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# Input shaping for travelling wave generation

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## Abstract

Travelling wave patterns observed in the movement of certain aquatic animals has motivated research in the modification of flow behavior, especially to deal with boundary layer separation in airplane wings. Research has shown that inducing travelling waves on the top surface of the wing can generate sufficient momentum to prevent boundary layer separation without increasing the drag. Due to this effect of propagating waves on the aerodynamics, generation of travelling waves on solid surfaces is being widely studied. Recently, methods such as two-mode excitation, active sink and impedance matching have shown promise in generation of uniform travelling waves in solids with the help of piezo electric actuators. Unfortunately, there are some challenges involved in the experimental application of these methods. Although these techniques have shown to be adequate in laboratory settings, they require laborious tuning procedures which do not guarantee desired trajectories and are followed in light of interference from unwanted modes and their transients. Some methods rely on selective mode excitation, which can cause interference from unwanted modes if the transient behavior of the system is not accounted for. Feed-forward input shaping control methods are proposed that augment the open-loop piezo actuation method (two-mode excitation) and provide a more robust method for generating uniform travelling waves. The input shaping control alters the reference signal such that the parasitic behavior of unnecessary modes is cancelled out. The combination of the mode suppression and selective mode excitation through input shaping is verified experimentally for generation of a smooth travelling waves in finite structures.

**Keywords:** travelling wave, input shaping, mode suppression, feed-forward control, piezo-electric actuation

## 1. Introduction

Boundary layer separation [20] is a phenomenon where the flow separates from the solid surface when it loses momentum. The flow separation is widely studied in the aerodynamic industry due to its negative effects on the lift and drag causing stall and buffeting. Traditionally, vortex generators are used to deal with flow separation. Vortex generators create a small vortex by removing a chunk of the slow moving boundary layer, which delays the flow separation [3] at the cost

of increased drag. Recently, it was discovered that inducing travelling waves [4] on the surface of the solid can generate sufficient momentum to deal with flow separation without increased drag. Since this discovery, there has been a boost in the research for generation and control of travelling waves on solids.

Despite numerous studies on the effects of travelling waves with respect to aerodynamics, the literature on controlling travelling waves over structures is somewhat limited. Methods such as two-mode excitation (TME) [11], active sink [16] and impedance matching [19] have shown promising results, but the open loop nature of these methods can create unsteady waves in practical application. Problems such as

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parasitic mode behavior [13] can lead to poor quality waves with higher standing wave components. Traditionally, closed-loop or feedback control methods can be used to deal with this problem. But, closed loop control on distributed parameter systems requires proper sensing and a lot of measurements to achieve good results. Sensing alone is a huge concern when dealing with distributed parameter systems, as the closed loop control requires the temporal information rather than spatial which is harder to measure.

Input shaping methods to suppress modes have been widely studied in the past [6, 12, 15]. In this paper such open loop input shaping methods are incorporated in the travelling wave generation methods in order to simplify the tracking of input signal while eliminating the parasitic mode behavior. The two-mode excitation method was augmented with the input shaping techniques to generate uniform travelling waves with low transients. A fixed-fixed spring steel beam with piezo electric patches was used for experimental verification of the performance of the input shaping methods.

The paper progresses by a detailed discussion on the effect of parasitic mode behavior and transients in section 3. An example with the traditional two-mode excitation method is shown to illustrate the problems caused due to mode interference. Section 4 discusses the input shaping methods, such as time delay filter (TDF) and periodic signal tracker (PST), for generating uniform travelling waves by augmenting them to two-mode actuation in an open loop setting. It further delves into the implementation of the time delay filter [12] and illustrates its performance in section 4.1. Section 4.2 implements the PST [18] which compensates for the difference in phase between the two input. The performance of the two input shapers is further verified through experimental tests discussed in section 5.

## 2. Travelling wave generation

A popular method for generating traveling waves in solids is two-mode excitation. In two-mode excitation two neighbouring modes are excited with the same frequency but with a phase difference to obtain a travelling wave. It is a preferred method due to easy implementation and configuration. Furthermore, it provides easy manoeuvrability of the travelling wave just by adjusting the magnitude and phase relationship of the supplied voltage. The configuration of the excitation frequency and the phase difference is extremely crucial in generating a good quality travelling wave. Conventionally the midpoint between the two modes is chosen as the excitation frequency and the phase of  $90^\circ$  is selected to achieve a good travelling wave [11].

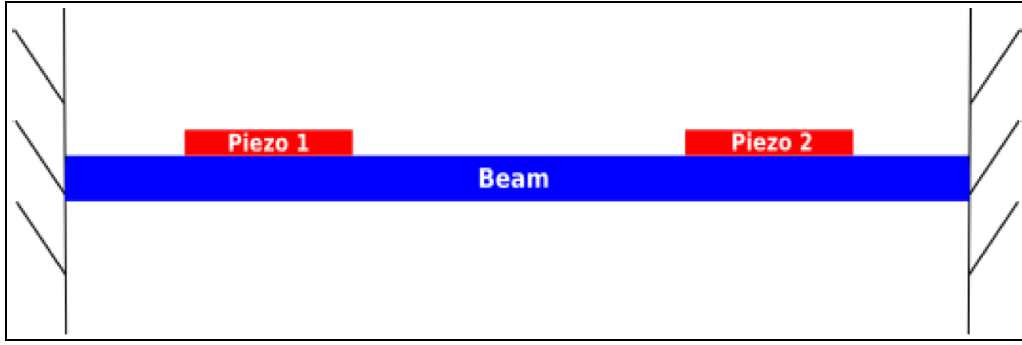
Figure 1 represents the travelling wave setup for a fixed-fixed beam. The two piezos are placed at the anti-nodes of the two modes closest to the excitation frequency to generate higher amplitudes [8]. The travelling waves generated from two-mode excitation are not pure travelling waves, i.e. the waves are a mixture of standing and travelling waves. The

ratio of travelling and standing wave components in the generated wave can be quantified by using the 2D-FFT [7] method developed by Hani-Bani *et al* [2] and therefore, the wave can be optimized by configuring the actuation frequency and the phase difference for higher ratio of travelling wave components as illustrated in [8]. Often the dynamic behaviour of two-mode excitation is ignored and only the steady state is studied. Negligence of the transients and parasitic mode behaviour can affect the quality and uniformity of the travelling wave. Section 3 further details the dynamic behaviour of the travelling wave obtained from the two-mode excitation and the effects of parasitic mode behaviour.

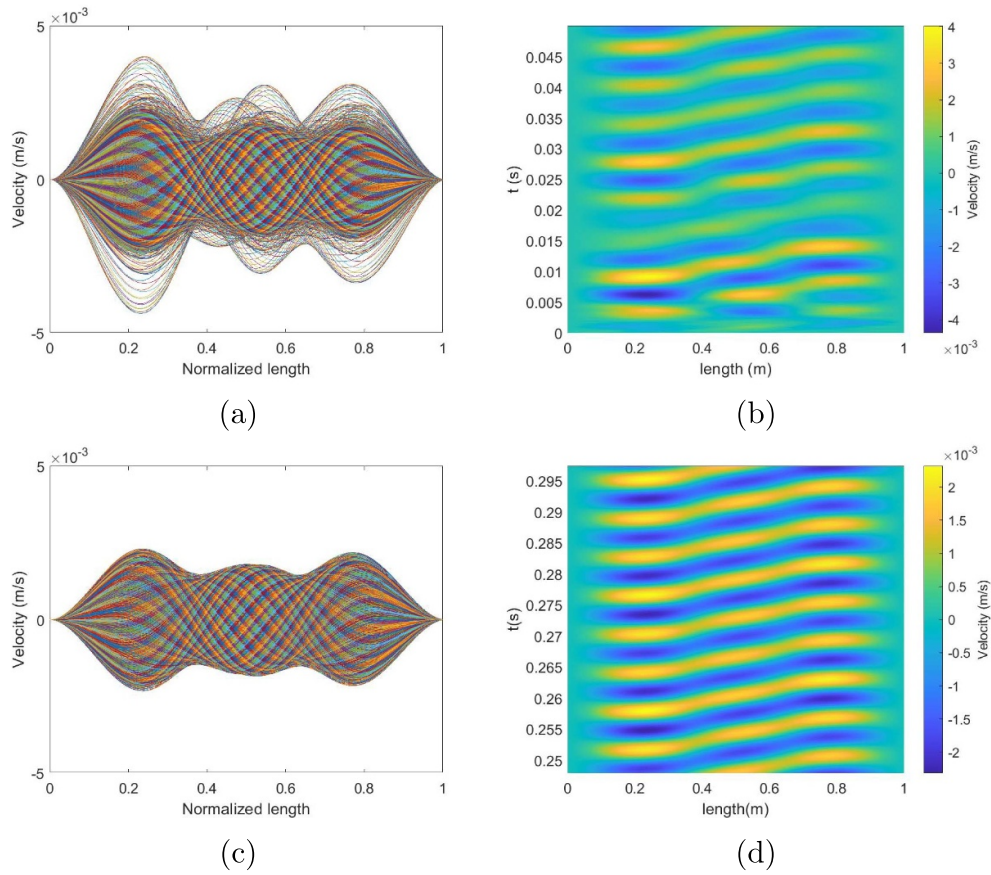
## 3. Parasitic mode behaviour

A major issue with open-loop piezo actuation methods is the parasitic interference of the unwanted modes. In the case of two-mode excitation where the excitation frequency lies between two modes, parasitic mode behaviour is a major concern as the interfering modes and the transients caused by them can lead to low quality of travelling waves. The existing methods for generation of travelling waves such as two-mode excitation and active sink do not account for the transients generated at the time of actuation. It can be debated that the transients that occur at the beginning of actuation do not affect the waves as they die out quickly. But, in reality the transients generated from traditional two mode excitation delay the travelling wave convergence due to prevailing standing wave components as demonstrated in figure 2(b). In travelling wave applications that deal with boundary layer separation and drag reduction, the travelling waves need to be maintained within a certain range of amplitudes for optimal results as addressed by Akbarzadeh *et al* [1]. Non-uniform waves with high transients can sometimes exceed these bounds leading to poor performance. To study the negative effects of parasitic mode behaviour and transients, dynamic and steady state response of two mode excitation on a fixed-fixed spring steel beam was simulated. The beam was modelled using Euler Bernoulli equations [9] with 4% damping. The actuators were excited at 160.8 Hz between the second (108.6 Hz) and third (213 Hz) natural frequency of the beam. Figure 2 depicts the dynamic (figures 2(a) and (b)) and steady state response (figures 2(c) and (d)) of two mode excitation. The steady-state two mode excitation has a smooth uniform travelling wave (figure 2(c)). But when the transients are accounted for the resulting travelling wave is distorted with significantly higher amplitudes (figure 2(a)).

A better way of observing the waves is from the heat maps, as represented in figures 2(b) and (d). The heat map represents the top view of the surface plot of the spatio-temporal displacement of a beam under actuation. A checkered pattern is observed when the actuation results in a standing wave. This is due to the isolated motion of the beam between the stationary nodes. In the case of a travelling wave, the nodes are absent and the displacement occurs in the form of a translational motion, hence the diagonal lines. The steady state



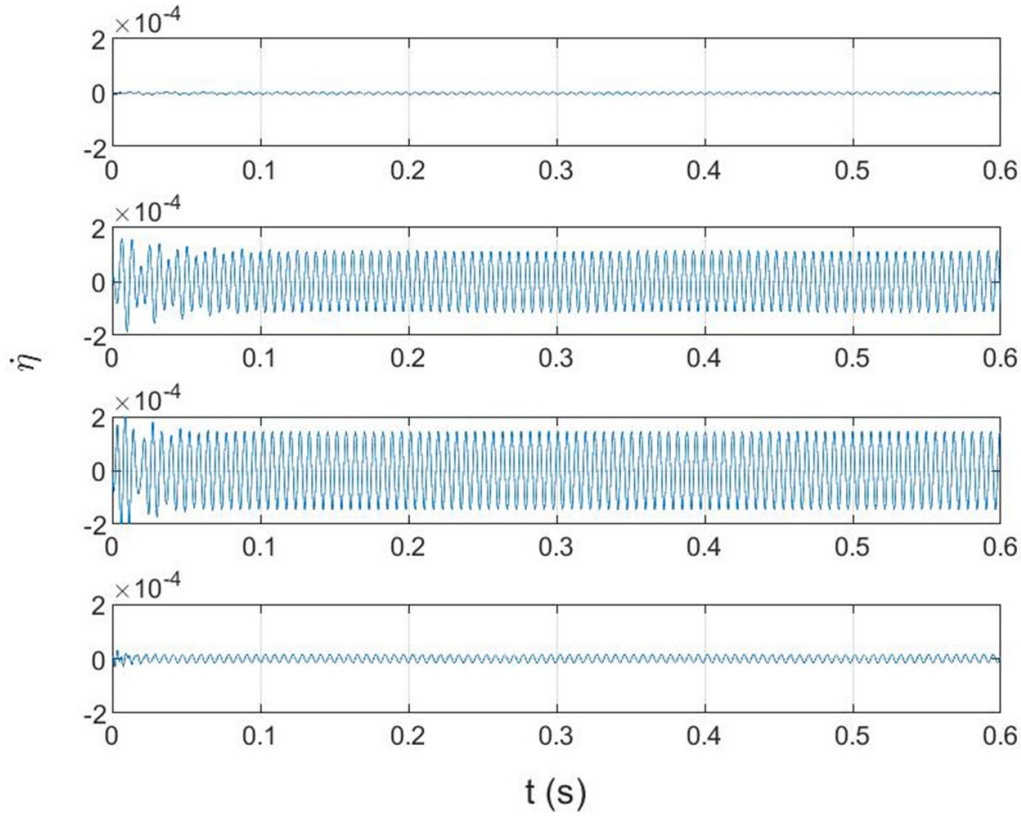
**Figure 1.** Beam setup for two mode excitation.



**Figure 2.** (a) Dynamic response, (b) transient heat map, (c) steady state response, (d) steady state heat map.

response (figure 2(d)) shows diagonal lines representing the uniform travelling waves. The transient response however has a mixture of diagonal lines and checkered pattern indicating a mixture of travelling and standing wave components. Therefore, presence of transients can also give rise to prevalent standing wave components. The excitation of the neighbouring modes can be observed in figure (3) in the form of temporal velocity. Even though the actuators were excited between the second and third mode, unwanted modes such as the fourth and first were also slightly excited. Of course, the second and the third mode have more significant contribution but so do transients associated with them. The modal amplitude of the

3rd mode is higher than that of the 2nd mode due to the placement of the actuators. The position of the actuators favours the 3rd mode more than the 2nd, which can also be observed from the shape of the waves in figures 2(a) and (c). The interference of these unwanted modes and the transients leads to unwanted oscillation in generated wave. The closer the modes are to the excitation frequency the higher the impact they have on the generated wave. Therefore, to counter the parasitic behaviour and the transients generated from them, open-loop techniques (two-mode excitation) can be augmented with input shaping techniques to cancel the poles of the beam such that the interference from the unwanted poles is eliminated.



**Figure 3.** (a) Temporal response of modes 1–4 from two-mode excitation.

#### 4. Input shaping

The ‘Posicast’ technique was developed by Smith [6] to generate non oscillatory response from a lightly damped system. The non-oscillatory response was realised through exciting two transient oscillations such that it resulted in constructive cancellations of oscillations. The input shaping methods [6, 18] presented in this section were developed for the same purpose of generating non-oscillatory response. The two-mode excitation method however, exploits the oscillatory behaviour to generate travelling wave, i.e., the response of the system has to be oscillatory. Therefore, the input shaping method is used to cancel the dominant modes (unwanted modes) such that the temporal response of the wave tracks the input signal from two piezos without the interference from unwanted modes. The implementation of the input shaper for a piezo actuated beam can be achieved by considering the temporal equation of the beam for necessary modes and designing an individual filter for those selected modes:

$$G_i(s) = \frac{\kappa_1 U_1(s) + \kappa_2 U_2(s)}{s^2 + 2\zeta\omega_{ni} + \omega_{ni}^2}, \quad (1)$$

where,

$$\kappa_i = e_{31}z_m b (W'_i(L_e) - W'_i(L_s)). \quad (2)$$

For  $i = 1, 2, \dots, \infty$ , equation (1) represents the temporal transfer function for a single mode of a piezo actuated beam

[4] with two-mode excitation.  $\kappa$  denotes the piezo parameters and  $U$  denotes the input voltages for the leading and trailing edge piezos in two-mode excitation. The parameter  $L_e$  and  $L_s$  represent the start and end indices of the piezo on the beam. The input shapers designed for individual modes are augmented in series with the input voltage (figure 4) for each piezo to cancel mode interference.

The suppression of the modes is achieved through implementing time delays based on the natural frequency and damping of the system. The time delays in the controller ensure that the zeros of the controller are located at the poles of the plant. As a result, the cancelled poles of the plant have no influence on the output. The feed-forward mechanism of input shaping alters the reference signal such that the oscillatory poles of the system are quiescent and therefore, the system response tracks the input signal. Compared to a closed loop technique, the input shaping method is easier to implement and does not require any sensing. Input shaping purely relies on the information of the system model to achieve the desired response. Two different types of input shapers were tested with the two-mode excitation method to achieve uniform travelling waves. The performance of the input shapers is showcased through simulation of a fixed-fixed spring steel beam model with 4% damping in order to be consistent with the experiments (section 5). The implementation and results of these input shapers are discussed in detail in the following sections.

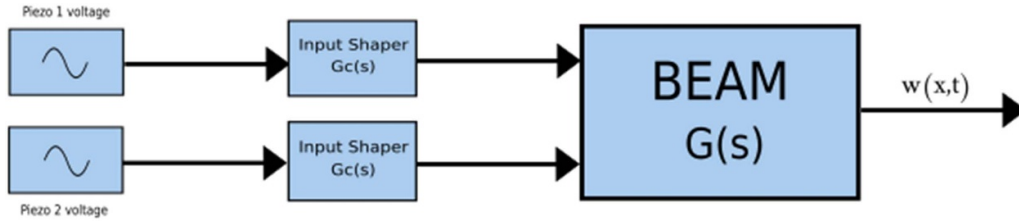


Figure 4. Input shaping control.

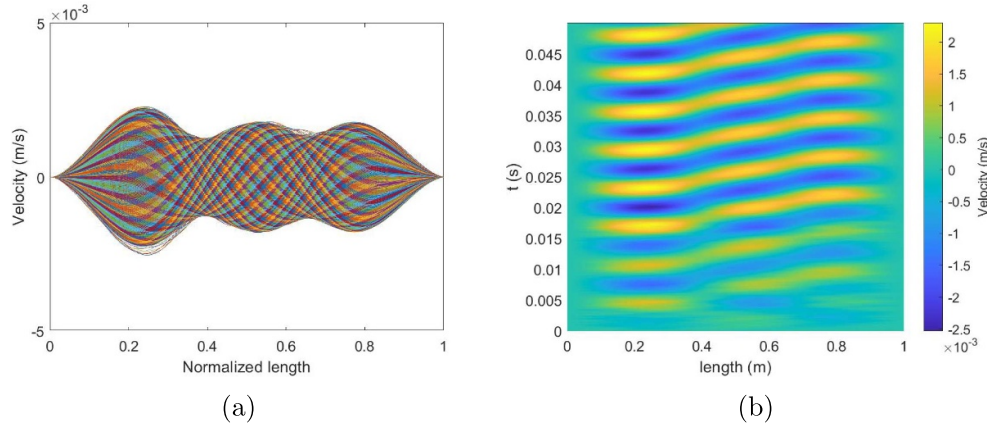


Figure 5. (a) Dynamic response of the time delay filter and (b) heat map of the time delay filter.

#### 4.1. Time delay filter

The time delay filter [12] is a generalisation of the ‘Posicast’ control. The normalised time delay filter for a stable lightly damped second order system, in frequency domain [13], can be represented as,

$$G_c(s) = \frac{A_0}{1 + A_0} + \frac{e^{-sT}}{1 + A_0}, \quad (3)$$

where  $A_0$  is the gain and  $T$  the delay time. The gain and the delay time of the controller can be determined such that the transfer function (equation (3)) of the time delay filter locates zeros at the location of the underdamped poles of the plant. This requirement ensures that the zeros of the input shaper cancel the poles of the system. The closed form solution for gain and time delay for a second order system can be denoted as,

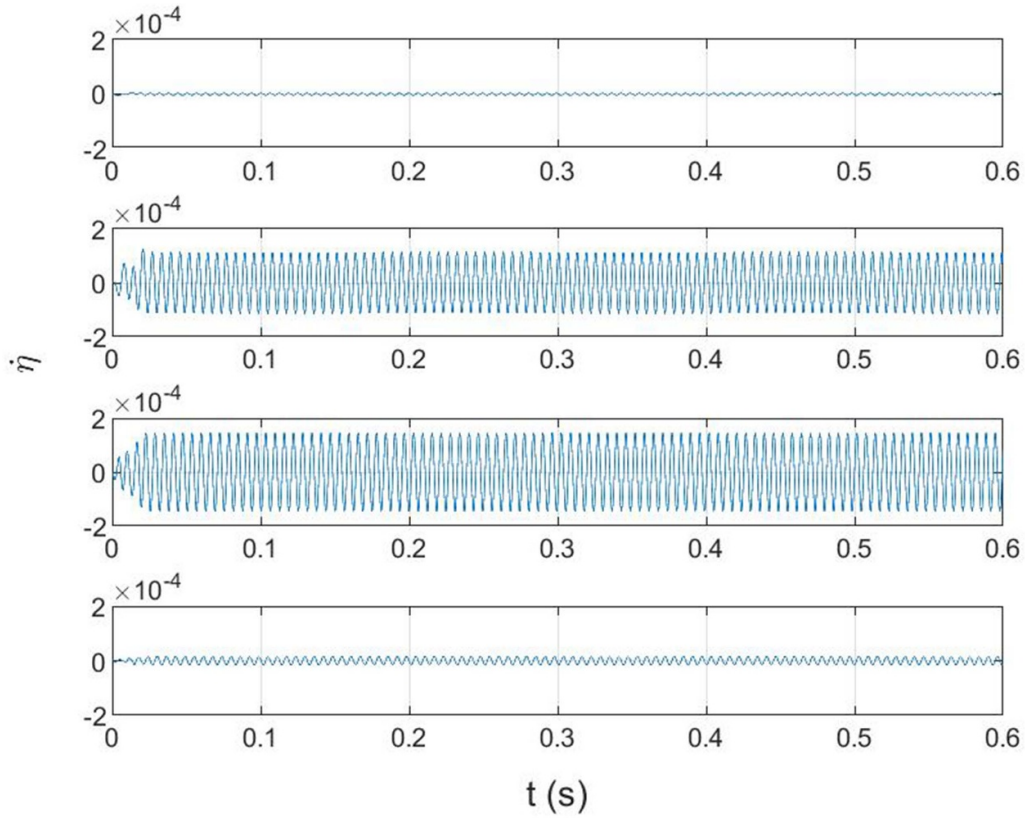
$$A_0 = e^{\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}, \quad (4)$$

$$T = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}, \quad (5)$$

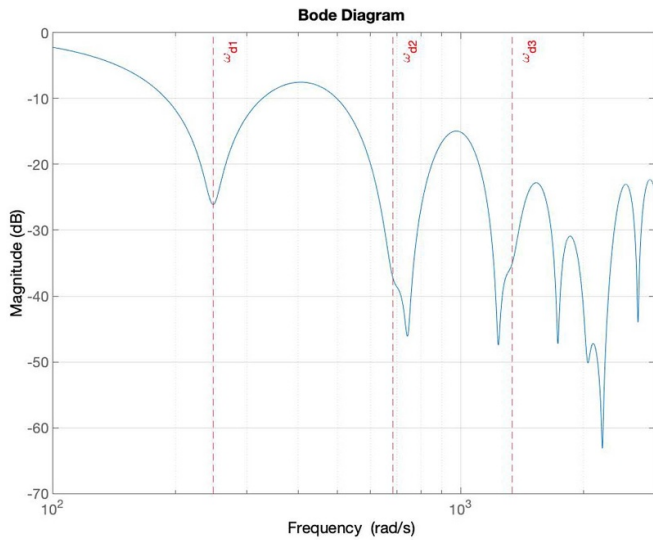
where  $\omega_n$  and  $\zeta$  are the natural frequencies and the damping ratio of the mode of interest. Additionally the gains of the time delay filter can be scaled to satisfy a DC gain of unity. Individual time delay filters for selective  $\omega_n$  and  $\zeta$  can be designed using equations (3)–(5) to suppress the corresponding modes and ensure the plant output tracks the input signal.

Time delay filters were designed to suppress the first four modes of a fixed-fixed spring steel beam. Conventionally, only mode 1 and mode 4 need to be cancelled but since the transients are needed to be accounted, all the modes with significant amplitudes are cancelled. The filters for the first four modes were cascaded in series for both inputs and the following results were obtained.

The two-mode excitation with time delay filter generates a more uniform travelling wave (figure 5(a)) than the traditional two-mode excitation. The time delay filter successfully suppresses the transients with slightly faster convergence to travelling waves (figures 5(a) and (b)) compared to the traditional TME shown in figures 2(a) and (b). The cancellation of the first four modes can be observed in figure 6, as the temporal response for the selected modes is suppressed and is significantly lower than that of the traditional two mode excitation (figure 3). The time delay filter successfully eliminates the parasitic behaviour of the modes by placing the zeros of the filter at the poles of the beam. The magnitude plot of the Bode diagram (figure 7) at the location of damped natural frequency is not exactly zero since the stable pole being cancelled is located to the left of the damped natural frequency in the complex domain. The controllers are designed to cancel the stable poles of the system, therefore the actual zeros of the time-delay filter are coincident with the underdamped poles of the beam. Since two consecutive modes are deliberately excited with a periodic signal to generate a travelling, the contribution of those mode is necessary. The placement of the zeros of the input shaper at the poles of the beam results in low energy transfer to the cancelled modes, ensuring suppression of the transients



**Figure 6.** Temporal response of modes 1–4 with TDF.



**Figure 7.** Frequency response of the TDF  $\zeta = 0.04$  ( $\omega_d$  corresponds to the damped natural frequencies of the beam).

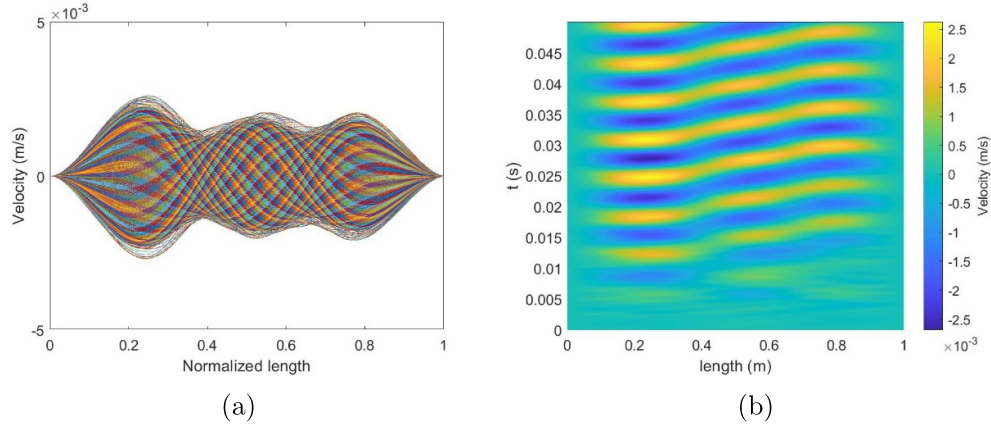
from those modes but not cancelling their contribution in generation of travelling wave generation. Therefore, augmenting two-mode excitation with time delay filters improves the quality of the travelling wave as the parasitic behaviour of the mode is eliminated. The traditional time delay filter ensures perfect tracking for step input but it does not guarantee the same for

periodic signals. The only requirements the time delay filter is designed for is the cancellation of poles and a DC gain of unity. In the case of two-mode excitation for optimal results the second input must have a phase difference of 90 degrees [4]. Although the TDF for the presented beam setup works efficiently, adjusting the location of the actuators can affect the performance of the TDF as the required phase difference for optimal travelling wave can change [8]. Therefore, additional phase constraints that accounts for the phase difference in the inputs must be implemented. Section 4.2 discusses in detail the inclusion of phase constraints in input shapers.

#### 4.2. Periodic signal tracker

Traditional input shapers lack the ability to generate a response with zero-phase error, which leads to residual transients from unwanted modes in the wave generated in section 4.1. In order to improve the performance of the input shaping a zero-phase error [17] and zero-magnitude error [10] design requirements must be considered. The periodic signal tracking control accounts for both the phase and the magnitude constraints in order to ensure perfect tracking of periodic signals for lightly damped systems. According to the Fourier series [5], the input voltage signal for the piezos can be represented as,

$$r(t) = \sum_{k=1}^{N_{\omega_k}} a_k \sin(\omega_k t + \phi_k), \quad (6)$$



**Figure 8.** (a) Dynamic response with periodic signal tracker. (b) PST heat map.

where  $N_{\omega_k}$  is the total number for harmonics,  $\omega$  is the actuation frequency and  $\phi_k$  is the phase shift. Therefore, for such input the periodic signal tracker can be parametrized as [18],

$$G_h(s) = \sum_{k=0}^{N_p+N_{\omega_k}-2} A_k e^{-skT}, \quad (7)$$

where  $N_p$  is the total number of poles of the system. The gains  $A_k$  and delays  $T$  can be determined by cancelling the poles of the system in addition with the phase and gain constraints denoted by,

$$|G_h(s = i\omega_k)| = |G(s = i\omega_k)|^{-1}, \quad (8)$$

$$\angle G_h(s = i\omega_k) + \angle G(s = i\omega_k) = \begin{cases} 0 & \text{if } \omega_k < \omega_n \\ \pi & \text{if } \omega_k > \omega_n \end{cases}. \quad (9)$$

The piezo beam can be modelled with multiple second order transfer functions. Therefore, an input shaper designed for each second order sub-system of distributed parameter system should suffice. From equation (7), the periodic signal tracker for a second order system with single sinusoidal reference can be depicted as,

$$G_h(s) = A_0 + A_1 e^{-sT} + A_2 e^{-2sT}, \quad (10)$$

where the closed form solution [18] of the gains and delay using the constraints in equations (8) and (9) along with the pole cancellation constraint is given as,

$$T = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}, \quad (11)$$

$$\begin{aligned} A_0 &= \Phi_0 A_2 \\ A_1 &= \Phi_1 A_2, \\ A_2^2 &= \Phi_2^2 \end{aligned} \quad (12)$$

where, for a delay term (a) of 1,

$$\Phi_1 = -\frac{\sin(2\omega T)(\omega_n^2 - \omega^2) + 2\zeta\omega\omega_n(\cos(2\omega T) - e^{2\zeta\omega_n T})}{\sin(\omega T)(\omega_n^2 - \omega^2) + 2\zeta\omega\omega_n(\cos(\omega T) - (-1)^a e^{\zeta\omega_n T})}, \quad (13)$$

$$\Phi_0 = -(-1)^a \Phi_1 e^{\zeta\omega_n T} - e^{2\zeta\omega_n T}, \quad (14)$$

$$\Phi_2^2 = \frac{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}{D + 2\Phi_0 \cos(2\omega T) + 2\Phi_1 \cos(\omega T)}, \quad (15)$$

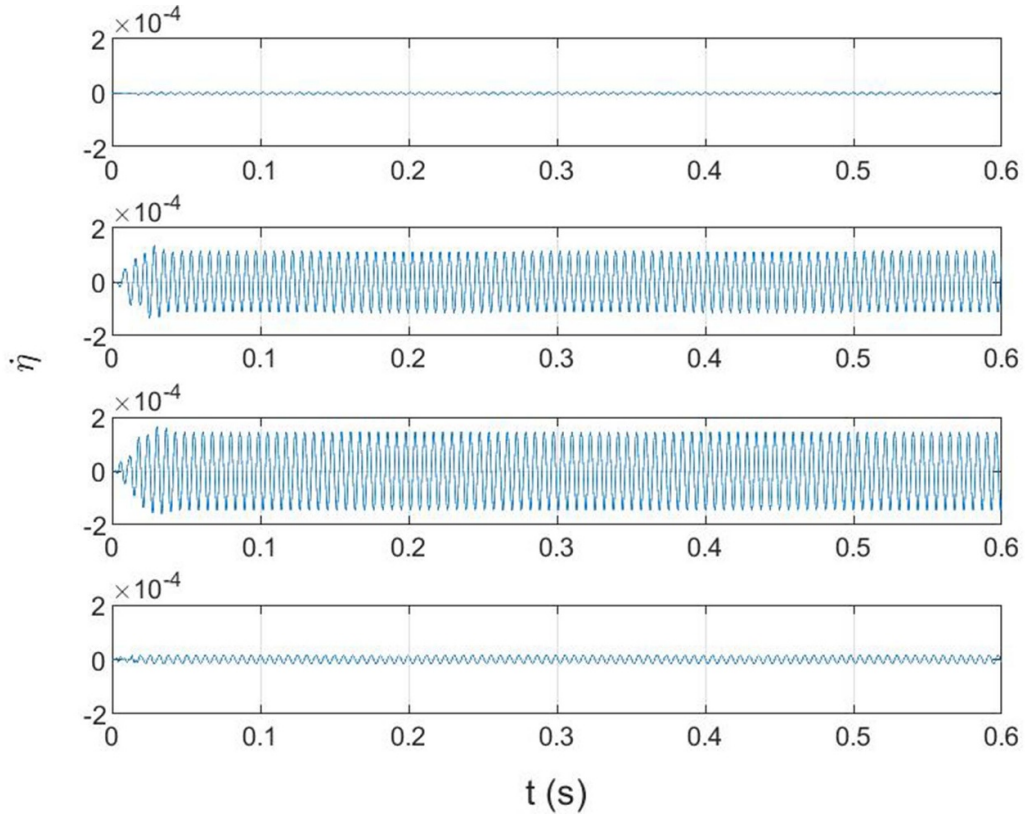
$$D = \omega_n^4 (1 + \Phi_0^2 + \Phi_1^2 + 2\Phi_0 \Phi_1 \cos(\omega T)).$$

Similar to the time delay filter the periodic signal trackers for the piezo inputs were designed for the first four modes of the beam and were implemented for both inputs. The travelling wave generated (figure 8) by incorporating the periodic signal tracker was uniform and converged around the same time as the time delay filter (figure 8(b)). The tracking of the input signal ensured that travelling waves are obtained with no significant transients. The pole cancellation constraints were satisfied in the same manner as the time delay filter. Moreover, the periodic signal tracker had slight overshoots due to the magnitude constraints (figure 9). The magnitude constraints of the periodic signal causes the input to have a slightly high amplitude initially and the phase constraints shifts the phase of inputs to compensate for the phase difference in inputs (figure 11). The generated wave has little to no interference from the unwanted modes and thus the wave is mostly a response of tracking the input for both piezos.

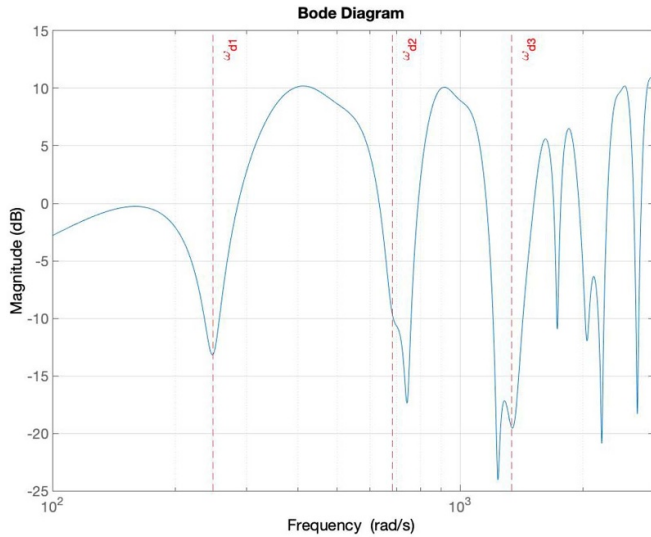
## 5. Experimental verification

The method for travelling wave generation from two mode excitation with input shaping was verified experimentally using a setup described in section 5.1. The time delay filter and the periodic signal tracker were designed based on the natural frequencies obtained from the modal analysis of the beam model (table 1). The damping ratio of 4% for the controller design was selected to ensure consistency with the simulations. Similar to input shapers in sections 4.1 and 4.2 the first four modes of the beam were suppressed.

The experimental results are presented with travelling to standing wave ratio, TSR, which is a value between 0 and 1 representing the quality of the traveling wave—0 being a



**Figure 9.** Temporal response of modes 1–4 with periodic signal tracker.



**Figure 10.** Frequency response with periodic signal tracker with  $\zeta = 0.04$  ( $\omega_d$  corresponds to the damped natural frequencies of the beam).

standing wave, 1 a pure traveling wave. The TSR is assigned based on analysis of the waveform's two dimensional Fast Fourier Transform, where one axis represents frequency, and the other wave number. It is understood that a standing wave is the superposition of forward and backward propagating waves, and that the distinction between forward and backward is sign

of the wave number  $k$  (equations (16) and (17)). By looking at the shifted 2D-FFT the nature of each wave can be deduced following this logic. The analysis looks specifically at the most dominant peaks in each quadrant, and calculates TSR based on the ratio of minimum and maximum peaks equation (18). The following sections describe the setup and the experimental procedures with a detailed discussion on the experimental results:

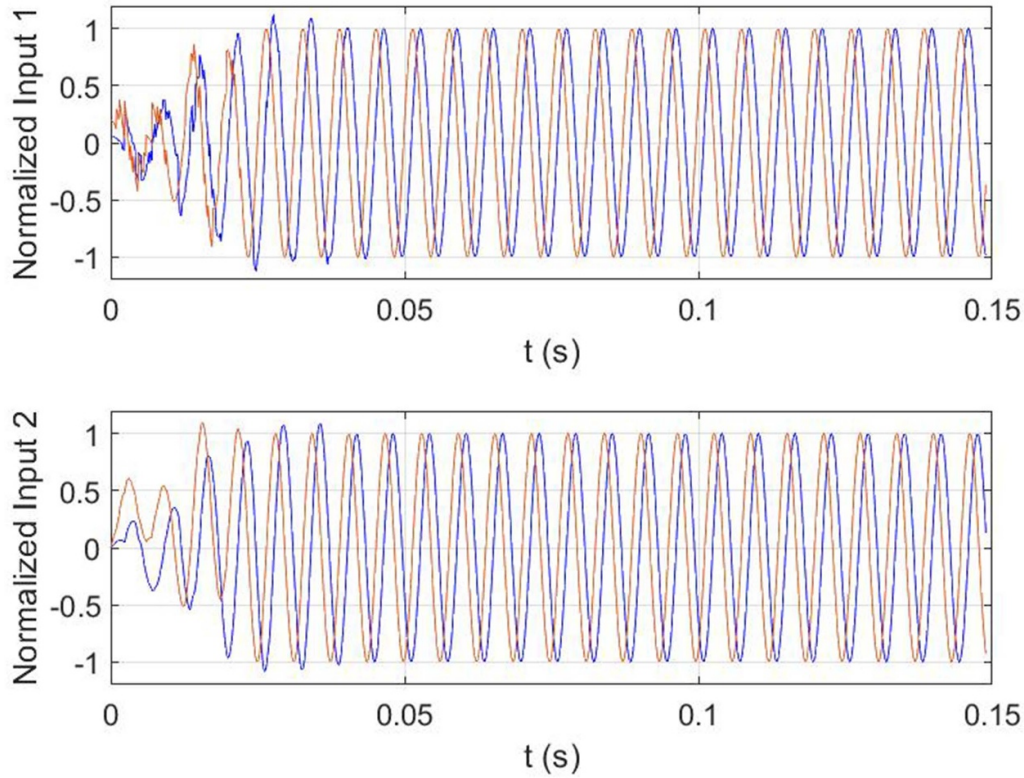
$$w_{fwd}(x, t) = A \exp i(\omega t - kx), \quad (16)$$

$$w_{bck}(x, t) = A \exp i(\omega t + kx), \quad (17)$$

$$TSR = 1 - \frac{\max(peaks)}{\min(peaks)}. \quad (18)$$

### 5.1. Experimental setup

The experimental procedure was performed with a  $228.6 \times 21 \times 0.4064$  mm spring steel beam. The beam was clamped on both ends, as depicted in figure 12(a), to ensure a fixed-fixed boundary condition. Two S128-H5FR-1107YB piezo-electric actuators from Piezo.com were used to excite the beam. The actuators were placed between the outermost anti-nodes of modes 2 and 3 (figure 12(b)) to ensure higher amplitude from these modes during experiments [8]. The input signals for the actuators were generated in SIMULINK. The



**Figure 11.** Actuator input for TDF (red) and PST (blue).

**Table 1.** Natural frequencies of the spring steel beam.

$\omega_{n1}$	$\omega_{n2}$	$\omega_{n3}$	$\omega_{n4}$
39.41 Hz	108.65 Hz	213 Hz	352.1 Hz

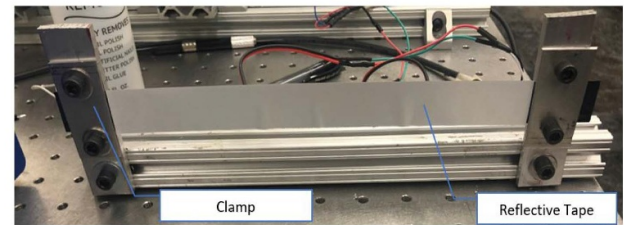
signals were then fed to the amplifiers, to attain a gain of 25 V, through a DAQ device.

In the case of traditional two mode excitation, two sine signals with frequency of 160.8 Hz and a phase difference of  $90^\circ$  were used as the actuating signals. For the time delay filter and the periodic signal tracker, the sine signals were passed through the input shaping transfer functions that were formulated based on the equations in sections 4.1 and 4.2. A Polytec Scanning Laser Doppler Vibrometer was used to measure the velocity across 126 points on the beam. A reflective tape was also added to the front surface of the beam to ensure high quality measurements with minimal noise.

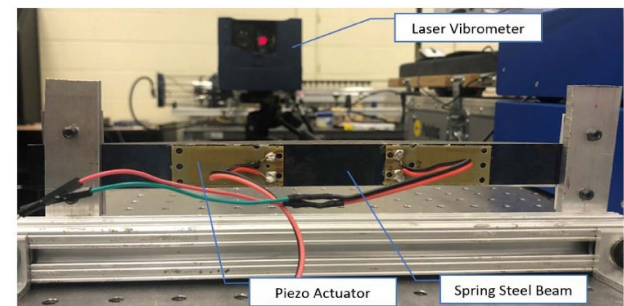
## 5.2. Experimental results

Each experiment was run for a time duration of 0.5 seconds. The data from all the 2D points on the beam was collected and interpolated in a 1D format to match the simulations. The following figure shows the experimental results from traditional two mode excitation.

As theorized in section 3, the dynamic response of the traditional two mode excitation has transients that go up 3 times the steady state amplitude. The transients die out after



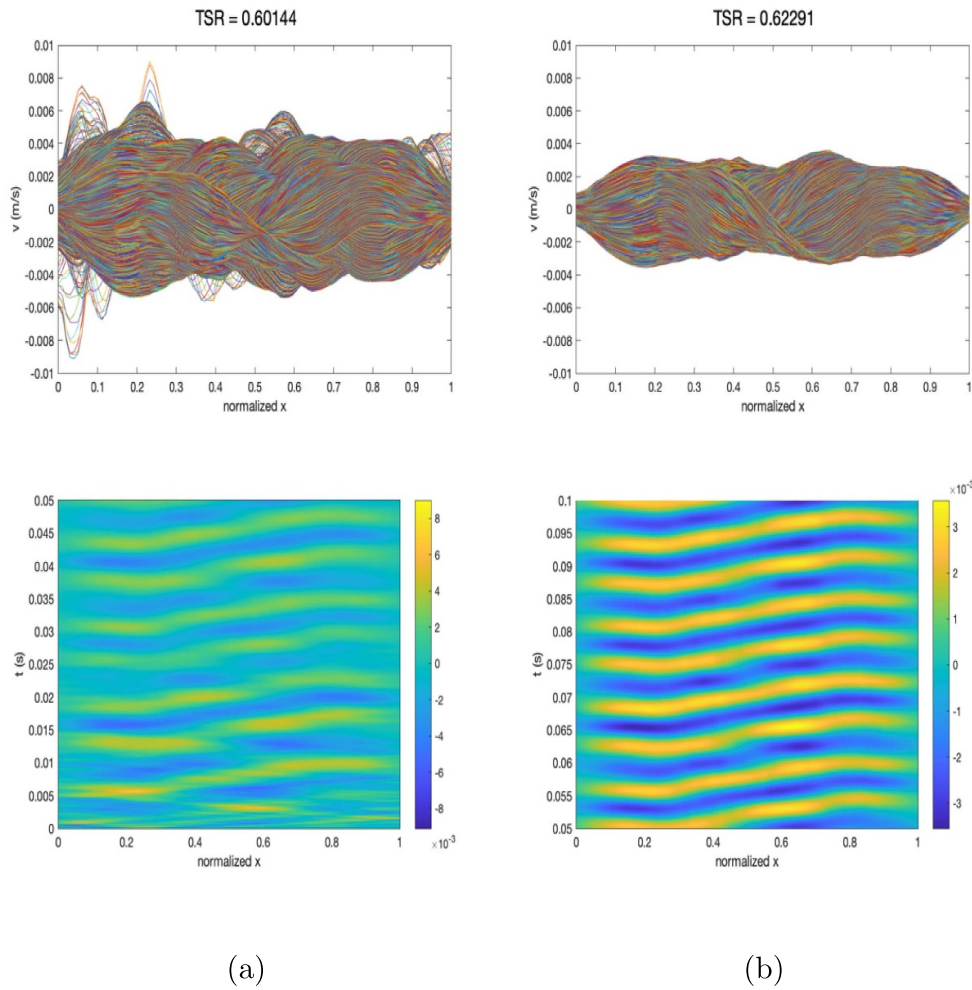
(a)



(b)

**Figure 12.** (a) Front view of the beam. (b) Back view of the beam.

0.05 seconds. In the first 0.05 seconds, a mixture of travelling and standing wave components can be observed in the dynamic response (figure 13(a) bottom). The transients and interference from the parasitic modes causes standing



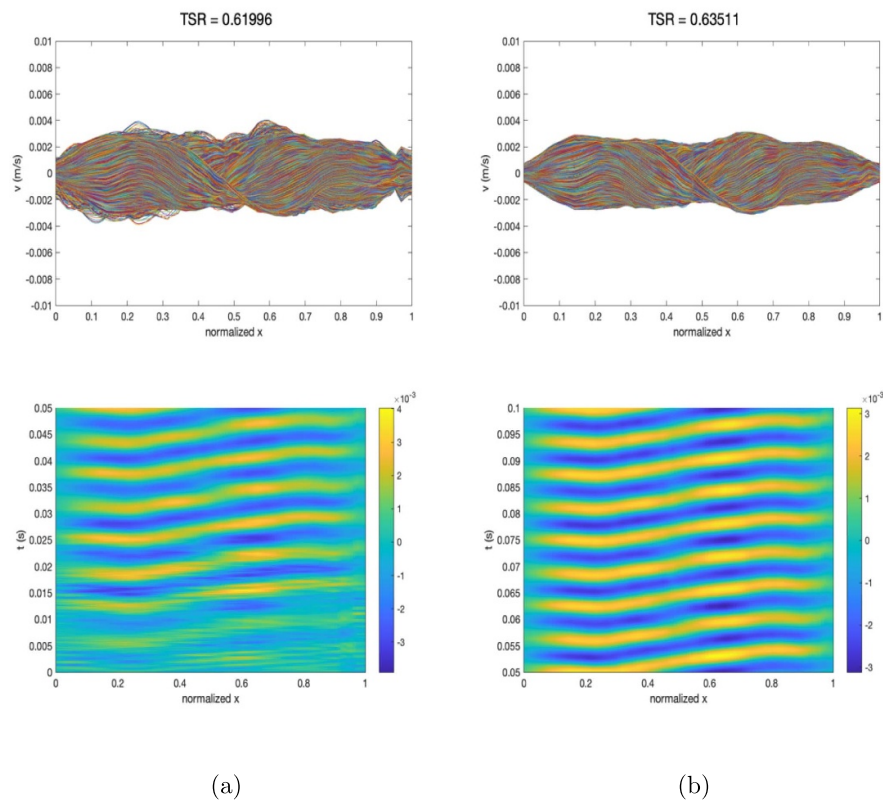
**Figure 13.** (a) Dynamic response of experimental TME. (b) Steady state response of experimental TME.

waves components to dominate upon actuation. As the transients die out, the travelling wave components becomes more prevalent and the amplitude of the wave settles down. The time delay filter augmented two mode excitation performs considerably well. Figure 14(a) shown the dynamic and steady state response two mode excitation augmented with time delay filter. Compared to the traditional two mode excitation the transients die out instantly upon actuation. The difference in the dynamic and the steady state amplitude is significantly low. The travelling wave converges within 0.015 seconds and very few standing wave components are observed upon actuation (figure 14(a) bottom). The time delay filter successfully cancels out the selected poles of the system thus eliminating the transients and the parasitic mode behaviour.

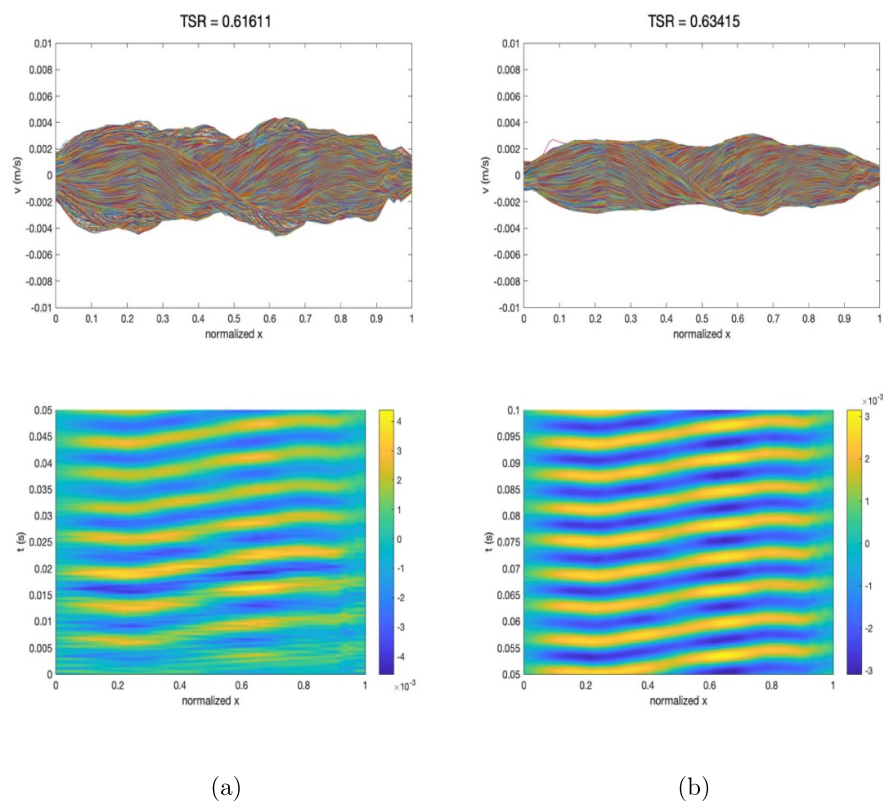
The two mode excitation augmented with periodic signal tracker performs slightly worse than the time delay filter in terms of suppressing the transients (figure 15). As mentioned in section 4.2, the magnitude constraint of the periodic signal tracker amplifies the input initially, therefore resulting in slightly higher transients than the time delay filter. Although the transients are high they settle down instantly and the

travelling wave converges within 0.015 s. In terms of mode suppression the periodic signal tracker performs similar to the time delay filter.

In terms of travelling to standing wave ratio (TSR), the first 0.05 seconds of two-mode excitation without input shaping show a higher ratio of standing wave components than others and this is because of overshoot and transient mode behaviour that the periodic signal tracker and time delay filter are able to eliminate. Beyond 0.05 s, the TSR is similar between all cases, as they have common steady forcing parameters. This can be seen qualitatively by analysing nodes, or absence of nodes, in the surface plots which show spatial-temporal evolution. The experimental results are consistent with the simulations and verify the advantages of input shaping in travelling wave generation. Although the input shapers were designed using analytically calculated parameters, they operate sufficiently. The zeros of the input shapers are located in close proximity of the actual poles of the beam, thereby resulting in minimal transients in the travelling wave. This goes to show that locating zeros at the location of the poles of the system or in their close proximity significantly suppresses



**Figure 14.** (a) Dynamic response of experimental TDF. (b) Steady state response of experimental TDF.



**Figure 15.** (a) Dynamic response of experimental PST. (b) Steady state response of experimental PST.

the modes. Geometric errors of material property can result in high variations between the true natural frequencies of the system and the analytical ones. In such case, the TDF and PST can prove inadequate in dealing with mode suppression [13]. Robust input shapers [14, 18] capable of locating multiple zeros, near the poles of the system, to compensate for the uncertainties in poles can be utilised. The robust input shapers are not showcased in the paper and further investigation in the uncertainty compensation using robust filters for travelling wave generation in solids is needed in the future.

## 6. Conclusion

Input shaping methods were incorporated in the two-mode excitation method to eliminate the parasitic mode behaviour and the transients in the generated travelling wave. These methods are feed-forward techniques that only require knowledge of the natural frequencies and proportional damping of the system. The time delay filter is designed to cancel the poles in order to suppress the modes of the system, but lacks the ability to track the phase of the inputs. The periodic signal tracker accounts for the shortcomings in the time delay filter by adding additional phase and magnitude constraints to perfectly track both input signals. The performance of both input shapers was studied on a fixed-fixed beam and was further verified through experiments. Both the time delay filter and the periodic signal tracker show similar performance with the same convergence time for a uniform travelling wave. The periodic signal tracker had slightly higher transients than the time delay filter due to the overcompensation from the magnitude constraints. Both input shapers were able to efficiently suppress the transients and eliminate the parasitic mode behaviour that was observed in the traditional two mode excitation method. In conclusion, augmented feed-forward methods are proposed that account for parasitic mode behaviour and transients in a distributed parameter system to generate higher quality travelling waves in finite structures.

## Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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