A Continuous-Discontinuous Galerkin Method for the Modeling and Simulation of Electromagnetic Multiscale Problems

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Many electromagnetic problems involve multiscale issues, which are usually induced by the size differences of the geometrical features in different parts of the objects under consideration. Typical examples of multiscale problems include the modeling and simulation of antenna arrays, integrated circuits, and electrical machines. To accurately and efficiently solve such problems, two challenges have to be properly addressed. The first challenge comes from the infamous low-frequency breakdown catastrophe, which refers to the fact that when applied to solve electrically small problems, the vector Helmholtz equation for the electrical field generates erroneous numerical solutions. As a result, the vector Helmholtz equation cannot be applied directly to solve multiscale problems where small features are unavoidable. The second challenge results from the geometrical discretization of the object. Due to the presence of both tiny geometrical features and the overall large-sized object, a tremendous number of unknowns need to be handled by a numerical method.

In the past few decades, several techniques have been developed to address the low-frequency breakdown issue. Based on the incomplete Helmholtz decomposition, a tree-cotree splitting technique [1] has been proposed to represent, separately, the purely gradient and rotational-like parts of the electric field, which scale differently when the frequency approaches zero. Another type of method seeks low-frequency solutions by reformulating the governing equations to solve for the magnetic vector and electric scalar potentials, rather than the electric field itself. Recently, a potential-based formulation was developed by incorporating all four Maxwell's equations and the current continuity equation [2], which has shown to be stable at all frequencies, and hence, highly suitable for performing multiscale simulations.

In this work, we describe a hybrid continuous-discontinuous Galerkin method to solve the all-frequency stable formulation developed in [2] for multiscale modeling and simulations. The discontinuous Galerkin method is known to be a very flexible numerical method in solving mathematical/physical equations of different types. With the use of the high-order geometrical discretization, the application of high-order basis functions, and the design of appropriate numerical fluxes, the discontinuous Galerkin method can achieve an optimal *hp* convergence, energy conservation, and spurious-free solutions. However, due to the introduction of discontinuous basis functions, the number of unknowns on the elemental interfaces is doubled, leading to a larger system matrix compared to the traditional continuous Galerkin method. It is, therefore, desirable to hybridize the discontinuous Galerkin with the continuous Galerkin method to enjoy the benefits of both. Based on the idea of the interior penalty [3], the hybrid continuous-discontinuous Galerkin method for the all-frequency stable formulation is designed, which minimizes the total number of unknowns in a given problem while allowing the basis functions to be discontinuous wherever needed. In this talk, the application and performance of the hybrid Galerkin method are demonstrated and investigated through several electromagnetic problems with low-frequency and multiscale features.

1. J. B. Manges and Z. J. Cendes, "Tree-cotree decompositions for first-order complete tangential vector finite elements," *Int. J. Numer. Methods Eng*, vol. 40, pp. 1667–1685, 1997.

2. S. Yan, "All-frequency stable potential-based formulation for electromagnetic modeling and simulation," in *Proc. IEEE Antennas Propag. Symp.*, Atlanta, GA, USA, July 2019.

3. D. N. Arnold, F. Brezzi, B. Cockburn, and L. D. Marini, "Unified Analysis of Discontinuous Galerkin Methods for Elliptic Problems," *SIAM J. Numer. Anal*, vol. 39, no. 5, pp. 1749–1779, 2002.