Letter

Nonreciprocal coupling in space-time modulated systems at exceptional points

Junfei Li,[1] Yun Jing,[2] and Steven A. Cummer[1,*]
[1]*Department of Electrical and Computer Engineering, Duke University, Durham, North Carolina 27708, USA*
[2]*Graduate Program in Acoustics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*

(Received 31 May 2021; revised 20 January 2022; accepted 14 March 2022; published 28 March 2022)

Exceptional points (EPs) are critical points in the parameter space of non-Hermitian systems, where two or more eigenvalues and eigenvectors simultaneously coalesce. The remarkable physics and behavior of waves at these EPs have raised considerable attention. Previous research has accessed EPs in parity-time (PT) symmetric systems through spatially modulated parameters. Using acoustics, this Letter demonstrates a different family of EPs in classical wave systems that emerge from coordinated modulation of mass density and loss/gain in time. This condition can create nonreciprocal coupling between arbitrary modes at the EPs, leading to exotic behaviors such as unilateral frequency conversion and linear amplification of waves that are unattainable at conventional time-invariant systems. Moreover, these phenomena can be attained with only loss, and acoustic gain via modal energy transfer is demonstrated in a loss-only system at such EPs. Our work marries time-varying systems with EPs, which could open new avenues for wave manipulation.

DOI: 10.1103/PhysRevB.105.L100304

Exceptional points (EPs) are branch-point singularities in the parameter space of a system. First introduced in the realm of quantum mechanics [1], the rich physics of exceptional points has sparked a wide interest in both optics [2–8] and acoustics [9–17]. By harnessing the parity-time (PT) symmetric systems, i.e., a particular family of non-Hermitian Hamiltonians, the existence of EPs has been demonstrated in various non-Hermitian systems, allowing for unprecedented applications [18–22].

To date, the vast majority of the EPs have been realized through static non-Hermitian systems, where the material parameters including loss/gain follow symmetric and antisymmetric distribution in space and do not change over time. In those PT-symmetric systems, the coupling strengths between the two modes are almost always identical. The emergence of time as a new degree of freedom has recently opened new and intriguing avenues for wave control. The modulation of system parameters in time and, more generally, space-time has enabled flourishing new physics, including mode transitions [23–28], unidirectional parametric amplification [27,29,30], nonreciprocal devices [31–38], magnetic-free circulators [39,40], topological insulators [41,42], and multifunctional nonreciprocal metasurfaces [23]. However, whether EPs exist in time modulated systems and what new physics EPs can bring to the time modulated systems remain largely unexplored.

In this Letter, by marrying time modulation with the concept of EP, we demonstrate a different class of EPs in space-time modulated systems. We illustrate such EPs in a time modulated and, more generally, space-time modulated acoustic system, in which carefully coordinated modulation of the density and loss/gain in time gives rise to the singularity of the system's Hamiltonian. At such EPs, mode coupling becomes unilateral, leading to exotic wave behavior such as nonreciprocal mode coupling and linear amplification of the participating mode, as opposed to the exponential growth in traditional gain media. Moreover, when a loss offset is introduced, we show that acoustic gain can be achieved in a purely lossy system through modal energy transfer. The theoretical findings are verified with independent finite-difference time-domain (FDTD) simulations. This physics at the space-time EPs could also be applied to other wave systems such as photonics and elastodynamics.

Accessing EP in space-time modulated systems. Let us consider a general space-time modulated system shown in Fig. 1. The system can be readily reduced to time-only modulation by enforcing a uniform modulation in space. The density and loss/gain in a one-dimensional dispersive waveguide are modulated in the traveling-wave manner:

$$\begin{aligned} \rho &= \rho_0[1 + m\cos(\Omega t - \beta x)], \\ \eta &= \eta_0 + n\eta_1 \sin(\Omega t - \beta x), \end{aligned} \tag{1}$$

where ρ_0 is the static density of air, η_0 is the loss offset, and η_1 represents the modulation over the loss factor. m and n are unitless numbers, denoting the modulation depth of the density and loss factor, respectively. Ω and β represent the frequency and momentum of the traveling-wave-like modulation. In such modulated systems, modes that are orthogonal in static systems can be coupled when the modulation satisfies the frequency-matching and phase-matching conditions. The pressure field p and velocity field v satisfy Newton's second law and Hooke's law,

$$\begin{aligned} -\frac{\partial p}{\partial x} &= \rho(t)\frac{\partial v}{\partial t} + \eta(t)v, \\ -\frac{\partial p}{\partial t} &= \kappa\frac{\partial v}{\partial x}. \end{aligned} \tag{2}$$

*cummer@ee.duke.edu

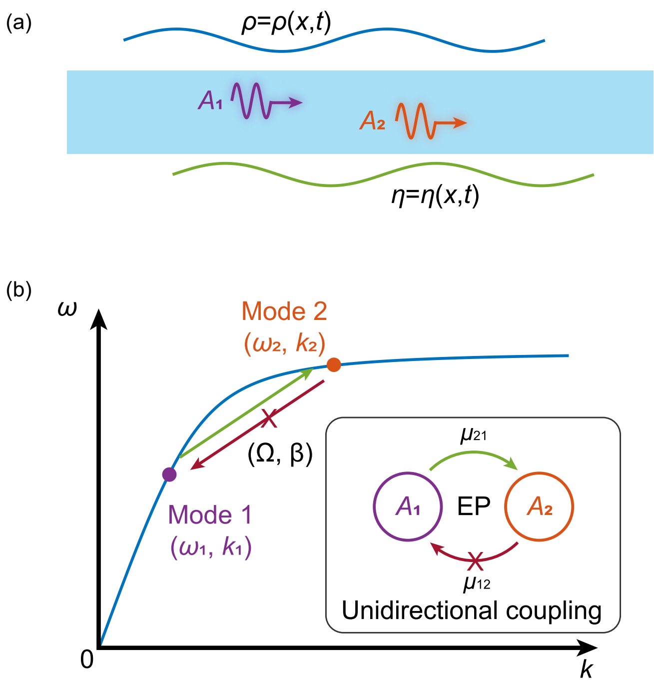

FIG. 1. Schematics of the space-time exceptional point (EP). (a) Two modes are coupled in a system where the density and loss/gain are modulated in space and time. (b) In the dispersive media, only the two target modes are participating in the coupling. At the EPs, the coupling between the two modes becomes nonreciprocal.

Considering only two modes involved in the coupling, the velocity field in the waveguide can be written as $v = A_1(x)e^{j(\omega_1 t-k_1x)} + A_2(x)e^{j(\omega_2 t-k_2x)}$, where $\omega_{1,2}$ and $k_{1,2}$ denote the angular frequency and wave number of the two modes. Defining $\psi(x) = [A_1(x), A_2(x)]^T$, the evolution of the two modes in such a space-time modulated waveguide can be derived (see Supplemental Material for full derivation [43]):

$$\partial_x\psi(x) = -jH\psi(x),$$
$$H = \begin{bmatrix} -j\gamma_1 & \mu_{12} \\ \mu_{21} & -j\gamma_2 \end{bmatrix} = \begin{bmatrix} -j\frac{\eta_0}{2\rho_0 c_1} & \frac{m\rho_0\omega_2 - n\eta_1}{4\rho_0 c_2} \\ \frac{m\rho_0\omega_1 + n\eta_1}{4\rho_0 c_1} & -j\frac{\eta_0}{2\rho_0 c_2} \end{bmatrix}. \quad (3)$$

Here, $c_{1,2} = \omega_{1,2}/k_{1,2}$ denotes the sound speed of the two coupled modes. By tuning the modulation depth of the density and loss, the coupling coefficient can be radically different. The eigenvalues of the Hamiltonian in Eq. (3) are

$$\lambda_\pm = -j\frac{\gamma_1+\gamma_2}{2} \pm \sqrt{\mu_{12}\mu_{21} - \left(\frac{\gamma_1-\gamma_2}{2}\right)^2}. \quad (4)$$

When $4\mu_{12}\mu_{21} = (\gamma_1-\gamma_2)^2$, the eigenvalues change from real to complex conjugate pairs and, at this transition point, the two eigenvalues coalesce to give an EP. Such an EP can be achieved by keeping the modulation depth of density fixed at $m = m_0$, while varying the modulation depth of loss n. For simplicity, let us first consider the case where the loss offset is 0, i.e., $\eta_0 = 0$. The eigenvalues can be simplified as $\lambda_\pm = \pm\sqrt{\mu_{12}\mu_{21}}$. We can immediately tell that the system reaches its EP when either μ_{12} or μ_{21} vanishes. Varying n renders the eigenvalues to transit between purely real and purely imaginary around two EPs $n_1 = m_0\rho_0\omega_2/\eta_1$ and $n_2 = -m_0\rho_0\omega_1/\eta_1$. Likewise, the EP can also be achieved by fixing n and varying m. The real and imaginary parts of the eigenvalues in the full m-n parameter space are shown in Fig. 2.

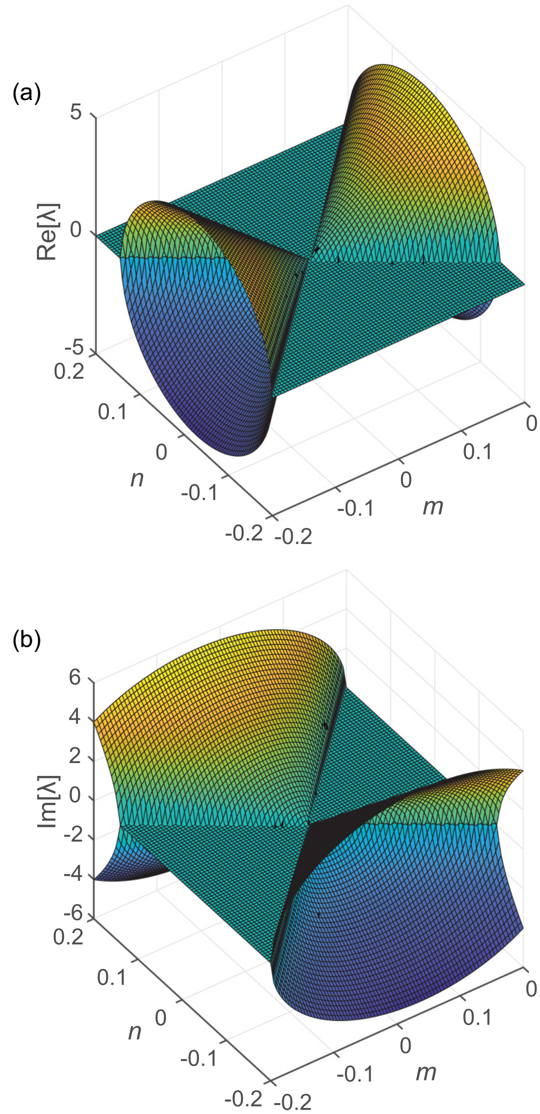

FIG. 2. Real and imaginary parts of the eigenvalues in the m-n parameter space. The two eigenvalues coalesce along the two lines of the EPs. The EPs can be achieved by fixing m and varying n, or by fixing n and varying m.

What happens at the space-time EP? When the parameters m, n are tuned to reach the EP, the two eigenvalues coalesce and the Hamiltonian cannot be diagonalized. As a result, the evolution operator cannot be calculated. In this case, the behavior of the two participating modes can be solved by examining Eq. (3). If we pick the EP at $n_1 = m_0\rho_0\omega_2/\eta_1$, the two modes satisfy

$$A_1(x) = A_1(0),$$
$$A_2(x) = -j\frac{m_0(\omega_1+\omega_2)}{4c_1}A_1(0)x + A_2(0). \quad (5)$$

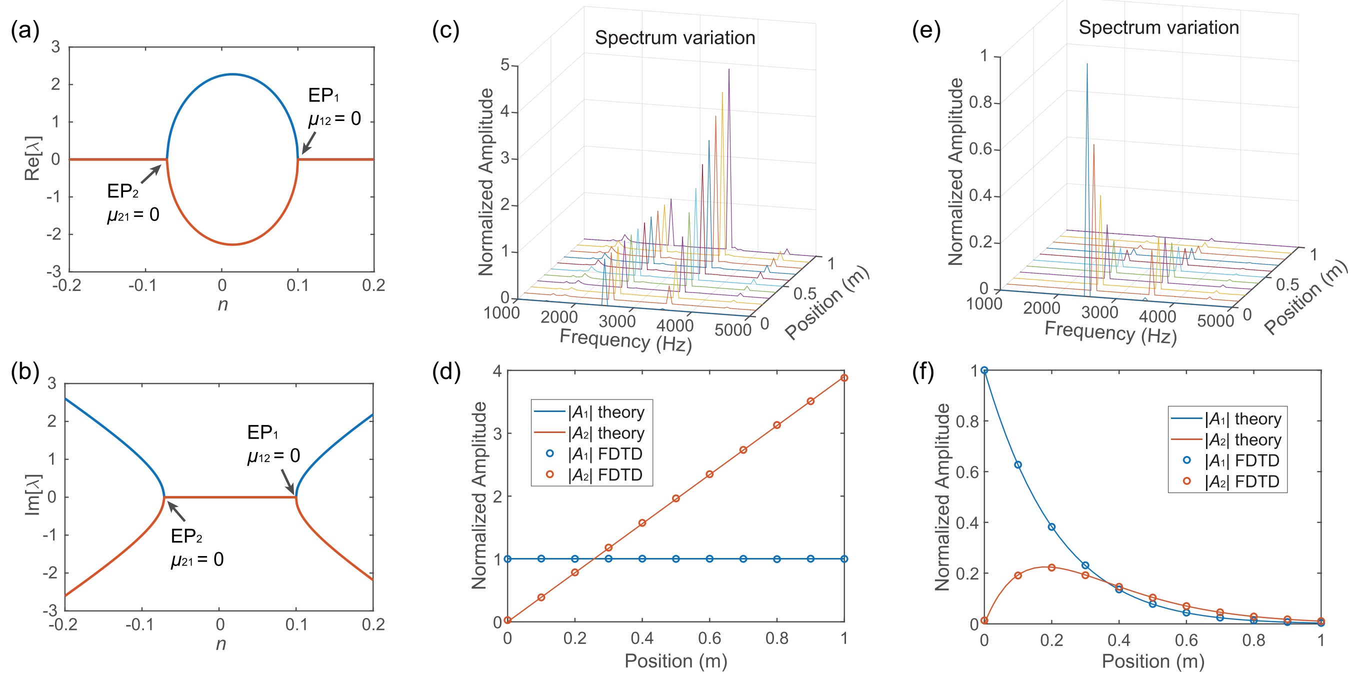

FIG. 3. Simulation of wave propagation at the exceptional point of the system. (a) Real and (b) imaginary part of the eigenvalues when fixing $m = 0.05$ and varying n. The two EPs correspond to $\mu_{12} = 0$ and $\mu_{21} = 0$, respectively. (c) Spectrum variation as the wave propagates in the system at the EP when $\eta_0 = 0$. (d) Good agreement can be found between theory and simulation. The nonreciprocal coupling at the EP gives rise to linear amplification. (e),(f) Spectrum and target mode variation at the EP when $\eta_0 = m\eta_1$. Within a short range, gain is achieved in a loss-only system. All the mode amplitudes are normalized by the input amplitude $|A_1(0)|$.

Although the two participating modes are coupled through space-time modulation at such an EP, the mode A_1 remains unchanged while propagating in the system independent of A_2. On the other hand, $|A_2|$ experiences a linear growth, with the growth rate depending on A_1. Such an exotic behavior happens because the mode coupling is nonreciprocal. Different from the conventional gain media that is typically exponential, the system provides a linear gain. Such a different property at EPs paves the way for designing robust linear wave amplifiers. It is noted here that these different and nonreciprocal behaviors are the results of time modulation, which cannot be achieved through conventional EP systems where parameters are modulated in space.

In most practical systems, it is easier to control loss instead of gain. In our system, we approach the EP in the loss-only system by imposing a nonzero loss offset η_0 so that η always remains positive during the modulation. In this case, nonzero γ_1 implies that the mode A_1 experiences an exponential decay while propagating in the system. For mode A_2, it will be generated and amplified in a short range. However, the growth rate decays with mode A_1 and is eventually dominated by loss. An interesting phenomenon in such a system is that within a short range, acoustic amplification can be achieved with loss only.

To verify the above findings, we numerically simulated the space-time modulated system using the finite-difference time-domain (FDTD) method implemented in MATLAB. We designed a dispersive waveguide so that only the two target modes are allowed by the space-time coupling. In the nondispersive case, more modes can be coupled through space-time modulation, and the corresponding results are summarized in the Supplemental Material [43]. In the FDTD simulation, the dispersion is introduced by imposing an ordinary differential equation to each grid point, while updating the velocity field. The system resembles a metamaterial composed of an air waveguide sideloaded with Helmholtz resonators with a resonance frequency of 3980 Hz. The density of air is 1.21 kg/m^3. The effective compressibility, and hence the sound speed, can be theoretically calculated [44]. The calculated dispersion relation of the one-dimensional waveguide follows Fig. 1(b). The modulated waveguide has a length of 1 m, connected to two nonmodulated regions with a length of 0.1 m on the input side and 10 m on the output side to eliminate reflection. Perfect matched layers are applied on both ends to reduce the unwanted reflection. To study the steady-state response of the system, the first 20 ms duration of the signals is discarded and Fourier transform is applied to analyze their spectra.

We pick the two arbitrary modes $(\omega_1, k_1) = (2\pi \times 2500\ \text{Hz}, 65.04\ \text{rad/m})$ for mode A_1, $(\omega_2, k_2) = (2\pi \times 3500\ \text{Hz}, 123.63\ \text{rad/m})$ for mode A_2. In the simulation, we approach the EP by fixing the density modulation depth $m = 0.1$ and varying the loss modulation depth n. For the case of $\eta_0 = 0$, the real part and imaginary part of the eigenvalue with varying n are plotted in Figs. 3(a) and 3(b). At two values $n_1 = 0.1$ and $n_2 = -0.0714$, the eigenvalues coalesce and the systems arrives at the EPs, corresponding to $\mu_{12} = 0$ and $\mu_{21} = 0$, respectively. We take the $\mu_{12} = 0$ case as an example and set $[A_1(0), A_2(0)] = [1, 0]$ as the input. The spectrum variation during the propagation is shown in Fig. 3(c). We can see that mode 1 does not change during propagation, while mode 2 is amplified linearly, as predicted by the theory. In the simulation, all the other modes are negligible thanks to the frequency dispersion. A comparison between the theoretical model and FDTD simulation is provided in Fig. 3(d), showing excellent agreement The results also validate the assumption that the envelopes of both waves are changing slowly.

Figures 3(c) and 3(d) showed nonreciprocal phonon transition at space-time EPs in a system containing no loss offset. However, acoustic gain in the media can be difficult to control in practice. Therefore, the study of space-time EP in loss-only systems becomes critical. In Figs. 3(e) and 3(f), we show the spectrum variation and the comparison between theory and simulation in a case where $\eta_0 = m\eta_1$ so the system contains only loss. In this case, we set $\eta_1 = \rho_0\omega_2 = 26\,609$ Pa s m^{-2} and $m = 0.1$. We can see that mode 1 decays exponentially, while mode 2 grows in a short range through modal energy transfer, until eventually the gain provided by mode 1 cannot support the growth of mode 2. Note here that in this case, mode 1 is only excited through one end of the media. When mode 1 is pumped across the whole modulated area, which is analogous to optical pumping in lasers, a constant growth of mode 2 can be observed. Nevertheless, it is interesting to show that at EP, the acoustic gain can be achieved even in a loss-only system.

In summary, we have unveiled the exceptional points in a time modulated system. We have shown that by modulating the density and loss factor in a coordinated manner, the system experiences a transition between the exact phase and broken phase. At the EP, the coupling between two modes at different frequencies becomes nonreciprocal, a property which is not found in conventional time-invariant PT symmetric systems. This nonreciprocal coupling gives rise to different phenomena such as nonreciprocal, linear amplification of the modes, paving the way for designing robust, linear, and controllable wave amplifiers. We have also shown that gain can be achieved in a loss-only system at the EPs. In many scenarios where space-time modulation is difficult to implement practically, a uniform modulation is preferred. Such requirement can be met by designing cavities supporting discrete energy levels or by picking modes with identical wave numbers. In the Supplemental Material [43], the case of a nonreciprocal interband phonon transition with only time modulation is demonstrated. In this case, time-reversal symmetry is broken and nonreciprocal wave behavior could be achieved without introducing any spatial bias.

The EPs in a time-varying system are not constrained to acoustics, but equally work for optics, and may open new routes for wave manipulation and communication with new functionalities. For practical realization, it is expected that the space-time EP can be verified with distributed transistors in active circuits, acoustic components with feedback [22], or acousto-optic devices [45]. One possible realization of a discrete resonator system is coupling two electroacoustic resonators through active circuits. The coupling strength and loss factor can be modulated by varactors and transistors, and, therefore, nonreciprocal mode transition can be experimentally observed. Elastic wave is also a promising platform since we can readily apply time modulation of stiffness through shunted piezoelectric patches with active circuits [34,38]. Due to the similarity between the Hamiltonian of a space-time modulated media and electron hopping described in the Jaynes-Cummings model, nonreciprocal coupling in the Hamiltonian is also expected to help control electron hopping between energy levels, which can potentially benefit quantum computing, quantum dot display, and lasers by suppressing spontaneous emission.

This work was supported by an Emerging Frontiers in Research and Innovation grant from the National Science Foundation (Grant No. 1641084) and a CMMI grant from the National Science Foundation (Grant No. 1951106). The authors would like to thank Nikhil JRK Gerard and Xiuyuan Peng for their useful discussion.

[1] T. Kato, Perturbation Theory for Linear Operators (Springer-Verlag, Berlin,1966).
[2] X. Yin and X. Zhang, Unidirectional light propagation at exceptional points, Nat. Mater. **12**, 175 (2013).
[3] Z.-P. Liu, J. Zhang, Ş. K. Özdemir, B. Peng, H. Jing, X.-Y. Lü, C.-W. Li, L. Yang, F. Nori, and Y.-x. Liu, Metrology with $\mathscr{PT}$-Symmetric Cavities: Enhanced Sensitivity Near the $\mathscr{PT}$-Phase Transition, Phys. Rev. Lett. **117**, 110802 (2016).
[4] J. Doppler, A. A. Mailybaev, J. Böhm, U. Kuhl, A. Girschik, F. Libisch, T. J. Milburn, P. Rabl, N. Moiseyev, and S. Rotter, Dynamically encircling an exceptional point for asymmetric mode switching, Nature (London) **537**, 76 (2016).
[5] L. Feng, R. El-Ganainy, and L. Ge, Non-Hermitian photonics based on parity–time symmetry, Nat. Photon. **11**, 752 (2017).
[6] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, Non-Hermitian physics and PT symmetry, Nat. Phys. **14**, 11 (2018).
[7] S. Longhi, Parity-time symmetry meets photonics: A new twist in non-Hermitian optics, Europhys. Lett. **120**, 64001 (2018).
[8] M.-A. Miri and A. Alu, Exceptional points in optics and photonics, Science **363**, 6422 (2019).
[9] X. Zhu, H. Ramezani, C. Shi, J. Zhu, and X. Zhang, PT-Symmetric Acoustics, Phys. Rev. X **4**, 031042 (2014).
[10] R. Fleury, D. Sounas, and A. Alù, An invisible acoustic sensor based on parity-time symmetry, Nat. Commun. **6**, 5905 (2015).
[11] K. Ding, G. Ma, M. Xiao, Z. Q. Zhang, and C. T. Chan, Emergence, Coalescence, and Topological Properties of Multiple Exceptional Points and their Experimental Realization, Phys. Rev. X **6**, 021007 (2016).
[12] C. Shi, M. Dubois, Y. Chen, L. Cheng, H. Ramezani, Y. Wang, and X. Zhang, Accessing the exceptional points of parity-time symmetric acoustics, Nature Commun. **7**, 11110 (2016).
[13] K. Ding, G. Ma, Z. Q. Zhang, and C. T. Chan, Experimental Demonstration of an Anisotropic Exceptional Point, Phys. Rev. Lett. **121**, 085702 (2018).
[14] W. Zhu, X. Fang, D. Li, Y. Sun, Y. Li, Y. Jing, and H. Chen, Simultaneous Observation of a Topological Edge State And Exceptional Point in an Open and Non-Hermitian Acoustic System, Phys. Rev. Lett. **121**, 124501 (2018).

[15] X. Wang, X. Fang, D. Mao, Y. Jing, and Y. Li, Extremely Asymmetrical Acoustic Metasurface Mirror at the Exceptional Point, Phys. Rev. Lett. **123**, 214302 (2019).
[16] H.-Z. Chen, T. Liu, H.-Y. Luan, R.-J. Liu, X.-Y. Wang, X.-F. Zhu, Y.-B. Li, Z.-M. Gu, S.-J. Liang, H. Gao *et al.*, Revealing the missing dimension at an exceptional point, Nat. Phys. **16**, 571 (2020).
[17] W. Tang, X. Jiang, K. Ding, Y.-X. Xiao, Z.-Q. Zhang, C. T. Chan, and G. Ma, Exceptional nexus with a hybrid topological invariant, Science **370**, 1077 (2020).
[18] W. Chen, Ş. K. Özdemir, G. Zhao, J. Wiersig, and L. Yang, Exceptional points enhance sensing in an optical microcavity, Nature (London) **548**, 192 (2017).
[19] J. Zhang, B. Peng, Ş. K. Özdemir, K. Pichler, D. O. Krimer, G. Zhao, F. Nori, Y.-x. Liu, S. Rotter, and L. Yang, A phonon laser operating at an exceptional point, Nat. Photon. **12**, 479 (2018).
[20] J. Wiersig, Prospects and fundamental limits in exceptional point-based sensing, Nat. Commun. **11**, 2454 (2020).
[21] A. Ghatak, M. Brandenbourger, J. van Wezel, and C. Coulais, Observation of non-Hermitian topology and its bulk-edge correspondence in an active mechanical metamaterial, Proc. Natl. Acad. Sci. USA **117**, 29561 (2020).
[22] L. Zhang, Y. Yang, Y. Ge, Y.-j. Guan, Q. Chen, Q. Yan, F. Chen, R. Xi, Y. Li, D. Jia *et al.*, Acoustic non-Hermitian skin effect from twisted winding topology, Nat. Commun. **12**, 6297 (2021).
[23] Y. Hadad, D. L. Sounas, and A. Alu, Space-time gradient metasurfaces, Phys. Rev. B **92**, 100304(R) (2015).
[24] G. Trainiti and M. Ruzzene, Non-reciprocal elastic wave propagation in spatiotemporal periodic structures, New J. Phys. **18**, 083047 (2016).
[25] P. A. Huidobro, E. Galiffi, S. Guenneau, R. V. Craster, and J. Pendry, Fresnel drag in space–time-modulated metamaterials, Proc. Natl. Acad. Sci. USA **116**, 24943 (2019).
[26] G. Trainiti, Y. Xia, J. Marconi, G. Cazzulani, A. Erturk, and M. Ruzzene, Time-Periodic Stiffness Modulation in Elastic Metamaterials for Selective Wave Filtering: Theory and Experiment, Phys. Rev. Lett. **122**, 124301 (2019).
[27] J. Li, C. Shen, X. Zhu, Y. Xie, and S. A. Cummer, Nonreciprocal sound propagation in space-time modulated media, Phys. Rev. B **99**, 144311 (2019).
[28] X. Xu, Q. Wu, H. Chen, H. Nassar, Y. Chen, A. Norris, M. R. Haberman, and G. Huang, Physical Observation of a Robust Acoustic Pumping in Waveguides with Dynamic Boundary, Phys. Rev. Lett. **125**, 253901 (2020).
[29] H. Lira, Z. Yu, S. Fan, and M. Lipson, Electrically Driven Nonreciprocity Induced by Interband Photonic Transition on a Silicon Chip, Phys. Rev. Lett. **109**, 033901 (2012).
[30] E. Galiffi, P. A. Huidobro, and J. B. Pendry, Broadband Nonreciprocal Amplification In Luminal Metamaterials, Phys. Rev. Lett. **123**, 206101 (2019).
[31] D. L. Sounas and A. Alù, Non-reciprocal photonics based on time modulation, Nat. Photon. **11**, 774 (2017).
[32] Y. Wang, B. Yousefzadeh, H. Chen, H. Nassar, G. Huang, and C. Daraio, Observation of Nonreciprocal Wave Propagation in a Dynamic Phononic Lattice, Phys. Rev. Lett. **121**, 194301 (2018).
[33] C. Caloz, A. Alu, S. Tretyakov, D. Sounas, K. Achouri, and Z.-L. Deck-Léger, Electromagnetic Nonreciprocity, Phys. Rev. Appl. **10**, 047001 (2018).
[34] Y. Chen, X. Li, H. Nassar, A. N. Norris, C. Daraio, and G. Huang, Nonreciprocal Wave Propagation in a Continuum-Based Metamaterial with Space-Time Modulated Resonators, Phys. Rev. Appl. **11**, 064052 (2019).
[35] J. Li, X. Zhu, C. Shen, X. Peng, and S. A. Cummer, Transfer matrix method for the analysis of space-time-modulated media and systems, Phys. Rev. B **100**, 144311 (2019).
[36] H. Nassar, B. Yousefzadeh, R. Fleury, M. Ruzzene, A. Alù, C. Daraio, A. N. Norris, G. Huang, and M. R. Haberman, Nonreciprocity in acoustic and elastic materials, Nat. Rev. Mater. **5**, 667 (2020).
[37] Q. Wu, H. Chen, H. Nassar, and G. Huang, Non-reciprocal Rayleigh wave propagation in space-time modulated surface, J. Mech. Phys. Solids **146**, 104196 (2021).
[38] Y. Xia, E. Riva, M. I. N. Rosa, G. Cazzulani, A. Erturk, F. Braghin, and M. Ruzzene, Experimental Observation of Temporal Pumping in Electromechanical Waveguides, Phys. Rev. Lett. **126**, 095501 (2021).
[39] D. L. Sounas, C. Caloz, and A. Alù, Giant non-reciprocity at the subwavelength scale using angular momentum-biased metamaterials, Nat. Commun. **4**, 2407 (2013).
[40] D.-W. Wang, H.-T. Zhou, M.-J. Guo, J.-X. Zhang, J. Evers, and S.-Y. Zhu, Optical Diode Made from a Moving Photonic Crystal, Phys. Rev. Lett. **110**, 093901 (2013).
[41] R. Fleury, A. B. Khanikaev, and A. Alù, Floquet topological insulators for sound, Nat. Commun. **7**, 11744 (2016).
[42] M. Oudich, Y. Deng, M. Tao, and Y. Jing, Space-time phononic crystals with anomalous topological edge states, Phys. Rev. Research **1**, 033069 (2019).
[43] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.105.L100304 for the detailed derivation of the system Hamiltonian, implementation of dispersive waveguide in FDTD, interband unidirectional phonon transition, and robustness analysis of space-time EP.
[44] N. Fang, D. Xi, J. Xu, M. Ambati, W. Srituravanich, C. Sun, and X. Zhang, Ultrasonic metamaterials with negative modulus, Nat. Mater. **5**, 452 (2006).
[45] E. A. Kittlaus, W. M. Jones, P. T. Rakich, N. T. Otterstrom, R. E. Muller, and M. Rais-Zadeh, Electrically driven acousto-optics and broadband non-reciprocity in silicon photonics, Nat. Photon. **15**, 43 (2021).