

# Experimental Measurements of the Wing Deformation and Force Production of a Real Monarch Butterfly Wing

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The annual migration of monarch butterflies can span over 4000 kilometers, the longest among insects. Little is known about how the monarchs are capable of this extended flight. This study is motivated by the hypothesis that the monarchs' flapping wing flight is enhanced by fluid-structure interactions of their relatively large wings. However, the effects of the wing flexibility on the aerodynamic force generation of the large, slowly flapping butterfly wings are inadequately understood. The main objective of this study is to examine the aeroelastic response of a real butterfly wing at a flapping amplitude around the free flight flapping amplitude of monarchs as a function of varying flapping frequencies. In this paper, the performance of a real monarch butterfly wing was tested by measuring the wing motion and lift at a flapping amplitude of 55 deg and a comparison was made to a 20 deg amplitude motion. Both cases produced a peak lift at 10.3 Hz, approximately the flapping frequency of a real monarch butterfly. For the 55 deg case, the maximum force produced was 8.4 mN, over double that of the 20 deg case, 3.8 mN. This is sufficient to overcome a butterfly's weight of 5 mN. For both cases, the pitch amplitude increased linearly as the frequency increased with the pitch amplitude at the peak lift being 15.4 deg and 7.5 deg for the 55 deg case and 20 deg case, respectively.

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## I. Introduction

Despite visually appearing structurally the same, the monarch butterfly is unique amongst all the species of the Nymphalidae family in its ability to travel vast distances, up to 4000 kilometers, and is easily the one of the most recognizable butterfly species in North America [1]. During its migration, it is presumed that monarchs use the low atmospheric density at high altitudes to their advantage to aid in soaring flight and reduce aerodynamic drag [2]. However, the detailed aerodynamic phenomena driving the three-month journey is yet unknown.

The wings of the monarch are flexible structures, allowing them to deform significantly during flight. This is significant as flexible flapping wings are capable of generating large forces with considerably less power consumption when compared to their rigid counterparts [3–19]. Monarch wings exhibit an anisotropic nature in that their spanwise flexibility differs from that of their chordwise flexibility [5,20,21] and likewise, the dorsal flexibility from the ventral [22–24]. This anisotropic nature is in large part due to the membrane and vein structure in the wing and one-way hinges that use resilin [25].

Annually, the monarch butterfly ventures thousands of kilometers to a never before visited winter camp, relying on food stops along the way, in the mountains of central Mexico [26–29]. There, they wait out their diapause before returning north for the summer [26,27,30,31]. Their migration is one of the more extraordinary natural phenomena amongst the butterfly species that exist in North America. Despite the hundreds of thousands that make the journey to Central Mexico, they all end their journey in the same location, which was not discovered until 1975 [32]. The overwintering camp in itself is also impressive as it requires ample food and water sources to feed hundreds of thousands of monarchs. It also must have shaded and sunny regions for thermoregulation, protection from wind gust, and the correct types of trees [33,34]. As such a habitat is complicated and rare, it is important to preserve them and to study and gather as much information of the monarch's unique species characteristics, navigational skills, and flight capabilities.

For years, the flight dynamics of insects has been of interest. In particular, they have been a source of inspiration and information for the development of micro flying robots, the study of low-Reynolds number vortex dynamics [7,9,13,17,18,20,35], and quantifying the efficiency of flexible wings [4,5,8–11,15,36]. Insects are capable of incredible maneuvering and lift generation as small, lightweight, streamlined organisms and having wing structures with high surface area. They are capable of stable flight aided by the coupling of their body undulation and wing flapping [13,15,20], which contributes to their energy preservation and thus the overall efficiency during their migrations [37]. It is implied that the material properties of the wing may impact flight efficiency as the fluid surrounding a deforming flapping wing is altered.

Research on the material properties of insect wings, in addition to their aerodynamics, can provide insight into their performance capabilities. The characteristics of a material dictate how a structure will interact with its surroundings or react to forces and are therefore, important to study. Material properties such as flexural stiffness and density for insect wings have previously been researched [22–24,38–40]. Combes and Daniel [22], Stepan [24], and Tanaka and Wood [23] found that the flexibility in insect wings when measured along the chordwise versus the spanwise direction differed as well as if the wing was dry versus wet [22,24]. Wainwright [39], Jensen and Weis-Fogh [40], and Song et al. [38] measured the densities of different insect wings. However, the monarch butterfly was not included amongst the species research. As monarchs are the only species in North America to migrate great distances, their flight capabilities are of particular interest.

Aerodynamic and viscoelastic damping can also dictate a structures response [41,42]. However, modeling and analyzing these effects can be difficult as this constitutes a closely coupled dynamical system [42]. The study of the wing motion and force generation of a monarch will aid in future model developments.

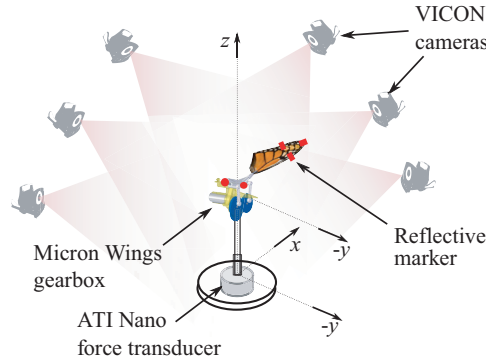
The outline of this paper is as follows. First, we present the methodology, developed in our previous work [43], to determine the wing angles and force generation of a monarch wing flapping in Section III.A. Next, we present the improvements made to the setup to determine the wing angles and force generation of a real monarch wing flapping at an amplitude neared to that of the monarch's free flight amplitude in Section III.B. We then discuss the resulting wing angles (Section IV.A) and force generation (Section IV.B) of a monarch wing flapping at an amplitude of 55 deg and provide a comparison to our previous work done at 20 deg.

## II. Methodology

### A. Deformation and Force Measurements a Real Butterfly Wing at a Flapping Amplitude of 20 deg [43]

The deformation and force measurement experimental setup for a real monarch butterfly wing consisted of an ATI Nano 17 Titanium Force Transducer and a set of VICON T40s motion capture cameras (Fig. 1). We mounted a Micron Wings 6 mm Ornithopter gearbox to the ATI Nano force transducer (Fig. 1) to generate the flapping motion with an amplitude of 20 deg.

The test stand was surrounded by six VICON T40s cameras. We placed three small reflective markers on the wing and two reference markers on the longitudinal axis of the flapper (Fig. 1). The three-dimensional positions of the markers were acquired at a sampling rate of 200 Hz using the VICON system for a duration of 5 s. Each reflective marker was  $6 \times 2.34$  mm in size. The total mass of the three markers was around 1.9% of the total mass of a monarch butterfly, and 52% of the mass of the individual wing. The force generated by the butterfly wing motion was recorded by the force transducer at a sampling rate of 200 Hz for 5 s. The VICON camera recording and the force transducer recording were simultaneously triggered such that the time histories of wing motion and forces were correlated. We previously used the same optical measuring technique to measure the three-dimensional wing kinematics and the body motion of freely flying monarch butterflies [44].

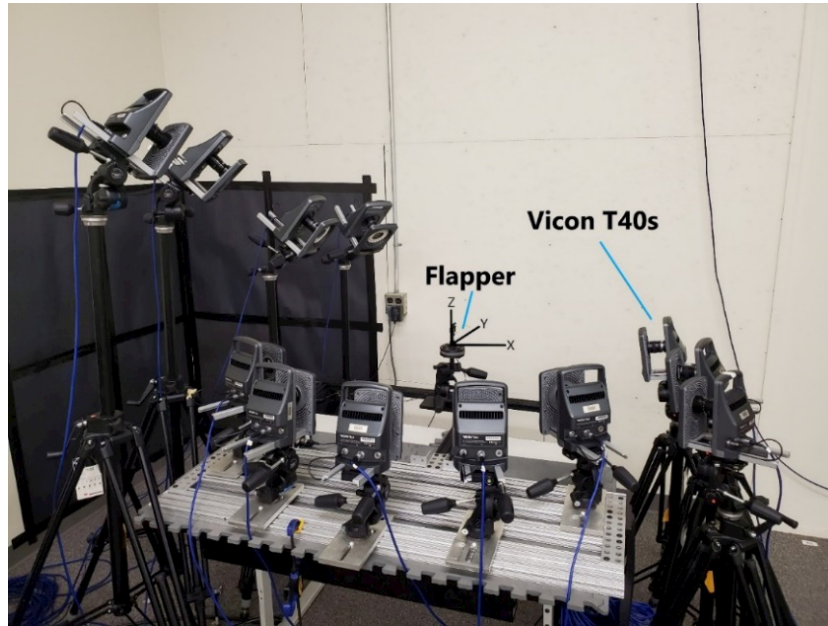


**Fig. 1. VICON system setup to record the deflections of a wing attached to a gearbox. The wing was attached to a force transducer which simultaneously recorded the forces produced during the flapping. From Twigg et al. [43] .**

The flapping, feathering and deviation angles of a real monarch wing were determined using the position of the three markers on the left wing and a reference marker on the gearbox. The flapping frequency was obtained by taking the Fast Fourier transform (FFT) of the time history of the flapping angle. The wing motion data were smoothed using a low pass filter with a cut off frequency of 30 Hz, which is nearly three times the monarch's flapping frequency. The recorded force data, which included higher frequency oscillations, were also filtered using a low pass filter with a cut off frequency of 30 Hz.

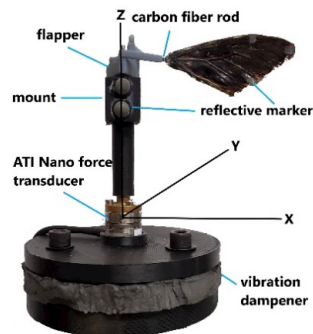
### B. Deformation and Force Measurements a Real Butterfly Wing at a Flapping Amplitude of 55 deg

Similar to the methods described in Section II.A., the experimental setup used to perform the deformation and force measurements for the high amplitude test consisted of a set of VICON T40s motion capture cameras (Fig. 2) and an ATI Nano 17 Titanium Force Transducer. To facilitate the larger flapping amplitudes and in-house flapper was designed and mounted on top of the ATI Nano Force Transducer. Furthermore, additional cameras were strategically added to the setup to account for both the larger flapping amplitude and pitching amplitude of the wing.



**Fig. 2. Improved experimental setup for recording the forces and wing motion at higher flapping amplitude. Wing motion was recorded using optical motion tracking with 12 VICON T40s cameras. A close-up view of the flapper assembly is shown in Fig. 3. The experimental environment was adjusted to reduce extraneous camera noise which enabled recording the wing motion at a higher sampling rate.**

The recording of the wing motion using VICON cameras and the forces using the force transducer were simultaneously triggered so that the resulting time histories can be correlated. The resulting three retro-reflective markers were placed on the wing and two additional reference markers were placed on the longitudinal axis of the flapper (Fig. 3). The three-dimensional positions of the markers were acquired at a sampling rate of 400 Hz using the VICON system for a duration of 2 s. The corresponding force generated by the butterfly wing motion was recorded by the ATI nano force transducer at a sampling rate of 1000 Hz for the same duration. The optical measurement technique used here was also used in our previous study to measure the three-dimensional wing kinematics and the body motion of freely flying monarch butterflies [44].



**Fig. 3. The assembly of the in-house mechanical flapper and the attached monarch wing. The flapper assembly is mounted on top the force transducer via an in-house 3D printed mount.**

The flapping, feathering, and deviation angles of a real monarch wing were calculated using the position of the three markers on the right wing and the two reference markers on the gearbox (Fig. 4). The flapping frequency was obtained by taking the FFT of the time history of the flapping angle. The wing motion data were smoothed using a low pass filter with a cut off frequency of 30 Hz. The recorded force data, which included higher frequency oscillations, were also filtered using a low pass filter with a cut off frequency of 30 Hz.



**Fig. 4. The assembly of the in-house mechanical flapper and the attached monarch wing. The reflective markers used for optical measurements are labeled. The flapper and the mount are painted black to reduce camera noise.**

To mimic the high flapping amplitudes seen in a monarch butterfly's flight, an in-house mechanical flapper was designed (Fig. 5). The mechanism consists of a two-stage gear reduction and a 4-bar linkage to convert rotary motion to flapping motion. The flapper was designed to output a peak-to-peak amplitude of 110 deg.



**Fig. 5. In-house mechanical flapper designed to mimic the flapping amplitude of monarch butterfly in free flight.**

### III. Results and Discussion

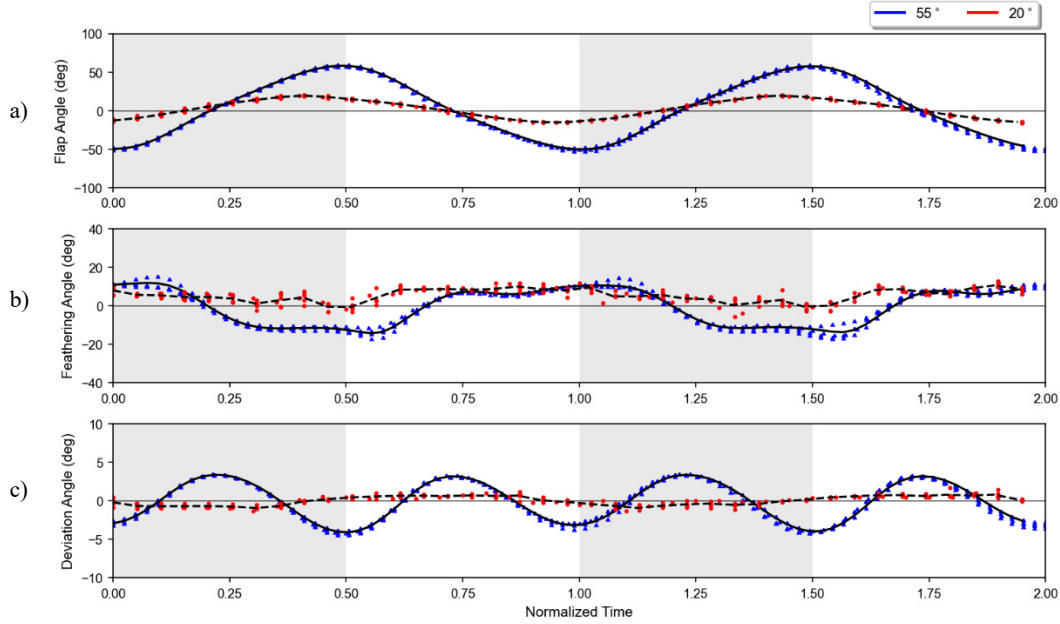
This section summarizes the results on the 55 deg flapping wing actuator and provides a comparison of the resulting wing motion and forces to the results of our previous work with a 20 deg flapping wing actuator [43]. Table 1 displays the morphological data for a real monarch wing.

**Table 1: Morphological data of the right forewing of a real monarch butterfly used in the measurements at flapping amplitude of 55 deg.**

| Wing mass<br>(g) | Span<br>( $\times 10^{-3}$ m) | Chord<br>( $\times 10^{-3}$ m) | Area<br>( $\times 10^{-4}$ m <sup>2</sup> ) |
|------------------|-------------------------------|--------------------------------|---------------------------------------------|
| 0.0194           | 53                            | 27                             | 9.42                                        |

#### A. Wing Angles

Figure 6 shows the wing angles for a single real butterfly wing tested at flapping amplitude of 55 deg, overlaid with the results of the real butterfly wing at 20 deg flapping amplitude. The average flapping frequency of each wing was approximately 10 Hz. Both wings were tested in a similar manner. Measurements for both wings were repeated six times for repeatability which are indicated by symbols in Fig. 6. The average time history is shown with a dotted line in Fig. 6. The minimal spread in the flapping angle between the repeated measurements at a flapping amplitude of 55 deg indicates the consistency in the performance of the new gearbox and wing. The feathering angle at higher flapping amplitude is sinusoidal with clearer and more distinct peaks compared to the low amplitude. Fig. 6 also shows that the deviation angle remains essentially zero, indicating the wing was not moving in an unanticipated direction.



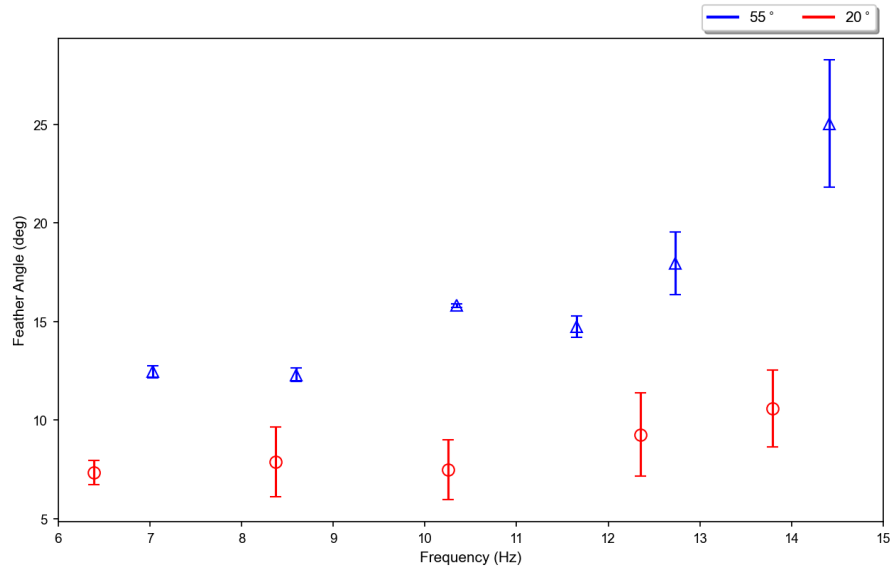
**Fig. 6. Comparison of wing kinematics at flapping amplitude of 20 deg (red) and 55 deg (blue) for a representative case. Time history of (a) flapping angle, (b) feathering angle, and (c) deviation angle of a real monarch butterfly wing mounted on the gearbox. The wing motion at 20 deg flapping amplitude corresponds to an input voltage of 1.1 V and motion at 55 deg corresponds to an input voltage of 0.7 V. The FFT analysis of the time history of flapping angle indicates that the flapping frequency is 10.3 Hz for both cases. The grey bars indicate the downstroke, and the white bars indicate the upstroke.**

Table 2 shows the average flapping frequency for a given input voltage of the test performed at both the 20 deg and 55 deg cases. It should be noted that despite the higher flapping amplitude, the 55 deg case required smaller input voltages to operate at the same flapping frequency. This is due to the 55 deg tests being ran with a more efficient motor.

**Table 2: Input voltage and corresponding flapping frequency measured for 20 deg and 55 deg flapping amplitudes. More efficient motor used at 55 deg amplitude required lower voltage to actuate the wing at a given flapping frequency.**

| Input Voltage (V) | Frequency (Hz) |        |
|-------------------|----------------|--------|
|                   | 20 deg         | 55 deg |
| 0.5               | -              | 7.0    |
| 0.6               | -              | 8.6    |
| 0.7               | 6.4            | 10.3   |
| 0.8               | -              | 11.7   |
| 0.9               | 8.4            | 12.7   |
| 1                 | -              | 14.4   |
| 1.1               | 10.3           | -      |
| 1.2               | -              | -      |
| 1.3               | 12.3           | -      |
| 1.4               | -              | -      |
| 1.5               | 13.8           | -      |

Figure 7 shows the average peak feathering amplitude at different flapping frequencies for both the 20 deg and 55 deg cases. In both cases, the feathering angle increases almost linearly with flapping frequency. This suggests that a faster wing motion results in a larger passive deformation. The peak lift occurs at 10.3 Hz for both the 20 and 55 deg cases. At this frequency, the average feathering angle for the 55 deg case is 15.4 deg, just over double that of the average feathering angle for the 20 deg case, 7.5 deg.



**Fig. 7. The average maximum feathering amplitude for 20 deg (red) and 55 deg (blue) flapping amplitudes per flapping frequency (n=6).**

The phase offsets for both the 20 deg and 55 deg cases are given in Table 3. Depending on the flapping frequency, the measured feathering angle can be non-sinusoidal, despite a sinusoidal flapping angle input. In cases where the flapping frequency was less than 10 Hz, the feathering angle displayed up to three distinct peaks. Two distinct peaks were present in cases between 10 Hz and 13 Hz, with the feathering output being nearly sinusoidal with only one distinct peak at cases above 14 Hz. The second peak was the prominent peak for all cases with more than one peak. The prominent peak is the peak with the greatest amplitude. The phase offset for the prominent peaks fell between 149 deg and 206 deg, with the notable exception for cases above 14 Hz, which had a phase offset of approximately 117°. This, in combination with the phase offset for the 20 deg cases falling between 156 deg and 188.5 deg, suggest that the timing of the max wing rotation reaches the stroke-end as the frequency increases and further delays when the wing flaps faster than at the Monarch flapping frequency.

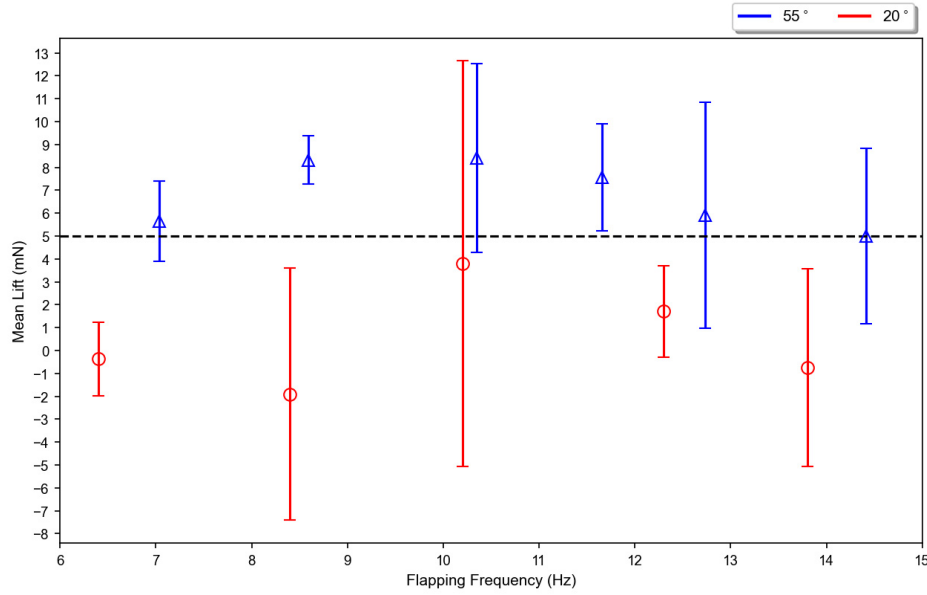
**Table 3: Average phase offset and corresponding average amplitude of each peak for a given input voltage and corresponding average frequency. The highlighted blocks represent the prominent peak and corresponding phase offset.**

| Flapping Amplitude (deg) | Input Voltage, (V) | Frequency, (Hz) | Phase Offset    |                 |                 | Feather Amplitude |                 |                 |
|--------------------------|--------------------|-----------------|-----------------|-----------------|-----------------|-------------------|-----------------|-----------------|
|                          |                    |                 | 1st peak, (deg) | 2nd peak, (deg) | 3rd peak, (deg) | 1st peak, (deg)   | 2nd peak, (deg) | 3rd peak, (deg) |
| 55                       | 0.5                | 7.1             | 101.0           | 149.0           | 212.6           | 3.5               | 12.2            | 11.4            |
|                          | 0.6                | 8.6             | 103.0           | 149.0           | 223.0           | 8.0               | 11.7            | 8.2             |
|                          | 0.7                | 10.3            | 115.6           | 174.6           | -               | 9.8               | 13.4            | -               |
|                          | 0.8                | 11.7            | 116.6           | 205.5           | -               | 10.2              | 12.9            | -               |
|                          | 0.9                | 12.7            | 124.8           | 199.1           | -               | 12.9              | 14.7            | -               |
|                          | 1                  | 14.4            | 117.1           | -               | -               | 19.4              | -               | -               |
| 20                       | 0.7                | 6.4             | -               | 161.2           | -               | -                 | 7.3             | -               |
|                          | 0.9                | 8.4             | -               | 176.3           | -               | -                 | 7.9             | -               |
|                          | 1.1                | 10.3            | -               | 156.6           | -               | -                 | 7.5             | -               |
|                          | 1.3                | 12.3            | -               | 175.6           | -               | -                 | 9.3             | -               |
|                          | 1.5                | 13.8            | -               | 188.5           | -               | -                 | 10.6            | -               |

## B. Force Generation

Figure 8 shows the mean lift measured from real butterfly wings at a flapping amplitude of 55 deg and at 20 deg [43]. Both curves show that the flapping frequency that the maximum force is generated is around 10 Hz, the flapping frequency of monarchs. At this frequency, the resulting mean lift for the 55 deg case is 8.4 mN, over double that of the mean lift generated by the 20 deg case, 3.8 mN. For the 55 deg case, this force, which is generated by one wing, is larger than the butterfly weight of 5 mN. For both cases, assuming the force generation of two wings is nearly double, this would provide the Monarch an excess amount of force that can be used to maneuver or overcome obstacles such as wing gust.

Inspection of the lift shown in Fig. 8 and the feathering amplitude in Table 3 suggests that there is no linear relationship between the two. The feathering angle, which is due to the flexibility of the wing, increases with the flapping frequency. However, the lift initially increases with the flapping frequency, but, then, reduces after reaching its peak at around 10 Hz. It is interesting to note that the peak lift occurs when the phase offset is around 180 deg, meaning that the feathering motion is half a stroke behind the flapping motion. Further modeling and analysis of the effects of wing flexibility is required to adequately understand this phenomenon.



**Fig. 8. Mean lift measured from real butterfly wings flapping at a flapping amplitude of 55 deg (blue) and 20 deg (red). Parabolic trendlines are shown as solid lines with the dashed line representing the Monarchs weight.**

#### IV. Summary

The purpose of this study was to determine aeroelastic properties of a monarch butterfly wing near monarch free flight flapping amplitudes. The knowledge gained from these values, feathering angle and phase offset, will allow future butterfly models to be more accurate, furthering future micro-air vehicle development. This study characterized real monarch butterfly wings by measuring the wing angles and force generation. This allowed us to better understand motion of a monarch wing for future creating of a bio-inspired MAV.

The wing angles of a monarch wing flapping at an amplitude of 55 deg, approximately the flapping amplitude of a monarch in free flight, 60 deg, were measured. The feathering amplitude was found to linearly increase with the increase of flapping frequency. This correlated well with the previous studies done at 20 deg and further shows that the faster the wing flaps, the more it passively deforms. The feathering amplitude at the peak lift frequency, 10.3 Hz, was found to be 15.4 deg, just over double that of the pitching amplitude of the 20 deg case, 7.4 deg, at the same frequency. The feathering angle time history was shown to be clearer and more distinct than the 20 deg case, showing that despite the sinusoidal input of the flapping angle, the feathering angle wasn't necessarily sinusoidal, suggesting that the feathering angle is an outcome of complex fluid-structure interaction dynamics. The phase offset of the prominent peak, however, was approximately 180 deg, similar to that of the 20 deg case. This further indicates that the wing reaches its max pitching amplitude near the mid-stroke.

We determined the forces generated by a real monarch wing at 55 deg flapping amplitudes and provided a comparison to previous test done at 20 deg. The wing produced a peak lift at 10.3 Hz, which corresponds to the flapping frequency of the monarch's standard flight. This correlated well with similar test done at 20 deg flapping amplitudes. The peak lift measured 8.4 mN, which is over double the peak lift generated at 20 deg flapping amplitudes. The lift produced, 5.6 mN, at the lowest tested frequency, 7.0 Hz, was more than sufficient to overcome a monarch's average weight, 5 mN. This indicated that a monarch is capable of sustaining flight even outside of its normal flapping frequency of 10 Hz. Furthermore, this lift was produced by one wing, therefore the lift generation of two wing would give the monarch an excess of lift to overcome gust or provide thrust for agile maneuvers.

#### Acknowledgments

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