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CONDENSED MATTER

Suppression of Magneto-Intersubband Resistance Oscillations by Large-Scale Fluctuations of the Intersubband Energy Splitting

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Low-temperature dependences of the amplitude of magneto-intersubband resistance oscillations (ΔR_{MISO}) on the magnetic field $B < 2$ T are studied in single GaAs quantum wells with the width (d_{SQW}) from 22 to 46 nm and two occupied quantum confinement subbands E_1 and E_2. It is established that additional damping appears in dependences of ΔR_{MISO} on $1/B$ in the studied quantum wells, which is explained by the effect of large-scale fluctuations of the intersubband splitting $\Delta_{12} = E_2 - E_1$ on the amplitude of oscillations ΔR_{MISO}. It is found that the suppression of oscillations ΔR_{MISO} with the increase in $1/B$ is more efficient in "narrow" quantum wells. This experimental fact makes it possible to suppose that the main origin of fluctuations of Δ_{12} in the studied narrow quantum wells is large-scale fluctuations of the well width d_{SQW}. An expression taking into account the role of large-scale fluctuations of Δ_{12} in the damping of ΔR_{MISO} is obtained. The comparison of theory and experiment has made it possible to determine the average amplitude of fluctuations of the intersubband splitting in the studied GaAs quantum wells.

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Modern molecular beam epitaxy makes it possible to grow selectively doped GaAs quantum wells where the two-dimensional (2D) electron gas has a low-temperature mobility of $\mu \sim 4400$ m^2/(V s) [1–4]. However, despite the advances made in growing technology and optimizing the design of high-mobility heterostructures, they are not ideal 2D electron systems. In particular, such structures contain large-scale fluctuations of the 2D electron gas density δn, which lead to the inhomogeneous broadening of Landau levels and are manifested in the nonlinearity of Dingle plots, i.e., dependences of the logarithm of the amplitude of Shubnikov–de Haas (SdH) oscillations on the inverse magnetic field [3]. The form of Dingle plots is determined by the temperature T and broadening mechanisms of Landau levels, and they are widely used to study the processes of scattering of the 2D electron gas in semiconductor heterostructures [5–8].

For the case of the homogeneous broadening of Landau levels, the dependence of the amplitude of SdH oscillations (ΔR_{SdH}) on the magnetic field B follows from the Lifshitz–Kosevich formula and is given by the expression [9, 10]

$$\Delta R_{\mathrm{SdH}} = 4R_0 X(T)\exp(-\pi/\omega_c\tau_q), \quad (1)$$

where R_0 is the resistance in the zero magnetic field, $X(T) = (2\pi^2 k_B T/\hbar\omega_c)/\sinh(2\pi^2 k_B T/\hbar\omega_c)$ is the thermal damping factor, $\omega_c = eB/m^*$ is the cyclotron frequency, m^* is the effective electron mass, and τ_q is the electron quantum lifetime. The dependence $\ln[\Delta R_{\mathrm{SdH}}/R_0 X(T)]$ on $1/B$ (Dingle plot) for the homogeneous broadening of Landau levels is linear, and its slope is determined by the τ_q value.

The effect of the inhomogeneous broadening caused by fluctuations δn on SdH oscillations in the 2D electron gas was studied in [11]. Fluctuations δn lead to different periods of SdH oscillations in different regions of the sample. As a result, the amplitude of SdH oscillations, which is averaged over the sample area, decreases with increasing $1/B$ faster than that in the homogeneous sample. In this case, the inhomogeneous damping factor appears as an additional factor on the right-hand side of Eq. (1), and the quantity ΔR_{SdH} is determined by the formula [11]

$$\Delta R_{\mathrm{SdH}} = 4R_0 X(T) \times \exp(-\pi/\omega_c\tau_q)\exp[-(\pi\delta\varepsilon_F/\hbar\omega_c)^2], \quad (2)$$

Table 1. Sample parameters: the width of the quantum well d_{SQW}, the total electron density n_{T}, the electron mobility μ, the intersubband energy splitting Δ_{12}, the average fluctuation of the intersubband energy splitting $\delta\Delta_{12}$, and the average fluctuation of the width of the quantum well δd_{SQW}

Sample number	d_{SQW} (nm)	n_{T} (10^{15} m^{-2})	μ (m^2/(V s))	Δ_{12} (meV)	$\delta\Delta_{12}$ (meV)	δd_{SQW} (nm)
1	22	10	121	21.7	0.103	0.055
2	26	8.1	119	15.1	0.066	0.045
3	30	6.8	233	9.95	0.058	0.053
4	36	8.4	162	4.76	0.042	0.066
5	46	8.4	158	1.45	0.036	0.51

where $\delta\varepsilon_{\mathrm{F}} = (\pi\hbar^2/m^*)\delta n$ is the characteristic (average) fluctuation of the Fermi energy corresponding to the δn value. It can be seen that the Dingle plot acquires an additive proportional to $1/B^2$ in addition to the term linear in $1/B$. This additive takes into account the inhomogeneous broadening of Landau levels caused by density fluctuations δn.

Owing to intersubband electron scattering, which becomes resonant when the Landau levels belonging to different subbands coincide with each other, magneto-intersubband (MIS) oscillations arise in magnetic field dependences of the resistance $R_{xx}(B)$ in multiband electron systems along with SdH oscillations [12–15]. In a two-subband electron system, the position of the maxima of MIS oscillations in a magnetic field is determined by the equality

$$E_2 - E_1 = k\hbar\omega_{\mathrm{c}}, \quad (3)$$

where E_1 and E_2 are the positions of the bottoms of the first and second subbands, respectively, and k is a positive integer. Magneto-intersubband oscillations, as well as SdH oscillations, are periodic in $1/B$, and their period is determined by the ratio $\Delta_{12}/\hbar\omega_{\mathrm{c}}$, where $\Delta_{12} = E_2 - E_1$ is the intersubband splitting.

The amplitude of MIS resistance oscillations for charge carriers in the quantum well with two occupied energy subbands is expressed by the equality [16–18]

$$\Delta R_{\mathrm{MISO}} = R_0 A_{\mathrm{MISO}} \exp(-2\pi/\omega_{\mathrm{c}}\tau_{\mathrm{q}}^{\mathrm{MISO}}), \quad (4)$$

where $A_{\mathrm{MISO}} = 2\tau_{\mathrm{tr}}/\tau_{12}^*$, τ_{tr} is the transport scattering time, τ_{12}^* is the effective intersubband scattering time, $\tau_{\mathrm{q}}^{\mathrm{MISO}} = 2\tau_{\mathrm{q1}}\tau_{\mathrm{q2}}/(\tau_{\mathrm{q1}}+\tau_{\mathrm{q2}})$, and τ_{q1} and τ_{q2} are the electron quantum lifetimes in the first and second subbands, respectively. It follows from Eq. (4) that the dependence $\ln(\Delta R_{\mathrm{MISO}}/R_0)$ on $1/B$ is linear, and its slope is determined by the $\tau_{\mathrm{q}}^{\mathrm{MISO}}$ value; i.e., Eq. (4) predicts the linear behavior of Dingle plots for MIS oscillations.

Dingle plots for SdH oscillations in the GaAs/AlGaAs heterojunction with two occupied subbands E_1 and E_2 were studied in [14]. It was shown that they are described by linear dependences, and the quantities τ_{q1} and τ_{q2}, which are calculated from their slope, are significantly different. Dingle plots for MIS oscillations were studied only in GaAs quantum wells with AlAs/GaAs lateral superlattice barriers [19–22]. It was also established that the dependences $\ln(\Delta R_{\mathrm{MISO}}/R_0)$ on $1/B$ in such heterostructures at $T < 10$ K are nonlinear. The origins for the discovered nonlinearity are still debatable. Here, we report the experimental results for dependences $\Delta R_{\mathrm{MISO}}(1/B)$ in GaAs quantum wells with the Δ_{12} value varying from 22.7 to 1.44 meV. These data are analyzed within a model that takes into account the role of large-scale fluctuations Δ_{12} in the suppression of MIS oscillations.

In this work, we studied symmetrically doped GaAs quantum wells with widths of 22, 26, 30, 36, and 46 nm. Short-period AlAs/GaAs superlattices were used as side barriers to quantum wells [23, 24]. The heterostructures were grown by molecular beam epitaxy on (100) GaAs substrates. Samples for magnetotransport measurements were Hall bars with the length $L = 450$ μm and width $W = 50$ μm equipped with Schottky field-effect gates. The studies were carried out at a temperature of $T = 4.2$ K in magnetic fields of $B < 2$ T. The resistances R_{xx} and R_{xy} were measured in a linear mode with an alternating electric current with a frequency of 888 Hz and an amplitude below 1 μA. The total electron density in quantum wells n_{T} was calculated from the resistance R_{xy} in a magnetic field of $B = 0.5$ T. The mobility μ was calculated from n_{T} and R_0. The parameters of studied samples are given in Table 1.

Figure 1 shows a typical $R_{xx}(B)/R_0$ dependence for a "narrow" quantum well ($d_{\mathrm{SQW}} = 22$ nm), which is a "single-layer" two-subband electron system. The Δ_{12} value in this case is mainly determined by the width of the quantum well. The classical positive magnetoresistance is manifested in the narrow quantum well in the fields of $B < 0.1$ T [20]; the MIS oscillations then follow, which coexist with SdH oscillations in the fields

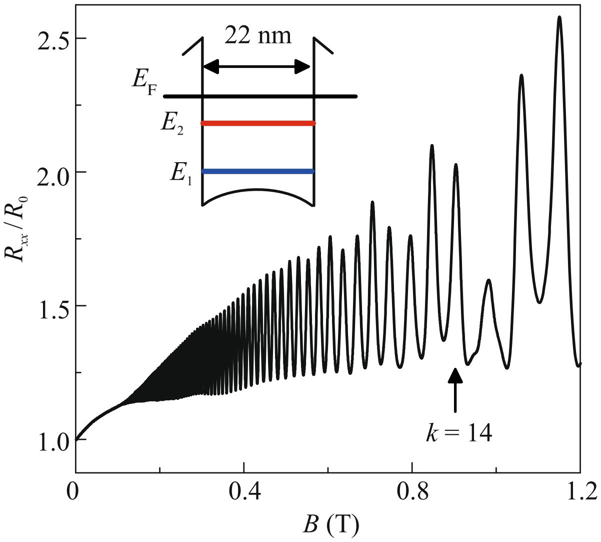

Fig. 1. (Color online) Magnetic field dependence of $R_{xx}(B)/R_0$ at $T = 4.2$ K for a narrow quantum well ($d_{SQW} = 22$ nm). The arrow indicates the position of the maximum for $k = 14$. The inset shows the schematic of the narrow quantum well.

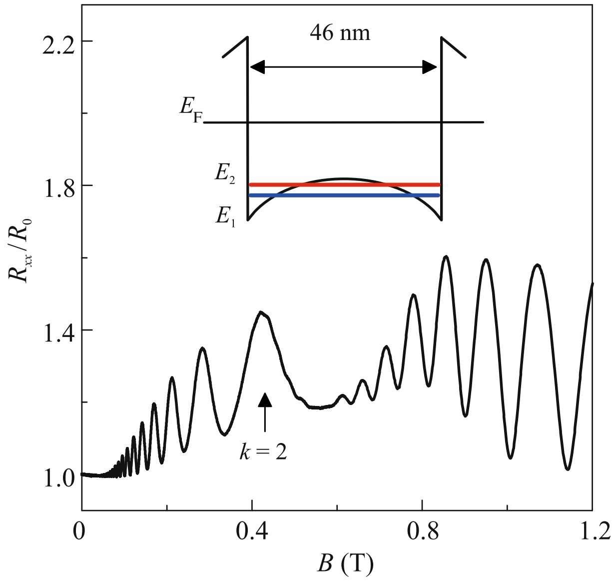

Fig. 2. (Color online) Magnetic field dependence of $R_{xx}(B)/R_0$ at $T = 4.2$ K for a wide quantum well ($d_{SQW} = 46$ nm). The arrow indicates the position of the maximum for $k = 2$. The inset shows the schematic image of the wide quantum well.

of $B > 0.5$ T. An insignificant modulation of the amplitude of MIS oscillations in the range from 0.1 to 0.5 T is associated with the interference of magnetophonon and MIS oscillations [25, 26]. The Fourier analysis of such dependences gives three frequencies corresponding to the periods of SdH oscillations in the first and second subbands, as well as the period of MIS oscillations. The electron densities in subbands in narrow quantum wells differ strongly; therefore, the quantum times τ_{q1} and τ_{q2} are not equal in the general case. In accordance with Eq. (4), the slope of the Dingle plot in this case is determined by the quantity $\tau_q^{MISO} = 2\tau_{q1}\tau_{q2}/(\tau_{q1}+\tau_{q2})$, and the $\Delta R_{MISO}/R_0$ value at $1/B = 0$ is A_{MISO}.

Figure 2 shows a typical $R_{xx}(B)/R_0$ dependence at $T = 4.2$ K for a "wide" quantum well ($d_{SQW} = 46$ nm), which is a "two-layer" two-subband electron system. The intersubband splitting in this case is mainly determined by the tunneling coupling between the electron layers separated by a smooth barrier arising due to the electrostatic repulsion of electrons to the heterointerfaces of the quantum well [27]. In wide quantum wells, in contrast to narrow ones, the classical positive magnetoresistance is not exhibited because the electron density and mobility are approximately the same in both subbands. In such a system, the quantum times in the subbands can be considered to be close, $\tau_{q1} \approx \tau_{q2}$. The slope of the Dingle plot for MIS oscillations in this case is determined by the value $\tau_q^{MISO} = \tau_q$, and the $\Delta R_{MISO}/R_0$ value at $1/B = 0$ is $A_{MISO} = 1$ [18].

Figure 3a demonstrates the behavior of oscillating components of experimental dependences $R_{xx}(1/B)/R_0$ for wide and narrow quantum wells in the region of $1/B > 4$ T^{-1}. Only MIS oscillations are present in this region. It can be seen that MIS oscillations decay faster with the increase in $1/B$ in the narrow quantum well. According to Eq. (4), this means that τ_q^{MISO} in the wide quantum well should be longer than that in the narrow one. Figure 3b shows the results of Fourier analysis of dependences $\Delta R_{xx}(1/B)/R_0$ in the region of $1/B > 4$ T^{-1}. In this region, there is only a peak for MIS oscillations whose frequencies are determined by the Δ_{12} value. It can be seen that the width of the peak for the narrow quantum well is larger than that for the wide one. This is in agreement with the assumption that τ_q^{MISO} in the narrow quantum well is smaller than that in the wide one. Figure 4 shows dependences $\Delta R_{MISO}(1/B)/R_0$ for the (a) wide and (b) narrow quantum wells. The experimental dependences are nonlinear, which is inconsistent with Eq. (4). This means that the different behaviors of dependences $\Delta R_{xx}(1/B)/R_0$ for the narrow and wide quantum wells cannot be explained only by the difference of τ_q^{MISO} values in them.

According to Eq. (3), the period of MIS oscillations is determined by the Δ_{12} value. If large-scale fluctuations of Δ_{12} are present in the system, they should lead to additional suppression of the amplitude

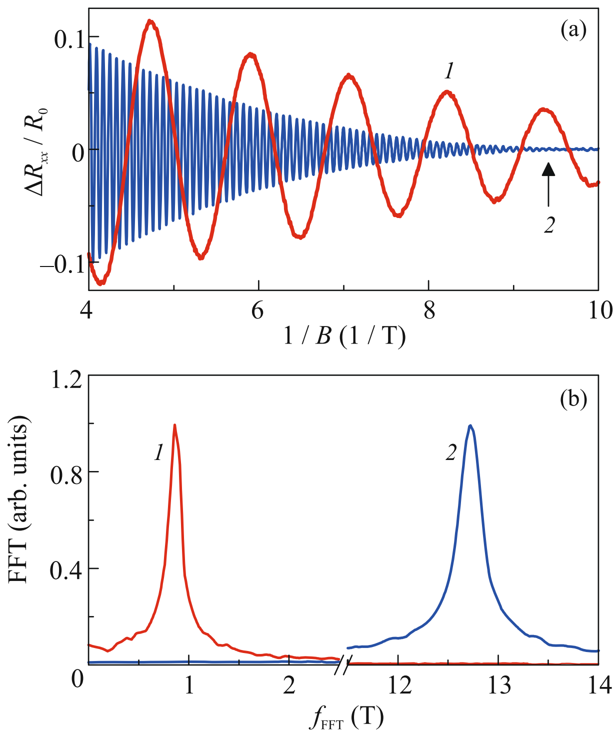

Fig. 3. (Color online) (a) Inverse magnetic field dependences of $\Delta R_{xx}(1/B)/R_0$ at $T = 4.2$ K for quantum wells with a width of (line *1*) 46 and (line *2*) 22 nm. (b) Fourier spectra of the dependences $\Delta R_{xx}(1/B)/R_0$ for quantum wells with a width of (line *1*) 46 and (line *2*) 22 nm.

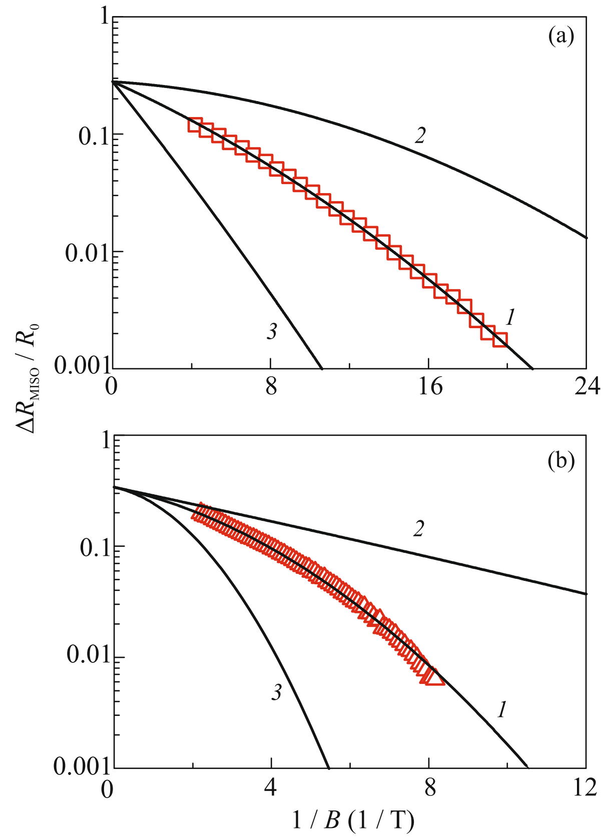

Fig. 4. (Color online) Inverse magnetic field dependences of $\Delta R_{\rm MISO}/R_0$ measured at $T = 4.2$ K for the quantum wells with the widths of $d_{\rm SQW}$ = (a, squares) 46 and (b, triangles) 22 nm in comparison with theoretical lines by Eq. (5) with the parameters (a) $A_{\rm MISO} = 0.28$, $\hbar/\delta\Delta_{12} =$ 18.5 ps, and $\tau_q^{\rm MISO}$ = (*1*) 14, (*2*) 100, and (*3*) 5 ps and (b) $A_{\rm MISO} = 0.34$, $\tau_q^{\rm MISO} = 14$ ps, and $\hbar/\delta\Delta_{12}$ = (*1*) 6.4, (*2*) 40, and (*3*) 3 ps.

of MIS oscillations because of averaging over the sample area. Following the approach proposed in [11], we assume that the distribution of large-scale fluctuations of Δ_{12} is Gaussian. In this case, the inhomogeneous damping factor appears as an additional factor on the left-hand side of Eq. (4), and the amplitude of MIS oscillations is expressed by the formula

$$\Delta R_{\rm MISO} = R_0 A_{\rm MISO} \exp(-2\pi/\omega_c \tau_q^{\rm MISO}) \times \exp[-(\pi\delta\Delta_{12}/\hbar\omega_c)^2], \quad (5)$$

where $\delta\Delta_{12}$ is the average amplitude of fluctuations of Δ_{12}. Formula (5) shows that Dingle plots for MIS oscillations are nonlinear when inhomogeneous broadening is taken into account.

It is seen in Fig. 4a that the theoretical dependence $\Delta R_{\rm MISO}(1/B)/R_0$ calculated by Eq. (5) and shown by line *1* is in good agreement with experimental data for the wide quantum well ($d_{\rm SQW}$ = 46 nm). Theoretical dependences *2* and *3* in this figure demonstrate the role of the quantity $\tau_q^{\rm MISO}$ in the suppression of the amplitude of MIS oscillations. When the $\tau_q^{\rm MISO}$ value, which describes the homogeneous (collisional) broadening of Landau levels, considerably exceeds the quantity $\hbar/\delta\Delta_{12}$, the suppression of $\Delta R_{\rm MISO}$ with increasing $1/B$ is mainly determined by large-scale fluctuations Δ_{12}. When $\tau_q^{\rm MISO} \ll \hbar/\delta\Delta_{12}$, the role of large-scale fluctuations $\delta\Delta_{12}$ in the suppression of the amplitude of MIS oscillations can be ignored.

Figure 4b demonstrates that the theoretical dependence for the narrow quantum well ($d_{\rm SQW}$ = 22 nm) shown by line *1* is mostly in agreement with the experimental data, but not in the entire studied range of inverse magnetic fields. In the range of $1/B > 7$ T^{-1}, values calculated by Eq. (5) slightly differ from experimental $\Delta R_{\rm MISO}$ values. We currently cannot explain this difference. Theoretical dependences *2* and *3* in this figure demonstrate the effect of large-scale fluctuations $\delta\Delta_{12}$ on the behavior of Dingle plots for MIS oscillations at the fixed $\tau_q^{\rm MISO}$ value.

Figure 5a demonstrates the effect of the gate voltage V_g on the behavior of dependences $R_{xx}(B)/R_0$. The application of the negative gate voltage V_g leads to an increase in the classical positive magnetoresistance, as well as to a decrease in the amplitude and an increase in the frequency of MIS oscillations. Such effect of V_g on the behavior of $R_{xx}(B)/R_0$ is quite expectable. First of all, the negative gate voltage V_g squeezes out X electrons localized in AlAs layers adjacent to the Si-δ-doped layer located between the Schottky gate and the quantum well [8]. The decrease in the concentration of compact dipoles, which are formed by positively charged donors in the Si-δ-doped layer and X electrons in AlAs layers, increases the scattering of electrons on the random potential of the dopant, which suppresses MIS oscillations. The increase in the frequency of MIS oscillations under the effect of V_g indicates the increase in Δ_{12} due to the "tilt" of the quantum well. Moreover, the negative gate voltage V_g increases the difference between the mobilities in the subbands, which is a reason for an increase in the classical positive magnetoresistance.

Figure 5b demonstrates the effect of V_g on the dependence of $\Delta R_{\rm MISO}/R_0$ on $1/B$. Experimental Dingle plots for the quantum well with the width of 26 nm for different V_g values are in complete agreement with theoretical dependences of $\Delta R_{\rm MISO}/R_0$ on $1/B$ calculated by Eq. (5). The observed agreement indicates that V_g in the studied quantum well changes only the $\tau_q^{\rm MISO}$ value, but does not affect the average amplitude of large-scale fluctuations of the intersubband energy splitting ($\delta\Delta_{12}$). The $\delta\Delta_{12}$ values obtained from the comparison of experimental and calculated dependences $\Delta R_{\rm MISO}(1/B)/R_0$ are given in Table 1. These data indicate that the quantity $\delta\Delta_{12}$ increases with the decrease in $d_{\rm SQW}$. Such behavior makes it possible to suppose that one of the main origins of large-scale fluctuations of Δ_{12} is the presence of large-scale fluctuations of $d_{\rm SQW}$.

It is reasonable to assume that the average value of width fluctuations ($\delta d_{\rm SQW}$) of the studied GaAs quantum wells, which have the same design of side barriers and were grown in the same technological modes, is the same for different $d_{\rm SQW}$ values. In this case, $\delta d_{\rm SQW}/d_{\rm SQW}$ increases with decreasing $d_{\rm SQW}$. Thus, large-scale fluctuations of $d_{\rm SQW}$ with the same average value lead to the higher $\delta\Delta_{12}$ values in the wells with the smaller width. This dependence of $\delta\Delta_{12}$ on $d_{\rm SQW}$ is really observed. There is no effect of V_g on the quantity $\delta\Delta_{12}$, despite the increase in Δ_{12} with $|V_g|$, which is in agreement with the fact that large-scale fluctuations $d_{\rm SQW}$ in narrow quantum wells are primarily responsible for the inhomogeneous broadening of Landau levels. Note that large-scale fluctuations of different parameters of semiconductor layers in selectively doped heterostructures, including $d_{\rm SQW}$, lead to the correlated change in the energies E_1 and E_2 in the quantum well. This means that $\delta\Delta_{12} = |\delta E_2 - \delta E_1|$ in principle may be zero.

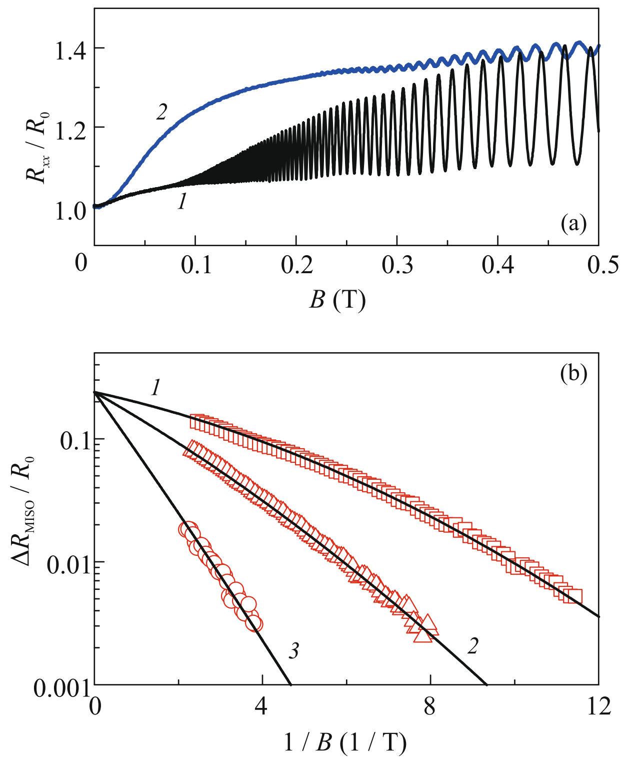

Fig. 5. (Color online) (a) Magnetic field dependences of $R_{xx}(B)/R_0$ measured for the quantum well with the width $d_{\rm SQW}$ = 26 nm at T = 4.2 K and the gate voltage V_g = (*1*) 0 and (*2*) −1.2 V. (b) Inverse magnetic field dependences of $\Delta R_{\rm MISO}/R_0$ measured for the quantum well with the width $d_{\rm SQW}$ = 26 nm at T = 4.2 K and the gate voltage V_g = (squares) 0, (triangles) −0.94, and (circles) −1.2 V in comparison with theoretical lines by Eq. (5) with the parameters $A_{\rm MISO} = 0.24$, $\hbar/\delta\Delta_{12} = 10$ ps, and $\tau_q^{\rm MISO}$ = (*1*) 14, (*2*) 5.4, and (*3*) 2.2 ps.

Such a situation is possible in wide quantum wells, since Δ_{12} in them is mainly determined by the tunnel coupling between the electron layers rather than by the $d_{\rm SQW}$ value. In this case, fluctuations of $d_{\rm SQW}$ should not lead to significant fluctuations $\delta\Delta_{12}$. However, the experiment shows that large-scale fluctuations Δ_{12} are also present in wide quantum wells, although their magnitude is much less than that in narrow ones.

In narrow wells with high barriers, the fluctuation of Δ_{12} caused by fluctuations of $d_{\rm SQW}$ is estimated as $\delta\Delta_{12} \sim 2\Delta_{12}\delta d_{\rm SQW}/d_{\rm SQW}$. It follows that large-scale fluctuations $d_{\rm SQW}$ lead to fluctuations Δ_{12} in narrow quantum wells, and in accordance with Eq. (5), this is a reason for the nonlinear behavior of Dingle plots for MIS oscillations.

The δd_{SQW} values calculated from the $\delta\Delta_{12}$ values are given in Table 1. It follows from Table 1 that $\delta d_{SQW} = 0.51$ nm for the wide quantum well does not agree with the δd_{SQW} values for narrower wells. This means that the physical origins of large-scale fluctuations of Δ_{12} are different in narrow and wide quantum wells. We believe that this behavior is natural because the splitting Δ_{12} in these wells is due to different physical mechanisms.

One of the origins of large-scale fluctuations of Δ_{12} in the studied wide quantum well can be the presence of large-scale "lakes" of X electrons, which can arise in AlAs layers adjacent to the Si-δ-doped layers. Since the spatial distribution of these lakes in the left- and right-hand barriers to GaAs quantum well is random, the $\delta\Delta_{12} = |\delta E_2 - \delta E_1|$ value in this case is nonzero, despite the symmetric location of δ-doping layers. The random distribution of large lakes in the left- and right-hand barriers leads to large-scale fluctuations of the tilt of the wide quantum well and, accordingly, to large-scale fluctuations of Δ_{12}.

To summarize, we have studied the effect of the inhomogeneous broadening of Landau levels on the amplitude of magneto-intersubband oscillations in quantum wells with two occupied energy subbands. It has been shown that large-scale fluctuations of the intersubband splitting are responsible for the nonlinearity of Dingle plots for MIS oscillations. An analytical expression has been obtained to take into account large-scale fluctuations of intersubband splitting in dependences of the amplitude of MIS oscillations on the inverse magnetic field. The comparison of theory and experiment has made it possible to determine the average amplitude of fluctuations of the intersubband splitting $\delta\Delta_{12}$ in the studied GaAs quantum wells with AlAs/GaAs lateral superlattice barriers.

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