Timely Broadcasting in Erasure Networks: Age-Rate Tradeoffs

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***Abstract*—The interplay between timeliness and rate efficiency is investigated in packet erasure broadcast channels with feedback. A scheduling framework is proposed in which coding actions, as opposed to users, are scheduled to attain desired tradeoffs between rate and age of information (AoI). This tradeoff is formalized by an upper bound on AoI as a function of the target rate constraints and two lower bounds: one as a function of the communication rate and one as a function of the arrival rate. Simulation results show that (i) surprisingly, coding can be beneficial in reducing AoI in the regime of moderate arrival rates even without rate constraints and the benefit increases with the number of users, and (ii) AoI increases with both the target rate constraint and the arrival rate when either is kept fixed, but decreases with them when they are set to be equal.**

I. Introduction

The technology of Internet of Things (IoT) provides a vision for integrating intelligence into cyber-physical systems using real-time applications. Timeliness is key for such applications and it has therefore emerged as a communication design criteria. There are, however, tradeoffs between timeliness and rate which we aim to investigate in broadcast networks.

Timeliness is measured using the metric of *Age of Information (AoI)*, as introduced in [1]. AoI captures, at the receiving side, how much time has passed since the generation time of the latest received packet. In the past decade, Age of information has been extensively investigated for status update systems [2]–[7]. From the aspect of scheduling, optimal transmission policies were proposed in [8]–[12] to optimize the overall age in wireless networks. The reader is referred to [13], [14] for a survey on the topic.

Rate efficiency is often provided by channel coding schemes over multiple realizations of the network and it comes at the cost of large delays. It is, therefore, not clear a-priori what types of tradeoffs exist between rate and timeliness. Prior works have mainly studied point to point channels [15]–[17]. In erasure channels, [18] proves that when the source alphabet and channel input alphabet have the same size, a Last-Come First-Serve (LCFS) policy with no coding is optimal. This is in contrast to channel coding schemes that provide rate efficiency by block coding. Considering erasure channels with FCFS M/G/1 queues, [19] finds an optimal block length for channel coding to minimize the average age and average peak age. In the context of broadcast packet erasure channels (BPECs) with feedback, coding is shown to be beneficial for age efficiency with two users [20]. In related work, [21], [22] design optimal precoding schemes to minimize AoI in a MIMO broadcast channel with multiple senders and receivers under FIFO channels without packet management. Reference [23] analyzes AoI in a multicast network with network coding.

In this work, we consider erasure networks and devise broadcast strategies that are efficient both in AoI and rate. The inherent tradeoff can be explained as follows. On the one hand, a higher rate can correspond to a smaller delay/AoI (both in the sense that queues get emptied faster and that fewer uses of the network are needed in total to transmit a fixed number of information bits). On the other hand, to achieve high rates, we need to wait for the arrival of other packets and change transmission priorities to facilitate coding, and this can lead to a larger AoI. To shed light on the above tradeoff, we build on our previous work [20] and consider an erasure wireless network with M users.

Motivated by the success of age-based scheduling in wireless networks, we propose a scheduling framework where we schedule various useful coding actions as opposed to the users. Within this framework, we can capture both rate efficiency and age efficiency. Coding is known to provide significant benefits compared to time sharing especially as the number of users M increases [24]–[27]. Our work shows, for the first time, that coding also provides benefits in terms of age and the gain increases by M sharply, especially when the generation rate is small and/or the channel erasure probability is large. More generally, we design deterministic coding policies that minimize the average AoI under given rate constraints.

The contributions of the work are summarized as follows: (i) We propose a novel framework of network AoI on the broadcast channels under transmission mechanism with coding (Section II). (ii) (Near-)optimal coding policies with uncoded and coded caching are proposed (Section III). Two general lower bounds and an upper bound are derived on EAoI for any transmission policy (Section IV, Theorem 2). The bounds are functions of generation rates, erasure probabilities and target rate constraints. (iii) Simulation results reveal that (a) coding is beneficial, and the benefits increase with the number of users; (b) a good approximation of proposed policies is obtained based on maximum clique size of information graph; (c) the tradeoff between rate and AoI exists, which implies that the

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system has to sacrifice AoI to achieve a higher rate.

II. System Model

The system model extends that of [20, Section VI] to M users. Transmission occurs in a wireless network which we model by a BPEC with M users. In the beginning of time slot k, a packet intended for user i is generated with probability θ_i. Let $G_i(k) = 1$ represent the corresponding event.

Every broadcast packet is received at user i with probability $1 - \epsilon_i$, $0 \leq \epsilon_i < 1$, and lost with probability ϵ_i. Erasure events at multiple users can be dependent in general. The *transmission delay* is assumed fixed and equal to one time slot. After each transmission, the transmitter receives ACK/NACK feedback from all receivers and can thus calculate and track the aging of information at each user. Let $d_i(k) = 1$ if user i decodes a packet of type i in time k, and $d_i(k) = 0$ otherwise.

If a packet is not received at its intended user, it can be cached by other user(s) that have received it. The cached packets can act as side information. Using the available feedback, the encoder can track the cached packets and exploit them as side information to form more efficient coded packets that are simultaneously useful for multiple users [24]–[27].

We call a packet *coded* if it is formed by combining more than one packets; otherwise we call it *uncoded*. A coded packet can be *fully decoded* by user i if user i extracts every uncoded packets combined within it upon successful delivery.

Depending on the available caching and coding capabilities, we can consider three class of policies: (i) policies that benefit from coding by caching uncoded packets, (ii) policies that benefit from coding by caching general (potentially coded) packets, and (iii) *time-sharing policies*, which schedule different users and perform no caching/coding [10], [11]. We investigate the first class in Section III and refer to them as coding policies with uncoded caching. The second class, referred to as coding policies with coded caching, is investigated in the long version of the work [28]. Time-sharing policies form benchmarks in our simulations. In this work, we consider coding policies with uncoded caching and linear network coding through XOR operations only. Coding over larger finite fields may impose larger decoding delay and is often practically less desirable.

A. A Virtual Network of Queues

The idea of caching and coding on the fly is to cache overheard packets at the users and track them using feedback at the encoder through a *network of virtual queues*. Let Q_i denote the queue of incoming packets for user i. If a packet chosen from Q_i is transmitted and received by its intended user, it is removed from the queue. If it is not received by its intended user i, but received by some other users, then the packet will be cached in the cache of those users (as side information) and tracked at a virtual queue at the encoder. Define $Q_{i,\mathcal{S}}$ as the virtual queue that tracks, *at the encoder*, uncoded packets for user i that are received *only* by the users in $\mathcal{S}$, where $\mathcal{S} \subset [M]\backslash i$. Note that $Q_i \left(= Q_{i,\varnothing}\right)$ is some sort of $Q_{i,\mathcal{S}}$. Queue $Q_{i,\mathcal{S}}$ contains two types of packets: packets from $Q_{i,\varnothing}$ that are cached (received or decoded) by the users in $\mathcal{S}$, and/or uncoded packets combined within coded packets which are fully decoded. The queues $Q_{i,\mathcal{S}}$ are defined so that the set of packets in them are disjoint.

Packets stored in the virtual queues at the encoder can form efficient coded packets that are simultaneously useful for multiple users. In this work, we consider linear network coding through XOR operations only. This is because for broadcast erasure channel with multiple unicast traffic, using simple coding operations leads to low decoding delay and is also practically desirable [29]. For example, consider a packet a in $Q_{1,\{2\}}$ and a packet b in $Q_{2,\{1\}}$. The XOR packet $p = a \oplus b$ is useful for both users 1 and 2 because user 2 has cached packet a and user 1 has cached packet b and they can therefore recover their desired packets by XORing packet p with their respective cached packet. More generally, consider a set of non-empty queues $\{Q_{\tau_i,\mathcal{S}_{\tau_i}}\}_{i=1}^{\ell}$ where τ_i is a user index ($\tau_i \in [M]$) and $\mathcal{S}_{\tau_i}$ is a subset of $[M]\backslash\tau_i$. Suppose the following condition holds:

$$\mathcal{S}_{\tau_i} \supset \{\cup_{j=1,j\neq i}^{\ell} \tau_j\} \qquad \forall i = 1, \ldots \ell. \tag{1}$$

XORing packets $a_i \in Q_{\tau_i,\mathcal{S}_{\tau_i}}$ leads to the coded packet

$$p = \bigoplus_{i=1}^{\ell} a_i \tag{2}$$

which is simultaneously decodable at all users $\{\tau_1, \ldots, \tau_\ell\}$. To view condition (1) alternatively, draw a *side information graph* $\mathcal{G}$ with nodes $V = \{1, \ldots, M\}$. Add an edge between nodes (i, j) if $Q_{i,\mathcal{S}_i}$ is non-empty for some set $\mathcal{S}_i$ that has j as an element. Condition (1) corresponds to the subgraph induced by nodes $\{\tau_1, \ldots, \tau_\ell\}$ forming a clique of size ℓ.

The *coding actions* we consider in this paper correspond to cliques on the side information graph (which has to be updated on the fly after each transmission). It is not difficult to see that maximal cliques are sufficient in this class . Among all possible maximal cliques (the number of which can generally be on the order of $3^{\frac{M}{3}}$ [30]), we aim to choose (schedule) one that leads to a coding action with the most benefit in terms of age and rate.

B. Age and Rate Efficiency

To capture the freshness of information, we use the metric of average *Age of Information* (AoI) defined in [13]. Denote $h_i(k)$ as the age of user i in time slot k. The age function $h_i(k)$ increases linearly in time when no delivery for user i occurs and drops with every delivery to a value that represents how old the received packet is. If an outdated packet (for user i) is received (meaning that a more recently generated packet is previously received at user i) then the outdated packet does not offer age reduction and $h_i(k)$ keeps increasing linearly.

Definition 1. *Denote the generation time of the packet received by user i in time slot k as $v_i(k)$. Assuming the initial state $h_i(0) = 1$, the age function $h_i(k)$ evolves as follows:*

$$h_i(k) = \begin{cases} \min\{h_i(k-1)+1, k - v_i(k)\} & d_i(k) = 1 \\ h_i(k-1)+1 & d_i(k) = 0. \end{cases}$$

The expected weighted sum of AoI (EAoI) at the users is thus given by $\mathbb{E}[J_K^\pi]$ where

$$J_K^\pi := \frac{1}{MK}\sum_{k=1}^{K}\sum_{i=1}^{M}\alpha_i h_i^\pi(k) \tag{3}$$

and $\alpha_1, \alpha_2, \cdots, \alpha_M$ are weights and the superscript π represents the communication policy. We are interested in minimizing EAoI under some constraints on the rate of communications. We define the *communication rate* to user i as the number of decoded packets (intended for user i) per time slot in the limit of time.

Let q_i be a strictly positive real value that represents the minimum rate requirement of node i. Without loss of generality, we assume that $\underline{q} = (q_1, q_2, \cdots, q_M)$ is in the capacity region. Similar to [9], we define the *long-term rate* of node i when policy π is employed as

$$r_i^\pi := \lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}\big[d_i^\pi(k)\big]. \tag{4}$$

Then, we express the *minimum rate constraint* of each individual node as

$$r_i^\pi \geq q_i, i = 1, 2, \cdots, M. \tag{5}$$

Ultimately, we seek to schedule the coding actions in order to achieve a judicious tradeoff between the EAoI and communication rate, as outlined below. Combining (3), (4) and (5), the objective is given by the following optimization problem:

$$\begin{aligned} J(\underline{q}) := \min_{\pi} \quad & \lim_{K\to\infty}\mathbb{E}[J_K^\pi] \\ s.t \quad & r_i^\pi \geq q_i, i = 1, 2, \cdots, M. \end{aligned} \tag{6}$$

III. Scheduling Coding Actions

In this section, we consider coding policies with uncoded caching, i.e., all cached packets are uncoded. We develop and analyze max-weight policies that schedule the coding actions to optimize (6). Each coding action can be described by a set of queues, each storing multiple packets. Packets stored at different queues can form desired coded packets corresponding to the chosen coding action. We allow packet management in each queue as it reduces age without impacting the rate.

Recall that $Q_{i,\mathcal{S}}$ is the queue that contains those packets of user i that are decoded only by the users in $\mathcal{S}$. Thus, if $a \in Q_{i,\mathcal{S}}$, for any $\mathcal{S}' \subset \mathcal{S}$ and $\mathcal{S}' \neq \mathcal{S}$, $a \notin Q_{i,\mathcal{S}'}$. In addition, if $a \in Q_{i,\varnothing}$, then $a \notin Q_{i,\mathcal{S}}$ for all $\mathcal{S} \neq \varnothing$. So the map from packets to queues is a surjection. From Section II-A, the encoder decides among the following actions, denoted by $A(k)$, and defined below:

- $A(k) = Q_{i,\varnothing}$: a packet is transmitted from $Q_{i,\varnothing}$;
- $A(k) = \oplus_{j=1}^{l} Q_{\tau_j,\mathcal{S}_{\tau_j}}$: a coded packet is transmitted that is formed by an XOR of l packets, one from each of the queues $Q_{\tau_1,\mathcal{S}_{\tau_1}}, Q_{\tau_2,\mathcal{S}_{\tau_2}}, \cdots, Q_{\tau_l,\mathcal{S}_{\tau_l}}$, where $S_{\tau_l} \not\ni \tau_l$ and users $\tau_1, \tau_2, \cdots, \tau_l$ form a *maximal* clique on the side information graph.

A. Encoder's Age of Information

To capture the aging of information at the encoder, we define the notion of AoI for each virtual queue. The following Lemma is proved in [28, Appendix B]

Lemma 1. *If $p_j \in Q_{i,\mathcal{S}}$ has the generation time k_j, $j \in \{1, 2\}$, and $k_2 > k_1$, then (encoding and) transmitting p_2 can not be worse than (encoding and) transmitting p_1 in terms of AoI.*

If $\mathcal{S} = \varnothing$, denote the AoI of $Q_{i,\varnothing}$ by $w_{i,\varnothing}(k)$, and the generation time of the most recent packet of that queue by k'. Based on Lemma 1, we define $w_{i,\varnothing}(k) = \min\{k - k', h_i(k)\}$ and $w_{i,\varnothing}(0) = h_i(0)$. This is to capture the fact that if $k - k' > h_i(k)$, then packets in $Q_{i,\varnothing}$ are older than the latest one recovered by user i, so packets in $Q_{i,\varnothing}$ are obsolete in terms of AoI in time slot k. The evolution of the AoI at the queue $Q_{i,\varnothing}$ is as follows: $w_{i,\varnothing}(k)$ drops to 0 if a new packet is generated; otherwise it increases by 1. Thus, the recursion of $w_{i,\varnothing}(k)$ is

$$w_{i,\varnothing}(k+1) = \begin{cases} 0 & G_i(k) = 1 \\ \min\{w_{i,\varnothing}(k)+1, h_i(k)+1\} & G_i(k) = 0. \end{cases} \tag{7}$$

Before defining the AoI of $Q_{i,\mathcal{S}}$, let $t_{i,\mathcal{S}}(k)$ be an indicator function as follows: $t_{i,\mathcal{S}}(k) = 1$ if the latest packet in $Q_{i,\mathcal{S}}$ is encoded and transmitted in time slot k, and is $t_{i,\mathcal{S}}(k) = 0$ otherwise. Now we consider the AoI of $Q_{i,\mathcal{S}}$ with $\mathcal{S} \neq \varnothing$. Denote the AoI of $Q_{i,\mathcal{S}}$ as $w_{i,\mathcal{S}}(k)$. Let the generation time of the latest packet in $Q_{i,\mathcal{S}}$ be k'. We define $w_{i,\mathcal{S}}(k) = \min\{k - k', h_i(k)\}$ and $Q_{i,\mathcal{S}}(0) = h_i(0)$. Then, $w_{i,\mathcal{S}}(k)$ increases by 1 unless $Q_{i,\mathcal{S}}$ is updated with a fresher packet. The content of $Q_{i,\mathcal{S}}$ change when packets move in other virtual queues at the encoder. For example, If packet $a \in Q_{i,\mathcal{S}}$ is recovered by other users in $\mathcal{I}$, $\mathcal{I} \cap \mathcal{S} = \varnothing$, then $a \in Q_{i,\mathcal{I}\cup\mathcal{S}}$ and $a \notin Q_{i,\mathcal{S}}$. The recursion of $w_{i,\mathcal{S}}(k)$ is given in [28, Eqn (14)]. From (7) and [28, Eqn (14)], the recursion of $h_i(k)$ is

$$h_i(k+1) = \begin{cases} w_{i,\mathcal{S}}(k)+1 & t_{i,\mathcal{S}}(k) = 1, d_i(k) = 1 \\ h_i(k)+1 & \text{otherwise.} \end{cases} \tag{8}$$

B. Age-Rate Max-Weight Scheduling

It is well established that coding actions can enhance the communication rate of broadcast channels [24], and may incur additional delays. To seek efficiency both in AoI and communication rate, similar to [9], [11], [20], we propose Age-Rate Max-Weight (ARM) policies to minimize EAoI in (6) under rate constraints.

We define the *age-gain* of queue $Q_{i,\mathcal{S}}$ (for user i), where $\mathcal{S} \subset [M]\backslash i$ as follows:

$$\delta_{i,\mathcal{S}}(k) = h_i(k) - w_{i,\mathcal{S}}(k). \tag{9}$$

The term $\delta_{i,\mathcal{S}}(k)$ quantifies how much the instantaneous user's age of information reduces upon successful delivery from the encoder's virtual queue $Q_{i,\mathcal{S}}$. If $Q_{i,\mathcal{S}}$ is empty or contains old packets, then , $\delta_{i,\mathcal{S}}(k) = 0$ by definition.

$A(k)$	Weights
$Q_{i,\varnothing}$	$(1-\epsilon_i)\Big(\beta_i\delta_{i,\varnothing}(k)+\lambda f_i(k)\Big)$
$\oplus_{u\in[l]}Q_{\tau_u,\mathcal{S}_{\tau_u}}$	$\sum_{u=1}^{l}\beta_{\tau_u}\delta_{\tau_u,\mathcal{S}_{\tau_u}}(k)(1-\epsilon_{\tau_u})$ $+\lambda\sum_{u=1}^{l}(1-\epsilon_{\tau_u})f_{\tau_u}(k)$

Fig. 1: Coding actions and their weights.

Let $x_i(k)$ be the throughput debt associated with node i at the beginning of slot k [9]. It evolves as follows:

$$x_i(k+1) = kq_i - \sum_{\tau=1}^{k} d_i^{\pi}(\tau). \tag{10}$$

The value of kq_i is the minimum average number of packets that node i should have decoded by slot $k+1$ and $\sum_{\tau=0}^{k} d_i^{\pi}(\tau)$ is the total number of recovered packets in the same interval. In fact, strong stability of the process $x_i^+(k)$ is sufficient to establish that the minimum rate constraint, $r_i^{\pi} \geq q_i$, is satisfied [9], [31, Theorem 2.8].

Define the encoder's state in time slot k as

$$S(k) = \Big(\{h_i(k)\}_i, \{w_{i,\mathcal{S}}(k)\}_{i,\mathcal{S}}, \{x_i(k)\}_i\Big),$$

and the Lyapunov function $L\big(S(k)\big)$ as

$$L\big(S(k)\big) = \sum_{i=1}^{M}\beta_i h_i(k) + \lambda\sum_{i=1}^{M}\left(x_i^+(k)\right)^2 \tag{11}$$

where $\beta_i, \lambda > 0$. Here, the quadratic function for $x_i(k)$ is to maximize the rate [8], [9], [11], and the linear function for $h_i(k)$ is to simplify the derivation. The one-slot Lyapunov Drift is defined as

$$\Theta(k) = \mathbb{E}\Big[L\big(S(k+1)\big) - L\big(S(k)\big)|S(k)\Big]. \tag{12}$$

Define the *rate-gain* of user i in time slot k as follows:

$$f_i(k) = \Big(\big(x_i(k)+q_i\big)^+\Big)^2 - \Big(\big(x_i(k)+q_i-1\big)^+\Big)^2. \tag{13}$$

Definition 2. *In each slot k, the ARM policy chooses the action that has the maximum weight in Table of Fig. 1.*

Remark 1. *When AoI is the only metric in decision making (i.e., $q_i = 0$ for all i), only the latest packets matter (see Lemma 1). We can thus assume that the buffer size of every queue is* 1 *and the stability region is* $\{\theta_i \leq 1, i = 1, 2, \cdots, M\}$.

Remark 2. *We have observed in simulations that a good approximation of the above ARM policy is obtained by choosing the maximal clique size l to be* 2. *This captures most of the gain with a much reduced complexity. The number of coding actions reduces from* 2^{M^2} *to* $2M(M-1)^3$.

Theorem 1. *The ARM policy defined in Definition 2 minimizes the one-slot Lyapunov Drift in each slot.*

Now we set to obtain an upper bound on AoI under rate constraints. We consider an upper bound with $M = 3$ in the symmetric system ($q_i = q$ and $\epsilon_i = \epsilon$ for $i \in [M]$), one can generalize to an upper bound with arbitrary M using the same idea.

Let $C^{uncoded}$ be the set of all tuples $\underline{q} = (q, q, q)$ for which $\{x_i^+(k)\}_{i=1}^3$ is strongly stabilized using the considered coding actions. We define a (symmetric) stationary randomized policy in which actions are chosen with probabilities:

$$\Pr(A(k) = Q_{i,\varnothing}) = \mu_{i,\varnothing} \tag{14}$$

$$\Pr\{A(k) = \oplus_{j\in[l]}Q_{\tau_j,\mathcal{S}_{\tau_j}}\} = \mu_{(\tau_1,\mathcal{S}_{\tau_1},\cdots,\tau_l,\mathcal{S}_{\tau_l})} \tag{15}$$

where we do not distinguish between different permutations on $\{\tau_j, \mathcal{S}_{\tau_j}\}_j$. By symmetry, let $\mu_{i,\varnothing} = \mu$, $\mu_{\tau_1,\{\tau_2\},\tau_2,\{\tau_1\}} = \zeta_1$, $\mu_{\tau_1,\{\tau_2\},\tau_2,\{\tau_1,\tau_3\}} = \zeta_2$, $\mu_{\tau_1,\{\tau_2,\tau_3\},\tau_2,\{\tau_1,\tau_3\}} = \zeta_3$ and $\mu_{\tau_1,\{\tau_2,\tau_3\},\tau_2,\{\tau_1,\tau_3\},\tau_3,\{\tau_1,\tau_2\}} = \zeta_4$.

Theorem 2. *For any* $\underline{q} \in C^{uncoded}$, $J(\underline{q})$ *is bounded by:*

$$\begin{aligned}
\min_{\mu}\quad & \frac{\frac{1}{3}\sum_{i=1}^3\alpha_i}{\theta} + \frac{\frac{1}{3}\sum_{i=1}^3\alpha_i}{\mu(1-\epsilon)} + \lambda \\
s.t\quad & 3\mu + 3\zeta_1 + 6\zeta_2 + 3\zeta_3 + \zeta_4 = 1 \\
& \mu(1-\epsilon^3) \geq q \\
& (\mu+\zeta_2)(1-\epsilon^2) + \zeta_1(1-\epsilon) \geq q \\
& \mu(1-2\epsilon^2+\epsilon^3) + 2\zeta_1(1-\epsilon) + 2\zeta_2(1-\epsilon^2) \geq q \\
& \big(\mu + 2\zeta_1 + 4\zeta_2 + 2\zeta_3 + \zeta_4\big)(1-\epsilon) \geq q, \\
& \mu \geq 0,\ \zeta_j \geq 0,\ j = 1,2,3,4.
\end{aligned} \tag{16}$$

Remark 3. *The upper bound reveals a tradeoff between the average AoI and the target rate constraints: as q increases (the feasible region of the linear program shrinks), our upper bound on age increases, capturing the tradeoff behavior that we ALSO observe in the simulations.*

IV. Lower Bound

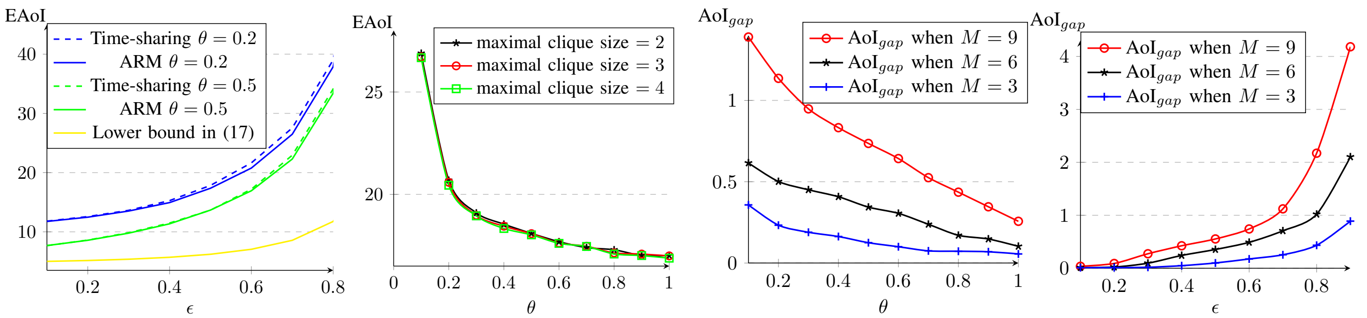
In prior works [9], [20], [32], lower bounds were found on AoI as a function of the communication rate. Similar to [32, Section III], we derive two lower bounds on the achievable age. The first lower bound is derived by assuming that there is always a fresh packet to be delivered. The second one assumes that all packets are delivered instantaneously upon arrival.

Theorem 3. *For any policy* π *with communication rate* r_i^{π}, *we have the following lower bounds on* $J^{\pi}(\underline{q})$ *in* (6)*:*

$$J^{\pi}(\underline{q}) \geq \frac{M}{2\sum_{i=1}^{M} r_i^{\pi}/\alpha_i} + \sum_{i=1}^{M}\frac{\alpha_i}{2M} \tag{17}$$

$$J^{\pi}(\underline{q}) \geq \frac{1}{M}\sum_{i=1}^{M}\frac{\alpha_i}{\theta_i} \tag{18}$$

In (17), as the communication rate increases, the lower bound on EAoI decreases. The rate terms r_i^{π} in (17) satisfy $r_i^{\pi} \geq q_i$, but it is not clear if we can replace them by q_i because (6) may admit its optimal solution at rates larger than the target values q_i. The high rate communication is indeed useful for age minimization. This is the reason why coding and caching can ultimately reduce age as shown in our work.



(a) EAoI as a function of ϵ and θ. (b) EAoI and maximal clique size. (c) AoI_{gap} vs. θ when $\epsilon = 0.6$. (d) AoI_{gap} vs. ϵ when $\theta = 0.2$.

Fig. 2: EAoI in different cases.

In (18), as θ_i increases, i.e., more fresh packets are generated, the lower bound on EAoI decreases capturing the importance of frequent updating. This bound is active in regimes where new packets are generated less frequently.

V. Numerical Results and Discussion

Finally, we seek to answer the questions that we raised in Section I through simulations. We assume symmetric networks with $\epsilon_i = \epsilon$, $q_i = q$, and $\theta_i = \theta$ for $i \in [M]$.

A. Benefits of Coding

We first consider the benefits of coding. The ARM policy and the time-sharing policy are compared in Figure 2a - Figure 2d. To eliminate the impact of rate, we consider the case defined in Remark 1, i.e., the buffer size of every queue is 1 and the stability region is $\{\theta \leq 1\}$. In Figure 2a - Figure 2d, we set $\lambda = 0$ and $\beta_i = \alpha_i = \lceil i/2 \rceil$. Figure 2a plots the EAoI for $M = 6$ users under the ARM and time-sharing policies, and againts the lower bound in (17). We observe that coding is indeed beneficial when the erasure probability ϵ is relatively large (≥ 0.6) and/or the arrival rate θ is relatively small (≤ 0.5). When θ is fixed, EAoI increases with ϵ.

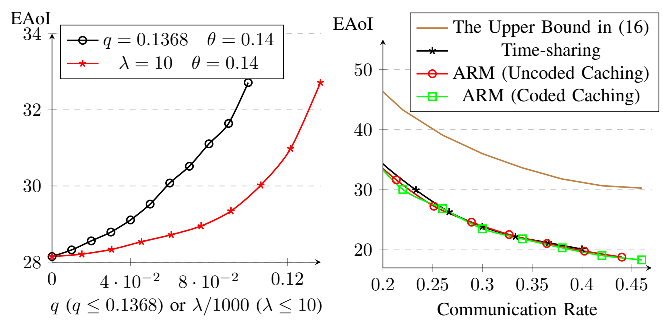
Next, we define AoI_{gap} as the gap between the EAoI under the ARM and time-sharing policies. The relationship between AoI_{gap} and θ (resp. ϵ) is provided in Figure 2c (resp. Figure 2d). In Figure 2c, we set $\epsilon = 0.6$. We observe that AoI_{gap} (the benfit of coding) decreases with the arrival rate θ. This is because the (expected) number of newly incoming packets increases with θ and the availability of fresh uncoded packets weakens the impact of coding actions.

In Figure 2d, we set $\theta = 0.2$. AoI_{gap} increases with ϵ. This is because erased packets can be cached and provide more coding opportunities. AoI_{gap} increases slowly when ϵ is small, and sharply when ϵ is large. In addition, from Figure 2c and Figure 2d, the benefits of coding increase with M.

B. Impact of Maximal Clique Size

The impact of maximal clique size is captured in Figure 2b. Let the buffer size of all (virtual) queues be 1 and set $M = 6$, $\lambda = 0$, $\epsilon = 0.6$ and $\beta_i = \alpha_i = \lceil i/2 \rceil$. The ARM policy with maximal clique sizes $\ell = 2, 3, 4$ are compared. We observe that $\ell = 2$ is a good approximation (see also Remark 2).

C. Tradeoff between Age and Rate



(a) EAoI vs. q or λ. (b) EAoI vs. rate.

Fig. 3: Tradeoffs between EAoI and rate.

We finally investigate the tradeoff between the AoI and rate. Set $M = 3$. The maximum sum-rate achievable with time sharing is around 0.4, with uncoded caching is around 0.44, and the channel capacity (with coded caching) is around 0.46. Setting $\beta_i = \alpha_i = i$ for $i \in [M]$ and $\epsilon = 0.6$ in the ARM policy, we first investigate the relationship between q and EAoI (the red star curve in Figure 3a). Now set $\theta = 0.14$, $\lambda = 10$, $q \in [0, 0.1368]$. EAoI increases with q implying that if the minimum required throughput becomes larger, the system has to sacrifice EAoI to satisfy the rate constraints. Next, the relationship between λ and EAoI is investigated (the black circle curve in Figure 3a). Let $\theta = 0.14$, $q = 0.1368$, $\lambda \in [0, 10]$. EAoI increases with λ. In other words, if the rate constraints become more important, then EAoI increases.

Finally, in Figure 3b, the EAoI is plotted as a function of the communication rate under the time-sharing policy as well as the ARM policy with uncoded and coded caching. This plot is obtained by setting $\beta_i = \alpha_i = i$, $\theta = q$, and $\lambda = 1$. We observe that EAoI decreases as rate increases. From the viewpoint of expectation, almost all packets are successfully delivered. The increase in theta implies more fresh packets are generated, and the increase in q (which equals to θ) implies more fresh packets are delivered. Thus, the EAoI decreases. The three policies have similar performances up to the rate they support. It appears that ARM with coded caching outperforms for rates close to the boundary of the capacity region.

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