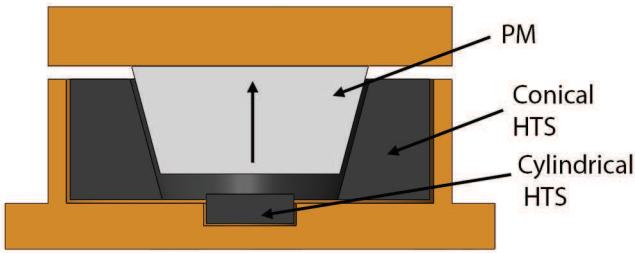


Fig. 10. A simple conical superconducting magnetic bearing, form similar to that presented in [75].

The design of the conically shaped SMB consists of a truncated cone (or conical frustum) shaped PM with two shaped YBCO high-temperature superconductors. Based on FEA and experimental results, the study concluded that when the permanent magnet rotor is under axial displacement, the assembly is capable of producing a higher and more consistent stiffness and levitation force over that of the conventional radial SMB design. Additionally, when under lateral displacement, the assembly was capable of producing double the amount of lateral force and four times the stiffness than that of a radial design.

The experimental data has shown promising results in suggesting a conically shaped SMB can be a viable option as an alternative geometry for designing SMB. However, using such a design could be difficult as the shape of the HTS is not commonly available and the research groups have suggested that further studies into its dynamics performance is needed.

C. Hybrid Magnetic Bearing

Another magnetic bearing configuration is a hybrid bearing design. The design is a combination of both PMB and SMB where the PMB provides the majority of the levitation force and the SMB provides the stabilizing force for the bearing under field-cooled condition. A hybrid bearing design can be visualized in Fig. 11 below.

Such a design has several advantages in comparison to standalone PMB and SMB designs. The stiffness of a hybrid bearing can be increased simply by increasing the amount of superconductor in between the permanent magnets. Moreover,

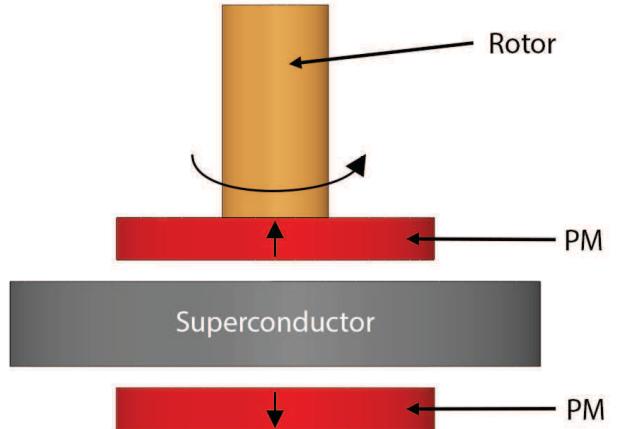


Fig. 11. A simple hybrid magnetic bearing design as shown in [1]. Majority of the levitation force is provided by the repulsive force between the permanent magnets and the stabilization is obtained from the superconducting magnet in between.

if the superconductor is above its critical temperature, the levitation force would still be present due to the force generated by the permanent magnets. As a result, only a stabilizing force is needed when not in operation [1]. It is also important to note that the permanent magnets inside a hybrid bearing can also be orientated in attraction to provide levitation force instead of repulsive levitation force.

Additional research into hybrid bearings has been conducted by Coombs and Campbell in [76]–[79]. In the paper, an FESS system was developed with an “Evershed” type hybrid bearing where the permanent magnets were positioned in a way that generates an attractive levitation force. Design studies that have investigated using alternating pole magnet configuration can also be found in [80], [81].

III. SYSTEM DESIGN

The design and analysis of PMBs and SMBs has been considered in independently. However they are of considerable interest for use together. PMBs provide passive levitation but cannot fully stabilize the system in all degrees of freedom. SMBs are commonly used to provide force in a direction, but also stabilization in one or more directions to provide fully passive system levitation and stabilization. These have been utilized in several flywheel energy storage systems [52], [53], [82]–[86].

One topology which is repeatedly seen in the literature uses a combination of radial PMBs or SMBs in conjunction with an axial lift SMB and is demonstrated in Fig. 12. One of the most prominent examples is the Boeing 5 kWh/ 3kW flywheel system design. The 164 kg composite flywheel rotor was fully suspended by an axial upper PMB and a lower axial SMB. The magnetizations are such that the upper stator acts in attraction with the SMB and repulsion for the lower planar SMB to generate lifting force [52], [53]. Similar flywheel design is also studied in [87], [88].

Another example where a radial SMB is utilized is the Japanese NEDO 10 kWh-Class FESS. The 425 kg FESS is supported by a radial SMB and two individual active magnetic

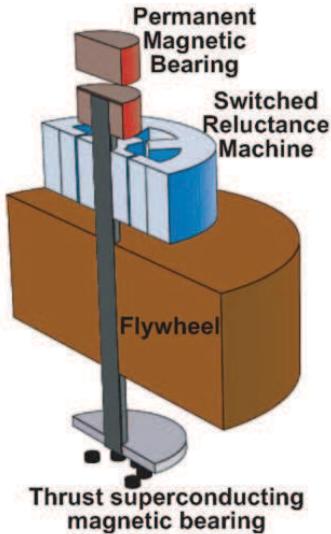


Fig. 12. A simple flywheel design supported with upper planar PMB and lower planar SMB [83].

bearings (AMB) and were able to achieve a rotational speed of approximately 15000 RPM [57], [62]. The radial SMB design is shown in Fig. 13 and 14.

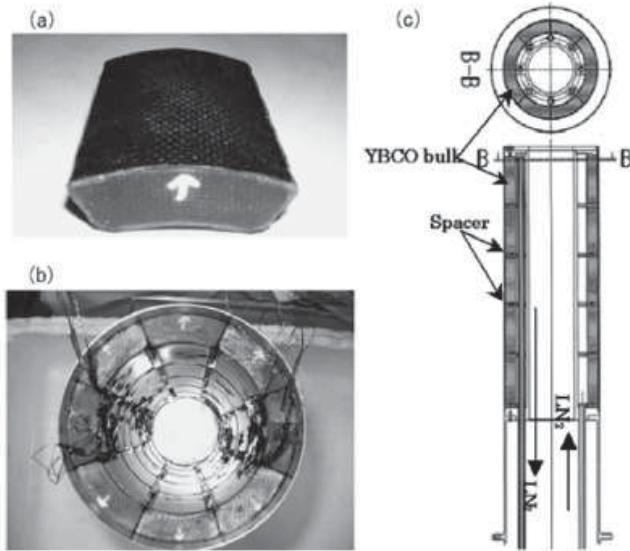


Fig. 13. a) A YBCO bulk superconductor tile used in the NEDO flywheel project. b) A fully constructed SMB stator with YBCO tiles. c) Section view of the SMB stator, with liquid nitrogen running in the center of the stator housing. [62].

IV. CONCLUSIONS

This paper presents a review of the methods for analyzing and designing magnetic bearing systems. It focuses primarily on passive PMB and SMB solutions, as well as the intersection of their design space with hybrid solutions. Types of analysis from analytical to FEA, design and optimization, as well as key conclusions are presented for each type. An overview of system design is presented which shows how systems are

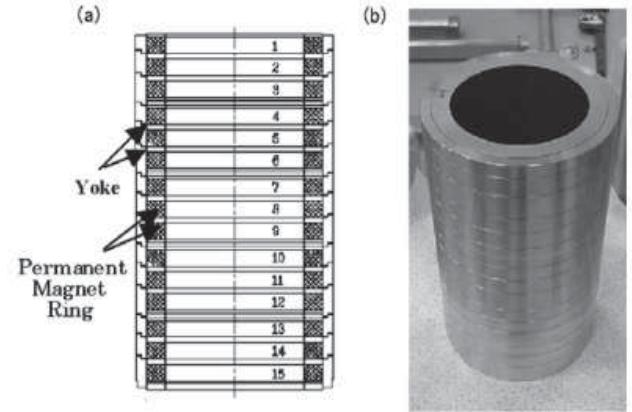


Fig. 14. a) Section view of the SMB outer rotor, stacked with layers of axially magnetized permanent magnets b) A fully assembled NEDO SMB outer rotor [62].

designed which utilize a combination of PMBs and SMBs to achieve full system, stable, passive levitation and bearings in 5 degrees of freedom.

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