Cordon-based Pricing Schemes for Mixed Networks: a Macroscopic Fundamental Diagram Approach

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ABSTRACT
The development of traffic models based on macroscopic fundamental diagram (MFD) enables many real-time control strategies for urban networks, including cordon-based pricing schemes. However, most existing MFD-based pricing strategies are designed only to optimize the traffic-related performance, without considering the revenue collected by operators. In this study, we investigate cordon-based pricing schemes for mixed networks with urban networks and freeways. In this system, heterogeneous commuters choose their routes based on the user equilibrium principle. There are two types of operational objectives for operating urban networks: (1) to optimize the urban network's performance, i.e., to maximize the outflux; and (2) to maximize the revenue for operators. To compare those two objectives, we first apply feedback control to design pricing schemes to optimize the urban network's performance. Then, we formulate an optimal control problem to obtain the revenue-maximization pricing scheme. With numerical examples, we illustrate the difference between those pricing schemes.

Keywords: Congestion pricing, feedback control, macroscopic fundamental diagram, optimal control, revenue

INTRODUCTION

Congestion pricing has been studied extensively both theoretically and practically to reduce traffic congestion in urban networks. In practice, single cordon-based pricing schemes have been designed and implemented in many cities, such as Stockholm and Milan. A hybrid scheme combining facility-based and cordon-based pricing has been implemented in Singapore, whereas a zone-based scheme has been applied in London. A comprehensive overview of nine congestion pricing practices is provided in (*1*). By imposing a fee to enter specific areas, congestion pricing can change commuters' behavior, including mode choice, route choice and departure time choice. A comprehensive review of congestion pricing methodologies and technologies is provided in (*2, 3*). Pigou (*4*) and Knight (*5*) were the first to advocate congestion pricing by arguing that an optimal charge should be implemented for the congested road to internalize the externality of vehicles and drive the system to an optimal state. The first-best pricing is proposed to reach a system optimal flow by setting the toll rate as the difference between marginal social cost and marginal private cost. Some assumptions to realize the first-best pricing include: (1) individual drivers choose the route based on utility maximization principle; (2) congestion pricing is applied to all relevant road segments in the network; (3) full information on all costs is available for both the operators and the drivers; (4) congestion pricing is technically feasible and the transaction costs are reasonably low (*6*). Therefore, the applications of first-best pricing models have been impractical despite the idealized theoretical basis. As a result, second-best pricing schemes have been proposed, in which tolls are only active in a subset of links. A mathematical program with equilibrium constraints (MPEC) has been formulated to derive the second-best pricing in (*7–12*). However, the solution to the MPEC for a large-scale dynamic network is computationally complex (*13*). In addition, one common assumption in those studies is that the traffic is in steady state, which may not capture traffic dynamics accurately.

To develop an effective congestion pricing scheme for urban networks, it is essential to understand the traffic dynamics at a network level. A macroscopic fundamental diagram (MFD) that relates the average speed and the average per-lane vehicle density in a road network was first proposed and calibrated for Ipswich in (*14*). Recently, the existence of MFD has been revealed in many cities (*15, 16*). The development of MFD models has enabled many real-time control strategies to improve network performance. For example, a model predictive control-based perimeter control is proposed in (*17–19*), and a dynamic routing strategy is developed in (*20, 21*).

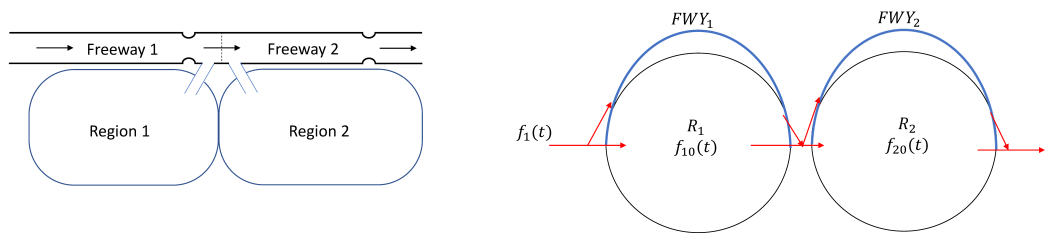
Recently, MFD-based pricing schemes are developed to reduce congestion. Geroliminis and Levinson (*22*) combined Vickrey's bottleneck model (*23*) with a MFD to derive an optimal fine toll when commuters are homogeneous. Results show that the proposed toll shortens the duration of the peak period, and is Pareto-efficient for every commuter. Zheng et al. (*24*) applied an integral controller to adjust the flat cordon-based tolls with an agent-based simulation of Zurich urban road network. Simoni et al. (*25*) derived two alternative cordon-based tolls, i.e., a step toll and a hybrid toll, by using the marginal cost pricing and a MFD, and applied to a case study of Zurich. Some other types of MFD-based congestion pricing schemes are proposed when considering the distribution of trip lengths, such as a usage-based toll (*26*), a distance-based toll (*27*), a joint distance and time toll and a joint distance and delay toll (JDDT) (*13*). Besides, Gu et al. (*28*) formulated two new high-dimensional toll level problems (TLPs) in a large-scale heterogeneously congested traffic network by integrating a linear JDDT with the MFD. Those two problems were solved by using surrogate-based optimization. Later, four state-of-the-art simulation-based optimization methods were applied and compared to solve the TLPs (*29*). Furthermore, recent studies extend the single-mode MFD to a bi-modal with cars and buses. Zheng et al. (*30*) proposed a proportional-integral controller to determine the area-based pricing for an urban network with cars and buses, and tested the performance with an agent-based simulation of the Sioux Fall network. Dantsuji et al. (*31*) proposed a simulation-based joint optimization framework to develop congestion pricing schemes and road space allocation plans based on the congestion costs represented by a multimodal MFD. In addition, Zheng and Geroliminis (*32*) designed an optimal VOT-based toll for a two-region bi-modal city when commuters differ in their income levels and value of times (VOTs).

Laval et al. (*33*) investigated the dynamic user equilibrium in a mixed network with two routes, an urban network modeled with a MFD and a freeway. However, to our knowledge, no study has investigated a mixed network that considers joint pricing and traffic dynamics in the combined system. In addition, in most MFD-based pricing schemes, commuters are assumed to have the same VOT or their VOTs have limited set of values. In this study, we aim to propose different pricing schemes for the urban networks when commuters differ in VOTs. Two popular operation objectives are explored: performance-optimization and revenue-maximization. The performance-optimization is realized by a feedback control approach, which is similar to (*34, 35*). The controllers do not directly determine the price, but estimate coefficients in the price model. The revenue-maximization pricing scheme is obtained by solving an optimal control problem. Note that the proposed framework is different from (*17, 18*) for several reasons: (1) the study site is different; and, (2) the utility function is different, since tolls are included in our paper. Also, this paper applies a feedback pricing scheme to optimize the performance, which is not considered in those studies.

The rest of this study is organized as follows. In Section 2, we describe the system dynamics, including traffic dynamics and route choice. In Section 3, we develop a feedback control approach and an optimal control approach to determine the prices for urban networks, considering two different operation objectives. In Section 4, we present numerical examples to show how proposed pricing schemes perform in various mixed networks. In Section 5, we conclude this study and provide future research topics.

SYSTEM DYNAMICS

An O-D pair is connected by mixed networks with urban networks and freeways. Figure 1 provide both realistic and schematic representation of a mixed network with two urban networks and two freeways. Each mixed network i is made up of one urban network and one freeway. The free-flow travel time on the freeway is longer than the urban network. In addition, commuters need to pay a fee to enter the urban networks, but they can use freeways for free.



(a) Realistic representation (b) Schematic representation

FIGURE 1: A mixed network with two urban networks and two freeways

A list of notations in provided in Table 1.

TABLE 1: A list of nations

$n_{iU}(t)$	Accumulation in urban network *i*	$\lambda_{iF}(0)$	Number of queueing vehicles on freeway *i*
C_{iF}	Capacity of freeway *i*	$t_{iF,ff}$	Free-flow travel time on freeway *i*
$w_{iU}(t)$	Travel time in urban network *i*	$w_{iF}(t)$	Travel time on freeway *i*
$f_i(t)$	External demand of mixed network *i*	$f_{i0}(t)$	Internal demand of urban network *i*
$f_{iU}(t)$	Demand of urban network *i*	$g_{iU}(t)$	Trip completion rate of urban network *i*
$f_{iF}(t)$	Influx of freeway *i*	$g_{iF}(t)$	Outflux of freeway *i*
μ	capacity drop ratio	β	Proportion of trips that end in urban network
$h_{iF}(t)$	Outflux of the point queue *i*	π^*	Average value of time (VOT) for commuters who make route choice
P_{iU}	Proportion of commuters that always choose urban network *i*	P_{iF}	Proportion of commuters that always choose freeway *i*
$\mathrm{Pr}_i(t)$	Proportion of commuters that choose urban network *i*	$\zeta_i(t)$	Residual accumulation in the urban network
$u_i(t)$	Price for urban network *i*	u_{max}	Maximum allowable price for urban network

Traffic dynamics

The traffic dynamics of each urban network depends on the sum of internal and external demand, and the outflux. The internal demand includes commuters who start their trips inside the urban network. They can finish trips either within or downstream of the urban network. The external demand is generated upstream of the mixed network. Those commuters can end trips in the urban network or travel through the network. We assume that there is no demand generated inside the freeway. Thus, each freeway 's traffic dynamics is determined by its external demand and supply.

Note that, although it is known that tolling may affect the commuters' behavior by shifting the demand to other periods, we assume that the exogenous total demand is not influenced by the toll, i.e., completely inelastic demand.

The urban traffic dynamics

We consider a homogeneous urban network *i* with length of L_{iU}. This network has a well-defined MFD, $q_{iU}(t) = Q(n_{iU}(t))$. As mentioned in (*36*), the MFD can be used to derive the network exit function (NEF), which expresses the flow rate of vehicles exiting the network, $g_{iU}(t)$, as a function of the total number of vehicles circulating in the network, $n_{iU}(t)$. If the trip length within an urban network, l_{iU}, is identical for all commuters,

$$g_{iU}(t) = G(n_{iU}(t)) = \frac{L_{iU}}{l_{iU}} Q(n_{iU}(t)), \tag{1}$$

and $n_{iU}(t)$ is captured by the following equation:

$$\frac{d}{dt} n_{iU}(t) = f_{iU}(t) - g_{iU}(t), \tag{2}$$

where $f_{iU}(t)$ is the demand of the urban network i, which can be internal or external. The average speed in the urban network, $v_{iU}(t)$, is computed by

$$v_{iU}(t) = \frac{g_{iU}(t))l_{iU}}{n_{iU}(t)}. \tag{3}$$

And the travel time in the urban network can be obtained by the ratio of the trip length and average speed.

The freeway dynamics

We apply the point queue model (PQM) to model traffic dynamics of a freeway, because it has been extensively applied to study the congestion effect of a bottleneck. As demonstrated in (*37*), a point queue is an approximation of the road by omitting the length of the road but retaining the influx and outflux. It can be derived as limits of two link-based queueing models: the link transmission model (*38*) and the link queue model (*39*). So, a point queue is sufficient for analyzing the total delay caused by queues. We choose the number of queueing vehicles ($\lambda_{iF}(t)$) as the state variable. Then, the dynamics of queue is described by the following ordinary differential equation:

$$\frac{d}{dt} \lambda_{iF}(t) = f_{iF}(t) - h_{iF}(t), \tag{4}$$

where $f_{iF}(t)$ is the demand of freeway i at time t. When no capacity drop exits, the outflux of the point queue is

$$h_{iF}(t) = \min\{f_{iF}(t) + \frac{\lambda_{iF}(t)}{\varepsilon}, C_{iF}\}, \tag{5}$$

where C_{iF} is the capacity of the freeway, and ε equals time step in discrete time.

In reality, the discharging rate drops to a value that is smaller than the downstream capacity, when a queue forms upstream to the bottleneck. Here, we apply a phenomenological model of capacity drop to calculate the discharging flow rate (*40*).

$$h_{iF}(t) = \min\{f_{iF}(t) + \frac{\lambda_{iF}(t)}{\varepsilon}, (1 - H(f_{iF}(t) + \frac{\lambda_{iF}(t)}{\varepsilon} - C_{iF})\mu)C_{iF}\}, \tag{6}$$

where $H(y)$ is the Heavyside function, equal to 0 when $y \le 0$ and 1 when $y > 0$. μ is the capacity drop ratio.

We denote $t_{iF,ff}$ as the free-flow travel time on the freeway. Then, the travel time on the freeway is

$$w_{iF}(t) = t_{iF,ff} + \frac{\lambda_{iF}(t)}{C_{iF}}. \tag{7}$$

Connection between multiple mixed networks
In this section, we show how the dynamics are modeled in a complex network with multiple urban networks and freeways. Here, we define a new variable: $g_{iF}(t)$. It is the outflux of freeway i. We assume that the queue forms at the beginning of each freeway, that is, commuters need to travel for an extra $t_{iF,ff}$ after leaving the queue to reach the end of the freeway. In this network, the relationship between $g_{iF}(t)$ and the outflux of the point queue ($h_{iF}(t)$) is

$$g_{iF}(t) = \begin{cases} 0 & t < t_{iF,ff} \\ h_{iF}(t - t_{iF,ff}) & t \geq t_{iF,ff} \end{cases} \tag{8}$$

It is straightforward that the total external demand of the first mixed network is $f_1(t)$. For any other network, the external demand is the sum of the outflux of the preceding network and freeway. In addition, we assume the trip completion rate inside each urban network i is $\beta g_{iU}(t)$ (β is a fixed value between 0 and 1). Then, $f_i(t) = g_{i-1U}(t)(1-\beta) + g_{i-1F}(t)$ $(i \geq 2)$. For example, for the second mixed network, the demand is a function with delay: when $t < t_{1F,ff}$, $f_2(t) = g_{1U}(t)$; when $t >= t_{1F,ff}$, $f_2(t) = g_{1U}(t)(1-\beta) + h_{1F}(t - t_{1F,ff})$.

Route choice

As mentioned earlier, commuters need to pay to the corresponding toll to enter an urban network, but they can use the freeway for free. Since commuters only pay the price when they enter, this pricing scheme is cordon-based.

In the original Wardrop's user equilibrium (UE) state (*41*), "the journey times on all the routes actually used are equal, and less than (or equal to) those which would be experienced by a single vehicle on any unused route". Here we extend the UE principle for individual vehicles choosing different routes based on generalized cost. We denote $w_{iF}(t)$ and $w_{iU}(t)$ as the travel time on freeway and the urban network in the mixed network, and $u_i(t)$ is the price for entering the urban network i. Then, if commuter i chooses the urban network at t, then

$$w_{iU}(t)\pi_i + u_i(t) \leq w_{iF}(t)\pi_i, \tag{9a}$$

if commuter j chooses the urban network i, then

$$w_{iF}(t)\pi_j \leq w_{iU}(t)\pi_j + u_i(t), \tag{9b}$$

where π_i is the VOT of commuter i. Without loss of generality, assuming that π_i (π_j) is a continuous random variable that follows a probability density function of $d(\pi)$ and $w_{iF}(t) > w_{iU}(t)$, we have the following conclusion:

Lemma 3.1. *The proportion of commuters choosing the urban network at t is given by*

$$Pr_i(t) = 1 - D\left(\frac{u_i(t)}{w_{iF}(t) - w_{iU}(t)}\right), \tag{10}$$

where $D(\cdot)$ is the cumulative distribution function of $d(\cdot)$.

Proof. From (9a), we can see that, for any commuter i choosing the urban network i, $\pi_i \geq \frac{u_i(t)}{w_{iF}(t)-w_{iU}(t)}$; for any commuter j choosing the urban network i, $\pi_j \leq \frac{u_i(t)}{w_{iF}(t)-w_{iU}(t)}$; Therefore, the proportion of commuters choosing the urban network i is given by (10). □

For example, if the VOTs follow the simplified variant of the Burr distribution (*35, 42*), the proportion of commuters choosing the urban network at t is

$$Pr_i(t) = \frac{1}{1+(\frac{u_i(t)}{\pi^*(w_{iF}(t)-w_{iU}(t))})^{\gamma}},$$

where π^* is the average VOT and γ is a shape parameter affecting the relative width of the VOT distribution. To capture the randomness in the choice model, the proportion is multiplied by a variable $\eta(t)$.

The complete dynamics

Inside a mixed network i, a proportion of P_{iF} commuters will always use the freeway, a proportion of P_{iU} commuters will always choose the urban network, and the remaining commuters make choose between two routes. In addition, there is an internal demand, $f_{i0}(t)$, inside urban network i. The traffic dynamics of the urban network and the freeway inside the mixed network are as follows:

$$\frac{d}{dt}n_{iU}(t) = f_{i0}(t) + f_i(t)P_{iU} + f_i(t)(1-P_{iU}-P_{iF})Pr_i(t)\eta(t) - g_{iU}(t), \tag{11a}$$

$$\frac{d}{dt}\lambda_{iF}(t) = \max\{f_i(t)P_{iF} + f_i(t)(1-P_{iU}-P_{iF})(1-Pr_i(t)\eta(t)) - C_{iF}, -\frac{\lambda_{iF}(t)}{\varepsilon}\}, \tag{11b}$$

TWO PRICING SCHEMES FOR URBAN NETWORKS

When designing pricing schemes, two objectives are commonly considered: system performance and revenue. In this section, we propose two pricing schemes for a single mixed network when the travel time on the freeway is longer than the urban network. First, we apply feedback control to obtain prices that maximize the outflux of the urban network. Then, we propose a pricing scheme to maximize the revenue for the urban network.

A feedback control approach

There are two types of equilibrium states, depending on the demand profile. When the demand is low, the maximum outflux is reached when all commuters choose to use the urban network. So, it is straightforward that the price should be set as 0 to attract as many commuters as possible. On the other hand, when the demand is high, the pricing scheme should keep the urban network's accumulation at its critical value. Next, we will discuss the latter case in detail. The results will provide some insights on the design of pricing schemes.

Solution of the control problem with high demand

We define a new variable $\zeta_i(t)$, which represents the residual accumulation in the urban network i. Mathematically,

$$\zeta_i(t) = n^*_{iU} - n_{1U}(t) - (f_{iU}(t) - G(n_{iU}(t)))\Delta t, \tag{12}$$

where $f_{iU}(t) = f_{i0}(t) + f_i(t)P_{iU} + f_i(t)(1-P_{iU}-P_{iF})Pr_i(t)\eta(t)$, n^*_{iU} is the critical accumulation for urban network i, and Δt is the time step.

Combining (10), we can obtain the following equation to calculate price:

$$u_i(t) = z(\frac{(n^*_{iU} - n_{iU}(t) - \zeta_i(t))/\Delta t + G(n_{iU}(t))) - f_{i0}(t) - f_i(t)P_{iU}}{f_i(t)(1-P_{iU}-P_{iF})\eta(t)})(w_{iF}(t) - w_{iU}(t)), \tag{13}$$

where $z(p)$ is the $100(1-p)$ th-percentile, defined by $p = 1 - D(z(p))$.

As an example, if the VOTs follow a Burr distribution, then the price would be

$$u_i(t) = \pi^*(\frac{f_i(t)(1-P_{iU}-P_{iF})\eta(t)+f_{i0}(t)+f_i(t)P_{iU}-(n^*_{iU}-n_{iU}(t)-\zeta_i(t))/\Delta t - G(n_{iU}(t)))}{(n^*_{iU}-n_{iU}(t)-\zeta_i(t))/\Delta t + G(n_{iU}(t)) - f_{i0}(t) - f_i(t)P_{iU}})^{1/\gamma}(w_{iF}(t)-w_{iU}(t)). \tag{14}$$

As another example, if the VOTs follow an exponential distribution: $F(x) = 1 - e^{\frac{-x}{\pi^*}}$, where π^* is the average VOT, we have

$$u_i(t) = \pi^* \ln(\frac{f_i(t)(1-P_{iU}-P_{iF})\eta(t)}{(n^*_{iU}-n_{iU}(t)-\zeta_i(t))/\Delta t + G(n_{iU}(t)) - f_{i0}(t) - f_i(t)P_{iU}})(w_{iF}(t)-w_{iU}(t)). \tag{15}$$

In both (14) and (15), the relationship between the residual accumulation and price can be written in the following form:

$$u_i(t) = A(\zeta_i(t))(w_{iF}(t) - w_{iU}(t)), \tag{16}$$

which can be considered as a general route choice model. $A(\zeta_i(t))$ represents the price for a unit of travel time difference when the residual accumulation is $\zeta_i(t)$. Note that, $A(\zeta_i(t))$ depends on the route choice mode and the corresponding parameters, which are unknown to the operators.

Design of controller

The block diagram of the control system for mixed network *i* is shown in Figure 2. The objective is to determine a pricing scheme for the peak period that maximizes the urban network's outflux via impacting commuters' route choice. To achieve the objective, the controller calculates the price based on the congestion level. The toll is then fed into the plant, which determines route choice and the traffic dynamics of the system. In this control system, $\zeta_i(t)$ is the state variable.

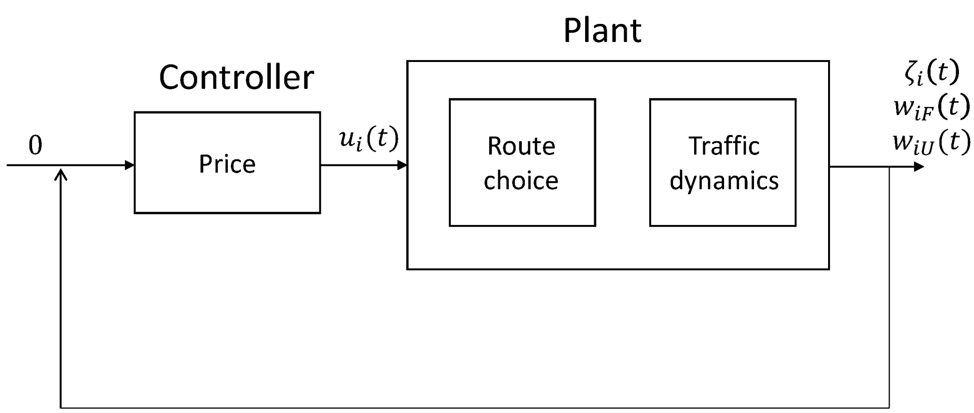


FIGURE 2: Block diagram of the control system

Figure 3 shows the design of the controller. Based on the analysis in the previous section, we can see that when the accumulation is around or over the critical value, the price is the product of a positive number and the travel time difference; otherwise, the price is 0. Then, the non-negative price $u_i(t)$ is updated as follows:

$$u_i(t) = \max\{\alpha_i(t)(w_{iF}(t) - w_{iU}(t)), 0\}, \tag{17}$$

where $\alpha_i(t)$ is determined by the following integral controller:

$$\frac{d}{dt}\alpha_i(t) = -K_i\zeta_i(t) \tag{18}$$

where K_i is a positive coefficient. That is, when the accumulation is higher than the optimal accumulation (i.e., $\zeta_i(t) < 0$), we should increase the price to reduce the demand of the urban network.

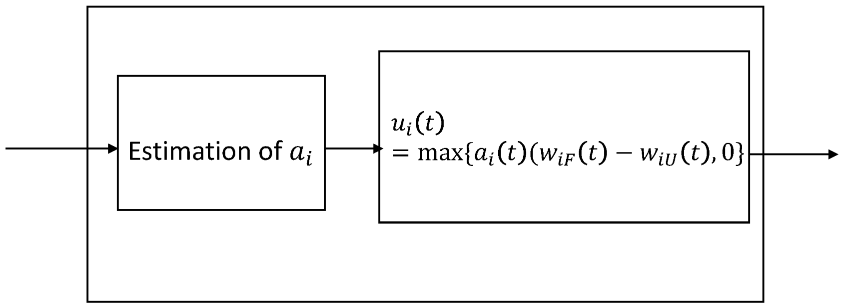


FIGURE 3: Design of the controller

An optimal control approach

In this section, we are interested in the revenue problem. We want to propose another pricing scheme that can maximize the operation revenue for the urban network over a time period. The demand of paying commuters at time t is composed of two groups. The first group includes commuters who always choose the urban network. The second group refers to commuters who make choice considering the travel time and dynamic tolls. So, the demand of paying commuters is $f_i(t)P_{iU} + f_i(t)(1 - P_{iU} - P_{iF})Pr_i(t)\eta(t)$. We apply an optimal control approach to obtain the optimal pricing scheme. Mathematically,

$$\max R = \sum_{n=1}^{N}\int_0^T (f_i(t)P_{iU} + f_i(t)(1 - P_{iU} - P_{iF})Pr_i(t)\eta(t))u_i(t)dt$$

subject to

$$\begin{aligned}
&\frac{d}{dt}n_{iU}(t) = f_{i0}(t) + f_i(t)P_{iU} + f_i(t)(1 - P_{iU} - P_{iF})Pr_i(t)\eta(t) - g_{iU}(t), \\
&\frac{d}{dt}\lambda_{iF}(t) = \max\{f_i(t)P_{iF} + f_i(t)(1 - P_{iU} - P_{iF})(1 - Pr_i(t)\eta(t)) - C_{iF}, -\frac{\lambda_{iF}(t)}{\varepsilon}\}, \\
&\quad Pr_i(t) = 1 - D\left(\frac{u_i(t)}{w_{iF}(t) - w_{iU}(t)}\right), \\
&\qquad 0 \le u_i(t) \le u_{max}
\end{aligned} \tag{19}$$

where u_{max} is the maximum allowable price for operating urban networks, which is determined by agencies.

CASE STUDY

Case study for a single mixed network

We first present the result for a single mixed network, as show in Figure 4. An approximate formula for the corresponding NEF has been given in (*43*).

$$G(n) = \begin{cases} 2.28 \times 10^{-8} n_1^3 - 8.62 \times 10^{-4} n_1^2 + 9.58 n_1 & 0 \leq n_1 < 14000, \\ 27331 - 1.38655(n_1 - 14000) & n_1 \geq 14000. \end{cases} \tag{20}$$

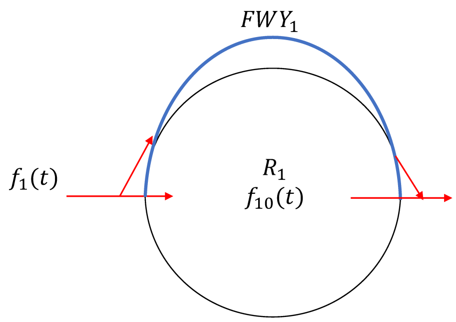


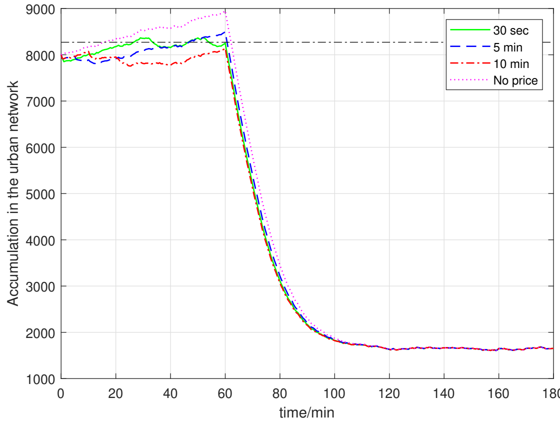
FIGURE 4: A mixed network with a freeway and an urban network

The maximum average outflux from is 33168 vph, which is achieved at an accumulation of 8271 veh. The maximum (jam) accumulation is 34,000 veh. The MFD, $Q(n_{1U}(t))$, is a scaled up version of the NEF formula - scaled up by the 2.3 km average trip length observed in Yokohama. When the outflux of the urban network is maximized, the travel time is 14.96 min. For the freeway, the capacity is $C_{1F} = 30$ veh/min, and $t_{1F,ff} = 15$ min. The capacity drop ratio is $\mu = 0.1$.

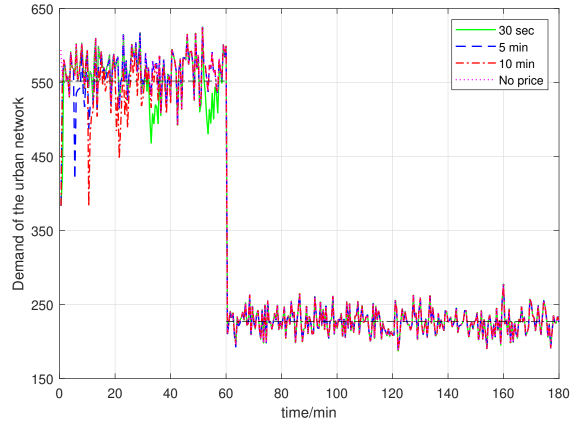
The initial accumulation in the urban network is 8000 veh, and the freeway is initially empty. For the first 60 minutes, $f_{10}(t)$ is Poisson with average of 50 veh/min, and $f(t)$ is Poisson with average of 560 veh/min. For the next 120 minutes, $f_{10}(t)$ is Poisson with average of 30 veh/min, and $f_1(t)$ is Poisson with average of 210 veh/min. In the lane choice model, $P_{1U} = 15\%$, $P_{1F} = 5\%$. $\eta(t)$ follows a truncated normal distribution with mean of 1 and variance of 0.04. Also, the value of $\eta(t)$ is between 0.9 and 1.1. The average VOT is $\pi^* = \$0.5/min$, which is the same as (*35, 44*). The Burr distribution is evaluated for $\gamma = 3$. The traffic dynamics are updated every 30 seconds.

The performance-optimization pricing scheme

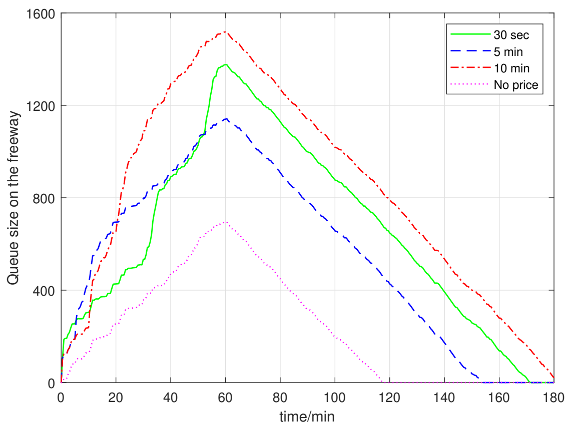
The coefficients in the controller is chosen as $K_1 = 1/1000$, and the initial guess of α_1 is $\alpha_1(0) = 0.5$. The results for the performance-optimization pricing scheme are presented in Figure 5. We test the performance when the prices are updated at three different frequencies (every 30 second, 5 minutes, and 10 minutes), We also provide the results when no price is implemented to the urban network.



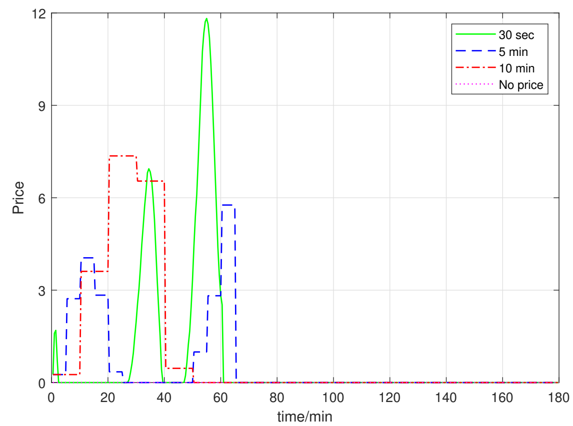
(a) Accumulation in the urban network



(b) Demand of the urban network



(c) Queue size on the freeway



(d) Price

FIGURE 5: Results for performance-optimization pricing scheme at different frequencies for updating price

In Figure 5a, the accumulation in the urban network drops when the initial price is applied to the system, because $\alpha_1(0)$ is high. After that, the accumulation starts to increase. When no price is implemented, the accumulation keeps increasing as long as the total demand is high. When the prices are updated every 30 seconds, the accumulation fluctuates closer to the critical value (marked by the horizontal dashed line) than the 5-minute case. The accumulation is always below the critical value, if the prices are updated every 10 minutes. When the demand is low, the accumulation first decreases dramatically and then fluctuates around 1645 veh in all four cases. Figure 5b shows the demand of the urban network. When the upstream demand is high, a larger update interval results in a larger fluctuation in the demand of the urban network. For example, the lowest demand of the urban network is 382 veh/min when the update frequency is set to be every 10 minutes. When the demand is low, $f_{1U}(t)$ becomes around 227 veh/min in all scenarios. As shown in Figure 5c, the queue size increases fast in the first because fewer commuters decide to use the urban network. The queue size keeps increasing as long as the external demand is high. When no price is applied, the maximum queue size on the freeway is 695 veh, which is the smallest in all cases. The longest queue (1429 veh) appears in the case where the prices are updated every 10 minutes. Meanwhile, if the price is not active, the queue is eliminated at $t = 118$ min, which is the fastest in those four scenarios. However, the queue cannot be eliminated at the end of the study period when the update frequency is every 10 minutes. In Figure 5d, the maximum prices are \$11.82, \$5.76, and \$7.35, respectively. When the demand is low, we just set the price to be 0 to attract more commuters to use the urban network.

We also compare the revenue and the average outflux of the urban network, and the maximum absolute residual accumulation (in high demand) in different pricing schemes in Table 2. The maximum absolute residual accumulation is 682 veh if no price is implemented. If we increase the duration of the high demand period, the value would keep increasing. When the prices are updated every 30 seconds, the urban network has the highest average trip completion rate when the upstream demand is high. When the prices are updated every 5 minutes, the average trip completion rate is 0.2 veh/min lower, and the maximum absolute residual accumulation is 38 veh than the 30-second case. If the update frequency is every 10 minutes, the urban network has the lowest average trip completion rate, but the highest revenue.

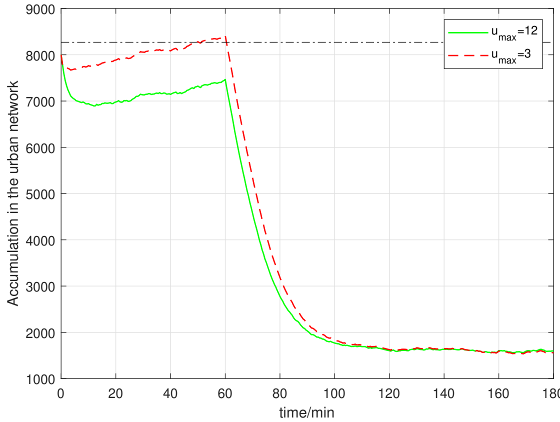
TABLE 2: Comparison of four pricing schemes

	Average trip completion rate of the urban network (veh/min)	Maximum absolute residual accumulation (veh)	Total revenue
no price	552.28	682	0
30 seconds	552.64	420	62,923
5 miutes	552.44	458	40,845
10 minutes	552.08	514	90,058

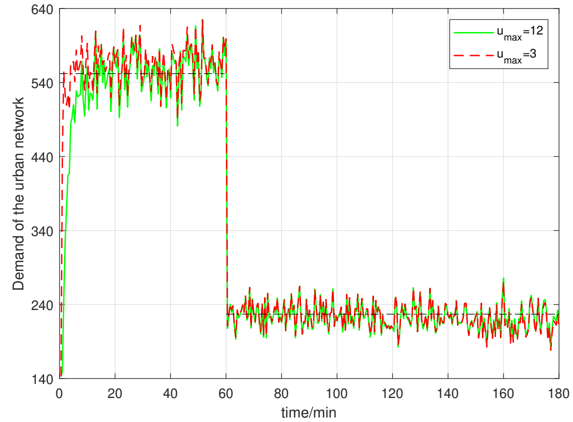
Based on the results in Figure 5 and some practical issues for implementing tolls, we will update the prices every five minute for the case study for two mixed networks later.

The revenue-maximization pricing scheme

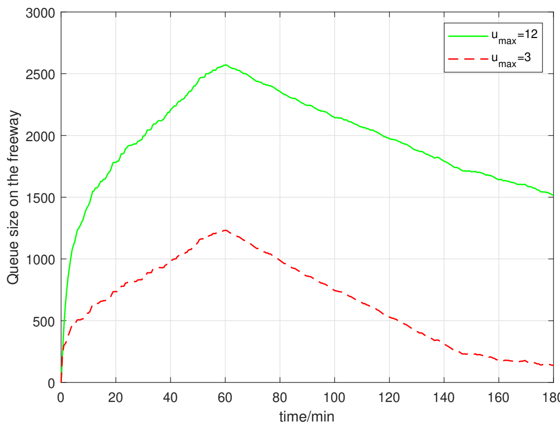
In this section, we test the impact of the maximum allowable price on the performance and revenue by setting two u_{max}: \$3 and \$12. In all four figures in Figure 6, the dashed line represents the results with $u_{max} = \$3$, and the solid line shows the results with $u_{max} = \$12$.



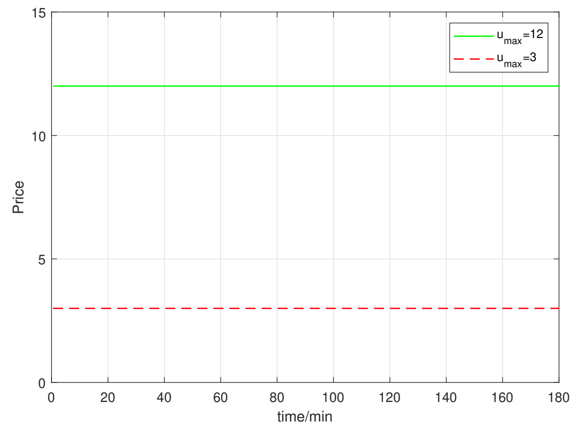
(a) Accumulation in the urban network



(b) Demand of the urban network



(c) Queue size on the freeway



(d) Price

FIGURE 6: Results for the revenue-maximizing pricing schemes under two maximum allowable prices

In Figure 6a, when $u_{max} = \$12$, the accumulation keeps decreasing in the first 12 minutes, until it reaches 6893 veh. After that, the accumulation starts to increase, as the demand (shown in Figure 6b) is higher than the outflux. At the end of the high demand period, the accumulation is 7467 veh. When the demand drops, the accumulation fluctuates around 1591 veh, which is lower than that in the performance-optimization pricing scheme. However, when $u_{max} = \$3$, the accumulation is higher than the critical value between 52 and 60 minutes, while it fluctuates around 1568 veh when the demand is low. In Figure 6b, the initial demands are both too low because of the high price. At the same time, when the demand is low, the demands are around 220 veh/min and 215 veh/min when$u_{max} = \$12$ and $u_{max} = \$3$, respectively. The queue size increases rapidly at first, since the external demand high and fewer commuters choose to pay to travel in the urban network. As more commuters choose the urban network, the queue size increases at a relatively lower speed, as shown in Figure 6c. When $u_{max} = \$12$, the maximum queue size on the freeway is 2571 veh. The queue size starts to drop as demand drops. At the end of the study period, the queue size is 1517 veh. The maximum queue size is 1231 veh when $u_{max} = \$3$. When the demand is low, the queue on the freeway cannot be eliminated either. Instead, the queue size decreases to 141 veh at the end of the study period. This shows that a higher maximum allowable would lead to a more congested condition on the freeway. In Figure 6d, the two prices are constant at their maximum allowable values respectively, which are independent of the demand profile. This is totally different from the performance-optimization pricing scheme.

In addition, at the end of the study period, the revenue is $\$161,356$ when $u_{max} = \$3$, and $\$628,900$ when $u_{max} = \$12$.

Comparing the results of those two types of schemes, we have the following conclusions: (1) the performance-optimization pricing scheme is dynamic when the demand is high, while the revenue-maximization pricing scheme is static; (2) the revenue-maximization pricing scheme creates much more revenue than the performance-optimization pricing scheme; and (3) the revenue-maximization pricing scheme leads to a poor traffic condition: very long queue on the freeway and low utilization of the urban network.

Thus, we think performance-optimization pricing schemes would be more helpful in practice. In the next section, we will test this pricing scheme in a more complex traffic network.

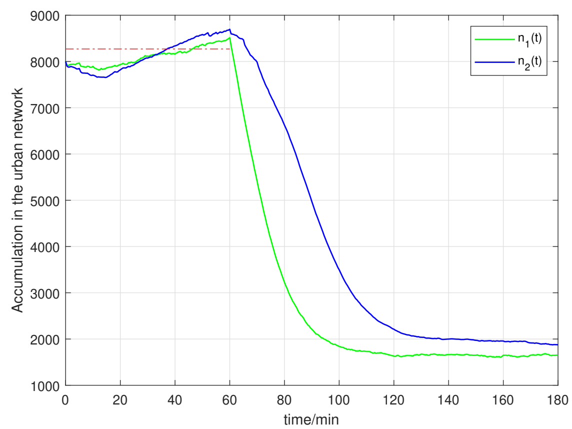
Case study for two mixed networks

In this section, we provide a case study for an extended mixed network, as shown in Figure 1. Both urban networks exhibit the same NEF in (20). The study period is 180 minutes, the traffic dynamics are updated every 30 seconds, and the prices are calculated every 5 minutes. A detailed description of traffic and operation parameters can be found in Table 3. Note that, we assume the demands follow Poisson distributions, and the value in the cells shows the average value. In addition, to include the randomness in the choice model, we set $\eta(t)$ follows a truncated normal distribution with mean of 1 and variance of 0.04, and its value is between 0.9 and 1.1. Same as the previous setup, the VOTs are Burr distributed with $\pi^* = \$0.5/\text{min}$ and $r = 3$.

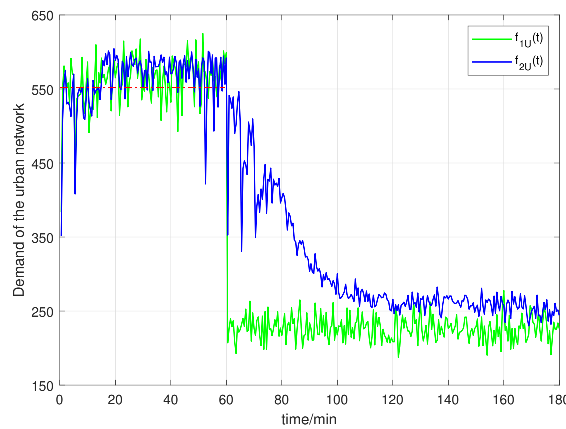
TABLE 3: Parameters for the two mixed networks

Fixed parameters			
P_{1U}	0.15	P_{1F}	0.05
P_{2U}	0.1	P_{2F}	0.02
C_{1F}	30 veh/min	C_{2F}	30 veh/min
$t_{1F,ff}$	15 min	$t_{2F,ff}$	15 min
K_1	1/500	K_2	1/750
μ	0.1	β	0.1
Initial Conditions			
$n_1(0)$	8000 veh	$\lambda_{1F}(0)$	0
$n_2(0)$	8000 veh	$\lambda_{2F}(0)$	0
$a_1(0)$	\$0.5/min	$a_2(0)$	\$0.5/min
Demand in the first 60 minutes		Demand in the remaining 120 minutes	
$f_1(t)$	560 veh/min	$f_1(t)$	210 veh/min
$f_{10}(t)$	50 veh/min	$f_{10}(t)$	75 veh/min
$f_{20}(t)$	30 veh/min	$f_{20}(t)$	40 veh/min

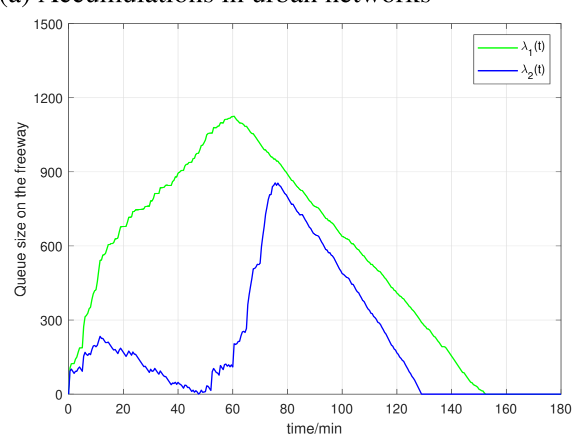
The numerical results are illustrated in Figure 7. In Figure 7a, for each network, the accumulation cross the optimal value just once when the upstream demand is high. When the demand becomes low, the accumulation in the first network fluctuates around 1646 veh. The accumulation in the second urban network is away from the optimal value in the first 15 minutes, because the external demand is relatively low. Then, it fluctuates around the optimal value. When the demand becomes low, the accumulation is around 1919 veh, which is higher than the first urban network. This can be explained by Figure 8: when the queue on the freeway is not eliminated in the first mixed network, the external demand of the second mixed network is higher. In Figure 7b, the total demands of those two urban networks have a similar trend when the demand is high, except that the demand of the second urban network is less stable. The second urban network has a higher demand when the upstream demand decreases. As shown in Figure 7c, the maximum and average queue size on the second freeway is smaller than that on the first freeway. Since the demand of the second freeway is relatively low when the price is 0, the queue length is eliminated at 130 minutes, which is earlier than the first freeway (153 minutes). In Figure 7d, the prices for urban networks are non-negative when the demands are high, and eventually become 0 when the demands are low. The maximum price for the second urban network ($7.58) is higher than the first one ($5.50), because the ne. The price for the second urban network does not become 0 immediately when $f_1(t)$ drops, because there is a delay in the outflux of the first freeway.



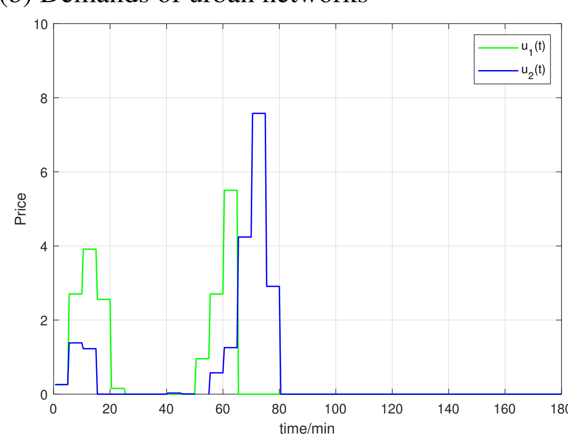
(a) Accumulations in urban networks



(b) Demands of urban networks



(c) Queue sizes on freeway



(d) Prices

FIGURE 7: Results for the extended network when prices are updated every 5 minutes

In addition, we present the external demands of each mixed network in Figure 8. For the first mixed network, the influx follows a Poisson distribution with average value of 560 veh/min and 210 veh/min for the high and low demand case, respectively. The external demand of the second mixed network in the first 15 minutes is about 497 veh/min, which equals the outflux to leave the freeway in the first mixed network (i.e., $t = 15$ min), $f_2(t)$ increases and keeps at 524 veh/min (since queue exists on the first freeway, the outflux of the freeway is at the dropped capacity). Then, $f_2(t)$ decreases as the external demand of the first mixed network drops. Note that, when $t > 168$ min, there is a larger fluctuation in $f_2(t)$, as highlighted in the figure. This is caused by the the randomness in $g_{1F}(t)$. When the queue is eliminated on the first freeway, $g_{1F}(t)$ varies between 8.8 and 28.9 veh/min.

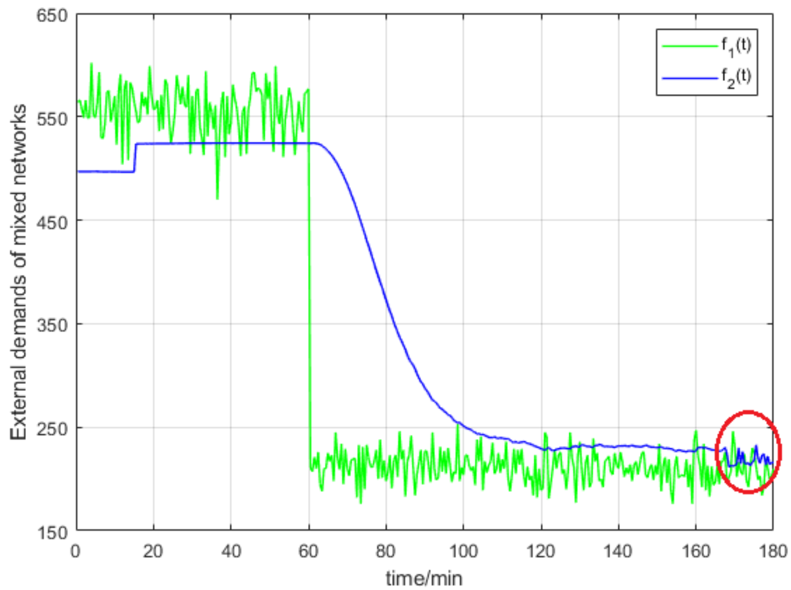


FIGURE 8: External demands of two mixed networks

CONCLUSIONS

In this study, we investigate cordon-based pricing schemes for mixed networks with urban networks and freeways. In most control strategies for urban networks, the operation objective is to optimize the urban network's performance, i.e., to maximize the outflux. In real-world application, another common operation objective is to maximize the operation revenue. To compare those two objectives, we first apply feedback control to design pricing schemes to optimize the urban network's performance. We also propose an optimal control problem to obtain the revenue-maximization pricing scheme. The differences between those pricing schemes are illustrated with numerical examples.

We first provide dynamics of the traffic system in Section 2. Each urban network has a well-defined MFD, and the traffic dynamics on the freeway is captured by a point queue model. Commuters with different VOTs choose their routes based on the UE principle. In Section 3, we consider two different operation objectives, and design pricing schemes for mixed networks. The first objective is to optimize the outflux of urban network. A feedback pricing scheme is proposed to reach the objective. The second pricing scheme aims to maximize the operation revenue, and it is formulated as an optimal control problem. Numerical examples are provided in Section 4. In order to obtain the maximum revenue, the price is at the maximum allowable value. However, the traffic condition is poor when the demand is high. When a high maximum allowable price is set, the queue size on the freeway is very long and the urban network is underutilized; when the maximum allowable price is low, the urban network is congested. Thus, the performance-optimization pricing scheme would be more practical in real-world operation. When discussing the performance-optimization pricing schemes, we compare the system performance when the prices are updated at three different frequencies. We also include a case study to show how the feedback pricing scheme works in an extended mixed network when we update the price every 5 minutes.

The following are some potential future research topics.

- In this study, we apply an optimal control approach when solving the revenue maximization problem. We will be interested in proposing MFD-based economic model predictive control schemes (*45*) to solve the problem in our future study.
- In this study, we optimize the performance of two networks separately, based on the traffic condition in each mixed network. We will be interested in addressing the two-region coordinated congestion pricing design problem.
- We will be interested in the distribution of trip lengths in an urban network, such as deterministic distributions of trip lengths (*46*) and constant trip length (*47*).
- When the trip distribution is introduced, we would like to propose a distance-based pricing strategy for the urban network (*26, 27*). We will be interested in comparing the performance of cordon-based and distance-based pricing schemes.
- We will also be interested in the departure time choice and related pricing schemes in mixed networks (*24, 30*).
- We will introduce new modes in the system, such as carpool and transit system (*48*).

AUTHOR CONTRIBUTIONS

The authors confirm contribution to the paper as follows: study conception and design: X. Wang and V. Gayah; analysis and interpretation of results: X. Wang, V. Gayah; draft manuscript preparation: X. Wang and V. Gayah. All authors reviewed the results and approved the final version of the manuscript.

DECLARATION OF CONFLICTING INTERESTS

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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REFERENCES

1. Gu, Z., Z. Liu, Q. Cheng, and M. Saberi, Congestion pricing practices and public acceptance: A review of evidence. *Case Studies on Transport Policy*, Vol. 6, No. 1, 2018, pp. 94–101.
2. Yang, H. and H.-J. Huang, *Mathematical and economic theory of road pricing*, 2005.
3. de Palma, A. and R. Lindsey, Traffic congestion pricing methodologies and technologies. *Transportation Research Part C: Emerging Technologies*, Vol. 19, No. 6, 2011, pp. 1377–1399.
4. Pigou, A. C., *The economics of welfare*. Macmillan and Co., London, 1920.
5. Knight, F. H., Some fallacies in the interpretation of social cost. *The Quarterly Journal of Economics*, Vol. 38, No. 4, 1924, pp. 582–606.
6. Emmerink, R., P. Nijkamp, and P. Rietveld, Is congestion pricing a first-best strategy in transport policy? A critical review of arguments. *Environment and Planning B: Planning and Design*, Vol. 22, No. 5, 1995, pp. 581–602.
7. Liu, L. N. and J. F. McDonald, Economic efficiency of second-best congestion pricing schemes in urban highway systems. *Transportation Research Part B: Methodological*, Vol. 33, No. 3, 1999, pp. 157–188.
8. Yang, H. and X. Zhang, Optimal toll design in second-best link-based congestion pricing. *Transportation Research Record*, Vol. 1857, No. 1, 2003, pp. 85–92.
9. Zhang, X. and H. Yang, The optimal cordon-based network congestion pricing problem. *Transportation Research Part B: Methodological*, Vol. 38, No. 6, 2004, pp. 517–537.
10. Meng, Q., Z. Liu, and S. Wang, Optimal distance tolls under congestion pricing and continuously distributed value of time. *Transportation Research Part E: Logistics and Transportation Review*, Vol. 48, No. 5, 2012, pp. 937–957.
11. Liu, Z., Q. Meng, and S. Wang, Speed-based toll design for cordon-based congestion pricing scheme. *Transportation Research Part C: Emerging Technologies*, Vol. 31, 2013, pp. 83–98.
12. Liu, Z., S. Wang, and Q. Meng, Optimal joint distance and time toll for cordon-based congestion pricing. *Transportation Research Part B: Methodological*, Vol. 69, 2014, pp. 81–97.
13. Gu, Z., S. Shafiei, Z. Liu, and M. Saberi, Optimal distance-and time-dependent area-based pricing with the Network Fundamental Diagram. *Transportation Research Part C: Emerging Technologies*, Vol. 95, 2018, pp. 1–28.
14. Godfrey, J., The mechanism of a road network. *Traffic Engineering & Control*, Vol. 8, No. 8, 1969.
15. Geroliminis, N. and C. F. Daganzo, Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. *Transportation Research Part B: Methodological*, Vol. 42, No. 9, 2008, pp. 759–770.
16. Wada, K., T. Akamatsu, Y. Hara, et al., An empirical analysis of macroscopic fundamental diagrams for Sendai road networks. *Interdisciplinary Information Sciences*, Vol. 21, No. 1, 2015, pp. 49–61.
17. Geroliminis, N., J. Haddad, and M. Ramezani, Optimal perimeter control for two urban regions with macroscopic fundamental diagrams: A model predictive approach. *IEEE Transactions on Intelligent Transportation Systems*, Vol. 14, No. 1, 2012, pp. 348–359.

18. Haddad, J., M. Ramezani, and N. Geroliminis, Cooperative traffic control of a mixed network with two urban regions and a freeway. *Transportation Research Part B: Methodological*, Vol. 54, 2013, pp. 17–36.
19. Yang, K., N. Zheng, and M. Menendez, Multi-scale perimeter control approach in a connected-vehicle environment. *Transportation research procedia*, Vol. 23, 2017, pp. 101–120.
20. Hajiahmadi, M., V. L. Knoop, B. De Schutter, and H. Hellendoorn, Optimal dynamic route guidance: A model predictive approach using the macroscopic fundamental diagram. In *16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013)*, IEEE, 2013, pp. 1022–1028.
21. Yildirimoglu, M., M. Ramezani, and N. Geroliminis, Equilibrium analysis and route guidance in large-scale networks with MFD dynamics. *Transportation Research Procedia*, Vol. 9, 2015, pp. 185–204.
22. Geroliminis, N. and D. M. Levinson, Cordon pricing consistent with the physics of overcrowding. In *Transportation and Traffic Theory 2009: Golden Jubilee*, Springer, 2009, pp. 219–240.
23. Vickrey, W. S., Congestion theory and transport investment. *The American Economic Review*, Vol. 59, No. 2, 1969, pp. 251–260.
24. Zheng, N., R. A. Waraich, K. W. Axhausen, and N. Geroliminis, A dynamic cordon pricing scheme combining the macroscopic fundamental diagram and an agent-based traffic model. *Transportation Research Part A: Policy and Practice*, Vol. 46, No. 8, 2012, pp. 1291–1303.
25. Simoni, M., A. Pel, R. A. Waraich, and S. Hoogendoorn, Marginal cost congestion pricing based on the network fundamental diagram. *Transportation Research Part C: Emerging Technologies*, Vol. 56, 2015, pp. 221–238.
26. Daganzo, C. F. and L. J. Lehe, Distance-dependent congestion pricing for downtown zones. *Transportation Research Part B: Methodological*, Vol. 75, 2015, pp. 89–99.
27. Lehe, L. J., Downtown tolls and the distribution of trip lengths. *Economics of transportation*, Vol. 11, 2017, pp. 23–32.
28. Gu, Z., S. T. Waller, and M. Saberi, Surrogate-based toll optimization in a large-scale heterogeneously congested network. *Computer-Aided Civil and Infrastructure Engineering*, Vol. 34, No. 8, 2019, pp. 638–653.
29. Gu, Z. and M. Saberi, Simulation-based optimization of toll pricing in large-scale urban networks using the network fundamental diagram: A cross-comparison of methods. *Transportation Research Part C: Emerging Technologies*, Vol. 122, 2021, p. 102894.
30. Zheng, N., G. Rérat, and N. Geroliminis, Time-dependent area-based pricing for multimodal systems with heterogeneous users in an agent-based environment. *Transportation Research Part C: Emerging Technologies*, Vol. 62, 2016, pp. 133–148.
31. Dantsuji, T., D. Fukuda, and N. Zheng, Simulation-based joint optimization framework for congestion mitigation in multimodal urban network: a macroscopic approach. *Transportation*, 2019, pp. 1–25.
32. Zheng, N. and N. Geroliminis, Area-based equitable pricing strategies for multimodal urban networks with heterogeneous users. *Transportation Research Part A: Policy and Practice*, Vol. 136, 2020, pp. 357–374.

33. Laval, J. A., L. Leclercq, and N. Chiabaut, Minimal parameter formulations of the dynamic user equilibrium using macroscopic urban models: Freeway vs city streets revisited. *Transportation research procedia*, Vol. 23, 2017, pp. 517–530.
34. Wang, X., W.-L. Jin, and Y. Yin, A Control Theoretic Approach to Simultaneously Estimate Average Value of Time and Determine Dynamic Price for High-Occupancy Toll Lanes. *IEEE Transactions on Intelligent Transportation Systems*, 2020.
35. Jin, W.-L., X. Wang, and Y. Lou, Stable dynamic pricing scheme independent of lane-choice models for high-occupancy-toll lanes. *Transportation Research Part B: Methodological*, Vol. 140, 2020, pp. 64–78.
36. Daganzo, C. F., Urban gridlock: Macroscopic modeling and mitigation approaches. *Transportation Research Part B: Methodological*, Vol. 41, No. 1, 2007, pp. 49–62.
37. Jin, W.-L., Point queue models: A unified approach. *Transportation Research Part B: Methodological*, Vol. 77, 2015, pp. 1–16.
38. Yperman, I., The link transmission model for dynamic network loading, 2007.
39. Jin, W.-L., A link queue model of network traffic flow. *Transportation Science*, 2021.
40. Jin, W.-L., Q.-J. Gan, and J.-P. Lebacque, A kinematic wave theory of capacity drop. *Transportation Research Part B: Methodological*, Vol. 81, 2015, pp. 316–329.
41. Wardrop, J. G., Some theoretical aspects of road traffic research. *Proceedings of the Institution of Civil Engineers*, Vol. 1, No. 3, 1952, pp. 325–378.
42. Gardner, L. M., H. Bar-Gera, and S. D. Boyles, Development and comparison of choice models and tolling schemes for high-occupancy/toll (HOT) facilities. *Transportation Research Part B: Methodological*, Vol. 55, 2013, pp. 142–153.
43. Daganzo, C. F., V. V. Gayah, and E. J. Gonzales, The potential of parsimonious models for understanding large scale transportation systems and answering big picture questions. *EURO Journal on Transportation and Logistics*, Vol. 1, No. 1-2, 2012, pp. 47–65.
44. Yin, Y. and Y. Lou, Dynamic tolling strategies for managed lanes. *Journal of Transportation Engineering*, Vol. 135, No. 2, 2009, pp. 45–52.
45. Sirmatel, I. I. and N. Geroliminis, Economic model predictive control of large-scale urban road networks via perimeter control and regional route guidance. *IEEE Transactions on Intelligent Transportation Systems*, Vol. 19, No. 4, 2017, pp. 1112–1121.
46. Fosgerau, M., Congestion in the bathtub. *Economics of Transportation*, Vol. 4, No. 4, 2015, pp. 241–255.
47. Arnott, R., A. Kokoza, and M. Naji, Equilibrium traffic dynamics in a bathtub model: A special case. *Economics of transportation*, Vol. 7, 2016, pp. 38–52.
48. Gonzales, E. J. and C. F. Daganzo, Morning commute with competing modes and distributed demand: user equilibrium, system optimum, and pricing. *Transportation Research Part B: Methodological*, Vol. 46, No. 10, 2012, pp. 1519–1534.