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Component actions of liberated $\mathcal{N} = 1$ supergravity and new Fayet-Iliopoulos terms in superconformal tensor calculus

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Abstract: We explicitly compute the component action of certain recently discovered new $\mathcal{N} = 1$ supergravity actions which enlarge the space of scalar potentials allowed by supersymmetry and also contain fermionic interaction terms that become singular when supersymmetry is unbroken. They are the "Liberated Supergravity" introduced by Farakos, Kehagias and Riotto, and supergravities with a new Kähler-invariant Fayet-Iliopoulos term proposed by Antoniadis, Chatrabhuti, Isono, and Knoops. This paper is complementary to our previous papers [*Phys. Rev. D* **103** (2021) 025008 and 105006], in which new constraints on the coupling constants of those new theories were found. In this paper we spell out many details that were left out of our previous papers.

Keywords: Supergravity Models, Superspaces

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1 Introduction

The results of LHC run 2 show no significant deviations from Standard Model predictions thereby pushing the energy scale of new physics — and in particular low-scale supersymmetry [1, 2]– to higher and higher values. This empirical observation combines with theoretical difficulties such as the eta problem that seems to plagues many supergravity models of inflation (see e.g. [3] for a review) to make alternative scenarios worth considering. One easy way out of all problems posed by supersymmetry is to assume that the world is not supersymmetric at all. On the other hand the high-energy completion of any field theory containing gravity provided by perturbative superstring theory implies that at some very high energy scale the world is supersymmetric (and 10-dimensional). In superstring theory the largest and best understood set of models of low-energy physics — such as Calabi-Yau or flux compactifications — are such that extra dimensions are compactified at energy scales which are much above the supersymmetry breaking scale. It is therefore reasonable to study four-dimensional models in which supersymmetry is broken at a high scale, e.g. at a scale $M_S \gg 1\,\text{TeV}$. While a large M_S means that supersymmetry does not help solving the Standard Model naturalness problem it does open up many new possibilities for writing consistent actions.

Among these possibilities, some recent proposals use explicitly supersymmetric terms that look at first sight singular, because when written in superspace they contain inverse powers of superfields. What makes possible to write such terms is that they in fact they depend at most on inverse powers *of the auxiliary fields*. So they are well defined when supersymmetry is broken. We will study in particular the theory called "liberated $\mathcal{N} = 1$ supergravity", proposed in ref. [10] and the new Fayet-Iliopoulos (FI) term introduced in [4–9]. These modifications of the standard supergravity action relax the highly constrained form of scalar potentials in supergravity. (recall that the scalar potential has the universal form $V = V_F + V_D$, $V_F = e^G(G_A G^{A\bar{B}} G_{\bar{B}} - 3)$ and V_D, for some function G of the chiral multipet scalars.). The cosmological applications to inflation of such models have been recently investigated in [5, 12].

Supergravity actions contain many nonrenormalizable operators that depend on the fermionic fields. Their existence defines an ultraviolet (UV) cutoff scale of the theory. While this property is generic in locally supersymmetic theories, it is not dangerous when the UV cutoff is the Planck scale M_{pl}, which is in any case the ultimate cutoff for supergravity, and of course also for Einstein gravity. In the models studied in refs. [11, 12] instead, the UV cutoff is connected to the supersymmetry breaking scale, so it becomes an important parameter that constraints the domain of validity of those models.

The main purpose of this work is to show a systematic analysis of how a cutoff scale of an effective supergravity theory is determined by its fermionic nonrenormalizable terms and what constraints on the cutoff arise from the nonrenormalizable terms. These fermionic nonrenormalizable Lagrangians are conventionally represented by $\mathcal{L}_F \sim \mathcal{C}^{(4-\delta)}\mathcal{O}_F^{(\delta)}$ where $\delta > 4$ is the mass dimension of the fermionic nonrenormalizable operators $\mathcal{O}_F^{(\delta)}$. Then, we find constraints on the expansion coefficients $\mathcal{C}^{(4-\delta)}$ because these coefficients must be suppressed by a cutoff scale Λ_{cut} of a theory for this to be valid as an effective field theory

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in range of some energies of interest below the cutoff; that is,

$$\mathcal{C}^{(4-\delta)} \lesssim \frac{1}{\Lambda_{\rm cut}^{\delta-4}}. \tag{1.1}$$

By taking this inequality into account, we are able to determine in what range of energies a low energy supergravity theory can be considered as an effective field theory. To do so, it is essential to explore all of the fermionic nonrenormalizable terms as we have just seen above. Checking these terms may be difficult if we use the superspace formulation as we have to fully calculate them and the superspace fields contain many more auxiliary components than needed for the off-shell closure of the supersymmetry algebra. However, we point out that using the superconformal tensor calculus enables us to compute both bosonic and fermionic terms in a systematical and economical way. Hence, in this work we take advantage of the superconformal formalism.

Since this paper is rather technical, the reader chiefly interested in finding the constraints on new supergravity terms can skip ahead to section 4. The paper is organized as follows. In section 2, we review how to embed liberated $\mathcal{N}=1$ supergravity in the superconformal tensor calculus, which was firstly introduced in ref. [11], and explicitly compute the component action of the liberated supergravity using the tensor calculus in detail.[1] In section 3, we review the component action of a new FI term, which is Kähler-invariant, proposed by Antoniadis, Chatrabhuti, Isono, and Knoops (ACIK).[2] In section 4, we recall the constraints on the liberated supergravity terms derived in [11] and derive the constraints on the new FI terms used in [12]. Both sets of constraints are based on an analysis of fermionic nonrenormalizable interactions. In section 5, we briefly summarize our findings.

2 Component action of liberated $\mathcal{N}=1$ supergravity in superconformal tensor calculus

In this section, we compute the component action of liberated $\mathcal{N}=1$ supergravity in superconformal tensor calculus [13–15]. We find that the superconformal Lagrangian of liberated $\mathcal{N}=1$ supergravity [10] can be written by a D-term

$$\mathcal{L}_{\rm NEW} \equiv \left[\mathcal{Y}^2 \frac{w^2\bar{w}^2}{T(\bar{w}^2)\bar{T}(w^2)}\mathcal{U}(Z,\bar{Z})\right]_D, \tag{2.1}$$

in which we define $\mathcal{Y} \equiv S_0\bar{S}_0 e^{-K/3}$ where S_0 is a conformal compensator with Weyl/chiral weights $(1,1)$ and K is a Kähler potential with the weights $(2,0)$. The notation for the other fields is as follows: $w^2 \equiv \frac{\mathcal{W}^2(K)}{\mathcal{Y}^2}$, $\bar{w}^2 \equiv \frac{\bar{\mathcal{W}}^2(K)}{\mathcal{Y}^2}$, $\mathcal{W}^2(K) \equiv \mathcal{W}_\alpha(K)\mathcal{W}^\alpha(K)$ where $\mathcal{W}_\alpha(K)$ is a field strength multiplet with respect to the Kähler potential; $\mathcal{U}(Z,\bar{Z})$ is a general function of matter multiplets Z, and $T(\bar{w}^2), \bar{T}(w^2)$ are chiral projection of $\bar{w}^2$ and its conjugate respectively.

[1] Section 2 provides the detail calculations of the liberated supergravity, which was omitted in ref. [11].

[2] We shall call the new FI term proposed by the authors as "ACIK-FI term" to distinguish it from other types FI terms. Section 3 also provides the detailed calculations of the ACIK-FI term, which were omitted in ref. [12].

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2.1 Embedding super-Weyl-Kähler transformations as an abelian gauge symmetry into the superconformal formalism

In liberated $\mathcal{N} = 1$ supergravity [10], a key idea is that the Super-Weyl-Kähler transformation can be promoted to an Abelian gauge symmetry. Liberated supergravity was constructed in [10] using the superspace formalism, where a Kähler transformation is introduced to compensate the variation of the action under a super-Weyl rescaling. In this work instead, we want to construct the equivalent liberated supergravity using the superconformal tensor calculus to analyze the fermionic interactions in a systematical and economical way. To do so, we introduce a conformal compensator multiplet, called S_0, removing the variation while maintaining the Kähler potential invariant under superconformal symmetry. Therefore, unlike the superspace formalism, it is essential to define such a gauge transformation independently of superconformal symmetry.

To find the Super-Weyl-Kähler transformations that are compatible with the superconformal formalism, we recall first the Super-Weyl-Kähler transformations that are used in the superspace formalism. A Kähler function $K(z, \bar{z})$, whose arguments have the vanishing Weyl/chiral weights, is defined up to a chiral gauge parameter Σ. The redefinition by Σ acts on the components of the Kähler multiplet as

$$K \to K + 6\Sigma + 6\bar{\Sigma}, \tag{2.2}$$

$$W \to We^{-6\Sigma}, \qquad \bar{W} \to \bar{W}e^{-6\bar{\Sigma}} \tag{2.3}$$

$$T \to e^{-4\Sigma+2\bar{\Sigma}}T, \qquad \bar{T} \to e^{2\Sigma-4\bar{\Sigma}}\bar{T}, \tag{2.4}$$

$$\mathcal{D}_\alpha K \to e^{\Sigma-2\bar{\Sigma}}\mathcal{D}_\alpha K, \qquad \mathcal{W}_\alpha \to e^{-3\Sigma}\mathcal{W}_\alpha, \qquad \mathcal{W}^2 \to e^{-6\Sigma}\mathcal{W}^2, \tag{2.5}$$

$$T(\bar{\mathcal{W}}^2) \to T(\bar{\mathcal{W}}^2)e^{-4\Sigma-4\bar{\Sigma}}, \tag{2.6}$$

$$\mathcal{D}^\alpha\mathcal{W}_\alpha \to e^{-2\Sigma-2\bar{\Sigma}}\mathcal{D}^\alpha\mathcal{W}_\alpha, \tag{2.7}$$

$$E \to Ee^{2\Sigma+2\bar{\Sigma}}, \qquad \mathcal{E} \to \mathcal{E}e^{6\Sigma}. \tag{2.8}$$

where E and $\mathcal{E}$ are the D- and F-term densities, respectively.

Next, it may be useful to recall the relation between the superspace and superconformal formalisms. The invariant actions from the superconformal formalism are identified with those from the superspace calculus as follows [16]:

$$[\mathcal{V}]_D = 2\int d^4\theta E\mathcal{V}, \tag{2.9}$$

$$[\mathcal{S}]_F = \int d^2\theta\mathcal{E}\mathcal{S} + \int d^2\bar{\theta}\bar{\mathcal{E}}\bar{\mathcal{S}}, \tag{2.10}$$

where $\mathcal{V}$ is a superconformal real multiplet with the Weyl/chirial weights $(2, 0)$ and $\mathcal{S}$ is a superconformal chiral multiplet with Weyl/chiral weights $(3, 3)$. To make the action invariant under the super-Weyl-Kähler transformations we should impose that the corresponding superconformal multiplets transform as

$$\mathcal{V} \to \mathcal{V}e^{-2\Sigma-2\bar{\Sigma}}, \qquad \mathcal{S} \to \mathcal{S}e^{-6\Sigma}. \tag{2.11}$$

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Instead of considering the Kähler transformation of the Kähler potential a superconformal compensator is introduced in the superconformal formalism to eliminate the variation of the action transformed by a super-Weyl rescaling (also called Howe-Tucker transformation [17]). Thus, the compensator must transform as

$$S_0 \to S_0 e^{-2\Sigma}, \qquad \bar{S}_0 \to \bar{S}_0 e^{-2\bar{\Sigma}}, \tag{2.12}$$

resulting in

$$S_0 \bar{S}_0 e^{-K/3} \to S_0 \bar{S}_0 e^{-K/3} e^{-2\Sigma - 2\bar{\Sigma}}, \tag{2.13}$$

where K is invariant under the superconformal symmetry (i.e. super-Weyl rescaling), so that the action can be invariant as desired.

At this point, differently from the usual story of the superconformal symmetry, we require a "Kähler transformation" of the Kähler potential in order to construct a "liberated" supergravity that is invariant under the same Super-Weyl-Kähler transformations as an abelian gauge symmetry used in [10]. Therefore, we assume that the superconformal compensators are inert under the super-Weyl-Kähler transformations

$$S_0 \to S_0, \qquad \bar{S}_0 \to \bar{S}_0, \tag{2.14}$$

while the Kähler potential *does* transform under the same transformations as above, namely as $K \to K + 6\Sigma + 6\bar{\Sigma}$, so that

$$\mathcal{Y} \to \mathcal{Y} e^{-2(\Sigma+\bar{\Sigma})}, \tag{2.15}$$

$$w^2 \equiv \frac{\mathcal{W}^2(K)}{(e^{-K/3} S_0 \bar{S}_0)^2} \to w^2 e^{-2\Sigma + 4\bar{\Sigma}}, \tag{2.16}$$

$$\bar{T}\left(w^2\right) \to \bar{T}\left(w^2 e^{-2\Sigma+4\bar{\Sigma}}\right) e^{2\Sigma - 4\bar{\Sigma}} = \bar{T}\left(w^2\right). \tag{2.17}$$

2.2 List of superconformal multiplets

In this section, we present all the superconformal multiplets of the liberated $\mathcal{N} = 1$ supergravity following the notations and multiplication laws used in [3, 15].

2.2.1 Kähler potential multiplet

Let us consider n physical chiral multiplets of matter $z^I \equiv \{z^I, P_L \chi^I, F^I\}$ where $I = 1, 2, 3, \cdots, n$ and their anti-chiral multiplets $\bar{z}^{\bar{I}} \equiv \{\bar{z}^{\bar{I}}, P_R \chi^{\bar{I}}, \bar{F}^{\bar{I}}\}$.[3] Then, according to the superconformal tensor calculus, the Kähler potential multiplet can be written as follows:

$$K(z, \bar{z}) = \{C_K, \mathcal{Z}_K, \mathcal{H}_K, \mathcal{K}_K, \mathcal{B}^K_\mu, \Lambda_K, \mathcal{D}_K\} \tag{2.18}$$

[3] The complex conjugates are $\bar{z}^{\bar{I}} \equiv (z^I)^*$, $\chi^{\bar{I}} \equiv (\chi^I)^C$, $\bar{\chi}^{\bar{I}} \equiv (\bar{\chi}^I)^C$, and $\bar{F}^{\bar{I}} \equiv (F^I)^*$ (The barred index is the complex conjugate index, so that the handedness of fermion becomes opposite, i.e. $(P_{L/R}\chi)^C = (P_{L/R})^C (\chi)^C = P_{R/L}(\chi)^C$.). The chiralities of fermion are specified as $\chi^I \equiv P_L \chi^I$ and $\chi^{\bar{I}} \equiv P_R \chi^{\bar{I}}$. The Majorana conjugates are $\overline{(P_{L/R}\chi)} = \bar{\chi} P_{L/R}$. (The handedness is preserved.)

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where

$$C_K = K(z,\bar{z}), \tag{2.19}$$

$$\mathcal{Z}_K = i\sqrt{2}(-K_I\chi^I + K_{\bar{I}}\chi^{\bar{I}}), \tag{2.20}$$

$$\mathcal{H}_K = -2K_I F^I + K_{I\bar{J}}\bar{\chi}^I\chi^J, \tag{2.21}$$

$$\mathcal{K}_K = -2K_{\bar{I}}\bar{F}^{\bar{I}} + K_{\bar{I}\bar{J}}\bar{\chi}^{\bar{I}}\chi^{\bar{J}} = \mathcal{H}_K^*, \tag{2.22}$$

$$\mathcal{B}_\mu^K = iK_I\mathcal{D}_\mu z^I - iK_{\bar{I}}\mathcal{D}_\mu\bar{z}^{\bar{I}} + iK_{I\bar{J}}\bar{\chi}^I\gamma_\mu\chi^{\bar{J}}, \tag{2.23}$$

$$\Lambda_K = P_L\Lambda_K + P_R\Lambda_K \tag{2.24}$$

$$P_L\Lambda_K = -\sqrt{2}iK_{\bar{I}J}[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] - \frac{i}{\sqrt{2}}K_{\bar{I}\bar{J}K}\chi^K\bar{\chi}^{\bar{I}}\chi^J, \tag{2.25}$$

$$P_R\Lambda_K = \sqrt{2}iK_{I\bar{J}}[(\not{\mathcal{D}}\bar{z}^{\bar{J}})\chi^I - F^I\chi^{\bar{J}}] + \frac{i}{\sqrt{2}}K_{I\bar{J}\bar{K}}\chi^{\bar{K}}\bar{\chi}^I\chi^{\bar{J}}, \tag{2.26}$$

$$\begin{aligned}\mathcal{D}_K = {} & 2K_{I\bar{J}}\Big(-\mathcal{D}_\mu z^I\mathcal{D}^\mu\bar{z}^{\bar{J}} - \frac{1}{2}\bar{\chi}^I P_L\not{\mathcal{D}}\chi^{\bar{J}} - \frac{1}{2}\bar{\chi}^{\bar{J}}P_R\not{\mathcal{D}}\chi^I + F^I\bar{F}^{\bar{J}}\Big) \\ & + K_{I\bar{J}\bar{K}}\Big(-\bar{\chi}^I\chi^J\bar{F}^{\bar{K}} + \bar{\chi}^I(\not{\mathcal{D}}z^J)\chi^{\bar{K}}\Big) + K_{\bar{I}\bar{J}K}\Big(-\bar{\chi}^{\bar{I}}\chi^{\bar{J}}F^K + \bar{\chi}^{\bar{I}}(\not{\mathcal{D}}\bar{z}^{\bar{J}})\chi^K\Big) \\ & + \frac{1}{2}K_{IJ\bar{K}\bar{L}}(\bar{\chi}^I P_L\chi^J)(\bar{\chi}^{\bar{K}}P_R\chi^{\bar{L}}).\end{aligned} \tag{2.27}$$

Here the covariant derivatives[4] of chiral multiplets of matter with the weights $(0,0)$ are given by

$$\mathcal{D}_a z^I = e_a^\mu\left[\partial_\mu z^I - \frac{1}{\sqrt{2}}\bar{\psi}_\mu\chi^I\right], \tag{2.28}$$

$$\mathcal{D}_a P_L\chi^I = e_a^\mu P_L\left[\left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - \frac{1}{2}b_\mu - \frac{3}{2}iA_\mu\right)\chi^I - \frac{1}{\sqrt{2}}(\not{\mathcal{D}}z^I + F^I)\psi_\mu\right]. \tag{2.29}$$

Note that the only bosonic contribution to $\mathcal{D}_K$ is given by

$$\mathcal{D}_K|_{\text{boson}} = 2K_{I\bar{J}}\left(-\partial_\mu z^I\partial^\mu\bar{z}^{\bar{J}} + F^I\bar{F}^{\bar{J}}\right) \equiv \tilde{\mathcal{F}} \tag{2.30}$$

[4] From eq. (16.34) in ref. [3] we find that for a general superconformal chiral multiplet $(z^I, P_L\chi^I, F^I)$ with Weyl/chiral weights $(w, c = w)$ and gauge symmetries with Killing vector fields k_A^I, the full superconformal covariant derivatives $\mathcal{D}_a$ are given by

$$\mathcal{D}_a z^I = e_a^\mu\left[(\partial_\mu - wb_\mu - wiA_\mu)z^I - \frac{1}{\sqrt{2}}\bar{\psi}_\mu\chi^I - A_\mu^A k_A^I\right],$$

$$\begin{aligned}\mathcal{D}_a P_L\chi^I = {} & e_a^\mu P_L\Big[\Big(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - (w+1/2)b_\mu + (w-3/2)iA_\mu\Big)\chi^I - \frac{1}{\sqrt{2}}(\not{\mathcal{D}}z^I + F^I)\psi_\mu \\ & - \sqrt{2}wz^I\phi_\mu - A_\mu^A\chi^J\partial_J k_A^I\Big].\end{aligned}$$

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and we especially denote this by $\tilde{\mathcal{F}}$. We also note that the $\tilde{\mathcal{F}}$ is positive definite up to terms containing spatial gradients; so, for small spatial gradients,

$$\mathcal{D}_K|_{\text{boson}} \equiv \tilde{\mathcal{F}} \sim 2K_{I\bar{J}}\left(\dot{z}^I\dot{\bar{z}}^{\bar{J}} + F^I\bar{F}^{\bar{J}}\right) > 0. \tag{2.31}$$

2.2.2 Compensator multiplet

Chiral compensators $S_0, \bar{S}_0$ with the Weyl/chiral weights $(1,1)$ are defined as follows. The chiral supermultiplets $S_0 = \{s_0, P_L\chi^0, F_0\}$ and $\bar{S}_0 = \{s_0^*, P_R\chi^0, F_0^*\}$ can be embedded into the superconformal formalism as

$$S_0 \equiv \left\{s_0, -i\sqrt{2}P_L\chi^0, -2F_0, 0, i\mathcal{D}_\mu s_0, 0, 0\right\}, \tag{2.32}$$

$$\bar{S}_0 \equiv \left\{s_0^*, i\sqrt{2}P_R\chi^0, 0, -2F_0^*, -i\mathcal{D}_\mu s_0^*, 0, 0\right\}. \tag{2.33}$$

Then, the composite real compensator $S_0\bar{S}_0$ is

$$S_0\bar{S}_0 = \{\mathcal{C}_0, \mathcal{Z}_0, \mathcal{H}_0, \mathcal{K}_0, \mathcal{B}_\mu^0, \Lambda_0, \mathcal{D}_0\}, \tag{2.34}$$

where[5]

$$\mathcal{C}_0 = s_0 s_0^*|_{0f}, \tag{2.35}$$

$$\mathcal{Z}_0 = i\sqrt{2}(-s_0^*P_L\chi^0 + s_0P_R\chi^0)|_{1f}, \tag{2.36}$$

$$\mathcal{H}_0 = -2s_0^*F^0|_{0f}, \tag{2.37}$$

$$\mathcal{K}_0 = -2s_0F_0^*|_{0f}, \tag{2.38}$$

$$\mathcal{B}_\mu^0 = is_0^*\mathcal{D}_\mu s_0 - is_0\mathcal{D}_\mu s_0^* + i\bar{\chi}_0\gamma_\mu P_R\chi^0 = is_0^*\partial_\mu s_0 - is_0\partial_\mu s_0^*|_{0f} + \cdots, \tag{2.39}$$

$$P_L\Lambda_0 = -\sqrt{2}i[(\not{\mathcal{D}}s_0^*)P_R\chi^0 - F_0^*P_L\chi^0] = -\sqrt{2}i[(\not{\partial}s_0^*)P_R\chi^0 - F_0^*P_L\chi^0]|_{1f} + \cdots, \tag{2.40}$$

$$P_R\Lambda_0 = \sqrt{2}i[(\not{\mathcal{D}}s_0)P_L\chi^0 - F_0P_R\chi^0] = \sqrt{2}i[(\not{\partial}s_0)P_L\chi^0 - F_0P_R\chi^0]|_{1f} + \cdots, \tag{2.41}$$

$$\mathcal{D}_0 = 2\left(-\mathcal{D}_\mu s_0\mathcal{D}^\mu s_0^* - \frac{1}{2}\bar{\chi}_0P_L\not{\mathcal{D}}\chi^0 - \frac{1}{2}\bar{\chi}_0P_R\not{\mathcal{D}}\chi^0 + F_0F_0^*\right) = 2(-\partial_\mu s_0\partial^\mu s_0^* + F_0F_0^*)|_{0f} + \cdots, \tag{2.42}$$

where the covariant derivatives of the conformal compensator $(s_0, P_L\chi^0, F^0)$ with Weyl/chiral weights $(1,1)$ are given by

$$\mathcal{D}_a s_0 = e_a^\mu\left(\partial_\mu s_0 - \frac{1}{\sqrt{2}}\bar{\psi}_\mu\chi^0\right), \tag{2.43}$$

$$\mathcal{D}_a P_L\chi^0 = e_a^\mu P_L\left[\left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - \frac{3}{2}b_\mu - \frac{1}{2}iA_\mu\right)\chi^0 - \frac{1}{\sqrt{2}}(\not{\mathcal{D}}s_0 + F^0)\psi_\mu - \sqrt{2}s_0\phi_\mu\right]. \tag{2.44}$$

The composite real compensator $\Upsilon \equiv S_0\bar{S}_0e^{-K/3}$

$$\Upsilon = \{\mathcal{C}_\Upsilon, \mathcal{Z}_\Upsilon, \mathcal{H}_\Upsilon, \mathcal{K}_\Upsilon, \mathcal{B}_\mu^\Upsilon, \Lambda_\Upsilon, \mathcal{D}_\Upsilon\} \tag{2.45}$$

[5] $P_{L,R}\gamma_{\text{odd indices}} = \gamma_{\text{odd indices}}P_{R,L}$.

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has components

$$\mathcal{C}_\Upsilon = \Upsilon = s_0\bar{s}_0 e^{-K/3}|_{0f}, \tag{2.46}$$

$$\begin{aligned}\mathcal{Z}_\Upsilon &= e^{-K/3}\mathcal{Z}_0 + \frac{i\sqrt{2}}{3}\Upsilon(K_I\chi^I - K_{\bar{I}}\chi^{\bar{I}}) \\ &= i\sqrt{2}\Upsilon\left(-\frac{1}{s_0}P_L\chi^0 + \frac{1}{s_0^*}P_R\chi^0 + \frac{1}{3}K_I\chi^I - \frac{1}{3}K_{\bar{I}}\chi^{\bar{I}}\right)|_{1f},\end{aligned} \tag{2.47}$$

$$\begin{aligned}\mathcal{H}_\Upsilon &= e^{-K/3}\mathcal{H}_0 + \Upsilon\left(\frac{2}{3}K_I F^I + \left(\frac{1}{9}K_I K_J - \frac{1}{3}K_{IJ}\right)\bar{\chi}^I\chi^J\right) - \frac{1}{2}[\bar{\mathcal{Z}}_0 P_L\mathcal{Z}_\Upsilon + \bar{\mathcal{Z}}_\Upsilon P_L\mathcal{Z}_0] \\ &= -2\Upsilon\left(\frac{F^0}{s_0} - \frac{1}{3}K_I F^I\right)\Big|_{0f} + \cdots,\end{aligned} \tag{2.48}$$

$$\begin{aligned}\mathcal{K}_\Upsilon &= e^{-K/3}\mathcal{H}_0^* + \Upsilon\left(\frac{2}{3}K_{\bar{I}}\bar{F}^{\bar{I}} + \left(\frac{1}{9}K_{\bar{I}}K_{\bar{J}} - \frac{1}{3}K_{\bar{I}\bar{J}}\right)\bar{\chi}^{\bar{I}}\chi^{\bar{J}}\right) - \frac{1}{2}[\bar{\mathcal{Z}}_0 P_R\mathcal{Z}_\Upsilon + \bar{\mathcal{Z}}_\Upsilon P_R\mathcal{Z}_0], \\ &= -2\Upsilon\left(\frac{F^{0*}}{s_0^*} - \frac{1}{3}K_{\bar{I}}\bar{F}^{\bar{I}}\right)\Big|_{0f} + \cdots,\end{aligned} \tag{2.49}$$

$$\begin{aligned}\mathcal{B}_\mu^\Upsilon &= e^{-K/3}\mathcal{B}_\mu^0 - i\Upsilon\left(\frac{1}{3}K_I\mathcal{D}_\mu z^I - \frac{1}{3}K_{\bar{I}}\mathcal{D}_\mu\bar{z}^{\bar{I}} - \left(\frac{1}{9}K_I K_{\bar{J}} - \frac{1}{3}K_{I\bar{J}}\right)\bar{\chi}^I\gamma_\mu\chi^{\bar{J}}\right) \\ &\quad + \frac{1}{4}i[\bar{\mathcal{Z}}_0\gamma_*\gamma_\mu\mathcal{Z}_\Upsilon + \bar{\mathcal{Z}}_\Upsilon\gamma_*\gamma_\mu\mathcal{Z}_0], \\ &= i\Upsilon\left(\frac{1}{s_0}\partial_\mu s_0 - \frac{1}{s_0^*}\partial_\mu s_0^* - \frac{1}{3}K_I\partial_\mu z^I + \frac{1}{3}K_{\bar{I}}\partial_\mu\bar{z}^{\bar{I}}\right)\Big|_{0f} + \cdots.\end{aligned} \tag{2.50}$$

$$\begin{aligned}\Lambda_\Upsilon &= e^{-K/3}\Lambda_0 - i\sqrt{2}\Upsilon\left(\left(\frac{1}{9}K_{\bar{I}}K_J - \frac{1}{3}K_{\bar{I}J}\right)[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J]\right. \\ &\quad + \frac{1}{2}\left(-\frac{1}{27}K_{\bar{I}}K_{\bar{J}}K_K + \frac{1}{9}(K_{\bar{I}\bar{J}}K_K + K_{\bar{I}K}K_{\bar{J}} + K_{\bar{I}}K_{\bar{J}K}) - \frac{1}{3}K_{\bar{I}\bar{J}K}\right)\chi^K\bar{\chi}^{\bar{I}}\chi^J \\ &\quad - \left(\frac{1}{9}K_I K_{\bar{J}} - \frac{1}{3}K_{I\bar{J}}\right)[(\not{\mathcal{D}}\bar{z}^{\bar{J}})\chi^I - F^I\chi^{\bar{J}}] \\ &\quad \left.-\frac{1}{2}\left(-\frac{1}{27}K_I K_J K_{\bar{K}} + \frac{1}{9}(K_{IJ}K_{\bar{K}} + K_{I\bar{K}}K_J + K_I K_{J\bar{K}}) - \frac{1}{3}K_{IJ\bar{K}}\right)\chi^{\bar{K}}\bar{\chi}^I\chi^{\bar{J}}\right) \\ &\quad + \frac{1}{2}\left([i\gamma_*\not{\mathcal{B}}_0 + \mathrm{Re}\mathcal{H}_0 - i\gamma_*\mathrm{Im}\mathcal{H}_0 - \not{\mathcal{D}}(s_0 s_0^*)]\mathcal{Z}_\Upsilon\right. \\ &\quad \left.+[i\gamma_*\not{\mathcal{B}}_\Upsilon + \mathrm{Re}\mathcal{H}_\Upsilon - i\gamma_*\mathrm{Im}\mathcal{H}_\Upsilon - \not{\mathcal{D}}(\Upsilon)]\mathcal{Z}_0\right), \\ &= -\sqrt{2}ie^{-K/3}[(\not{\partial}s_0^*)P_R\chi^0 - F_0^* P_L\chi^0] + \sqrt{2}ie^{-K/3}[(\not{\partial}s_0)P_L\chi^0 - F_0 P_R\chi^0] \\ &\quad -i\sqrt{2}\Upsilon\left(\left(\frac{1}{9}K_{\bar{I}}K_J - \frac{1}{3}K_{\bar{I}J}\right)[(\not{\partial}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] - \left(\frac{1}{9}K_I K_{\bar{J}} - \frac{1}{3}K_{I\bar{J}}\right)[(\not{\partial}\bar{z}^{\bar{J}})\chi^I - F^I\chi^{\bar{J}}]\right) \\ &\quad + \frac{1}{2}\left[\left\{i\gamma_*(is_0^*\not{\partial}s_0 - is_0\not{\partial}s_0^*) + \mathrm{Re}[-2s_0^* F^0] - i\gamma_*\mathrm{Im}[-2s_0^* F^0] - \not{\partial}(s_0 s_0^*)\right\}\right.\end{aligned}$$

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$$\times\left\{i\sqrt{2}\Upsilon\left(-s_0^{-1}P_L\chi^0+s_0^{-1*}P_R\chi^0+\frac{1}{3}K_I\chi^I-\frac{1}{3}K_{\bar{I}}\chi^{\bar{I}}\right)\right\}$$

$$+\left\{i\gamma_*\left(i\Upsilon\left(s_0^{-1}\not{\partial}s_0-s_0^{-1*}\not{\partial}s_0^*-\frac{1}{3}K_I\not{\partial}z^I+\frac{1}{3}K_{\bar{I}}\not{\partial}\bar{z}^{\bar{I}}\right)\right)+\mathrm{Re}\left[-2\Upsilon\left(F^0s_0^{-1}-\frac{1}{3}K_IF^I\right)\right]\right.$$

$$\left.\left.-i\gamma_*\left(-2\Upsilon(F^0s_0^{-1})-\frac{1}{3}K_IF^I\right)-\not{\partial}\Upsilon\right\}\left\{i\sqrt{2}(-s_0^*P_L\chi^0+s_0P_R\chi^0)\right\}\right]\Big|_{1f}+\cdots.$$

$$\mathcal{D}_\Upsilon = e^{-K/3}\mathcal{D}_0+\Upsilon\left[2\left(\frac{1}{9}K_IK_{\bar{J}}-\frac{1}{3}K_{I\bar{J}}\right)\left(-\mathcal{D}_\mu z^I\mathcal{D}^\mu\bar{z}^{\bar{J}}-\frac{1}{2}\bar{\chi}^I\not{P}_L\not{\mathcal{D}}\chi^{\bar{J}}\right.\right.$$

$$\left.-\frac{1}{2}\bar{\chi}^{\bar{J}}\not{P}_R\not{\mathcal{D}}\chi^I+F^I\bar{F}^{\bar{J}}\right)+\left(-\frac{1}{27}K_IK_JK_{\bar{K}}+\frac{1}{9}(K_{IJ}K_{\bar{K}}+K_{I\bar{K}}K_J+K_IK_{J\bar{K}})\right.$$

$$\left.-\frac{1}{3}K_{IJ\bar{K}}\right)\left(-\bar{\chi}^I\chi^JF^{\bar{K}}+\bar{\chi}^I(\not{\mathcal{D}}z^J)\chi^{\bar{K}}\right)$$

$$+\left(-\frac{1}{27}K_{\bar{I}}K_{\bar{J}}K_K+\frac{1}{9}(K_{\bar{I}\bar{J}}K_K+K_{\bar{I}K}K_{\bar{J}}+K_{\bar{I}}K_{\bar{J}K})-\frac{1}{3}K_{\bar{I}\bar{J}K}\right)$$

$$\times\left(-\bar{\chi}^{\bar{I}}\chi^{\bar{J}}F^K+\bar{\chi}^{\bar{I}}(\not{\mathcal{D}}\bar{z}^{\bar{J}})\chi^K\right)+\frac{1}{2}\left\{-\frac{1}{3}K_{IJ\bar{K}\bar{L}}+\frac{1}{81}K_IK_JK_{\bar{K}}K_{\bar{L}}\right.$$

$$-\frac{1}{27}[K_{IJ}K_{\bar{K}}K_{\bar{L}}+K_{I\bar{K}}K_JK_{\bar{L}}+K_IK_{J\bar{K}}K_{\bar{L}}+K_{I\bar{L}}K_JK_{\bar{K}}+K_IK_{J\bar{L}}K_{\bar{K}}+K_IK_JK_{\bar{K}\bar{L}}]$$

$$\left.+\frac{1}{9}[K_{IJ\bar{K}}+K_{IJ\bar{L}}K_{\bar{K}}+K_{IJ}K_{\bar{K}\bar{L}}+K_{I\bar{K}\bar{L}}K_J+K_{I\bar{K}}K_{J\bar{L}}+K_{I\bar{L}}K_{J\bar{K}}+K_IK_{J\bar{K}\bar{L}}]\right\}$$

$$\left.\times(\bar{\chi}^I\not{P}_L\chi^J)(\bar{\chi}^{\bar{K}}\not{P}_R\chi^{\bar{L}})\right]$$

$$+\frac{1}{2}(\mathcal{H}_0\mathcal{H}_\Upsilon^*+\mathcal{H}_0^*\mathcal{H}_\Upsilon-2\mathcal{B}_\mu^0\mathcal{B}_\Upsilon^\mu-2\mathcal{D}(s_0s_0^*)\cdot\mathcal{D}e^{-K/3}-2\bar{\Lambda}_0\mathcal{Z}_\Upsilon-2\bar{\Lambda}_\Upsilon\mathcal{Z}_0-\bar{\mathcal{Z}}_0\not{\mathcal{D}}\mathcal{Z}_\Upsilon-\bar{\mathcal{Z}}_\Upsilon\not{\mathcal{D}}\mathcal{Z}_0).$$

$$=\left[e^{-K/3}2(-\partial_\mu s_0\partial^\mu s_0^*+F_0F_0^*)+2\Upsilon\left(\frac{1}{9}K_IK_{\bar{J}}-\frac{1}{3}K_{I\bar{J}}\right)(-\partial_\mu z^I\partial^\mu\bar{z}^{\bar{J}}+F^I\bar{F}^{\bar{J}})\right.$$

$$+\frac{1}{2}(-2s_0^*F^0)\left(-2\Upsilon\left(\frac{F^{0*}}{s_0^*}-\frac{1}{3}K_{\bar{I}}\bar{F}^{\bar{I}}\right)\right)+\frac{1}{2}(-2s_0F^{0*})\left(-2\Upsilon\left(\frac{F^0}{s_0}-\frac{1}{3}K_IF^I\right)\right)$$

$$-(is_0^*\partial_\mu s_0-is_0\partial_\mu s_0^*)\left(i\Upsilon\left(\frac{1}{s_0}\partial^\mu s_0-\frac{1}{s_0^*}\partial^\mu s_0^*-\frac{1}{3}K_I\partial^\mu z^I+\frac{1}{3}K_{\bar{I}}\partial^\mu\bar{z}^{\bar{I}}\right)\right)$$

$$\left.-(s_0^*\partial s_0+s_0\partial s_0^*)\left(-\frac{1}{3}e^{-K/3}(K_I\partial z^I+K_{\bar{I}}\partial\bar{z}^{\bar{I}})\right)\right]_{0f}+\cdots. \tag{2.51}$$

The conventional superconformal gauge is defined by the choice

$$\mathcal{C}_\Upsilon = \Upsilon = 1, \tag{2.52}$$

$$\mathcal{Z}_\Upsilon = 0 \implies P_L\chi^0-\frac{1}{3}e^{K/6}K_IP_L\chi^I=0, \tag{2.53}$$

$$b_\mu = 0. \tag{2.54}$$

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2.2.3 $\mathcal{W}^2(K) \equiv \mathcal{W}_\alpha(K)\mathcal{W}^\alpha(K)$ composite chiral multiplet: (Weyl/Chiral) weights = (3, 3)

Let us define

$$\mathcal{W}^2(K) \equiv \{\mathcal{C}_W, \mathcal{Z}_W, \mathcal{H}_W, \mathcal{K}_W, \mathcal{B}^W_\mu, \Lambda_W, \mathcal{D}_W\}, \tag{2.55}$$

$$\bar{\mathcal{W}}^2(K) \equiv \{\mathcal{C}^*_W, \mathcal{Z}^C_W, \mathcal{K}^*_W, \mathcal{H}^*_W, (\mathcal{B}^W_\mu)^*, \Lambda^C_W, \mathcal{D}^*_W\} \tag{2.56}$$

where

$$\begin{aligned}
\mathcal{C}_W &= \bar{\Lambda}_K P_L \Lambda_K \\
&= -2\Big[K_{\bar{I}'J'}\bar{\chi}^{\bar{I}'}\not{\partial} z^{J'} K_{\bar{I}J}\not{\partial} z^J \chi^{\bar{I}} - K_{\bar{I}'J'}\bar{\chi}^{\bar{I}'}\not{\partial} z^{J'} K_{\bar{I}J}\bar{F}^{\bar{I}}\chi^J \\
&\quad -K_{\bar{I}'J'}\bar{F}^{\bar{I}'}\bar{\chi}^{J'} K_{\bar{I}J}\not{\partial} z^J \chi^{\bar{I}} + K_{\bar{I}'J'}\bar{F}^{\bar{I}'}\bar{\chi}^{J'} K_{\bar{I}J}\bar{F}^{\bar{I}}\chi^J\Big]\Big|_{2f} + \cdots,
\end{aligned} \tag{2.57}$$

$$\mathcal{Z}_W = -i2P_L\left(-\frac{1}{2}\gamma\cdot\hat{F}_K + i\mathcal{D}_K\right)\Lambda_K = -2\sqrt{2}i\tilde{\mathcal{F}}K_{\bar{I}J}P_L[\not{\partial} z^J\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J]\Big|_{1f} + \cdots,$$

$$\mathcal{H}_W = -2(2\bar{\Lambda}_K P_L \not{\mathcal{D}}\Lambda_K + \hat{F}^-_K\cdot\hat{F}^-_K - \mathcal{D}^2_K) = 2\tilde{\mathcal{F}}^2|_{0f} + \cdots \equiv -2F^W, \tag{2.58}$$

$$\mathcal{K}_W = 0, \tag{2.59}$$

$$\mathcal{B}^W_\mu = i\mathcal{D}_\mu(\bar{\Lambda}_K P_L\Lambda_K) = i\partial_\mu(\bar{\Lambda}_K P_L\Lambda_K)|_{2f} + \cdots, \tag{2.60}$$

$$\Lambda_W = 0, \tag{2.61}$$

$$\mathcal{D}_W = 0. \tag{2.62}$$

Note that $\mathcal{C}_W = \mathcal{C}_W|_{2f} + \cdots, \mathcal{Z}_W = \mathcal{Z}_W|_{1f} + \cdots, \mathcal{H}_W = \mathcal{H}_W|_{0f} + \cdots, \mathcal{K}_W = 0, \mathcal{B}^W_\mu = \mathcal{B}^W_\mu|_{2f} + \cdots, \Lambda_W = \mathcal{D}_W = 0$, and

$$\mathcal{C}_{\bar{W}} = \mathcal{C}^*_W = \bar{\Lambda}_K P_R \Lambda_K, \tag{2.63}$$

$$\mathcal{Z}_{\bar{W}} = \mathcal{Z}^C_W = i2P_R\left(-\frac{1}{2}\gamma\cdot\hat{F}_K - i\mathcal{D}_K\right)\Lambda_K, \tag{2.64}$$

$$\mathcal{H}_{\bar{W}} = \mathcal{K}^*_W = 0, \tag{2.65}$$

$$\mathcal{K}_{\bar{W}} = \mathcal{H}^*_W = -2(2\bar{\Lambda}_K P_R\not{\mathcal{D}}\Lambda_K + \hat{F}^+_K\cdot\hat{F}^+_K - \mathcal{D}^2_K) \equiv -2\bar{F}^{\bar{W}}, \tag{2.66}$$

$$\mathcal{B}^{\bar{Q}}_\mu = (\mathcal{B}^W_\mu)^* = -i\mathcal{D}_\mu(\bar{\Lambda}_K P_R\Lambda_K), \tag{2.67}$$

$$\Lambda_{\bar{W}} = \Lambda^C_W = 0, \tag{2.68}$$

$$\mathcal{D}_{\bar{W}} = \mathcal{D}^*_W = 0. \tag{2.69}$$

We also defined:

$$\hat{F}^K_{ab} \equiv e^\mu_a e^\nu_b(2\partial_{[\mu}\mathcal{B}^K_{\nu]} + \bar{\psi}_{[\mu}\gamma_{\nu]}\Lambda_K), \tag{2.70}$$

$$\tilde{\hat{F}}_{ab} \equiv -i\frac{1}{2}\varepsilon_{abcd}\hat{F}^{cd}, \qquad \hat{F}^\pm_{ab} \equiv \frac{1}{2}(\hat{F}_{ab} \pm \tilde{\hat{F}}_{ab}), \qquad (\hat{F}^\pm_{ab})^* = \hat{F}^\mp_{ab} \tag{2.71}$$

$$\not{\mathcal{D}}\Lambda_K \equiv \gamma\cdot\mathcal{D}\Lambda_K, \tag{2.72}$$

$$\mathcal{D}_\mu\Lambda_K \equiv \left(\partial_\mu - \frac{3}{2}b_\mu + \frac{1}{4}\omega^{ab}_\mu\gamma_{ab} - \frac{3}{2}i\gamma_*\mathcal{A}_\mu\right)\Lambda_K - \left(\frac{1}{4}\gamma^{ab}\hat{F}^K_{ab} + \frac{1}{2}i\gamma_*\mathcal{D}_K\right)\psi_\mu. \tag{2.73}$$

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Next, we shall consider a gauge fixing that is equivalent to the gauge condition given by $\eta = 0$ in ref. [10]. In fact, this can be obtained by imposing $\Lambda_K = 0$. We will call this gauge the "Liberated SUGRA gauge". Then, the only non-vanishing superconformal components of the multiplets $\mathcal{W}^2(K), \bar{\mathcal{W}}^2(K)$ in this gauge are given by $\mathcal{H}_W = -2(\hat{F}_K^- \cdot \hat{F}_K^- - \mathcal{D}_K^2)$ and $\mathcal{K}_{\bar{W}} = -2(\hat{F}_K^+ \cdot \hat{F}_K^+ - \mathcal{D}_K^2) = \mathcal{H}_W^*$.

Another representation of the chiral multiplet is

$$\begin{aligned}
\mathcal{W}^2(K) &= \Big(\mathcal{C}_W, \frac{i}{\sqrt{2}}\mathcal{Z}_W, -\frac{1}{2}\mathcal{H}_W\Big) \\
&= \Big(\bar{\Lambda}_K P_L \Lambda_K,\ \sqrt{2} P_L(-\gamma\cdot\hat{F}_K + 2i\mathcal{D}_K)\Lambda_K,\ 2\bar{\Lambda}_K P_L \not{\mathcal{D}}\Lambda_K + \hat{F}_K^- \cdot \hat{F}_K^- - \mathcal{D}_K^2\Big) \\
&\equiv \Big(X^W, P_L\chi^W, F^W\Big), \qquad (2.74) \\
\bar{\mathcal{W}}^2(K) &= \Big(\mathcal{C}_W^*, -\frac{i}{\sqrt{2}}\mathcal{Z}_W^C, -\frac{1}{2}\mathcal{H}_W^*\Big) \\
&= \Big(\bar{\Lambda}_K P_R \Lambda_K,\ \sqrt{2} P_R(-\gamma\cdot\hat{F}_K - 2i\mathcal{D}_K^*)\Lambda_K^C,\ 2\bar{\Lambda}_K P_R \not{\mathcal{D}}\Lambda_K + \hat{F}_K^+ \cdot \hat{F}_K^+ - (\mathcal{D}_K^*)^2\Big) \\
&\equiv \Big(\bar{X}^{\bar{W}}, P_R\chi^{\bar{W}}, \bar{F}^{\bar{W}}\Big) \qquad (2.75)
\end{aligned}$$

We will also need the following definitions:

$$\begin{aligned}
X^W = \bar{\Lambda}_K P_L \Lambda_K \equiv W = &-2K_{\bar{I}J}[\bar{\chi}^J(\overline{\not{\mathcal{D}} z^{\bar{I}}}) - \bar{F}^{\bar{I}}\bar{\chi}^J]K_{\bar{I}'J'}[(\not{\mathcal{D}} z^{\bar{I}'})\chi^{J'} - F^{J'}\chi^{\bar{I}'}] \\
&-K_{\bar{I}J}[\bar{\chi}^J(\overline{\not{\mathcal{D}} z^{\bar{I}}}) - \bar{F}^{\bar{I}}\bar{\chi}^J]K_{\bar{I}'\bar{J}'K'}[\chi^{K'}\bar{\chi}^{\bar{I}'}\chi^{\bar{J}'}] \\
&-K_{\bar{I}\bar{J}K}[\bar{\chi}^{\bar{J}}\chi^{\bar{I}}\bar{\chi}^K]K_{\bar{I}'J'}[(\not{\mathcal{D}} z^{\bar{I}'})\chi^{J'} - F^{J'}\chi^{\bar{I}'}] \\
&-\frac{1}{2}K_{\bar{I}\bar{J}K}[\bar{\chi}^{\bar{J}}\chi^{\bar{I}}\bar{\chi}^K]K_{\bar{I}'\bar{J}'K'}[\chi^{K'}\bar{\chi}^{\bar{I}'}\chi^{\bar{J}'}], \qquad (2.76) \\
\bar{X}^{\bar{W}} = \bar{\Lambda}_K P_R \Lambda_K \equiv \bar{W} = &(\bar{\Lambda}_K P_L \Lambda_K)^C, \qquad (2.77)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{D}_K P_L \Lambda_K = &\Big[2K_{I\bar{J}}\Big(-\mathcal{D}_\mu z^I \mathcal{D}^\mu \bar{z}^{\bar{J}} - \frac{1}{2}\bar{\chi}^I P_L \not{\mathcal{D}}\chi^{\bar{J}} - \frac{1}{2}\bar{\chi}^{\bar{J}} P_R \not{\mathcal{D}}\chi^I + F^I\bar{F}^{\bar{J}}\Big) \\
&+K_{IJ\bar{K}}\Big(-\bar{\chi}^I\chi^J\bar{F}^{\bar{K}} + \bar{\chi}^I(\not{\mathcal{D}} z^J)\chi^{\bar{K}}\Big) + K_{\bar{I}\bar{J}K}\Big(-\bar{\chi}^{\bar{I}}\chi^{\bar{J}}F^K + \bar{\chi}^{\bar{I}}(\not{\mathcal{D}}\bar{z}^{\bar{J}})\chi^K\Big) \\
&+\frac{1}{2}K_{IJ\bar{K}\bar{L}}(\bar{\chi}^I P_L\chi^J)(\bar{\chi}^{\bar{K}} P_R\chi^{\bar{L}})\Big] \\
&\times\Big[-\sqrt{2}iK_{\bar{I}'J'}[(\not{\mathcal{D}} z^{J'})\chi^{\bar{I}'} - \bar{F}^{\bar{I}'}\chi^{J'}] - \frac{i}{\sqrt{2}}K_{\bar{I}'\bar{J}'K'}\chi^{K'}\bar{\chi}^{\bar{I}'}\chi^{J'}\Big]. \qquad (2.78) \\
\mathcal{D}_K^2 = &\Big[2K_{I\bar{J}}\Big(-\mathcal{D}_\mu z^I \mathcal{D}^\mu \bar{z}^{\bar{J}} - \frac{1}{2}\bar{\chi}^I P_L \not{\mathcal{D}}\chi^{\bar{J}} - \frac{1}{2}\bar{\chi}^{\bar{J}} P_R \not{\mathcal{D}}\chi^I + F^I\bar{F}^{\bar{J}}\Big) \\
&+K_{IJ\bar{K}}\Big(-\bar{\chi}^I\chi^J\bar{F}^{\bar{K}} + \bar{\chi}^I(\not{\mathcal{D}} z^J)\chi^{\bar{K}}\Big) + K_{\bar{I}\bar{J}K}\Big(-\bar{\chi}^{\bar{I}}\chi^{\bar{J}}F^K + \bar{\chi}^{\bar{I}}(\not{\mathcal{D}}\bar{z}^{\bar{J}})\chi^K\Big) \\
&+\frac{1}{2}K_{IJ\bar{K}\bar{L}}(\bar{\chi}^I P_L\chi^J)(\bar{\chi}^{\bar{K}} P_R\chi^{\bar{L}})\Big]^2. \qquad (2.79)
\end{aligned}$$

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2.2.4 $w^2, \bar{w}^2$ composite complex multiplets: (Weyl/Chiral) weights $= (-1, \pm 3)$

$$w^2 \equiv \frac{\mathcal{W}^2(K)}{\Upsilon^2} = \{\mathcal{C}_w, \mathcal{Z}_w, \mathcal{H}_w, \mathcal{K}_w, \mathcal{B}^w_\mu, \Lambda_w, \mathcal{D}_w\}, \tag{2.80}$$

$$\bar{w}^2 \equiv \frac{\bar{\mathcal{W}}^2(K)}{\Upsilon^2} = \{\mathcal{C}_{\bar{w}}, \mathcal{Z}_{\bar{w}}, \mathcal{H}_{\bar{w}}, \mathcal{K}_{\bar{w}}, \mathcal{B}^{\bar{w}}_\mu, \Lambda_{\bar{w}}, \mathcal{D}_{\bar{w}}\} \tag{2.81}$$

where

$$\mathcal{C}_w = \frac{\mathcal{C}_W}{\Upsilon^2}, \tag{2.82}$$

$$\mathcal{Z}_w = \frac{1}{\Upsilon^2}\mathcal{Z}_W - 2\frac{\mathcal{C}_W}{\Upsilon^3}\mathcal{Z}_\Upsilon, \tag{2.83}$$

$$\mathcal{H}_w = \frac{1}{\Upsilon^2}\mathcal{H}_W - 2\frac{\mathcal{C}_W}{\Upsilon^3}\mathcal{H}_\Upsilon - \frac{1}{2}\Big[-4\frac{1}{\Upsilon^3}(\bar{\mathcal{Z}}_W P_L \mathcal{Z}_\Upsilon + \bar{\mathcal{Z}}_\Upsilon P_L \mathcal{Z}_W) + 6\frac{\mathcal{C}_W}{\Upsilon^4}\bar{\mathcal{Z}}_\Upsilon P_L \mathcal{Z}_\Upsilon\Big], \tag{2.84}$$

$$\mathcal{K}_w = \frac{1}{\Upsilon^2}\mathcal{K}_W - 2\frac{\mathcal{C}_W}{\Upsilon^3}\mathcal{K}_\Upsilon - \frac{1}{2}\Big[-4\frac{1}{\Upsilon^3}(\bar{\mathcal{Z}}_W P_R \mathcal{Z}_\Upsilon + \bar{\mathcal{Z}}_\Upsilon P_R \mathcal{Z}_W) + 6\frac{\mathcal{C}_W}{\Upsilon^4}\bar{\mathcal{Z}}_\Upsilon P_R \mathcal{Z}_\Upsilon\Big], \tag{2.85}$$

$$\mathcal{B}^w_\mu = \frac{1}{\Upsilon^2}\mathcal{B}^W_\mu - 2\frac{\mathcal{C}_W}{\Upsilon^3}\mathcal{B}^\Upsilon_\mu + \frac{1}{2}i\Big[-4\frac{1}{\Upsilon^3}(\bar{\mathcal{Z}}_W P_L \gamma_\mu \mathcal{Z}_\Upsilon + \bar{\mathcal{Z}}_\Upsilon P_L \gamma_\mu \mathcal{Z}_W) + 6\frac{\mathcal{C}_W}{\Upsilon^4}\bar{\mathcal{Z}}_\Upsilon P_L \gamma_\mu \mathcal{Z}_\Upsilon\Big], \tag{2.86}$$

$$\begin{aligned}\Lambda_w = {} & \frac{1}{\Upsilon^2}\Lambda_W - 2\frac{\mathcal{C}_W}{\Upsilon^3}\Lambda_\Upsilon + \frac{1}{2}\Big[-\frac{2}{\Upsilon^3}(i\gamma_* \not{\mathcal{B}}_W + P_L \mathcal{K}_W + P_R \mathcal{H}_W - \not{\mathcal{D}}\mathcal{C}_W)\mathcal{Z}_\Upsilon \\ & -\frac{2}{\Upsilon^3}(i\gamma_* \not{\mathcal{B}}_\Upsilon + P_L \mathcal{K}_\Upsilon + P_R \mathcal{H}_\Upsilon - \not{\mathcal{D}}\mathcal{C}_\Upsilon)\mathcal{Z}_W + 6\frac{\mathcal{C}_W}{\Upsilon^4}(i\gamma_* \not{\mathcal{B}}_\Upsilon + P_L \mathcal{K}_\Upsilon + P_R \mathcal{H}_\Upsilon - \not{\mathcal{D}}\mathcal{C}_\Upsilon)\mathcal{Z}_\Upsilon\Big], \\ & -\frac{1}{4}\Big[\frac{6}{\Upsilon^4}\frac{3!}{2!}\mathcal{Z}_{(W}\bar{\mathcal{Z}}_\Upsilon \mathcal{Z}_{\Upsilon)} - 24\frac{\mathcal{C}_W}{\Upsilon^5}\mathcal{Z}_\Upsilon \bar{\mathcal{Z}}_\Upsilon \mathcal{Z}_\Upsilon\Big],\end{aligned} \tag{2.87}$$

$$\begin{aligned}\mathcal{D}_w = {} & \frac{1}{\Upsilon^2}\mathcal{D}_W - 2\frac{\mathcal{C}_W}{\Upsilon^3}\mathcal{D}_\Upsilon + \frac{1}{2}\Big[-\frac{2}{\Upsilon^3}2!(\mathcal{K}_{(W}\mathcal{H}_{\Upsilon)} - \mathcal{B}_{(W}\cdot\mathcal{B}_{\Upsilon)} - \mathcal{D}\mathcal{C}_{(W}\cdot\mathcal{D}\mathcal{C}_{\Upsilon)} - 2\bar{\Lambda}_{(W}\mathcal{Z}_{\Upsilon)} - \bar{\mathcal{Z}}_{(W}\not{\mathcal{D}}\mathcal{Z}_{\Upsilon)}) \\ & +6\frac{\mathcal{C}_W}{\Upsilon^4}(\mathcal{K}_\Upsilon\mathcal{H}_\Upsilon - \mathcal{B}_\Upsilon\cdot\mathcal{B}_\Upsilon - \mathcal{D}\mathcal{C}_\Upsilon\cdot\mathcal{D}\mathcal{C}_\Upsilon - 2\bar{\Lambda}_\Upsilon\mathcal{Z}_\Upsilon - \bar{\mathcal{Z}}_\Upsilon\not{\mathcal{D}}\mathcal{Z}_\Upsilon)\Big] \\ & -\frac{1}{4}\Big[\frac{6}{\Upsilon^4}\frac{3!}{2!}\bar{\mathcal{Z}}_{(W}(i\gamma_* \not{\mathcal{B}}_\Upsilon + P_L\mathcal{K}_\Upsilon + P_R\mathcal{H}_\Upsilon)\mathcal{Z}_{\Upsilon)} - 24\frac{\mathcal{C}_W}{\Upsilon^5}\bar{\mathcal{Z}}_\Upsilon(i\gamma_* \not{\mathcal{B}}_\Upsilon + P_L\mathcal{K}_\Upsilon + P_R\mathcal{H}_\Upsilon)\mathcal{Z}_\Upsilon\Big], \\ & +\frac{1}{8}\Big[-\frac{24}{\Upsilon^5}\frac{4!}{3!}\bar{\mathcal{Z}}_{(W}P_L\mathcal{Z}_\Upsilon\bar{\mathcal{Z}}_\Upsilon P_R\mathcal{Z}_{\Upsilon)} + 120\frac{\mathcal{C}_W}{\Upsilon^6}\bar{\mathcal{Z}}_\Upsilon P_L\mathcal{Z}_\Upsilon\bar{\mathcal{Z}}_\Upsilon P_R\mathcal{Z}_\Upsilon\Big].\end{aligned} \tag{2.88}$$

Note that we have to insert $\mathcal{K}_W = \Lambda_W = \mathcal{D}_W = 0$ and $\mathcal{H}_{\bar{W}} = \Lambda_{\bar{W}} = \mathcal{D}_{\bar{W}} = 0$ as given in eqs. (2.62) and (2.69). In addition, the complex conjugate multiplet $\bar{w}^2$ can be obtained by taking a replacement $Q \to \bar{Q}$ in the above expressions.

In the Liberated SUGRA gauge ($\Lambda_K = 0$), the non-vanishing components are given by

$$\mathcal{H}_w|_{\Lambda_K=0} = \frac{\mathcal{H}_W}{\Upsilon^2}, \qquad \mathcal{K}_{\bar{w}}|_{\Lambda_K=0} = \frac{\mathcal{K}_{\bar{W}}}{\Upsilon^2} \tag{2.89}$$

$$\Lambda_w|_{\Lambda_K=0} = -\frac{1}{\Upsilon^3}P_R\mathcal{H}_W\mathcal{Z}_\Upsilon, \qquad \Lambda_{\bar{w}}|_{\Lambda_K=0} = -\frac{1}{\Upsilon^3}P_L\mathcal{K}_{\bar{W}}\mathcal{Z}_\Upsilon \tag{2.90}$$

$$\mathcal{D}_w|_{\Lambda_K=0} = -\frac{1}{\Upsilon^3}\mathcal{K}_\Upsilon\mathcal{H}_W - \frac{3}{2\Upsilon^4}\bar{\mathcal{Z}}P_R\mathcal{H}_W\mathcal{Z}_\Upsilon, \quad \mathcal{D}_{\bar{w}}|_{\Lambda_K=0} = -\frac{1}{\Upsilon^3}\mathcal{K}_{\bar{W}}\mathcal{H}_\Upsilon - \frac{3}{2\Upsilon^4}\bar{\mathcal{Z}}P_L\mathcal{K}_{\bar{W}}\mathcal{Z}_\Upsilon. \tag{2.91}$$

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Furthermore, with the superconformal gauge choice $\mathcal{Z}_\Upsilon = 0$, the non-vanishing components are

$$\mathcal{H}_w|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = \frac{\mathcal{H}_W}{\Upsilon^2}, \qquad \mathcal{K}_{\bar{w}}|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = \frac{\mathcal{K}_{\bar{W}}}{\Upsilon^2} \tag{2.92}$$

$$\mathcal{D}_w|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = -\frac{1}{\Upsilon^3}\mathcal{K}_\Upsilon\mathcal{H}_W, \qquad \mathcal{D}_{\bar{w}}|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = -\frac{1}{\Upsilon^3}\mathcal{K}_{\bar{W}}\mathcal{H}_\Upsilon. \tag{2.93}$$

2.2.5 $T(\bar{w}^2), \bar{T}(w^2)$ chiral projection multiplets: (Weyl/Chiral) weights = (0, 0)

$$T(\bar{w}^2) = \left(-\frac{1}{2}\mathcal{K}_{\bar{w}}, -\frac{1}{2}\sqrt{2}iP_L(\not{\mathcal{D}}\mathcal{Z}_{\bar{w}} + \Lambda_{\bar{w}}), \frac{1}{2}(\mathcal{D}_{\bar{w}} + \Box^C\mathcal{C}_{\bar{w}} + i\mathcal{D}_a\mathcal{B}^a_{\bar{w}})\right), \tag{2.94}$$

$$\bar{T}(w^2) = \left(-\frac{1}{2}\mathcal{K}^*_{\bar{w}}, \frac{1}{2}\sqrt{2}iP_R(\not{\mathcal{D}}\mathcal{Z}^C_{\bar{w}} + \Lambda^C_{\bar{w}}), \frac{1}{2}(\mathcal{D}^*_{\bar{w}} + \Box^C\mathcal{C}^*_{\bar{w}} - i\mathcal{D}_a(\mathcal{B}^a_{\bar{w}})^*)\right). \tag{2.95}$$

The superconformal setting for this and its complex conjugate are then given by

$$\begin{aligned} T \equiv T(\bar{w}^2) &= \{\mathcal{C}_T, \mathcal{Z}_T, \mathcal{H}_T, \mathcal{K}_T, \mathcal{B}^T_\mu, \Lambda_T, \mathcal{D}_T\}, \\ \bar{T} \equiv \bar{T}(w^2) &= \{\mathcal{C}_{\bar{T}}, \mathcal{Z}_{\bar{T}}, \mathcal{H}_{\bar{T}}, \mathcal{K}_{\bar{T}}, \mathcal{B}^{\bar{T}}_\mu, \Lambda_{\bar{T}}, \mathcal{D}_{\bar{T}}\}, \end{aligned} \tag{2.96}$$

where

$$\mathcal{C}_T = -\frac{1}{2}\mathcal{K}_{\bar{w}} = -\frac{1}{2\Upsilon^2}\mathcal{K}_{\bar{W}} + \frac{\mathcal{C}_{\bar{W}}}{\Upsilon^3}\mathcal{K}_\Upsilon + \frac{1}{4}\Big[-4\frac{1}{\Upsilon^3}(\bar{\mathcal{Z}}_{\bar{W}}P_R\mathcal{Z}_\Upsilon + \bar{\mathcal{Z}}_\Upsilon P_R\mathcal{Z}_{\bar{W}}) + 6\frac{\mathcal{C}_{\bar{W}}}{\Upsilon^4}\bar{\mathcal{Z}}_\Upsilon P_R\mathcal{Z}_\Upsilon\Big],$$

$$\mathcal{Z}_T = -P_L(\not{\mathcal{D}}\mathcal{Z}_{\bar{w}} + \Lambda_{\bar{w}}), \tag{2.97}$$

$$\mathcal{H}_T = -(\mathcal{D}_{\bar{w}} + \Box^C\mathcal{C}_{\bar{w}} + i\mathcal{D}_a\mathcal{B}^a_{\bar{w}}), \tag{2.98}$$

$$\mathcal{K}_T = 0, \tag{2.99}$$

$$\mathcal{B}^T_\mu = -\frac{1}{2}i\mathcal{D}_\mu\mathcal{K}_{\bar{w}}, \tag{2.100}$$

$$\Lambda_T = 0, \tag{2.101}$$

$$\mathcal{D}_T = 0. \tag{2.102}$$

and

$$\mathcal{C}_{\bar{T}} = -\frac{1}{2}\mathcal{K}^*_{\bar{w}}, \tag{2.103}$$

$$\mathcal{Z}_{\bar{T}} = -P_R(\not{\mathcal{D}}\mathcal{Z}^C_{\bar{w}} + \Lambda^C_{\bar{w}}), \tag{2.104}$$

$$\mathcal{H}_{\bar{T}} = 0, \tag{2.105}$$

$$\mathcal{K}_{\bar{T}} = -(\mathcal{D}^*_{\bar{w}} + \Box^C\mathcal{C}^*_{\bar{w}} - i\mathcal{D}_a(\mathcal{B}^a_{\bar{w}})^*), \tag{2.106}$$

$$\mathcal{B}^{\bar{T}}_\mu = \frac{1}{2}i\mathcal{D}_\mu\mathcal{K}^*_{\bar{w}}, \tag{2.107}$$

$$\Lambda_{\bar{T}} = 0, \tag{2.108}$$

$$\mathcal{D}_{\bar{T}} = 0. \tag{2.109}$$

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In the Liberated SUGRA gauge, the non-vanishing components are given by

$$\mathcal{C}_T|_{\Lambda_K=0} = -\frac{1}{2}\mathcal{K}_{\bar{w}}, \tag{2.110}$$

$$\mathcal{Z}_T|_{\Lambda_K=0} = -P_L\Lambda_{\bar{w}}, \tag{2.111}$$

$$\mathcal{H}_T|_{\Lambda_K=0} = -\mathcal{D}_{\bar{w}}, \tag{2.112}$$

$$\mathcal{B}^T_\mu|_{\Lambda_K=0} = -\frac{1}{2}i\mathcal{D}_\mu\mathcal{K}_{\bar{w}}. \tag{2.113}$$

With the superconformal gauge choice $\mathcal{Z}_\Upsilon = 0$, we have

$$\mathcal{C}_T|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = -\frac{1}{2}\mathcal{K}_{\bar{w}}, \qquad \mathcal{C}_{\bar{T}}|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = -\frac{1}{2}\mathcal{H}_w = -\frac{1}{2}\mathcal{K}^*_{\bar{w}} \tag{2.114}$$

$$\mathcal{H}_T|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = -\mathcal{D}_{\bar{w}}, \qquad \mathcal{K}_{\bar{T}}|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = -\mathcal{D}_w = -\mathcal{D}^*_{\bar{w}} \tag{2.115}$$

$$\mathcal{B}^T_\mu|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = -\frac{1}{2}i\mathcal{D}_\mu\mathcal{K}_{\bar{w}}, \qquad \mathcal{B}^{\bar{T}}_\mu|_{\Lambda_K=0,\mathcal{Z}_\Upsilon=0} = \frac{1}{2}i\mathcal{D}_\mu\mathcal{H}_w = \frac{1}{2}i\mathcal{D}_\mu\mathcal{K}^*_{\bar{w}}. \tag{2.116}$$

2.2.6 Final form of the composite multiplet of liberated $\mathcal{N}=1$ supergravity

To construct the final composite multiplet which will give the new term in the action of liberated supergravity, we first collect the following chiral multiplets:

$$\Big(X^i \equiv \{S_0, z^I, T(\bar{w}^2), \mathcal{W}^2(K)\},\ \chi^i \equiv \{\chi^0, \chi^I, \chi^T, \chi^W\},\ F^i \equiv \{F^0, F^I, F^T, F^W\}\Big), \tag{2.117}$$

where

$$S_0 = \left(s_0, P_L\chi^0, F^0\right), \qquad \bar{S}_0 = \left(s_0^*, P_R\chi^0, F^{0*}\right), \tag{2.118}$$

$$z^I = \left(z^I, P_L\chi^I, F^I\right), \qquad \bar{z^I} = \left(\bar{z}^{\bar{I}}, P_R\chi^{\bar{I}}, \bar{F}^{\bar{I}}\right), \tag{2.119}$$

$$\begin{aligned}\mathcal{W}^2(K) &= \left(\bar{\Lambda}_K P_L\Lambda_K,\ \sqrt{2}P_L(-\gamma\cdot\hat{F}_K + 2i\mathcal{D}_K)\Lambda_K,\ 2\bar{\Lambda}_K P_L\not{\mathcal{D}}\Lambda_K + \hat{F}^-_K\cdot\hat{F}^-_K - \mathcal{D}^2_K\right)\\ &\equiv \left(X^W, P_L\chi^W, F^W\right),\end{aligned} \tag{2.120}$$

$$\begin{aligned}\bar{\mathcal{W}}^2(K) &= \left(\bar{\Lambda}_K P_R\Lambda_K,\ \sqrt{2}P_R(-\gamma\cdot\hat{F}_K - 2i\mathcal{D}^*_K)\Lambda^C_K,\ 2\bar{\Lambda}_K P_R\not{\mathcal{D}}\Lambda_K + \hat{F}^+_K\cdot\hat{F}^+_K - (\mathcal{D}^*_K)^2\right)\\ &\equiv \left(\bar{X}^{\bar{W}}, P_R\chi^{\bar{W}}, \bar{F}^{\bar{W}}\right),\end{aligned} \tag{2.121}$$

$$\begin{aligned}T(\bar{w}^2) &= \left(-\frac{1}{2}\mathcal{K}_{\bar{w}}, -\frac{1}{2}\sqrt{2}iP_L(\not{\mathcal{D}}\mathcal{Z}_{\bar{w}} + \Lambda_{\bar{w}}), \frac{1}{2}(\mathcal{D}_{\bar{w}} + \Box^C\mathcal{C}_{\bar{w}} + i\mathcal{D}_a\mathcal{B}^a_{\bar{w}})\right) \equiv \left(X^T, P_L\chi^T, F^T\right),\\ \bar{T}(w^2) &= \left(-\frac{1}{2}\mathcal{K}^*_{\bar{w}}, \frac{1}{2}\sqrt{2}iP_R(\not{\mathcal{D}}\mathcal{Z}^C_{\bar{w}} + \Lambda^C_{\bar{w}}), \frac{1}{2}(\mathcal{D}^*_{\bar{w}} + \Box^C\mathcal{C}^*_{\bar{w}} - i\mathcal{D}_a(\mathcal{B}^a_{\bar{w}})^*)\right) \equiv \left(\bar{X}^{\bar{T}}, P_R\chi^{\bar{T}}, \bar{F}^{\bar{T}}\right).\end{aligned} \tag{2.122}$$

Then, the lowest component of the final composite real multiplet is

$$\begin{aligned}N &\equiv \Upsilon^2\frac{w^2\bar{w}^2}{T(\bar{w}^2)\bar{T}(w^2)}\mathcal{U} = \Upsilon^{-2}\frac{\mathcal{W}^2(K)\bar{\mathcal{W}}^2(K)}{T(w^2)\bar{T}(\bar{w}^2)}\mathcal{U}\\ &= (X^0\bar{X}^0e^{-K(z^I,\bar{z}^{\bar{I}})/3})^{-2}X^W\bar{X}^{\bar{W}}(X^T)^{-1}(\bar{X}^{\bar{T}})^{-1}\mathcal{U}(z^I,\bar{z}^{\bar{I}}).\end{aligned} \tag{2.123}$$

Then, since N is now expressed as a function of superconformal chiral multiplets, we can represent the full off-shell expression of the superconformal Lagrangian of the liberated $\mathcal{N}=1$ supergravity via eq. (17.19) of Freedman and Van Proeyen [3] as follows:

$$\begin{aligned}
\mathcal{L}_{\rm NEW} &\equiv [\mathbb{N}]_D e^{-1} \\
&= \frac{1}{2}\mathcal{D}_N - \frac{1}{4}\bar{\psi}\cdot\gamma i\gamma_*\Lambda_N - \frac{1}{6}\mathcal{C}_N R(\omega) + \frac{1}{12}\left(\mathcal{C}_N\bar{\psi}_\mu\gamma^{\mu\rho\sigma} - i\bar{\mathcal{Z}}_N\gamma^{\rho\sigma}\gamma_*\right)R'_{\rho\sigma}(Q) \\
&\quad + \frac{1}{8}\varepsilon^{abcd}\bar{\psi}_a\gamma_b\psi_c\left(\mathcal{B}_d^N - \frac{1}{2}\bar{\psi}_d\mathcal{Z}_N\right), \qquad (2.124) \\
&= N_{i\bar{j}}\left(-\mathcal{D}_\mu X^i\mathcal{D}^\mu\bar{X}^{\bar{j}} - \frac{1}{2}\bar{\chi}^i\not{\mathcal{D}}\chi^{\bar{j}} - \frac{1}{2}\bar{\chi}^{\bar{j}}\not{\mathcal{D}}\chi^i + F^i\bar{F}^{\bar{j}}\right) \\
&\quad + \frac{1}{2}\left[N_{ij\bar{k}}\left(-\bar{\chi}^i\chi^j\bar{F}^{\bar{k}} + \bar{\chi}^i(\not{\mathcal{D}}X^j)\chi^{\bar{k}}\right) + \text{h.c.}\right] + \frac{1}{4}N_{ij\bar{k}\bar{l}}\bar{\chi}^i\chi^j\bar{\chi}^{\bar{k}}\chi^{\bar{l}} \\
&\quad + \left[\frac{1}{2\sqrt{2}}\bar{\psi}\cdot\gamma\left(N_{i\bar{j}}F^i\chi^{\bar{j}} - N_{i\bar{j}}\not{\mathcal{D}}\bar{X}^{\bar{j}}\chi^i - \frac{1}{2}N_{ij\bar{k}}\chi^{\bar{k}}\bar{\chi}^i\chi^j\right)\right. \\
&\quad \left. + \frac{1}{8}i\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\left(N_i\mathcal{D}_\sigma X^i + \frac{1}{2}N_{i\bar{j}}\bar{\chi}^i\gamma_\sigma\chi^{\bar{j}} + \frac{1}{\sqrt{2}}N_i\bar{\psi}_\sigma\chi^i\right) + \text{h.c.}\right] \\
&\quad + \frac{1}{6}N\left(-R(\omega) + \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}R'_{\nu\rho}(Q)\right) - \frac{1}{6\sqrt{2}}\left(N_i\bar{\chi}^i + N_{\bar{i}}\bar{\chi}^{\bar{i}}\right)\gamma^{\mu\nu}R'_{\mu\nu}(Q), \qquad (2.125)
\end{aligned}$$

where $i,j = 0, I, W, T$ and $\bar{j},\bar{k},\bar{l} = \bar{0},\bar{I},\bar{W},\bar{T}$ and

$$R_{\mu\nu ab}(\omega) \equiv \partial_\mu\omega_{\nu ab} - \partial_\nu\omega_{\mu ab} + \omega_{\mu ac}\omega_\nu{}^c{}_b - \omega_{\nu ac}\omega_\mu{}^c{}_b, \qquad (2.126)$$

$$R'_{\mu\nu}(Q) \equiv 2\left(\partial_{[\mu} + \frac{1}{2}b_{[\mu} - \frac{3}{2}A_{[\mu}\gamma_* + \frac{1}{4}\omega_{[\mu}^{ab}(e,b,\psi)\gamma_{ab}\right)\psi_{\nu]}, \qquad (2.127)$$

$$\omega_\mu^{ab}(e,b,\psi) = \omega_\mu^{ab}(e,b) + \frac{1}{2}\psi_\mu\gamma^{[a}\psi^{b]} + \frac{1}{4}\bar{\psi}^a\gamma_\mu\psi^b. \qquad (2.128)$$

2.3 Bosonic Lagrangians

The bosonic contribution to the scalar potential can be found from the term $\mathcal{D}_N$. Using the above results and eq. (2.125), we find the equivalent bosonic contribution to the scalar potential as follows:

$$\mathcal{L}_B \supset N_{i\bar{j}}F^i\bar{F}^{\bar{j}} \sim N_{W\bar{W}}F^W\bar{F}^{\bar{W}} = \Upsilon^{-2}\frac{1}{\mathcal{C}_T\mathcal{C}_{\bar{T}}}\mathcal{U}F^W\bar{F}^{\bar{W}}. \qquad (2.129)$$

Since $\mathcal{C}_T = -\frac{1}{2}\mathcal{K}_{\bar{w}} \sim -\frac{1}{2}\frac{\mathcal{K}_{\bar{W}}}{\Upsilon^2} = -\frac{1}{2}\frac{(-2\bar{F}^{\bar{W}})}{\Upsilon^2} = \frac{\bar{F}^{\bar{W}}}{\Upsilon^2}$ and $\mathcal{C}_{\bar{T}} = \frac{F^W}{\Upsilon^2}$, we have

$$\mathcal{L}_B \supset \Upsilon^{-2}\frac{\Upsilon^2\Upsilon^2}{\bar{F}^{\bar{W}}F^W}\mathcal{U}F^W\bar{F}^{\bar{W}} = \Upsilon^2\mathcal{U}. \qquad (2.130)$$

In the superconformal gauge $\Upsilon = 1$, we get

$$V_{\rm NEW} = \mathcal{U}. \qquad (2.131)$$

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2.4 Fermionic Lagrangians

In this section, we investigate fermionic terms in the liberated $\mathcal{N}=1$ supergravity Lagrangian. We focus in particular on the matter-coupling and on the most divergent fermionic terms, in order to explore interesting interactions and check the limits of validity of the liberated $\mathcal{N}=1$ supergravity as an effective theory.

First of all, let us recall that the final composite multiplet N in terms of the superconformal chiral multiplets $\{S_0, z^I, W \equiv \mathcal{W}^2(K), T \equiv T(\bar{w}^2)\}$ are:

$$N = \left(s_0 s_0^* e^{-K(z^I,\bar{z}^{\bar{I}})/3}\right)^{-2} \frac{W\bar{W}}{T\bar{T}} \mathcal{U}(z^I, \bar{z}^{\bar{I}}). \tag{2.132}$$

We we also denote their lowest components as $W \equiv \bar{\Lambda}_K P_L \Lambda_K$, $\bar{W} \equiv \bar{\Lambda}_K P_R \Lambda_K$, $T \equiv \mathcal{C}_T$, and $\bar{T} \equiv \mathcal{C}_{\bar{T}}$. Note that the final composite multiplet consists of the four superconformal chiral multiplets only. Remember that $\mathcal{C}_T \sim \frac{\bar{F}^{\bar{W}}}{\Upsilon^2}$ and $F^W \propto \mathcal{D}_K^2|_{\rm boson} \equiv \tilde{\mathcal{F}}^2$.

Generically, the matter couplings are found from the following contributions:

$$\begin{aligned}
\mathcal{L}_F^{\rm matter}|_{\psi=0} &= N_{i\bar{j}}\Big(- \mathcal{D}_\mu X^i \mathcal{D}^\mu \bar{X}^{\bar{j}} - \frac{1}{2}\bar{\chi}^i \not{\mathcal{D}} \chi^{\bar{j}} - \frac{1}{2}\bar{\chi}^{\bar{j}} \not{\mathcal{D}} \chi^i + F^i \bar{F}^{\bar{j}} \Big) - \frac{N}{6} R(\omega)|_{\psi=0} \\
&\quad + \frac{1}{2}\Big[N_{ij\bar{k}} \Big(- \bar{\chi}^i \chi^j \bar{F}^{\bar{k}} + \bar{\chi}^i (\not{\mathcal{D}} X^j) \chi^{\bar{k}} \Big) + \text{h.c.} \Big] + \frac{1}{4} N_{ij\bar{k}\bar{l}} \bar{\chi}^i \chi^j \bar{\chi}^{\bar{k}} \chi^{\bar{l}} \Big|_{\psi=0}, \\
&= \mathcal{L}_{F1} + \mathcal{L}_{F2} + \bar{\mathcal{L}}_{F2} + \mathcal{L}_{F3} + (\mathcal{L}_{F4} + \mathcal{L}_{F5} + \text{h.c.}) + \mathcal{L}_{F6} + \mathcal{L}_{F7}
\end{aligned} \tag{2.133}$$

where

$$\begin{aligned}
\mathcal{D}_\mu X^i|_{\psi=0} &= (\partial_\mu - w_i b_\mu - w_i A_\mu) X^i, \\
\mathcal{D}_\mu P_L \chi^i|_{\psi=0} &= \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - (w_i + 1/2) b_\mu + (w_i - 3/2) i A_\mu \right) P_L \chi^i - \sqrt{2} w_i X^i P_L \phi_\mu.
\end{aligned}$$

Note that the matter couplings of fermions can be classified into seven types:

$$\mathcal{L}_{F1} \equiv -N_{i\bar{j}} \mathcal{D}_\mu X^i \mathcal{D}^\mu \bar{X}^{\bar{j}}\Big|_{\psi=0}, \tag{2.134}$$

$$\mathcal{L}_{F2} \equiv -\frac{1}{2} N_{i\bar{j}} \bar{\chi}^i \not{\mathcal{D}} \chi^{\bar{j}}\Big|_{\psi=0}, \tag{2.135}$$

$$\mathcal{L}_{F3} \equiv -N_{i\bar{j}} F^i \bar{F}^{\bar{j}}\Big|_{\psi=0}, \tag{2.136}$$

$$\mathcal{L}_{F4} \equiv -\frac{1}{2} N_{ij\bar{k}} \bar{\chi}^i \chi^j \bar{F}^{\bar{k}}\Big|_{\psi=0}, \tag{2.137}$$

$$\mathcal{L}_{F5} \equiv \frac{1}{2} N_{ij\bar{k}} \bar{\chi}^i (\not{\mathcal{D}} X^j) \chi^{\bar{k}}\Big|_{\psi=0}, \tag{2.138}$$

$$\mathcal{L}_{F6} \equiv \frac{1}{4} N_{ij\bar{k}\bar{l}} \bar{\chi}^i \chi^j \bar{\chi}^{\bar{k}} \chi^{\bar{l}}\Big|_{\psi=0}, \tag{2.139}$$

$$\mathcal{L}_{F7} \equiv -\frac{N}{6} R(\omega)|_{\psi=0}. \tag{2.140}$$

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The derivatives of the N are given in general by

$$\begin{aligned}
N^{(r=q+p+m+k)}_{q,p,m,k} &= (\partial_0^q \partial_W^p \partial_T^m \partial_I^k N) \\
&= \Big[(\partial_0^q \partial_I^{(k-n)} \Upsilon^{-2})_{(N)} (\Upsilon^2)_{(\mathcal{C}_T)} (\Upsilon^2)_{(\mathcal{C}_{\bar{T}})} \\
&\quad \times (\Upsilon^{2m_1})_{(\partial_T^{m_1}\mathcal{C}_T)} (\Upsilon^{2m_2})_{(\partial_{\bar{T}}^{m_2}\mathcal{C}_{\bar{T}})} (W)^{1-p_1} (\bar{W})^{1-p_2} \mathcal{U}^{(n)} \Big] \\
&\quad \Big/ \Big[(\tilde{\mathcal{F}}^2)_{(\mathcal{C}_T)} (\tilde{\mathcal{F}}^2)_{(\mathcal{C}_{\bar{T}})} (\tilde{\mathcal{F}}^{2m_1})_{(\partial_T^{m_1}\mathcal{C}_T)} (\tilde{\mathcal{F}}^{2m_2})_{(\partial_{\bar{T}}^{m_2}\mathcal{C}_{\bar{T}})} \Big]. \\
&= (\partial_0^q \partial_I^{(k-n)} \Upsilon^{-2}) \Upsilon^{4+2m_1+2m_2} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{2+2+2m_1+2m_2}} W^{1-p_1} \bar{W}^{1-p_2} \\
&= (\partial_0^q \partial_I^{(k-n)} \Upsilon^{-2}) \Upsilon^{4+2m} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}} W^{1-p_1} \bar{W}^{1-p_2},
\end{aligned} \tag{2.141}$$

where $\tilde{\mathcal{F}} \equiv 2K_{I\bar{J}} \Big(-\partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}} + F^I \bar{F}^{\bar{J}} \Big)$; $\mathcal{U}^{(n)}$ $(0 \leq n \leq 4)$ is the n-th derivative of the function $\mathcal{U}(z^I, \bar{z}^{\bar{I}})$ with respect to $z^I, \bar{z}^I$, which are the lowest component of the matter chiral multiplets; $q = q_1 + q_2$ where q_1 (q_2) is the order of the derivative with respect to the compensator scalar s_0 (s_0^*); $p = p_1 + p_2$ where p_1 (p_2) is the order of the derivative with respect to the field strength multiplet scalar W $(\bar{W})$; $m = m_1 + m_2$ where m_1 (m_2) is the order of the derivative with respect to the chiral projection multiplet scalar $T(\bar{w}^2)$ $(\bar{T}(w^2))$; k is the order of the derivative with respect to the matter multiplet scalar z^I; n is the order of the derivative acting on the new term $\mathcal{U}$ with respect to the matter multiplet; q is the total order of derivative with respect to the compensator scalars s_0 and s_0^*. An explicit form of the derivatives of N is given in appendix A.

The mass dimension[6] of the derivatives of the N is $[N^{(r=q+p+m+k)}_{q,p,m,k}] = -3p - 4m - 2$. This implies that the mass dimension of the operator coupled to $N^{(r=q+p+m+k)}_{q,p,m,k}$ must be equal to $3p + 4m + 2$.

Now, let us focus on the case such that $q = 0$ and $k = n$ which gives the most singular fermionic terms in the limit that the D-term vanishes. The most singular terms are those that contain the highest power of the auxiliary field D in the denominator and therefore are the nonrenormalizable operators associated with the smallest UV cutoff mass scale. That is, we consider that the matter scalar derivatives act only on the new term $\mathcal{U}$ and there are no the derivatives with respect to the compensator scalar. Then, we have

$$N^{(r=p+m+n)}_{p,m,n} = \Upsilon^{2+2m} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}} W^{1-p_1} \bar{W}^{1-p_2}. \tag{2.142}$$

In particular, since $r = p + m + n$ $(0 \leq r \leq 4)$, it reduces to

$$N^{(r)}_{i\ldots l} = N^{(r)}_{p,m,n} = \Upsilon^{2(1+r-n-p)} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{2(2+r-n-p)}} W^{1-p_1} \bar{W}^{1-p_2}. \tag{2.143}$$

[6]The mass dimensions of the multiplets' lowest components are $[s_0] = 1, [z^I] = 0, [W] = 3, [T] = 0$, which gives $[\tilde{\mathcal{F}}] = 2$.

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Remember that we called r the total order of the derivatives acting on the N. Here n is the number of derivatives acting on the matter scalars in the new term $\mathcal{U}$; they produce $\mathcal{U}^{(n)}$. Finally, $p = p_1 + p_2$ is the sum of the number of derivatives w.r.t. the multiplets $(W, \bar{W})$ acting on $\mathcal{U}$.

2.4.1 Structure of the fermionic components

Next, let us explore the detailed structure of the chiral fermions of the superconformal multiplets. First of all, the compensator and matter fermions are given by

$$\chi^0 = P_L\chi^0, \qquad \chi^I = P_L\chi^I. \tag{2.144}$$

Note that χ^0 and χ^I are fundamental fermions, and later in the S-gauge, the compensator chiral fermions will be replaced by the matter ones according to: $P_L\chi^0 = \frac{1}{3}s_0K_IP_L\chi^I$ and $P_R\chi^0 = \frac{1}{3}s_0^*K_{\bar{I}}P_R\chi^{\bar{I}}$.

On the contrary, the other fermions χ^W and χ^T are composite. Hence, we need to find their specific structure.

First, the $\mathcal{W}(K)^2$-multiplet fermion, say χ^W, is found to be

$$\begin{aligned}
\chi^W &= P_L\chi^W = \sqrt{2}P_L(-\gamma\cdot\hat{F}_K + 2i\mathcal{D}_K)\Lambda_K \\
&= -2\sqrt{2}P_L\gamma^{\mu\nu}\partial_{[\mu}\mathcal{B}^K_{\nu]}\Lambda_K - \sqrt{2}P_L\gamma^{\mu\nu}\bar{\psi}_{[\mu}\gamma_{\nu]}\Lambda_K\Lambda_K + 2\sqrt{2}iP_L\mathcal{D}_K\Lambda_K \\
&= -2\sqrt{2}P_L\gamma^{\mu\nu}\partial_{[\mu}(iK_I\mathcal{D}_{\nu]}z^I - iK_{\bar{I}}\mathcal{D}_{\nu]}\bar{z}^{\bar{I}} + iK_{I\bar{J}}\bar{\chi}^I\gamma_{\nu]}\chi^{\bar{J}})\Lambda_K + 2\sqrt{2}iP_L\mathcal{D}_K\Lambda_K \\
&= 2i\gamma^{\mu\nu}\partial_{[\mu}\left(K_I\bar{\psi}_{\nu]}\chi^I - K_{\bar{I}}\bar{\chi}^{\bar{I}}\psi_{\nu]} - \sqrt{2}K_{I\bar{J}}\bar{\chi}^I\gamma_{\nu]}\chi^{\bar{J}}\right)P_L\Lambda_K \\
&\quad +2\sqrt{2}i\Big[2K_{I\bar{J}}\Big(-\mathcal{D}_\mu z^I\mathcal{D}^\mu\bar{z}^{\bar{J}} - \frac{1}{2}\bar{\chi}^IP_L\not{\mathcal{D}}\chi^{\bar{J}} - \frac{1}{2}\bar{\chi}^{\bar{J}}P_R\not{\mathcal{D}}\chi^I + F^I\bar{F}^{\bar{J}}\Big) \\
&\quad +K_{IJ\bar{K}}\Big(-\bar{\chi}^I\chi^J\bar{F}^{\bar{K}} + \bar{\chi}^I(\not{\mathcal{D}}z^J)\chi^{\bar{K}}\Big) + K_{\bar{I}\bar{J}K}\Big(-\bar{\chi}^{\bar{I}}\chi^{\bar{J}}F^K + \bar{\chi}^{\bar{I}}(\not{\mathcal{D}}\bar{z}^{\bar{J}})\chi^K\Big) \\
&\quad +\frac{1}{2}K_{IJ\bar{K}\bar{L}}(\bar{\chi}^IP_L\chi^J)(\bar{\chi}^{\bar{K}}P_R\chi^{\bar{L}})\Big]P_L\Lambda_K \\
&= 2\sqrt{2}i\tilde{\mathcal{F}}(P_L\Lambda_K)_{1f} + \cdots + 7 \text{ fermions} \\
&= 2\sqrt{2}i\tilde{\mathcal{F}}\left(-\sqrt{2}iK_{\bar{I}J}[(\not{\mathcal{D}}z^J)_{0f}\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J]\right) + \cdots + 7 \text{ fermions} \\
&= 4\tilde{\mathcal{F}}K_{\bar{I}J}[(\not{\partial}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] + \cdots + 7 \text{ fermions},
\end{aligned} \tag{2.145}$$

where $P_L\Lambda_K = -\sqrt{2}iK_{\bar{I}J}[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] - \frac{i}{\sqrt{2}}K_{\bar{I}\bar{J}K}\chi^K\bar{\chi}^{\bar{I}}\chi^J$. Note that the composite fermion χ^W contains powers of the matter fermions ranging from one to seven, and it is nonvanishing on-shell.

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Second, the chiral projection multiplet $T(\bar{w}^2)$ fermions are found to be as follows:

$$\begin{aligned}
\chi^T &= P_L\chi^T = -\frac{i}{\sqrt{2}}P_L(\not{\mathcal{D}}\mathcal{Z}_{\bar{w}} + \Lambda_{\bar{w}}) = -\frac{i}{\sqrt{2}}\left[\frac{(\not{\mathcal{D}}\mathcal{Z}_{\bar{W}})}{\Upsilon^2} - \frac{(\not{\mathcal{D}}\Upsilon)\mathcal{Z}_{\bar{W}}}{\Upsilon^3} - 2\frac{\mathcal{C}_{\bar{W}}}{\Upsilon^3}P_L\Lambda_\Upsilon - \frac{i\not{\mathcal{B}}_\Upsilon\mathcal{Z}_{\bar{W}}}{\Upsilon^3}\right]_{\mathcal{Z}_\Upsilon=0} \\
&= \frac{1}{\Upsilon^2}\left[\not{\mathcal{D}}\chi^{\bar{W}} - \frac{(\not{\mathcal{D}}\Upsilon)\chi^{\bar{W}}}{\Upsilon} - 2\frac{\bar{\Lambda}_K P_R\Lambda_K}{\Upsilon}P_L\Lambda_\Upsilon - \frac{i\not{\mathcal{B}}_\Upsilon\chi^{\bar{W}}}{\Upsilon}\right]_{\mathcal{Z}_\Upsilon=0} \\
&= \frac{1}{\Upsilon^2}\left[(\not{\mathcal{D}}\chi^{\bar{W}})_{1f} - \frac{[(\not{\mathcal{D}}\Upsilon)_{0f} + i(\not{\mathcal{B}}_\Upsilon)_{0f}]}{\Upsilon}(\chi^{\bar{W}})_{1f}\right] + \cdots + 9 \text{ fermions}, \qquad (2.146) \\
&= \frac{1}{\Upsilon^2}\Bigg[4(\not{\partial}\tilde{\mathcal{F}})K_{\bar{I}J}[(\not{\partial}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] \\
&\quad - \left(\frac{2}{s_0^*}\not{\partial}s_0^* - \frac{2}{3}K_{\bar{K}}\not{\partial}\bar{z}^{\bar{K}} - 2\gamma^\mu(b_\mu + iA_\mu)\right)4\tilde{\mathcal{F}}K_{\bar{I}J}[(\not{\partial}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J]\Bigg]_{1f} + \cdots + 9 \text{ fermions}, \\
&= \frac{4}{\Upsilon^2}(\not{\partial}\tilde{\mathcal{F}})K_{\bar{I}J}[(\not{\partial}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] + (\chi^T)_{1f}|_{\tilde{\mathcal{F}}^1} + \cdots + 9 \text{ fermions}, \qquad (2.147)
\end{aligned}$$

where $\tilde{\mathcal{F}} \equiv 2K_{I\bar{J}}\left(-\partial_\mu z^I\partial^\mu\bar{z}^{\bar{J}} + F^I\bar{F}^{\bar{J}}\right)$; "$|_{\tilde{\mathcal{F}}^1}$" denotes the terms proportional to $\tilde{\mathcal{F}}^1$, and

$$\begin{aligned}
\not{\partial}\tilde{\mathcal{F}} &= 2(\not{\partial}K_{I\bar{J}})\left(-\partial_\mu z^I\partial^\mu\bar{z}^{\bar{J}} + F^I\bar{F}^{\bar{J}}\right) \\
&\quad + 2K_{I\bar{J}}\left(-(\not{\partial}\partial_\mu z^I)\partial^\mu\bar{z}^{\bar{J}} - \partial_\mu z^I(\not{\partial}\partial^\mu\bar{z}^{\bar{J}}) + (\not{\partial}F^I)\bar{F}^{\bar{J}} + F^I(\not{\partial}\bar{F}^{\bar{J}})\right). \qquad (2.148)
\end{aligned}$$

Note that χ^T is also nonvanishing on-shell.

In the above calculation we have used the formula:

$$\begin{aligned}
\mathcal{D}_\mu P_L\chi^W &= \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - \frac{7}{2}b_\mu + \frac{3}{2}iA_\mu\right)P_L\chi^W - \frac{1}{\sqrt{2}}P_L(\not{\mathcal{D}}W + F^W)\psi_\mu \\
&\quad - 3\sqrt{2}WP_L\phi_\mu. \qquad (2.149) \\
& \qquad (2.150)
\end{aligned}$$

to find

$$\begin{aligned}
(\not{\mathcal{D}}\chi^W)_{1f} &= (\not{\mathcal{D}}P_L\chi^W)_{1f} \\
&= \gamma^\mu\left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - \frac{7}{2}b_\mu + \frac{3}{2}iA_\mu\right)(P_L\chi^W)_{1f} + \frac{1}{\sqrt{2}}P_L(F^W)_{0f}\gamma^\mu\psi_\mu \\
&= \not{\partial}(P_L\chi^W)_{1f} + \gamma^\mu\left(\frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - \frac{7}{2}b_\mu + \frac{3}{2}iA_\mu\right)(P_L\chi^W)_{1f} + \frac{1}{\sqrt{2}}P_L\tilde{\mathcal{F}}^2\gamma^\mu\psi_\mu \\
&= 4(\not{\partial}\tilde{\mathcal{F}})K_{\bar{I}J}[(\not{\partial}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] + 4\tilde{\mathcal{F}}(\not{\partial}K_{\bar{I}J})[(\not{\partial}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] \\
&\quad + 4\tilde{\mathcal{F}}K_{\bar{I}J}[(\Box z^J)\chi^{\bar{I}} + \not{\partial}z^J\not{\partial}\chi^{\bar{I}} - (\not{\partial}\bar{F}^{\bar{I}})\chi^J - \bar{F}^{\bar{I}}(\not{\partial}\chi^J)] \\
&\quad + \gamma^\mu\left(\frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - \frac{7}{2}b_\mu + \frac{3}{2}iA_\mu\right)(4\tilde{\mathcal{F}}K_{\bar{I}J}[(\not{\partial}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J]) + \frac{1}{\sqrt{2}}P_L\tilde{\mathcal{F}}^2\gamma^\mu\psi_\mu \\
&= 4(\not{\partial}\tilde{\mathcal{F}})K_{\bar{I}J}[(\not{\partial}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] + (\not{\mathcal{D}}\chi^W)_{1f}|_{\tilde{\mathcal{F}}^1} + (\not{\mathcal{D}}\chi^W)_{1f}|_{\tilde{\mathcal{F}}^2}. \qquad (2.151)
\end{aligned}$$

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where $F^W = 2\bar{\Lambda}_K P_L \not{\mathcal{D}} \Lambda_K + \hat{F}_K^- \cdot \hat{F}_K^- - \mathcal{D}_K^2$, $(\hat{F}_K^-)_{0f} = 0$, and $(F^W)_{0f} = -(\mathcal{D}_K^2)_{0f} = -\tilde{\mathcal{F}}^2$, and

$$\begin{aligned}
(\not{\mathcal{D}}\chi^W)_{1f} &= (\not{\mathcal{D}} P_L \chi^W)_{1f} = 4(\not{\mathcal{D}}\tilde{\mathcal{F}})K_{\bar{I}J}[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] + 4\tilde{\mathcal{F}}(\not{\mathcal{D}}K_{\bar{I}J})[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] \\
&\quad + 4\tilde{\mathcal{F}}K_{\bar{I}J}[(\not{\mathcal{D}}\not{\mathcal{D}}z^J)\chi^{\bar{I}} + \not{\mathcal{D}}z^J\not{\mathcal{D}}\chi^{\bar{I}} - (\not{\mathcal{D}}\bar{F}^{\bar{I}})\chi^J - \bar{F}^{\bar{I}}(\not{\mathcal{D}}\chi^J)] \\
&= 4(\not{\mathcal{D}}\tilde{\mathcal{F}})K_{\bar{I}J}[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] + 4\tilde{\mathcal{F}}(\not{\mathcal{D}}K_{\bar{I}J})[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] \\
&\quad + 4\tilde{\mathcal{F}}K_{\bar{I}J}[(\Box z^J)\chi^{\bar{I}} + \not{\mathcal{D}}z^J\not{\mathcal{D}}\chi^{\bar{I}} - (\not{\mathcal{D}}\bar{F}^{\bar{I}})\chi^J - \bar{F}^{\bar{I}}(\not{\mathcal{D}}\chi^J)] \\
&\approx 4(\not{\mathcal{D}}\tilde{\mathcal{F}})K_{\bar{I}J}[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] + 4\tilde{\mathcal{F}}(\not{\mathcal{D}}K_{\bar{I}J})[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] \\
&\quad - 4\tilde{\mathcal{F}}K_{\bar{I}J}(\not{\mathcal{D}}\bar{F}^{\bar{I}})\chi^J. \qquad (2.152)
\end{aligned}$$

Here $\approx$ means equality up to terms proportional to the equations of motion of free massless matter fields. Such terms produce only terms that contain additional factors of the matter field masses in the numerator and therefore give rise to either renormalizable operators or nonrenormalizable operators weighted by a mass scale higher than that associated to those terms that do not vanish on shell. The details of the above calculation is given in appendix A.1.

Finally, we present here the chiral fermions of the superconformal multiplets up to multiple fermion terms.

$$\chi^i = \begin{cases} \chi^0, \\ \chi^I, \\ \chi^W = 4\tilde{\mathcal{F}}K_{\bar{I}J}[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] + \cdots 7 \text{ fermions}, \\ \chi^T = \frac{1}{\Upsilon^2}\Big[\, 4(\not{\mathcal{D}}\tilde{\mathcal{F}})K_{\bar{I}J}[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] - \Big(\frac{2}{s_0^*}\not{\mathcal{D}}s_0^* - \frac{2}{3}K_{\bar{K}}\not{\mathcal{D}}\bar{z}^{\bar{K}} - 2\gamma^\mu(b_\mu + iA_\mu)\Big) \\ \qquad\qquad \times 4\tilde{\mathcal{F}}K_{\bar{I}J}[(\not{\mathcal{D}}z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] \,\Big]_{1f} + \cdots + 9 \text{ fermions}, \end{cases}$$

where

$$\begin{aligned}
\tilde{\mathcal{F}} &= 2K_{I\bar{J}}\left(-\partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}} + F^I \bar{F}^{\bar{J}}\right) \\
\not{\mathcal{D}}\tilde{\mathcal{F}} &= 2(\not{\mathcal{D}}K_{I\bar{J}})\left(-\partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}} + F^I \bar{F}^{\bar{J}}\right) \\
&\quad + 2K_{I\bar{J}}\left(-(\not{\mathcal{D}}\partial_\mu z^I)\partial^\mu \bar{z}^{\bar{J}} - \partial_\mu z^I(\not{\mathcal{D}}\partial^\mu \bar{z}^{\bar{J}}) + (\not{\mathcal{D}}F^I)\bar{F}^{\bar{J}} + F^I(\not{\mathcal{D}}\bar{F}^{\bar{J}})\right).
\end{aligned}$$

Notice that none of the chiral fermions χ^i vanish on-shell, and that only χ^T dependens on $\tilde{\mathcal{F}}$ and includes the factor of Υ^{-2}. All of these properties affect the mass dimension of the expansion coefficients of the nonrenormalizable Lagrangians.

We finally expand W and $\bar{W}$ as follows:

$$\begin{aligned}
W &= -2K_{\bar{\imath}J}[\bar{\chi}^J(\overline{\not{\mathcal{D}}\bar{z}^{\bar{\imath}}}) - \bar{F}^{\bar{\imath}}\bar{\chi}^J]K_{\bar{\imath}'J'}[(\not{\mathcal{D}}\bar{z}^{\bar{\imath}'})\chi^{J'} - F^{J'}\chi^{\bar{\imath}'}] \\
&\quad - K_{\bar{\imath}J}[\bar{\chi}^J(\overline{\not{\mathcal{D}}\bar{z}^{\bar{\imath}}}) - \bar{F}^{\bar{\imath}}\bar{\chi}^J]K_{\bar{\imath}'\bar{\jmath}'K'}[\chi^{K'}\bar{\chi}^{\bar{\imath}'}\chi^{\bar{\jmath}'}] \\
&\quad - K_{\bar{\imath}\bar{\jmath}K}[\bar{\chi}^{\bar{\jmath}}\chi^{\bar{\imath}}\bar{\chi}^K]K_{\bar{\imath}'J'}[(\not{\mathcal{D}}\bar{z}^{\bar{\imath}'})\chi^{J'} - F^{J'}\chi^{\bar{\imath}'}] \\
&\quad - \frac{1}{2}K_{\bar{\imath}\bar{\jmath}K}[\bar{\chi}^{\bar{\jmath}}\chi^{\bar{\imath}}\bar{\chi}^K]K_{\bar{\imath}'\bar{\jmath}'K'}[\chi^{K'}\bar{\chi}^{\bar{\imath}'}\chi^{\bar{\jmath}'}], \qquad (2.153) \\
\bar{W} &= (W)^*. \qquad (2.154)
\end{aligned}$$

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Notice that W and $\bar{W}$ can be represented by products of two, four, and six fundamental fermions.

2.5 Additional gauge fixing for physical theory in the liberated supergravity

First of all, we consider the conventional superconformal gauge which is chosen by

$$\mathcal{C}_\Upsilon = \Upsilon = 1 \Longleftrightarrow s_0\bar{s}_0 e^{-K/3} = 1, \tag{2.155}$$

$$\mathcal{Z}_\Upsilon = 0 \implies P_L\chi^0 - \frac{1}{3}s_0 K_I P_L\chi^I = 0 \quad \& \quad P_R\chi^0 - \frac{1}{3}s_0^* K_{\bar{I}} P_R\chi^{\bar{I}} = 0, \tag{2.156}$$

$$s_0 = \bar{s}_0 \implies s_0 = \bar{s}_0 = e^{K/6}, \tag{2.157}$$

$$b_\mu = 0. \tag{2.158}$$

Note that the first condition is the D-gauge fixing which gets us to the Einstein frame; the second one is the improved S-gauge fixing; the third one is the A-gauge fixing; the last one is the K-gauge fixing.

To compare our results with the formulation of liberated supergravity in [10], we choose a gauge given by

$$\Lambda_K = 0 \Longleftrightarrow \chi^W = \chi^T = 0. \tag{2.159}$$

In both the conventional superconformal gauge (2.156), (2.157), (2.158) and in (2.159), the relevant multiplets are

$$S_0 = \left(e^{K/6}, \frac{1}{3}e^{K/6}K_I P_L\chi^I, F^0\right), \qquad \bar{S}_0 = \left(e^{K/6}, \frac{1}{3}e^{K/6}K_{\bar{I}} P_R\chi^{\bar{I}}, F^{0*}\right), \tag{2.160}$$

$$z^I = \left(z^I, P_L\chi^I, F^I\right), \qquad \bar{z}^{\bar{I}} = \left(\bar{z}^{\bar{I}}, P_R\chi^{\bar{I}}, \bar{F}^{\bar{I}}\right), \tag{2.161}$$

$$\mathcal{W}^2(K) = \left(W, P_L\chi^W, F^W\right) = \left(0, 0, \hat{F}_K^- \cdot \hat{F}_K^- - \mathcal{D}_K^2\right), \tag{2.162}$$

$$\bar{\mathcal{W}}^2(K) = \left(\bar{W}, P_R\chi^{\bar{W}}, \bar{F}^{\bar{W}}\right) = \left(0, 0, \hat{F}_K^+ \cdot \hat{F}_K^+ - (\mathcal{D}_K^*)^2\right), \tag{2.163}$$

$$T(\bar{w}^2) = \left(T, P_L\chi^T, F^T\right) = \left(-\frac{1}{2}\mathcal{K}_{\bar{w}}, 0, \frac{1}{2}\mathcal{D}_{\bar{w}}\right), \tag{2.164}$$

$$\bar{T}(w^2) = \left(\bar{X}^{\bar{T}}, P_R\chi^{\bar{T}}, \bar{F}^{\bar{T}}\right) = \left(-\frac{1}{2}\mathcal{K}^*_{\bar{w}}, 0, \frac{1}{2}\mathcal{D}^*_{\bar{w}}\right). \tag{2.165}$$

Let us further observe that all the terms coupled to N and its derivatives with respect to $0, I, T$ vanish because N contains the product of $W\bar{W}$, which is zero in the liberated gauge, i.e. $W = \bar{W} = 0$. We see that all the fermionic terms coupled to the derivatives of N with respect to W or T vanish as well, because these terms always couple to the fermions χ^W and χ^T which vanish in the gauge. The only non-vanishing contribution is given by only the bosonic term, $N_{W\bar{W}}F^W\bar{F}^{\bar{W}}$ which gives us the new term $\mathcal{U}$ to the scalar potential. Therefore, in both gauges, the liberated supergravity Lagrangian is specified by

$$\mathcal{L}_{\text{Lib}} = \mathcal{L}_{\text{SUGRA}} + \mathcal{L}_{\text{NEW}}, \tag{2.166}$$

where $\mathcal{L}_{\text{SUGRA}}$ is the standard supergravity Lagrangian which contains the auxiliary fields F^0, F^I and $\mathcal{L}_{\text{NEW}} = \mathcal{U}(z^I, \bar{z}^{\bar{I}})$. Then, with this Lagrangian, after solving the equations of

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motion for the auxiliary fields, we can obtain the usual supergravity action in addition to the new term.

3 Component action of new FI term in superconformal tensor calculus

In this section, we review the component action of a new, Kähler invariant Fayet-Iliopoulos term proposed by Antoniadis, Chatrabhuti, Isono, and Knoops [4], using again the superconformal tensor calculus. The full Lagrangian with the new FI terms is given by

$$\begin{aligned}\mathcal{L} = &-3[S_0\bar{S}_0 e^{-K(z,\bar{z})}]_D + \left([S_0^3 W(z)]_F - \frac{1}{4}[\bar{\lambda}P_L\lambda]_F + \text{h.c.}\right) \\ &-\xi\left[(S_0\bar{S}_0 e^{-K(z,\bar{z})})^{-3}\frac{(\bar{\lambda}P_L\lambda)(\bar{\lambda}P_R\lambda)}{T(\bar{w}'^2)\bar{T}(w'^2)}(V)_D\right]_D,\end{aligned} \tag{3.1}$$

where the last term coupled to the parameter ξ corresponds to the new FI terms.

3.1 Component action of the new FI term

We consider matter chiral multiplets Z^i, the chiral compensator S_0, a real multiplet V, and another real multiplet $(V)_D$, whose lowest component is the auxiliary D term of the real multiplet V. Their superconformal multiplets are given as follows:

$$V = \{0,0,0,0,A_\mu,\lambda,D\} \text{ in the Wess-Zumino gauge, i.e. } v=\zeta=\mathcal{H}=0, \tag{3.2}$$
$$Z^i = \left(z^i, -i\sqrt{2}P_L\chi^i, -2F^i, 0, +i\mathcal{D}_\mu z^i, 0, 0\right) = \{z^i, P_L\chi^i, F^i\}, \tag{3.3}$$
$$\bar{Z}^{\bar{i}} = \left(\bar{z}^{\bar{i}}, +i\sqrt{2}P_R\chi^{\bar{i}}, 0, -2\bar{F}^{\bar{i}}, -i\mathcal{D}_\mu\bar{z}^{\bar{i}}, 0, 0\right) = \{\bar{z}^{\bar{i}}, P_R\chi^{\bar{i}}, \bar{F}^{\bar{i}}\}, \tag{3.4}$$
$$S_0 = \left(s_0, -i\sqrt{2}P_L\chi^0, -2F_0, 0, +i\mathcal{D}_\mu s_0, 0, 0\right) = \{s_0, P_L\chi^0, F_0\}, \tag{3.5}$$
$$\bar{S}_0 = \left(\bar{s}_0, +i\sqrt{2}P_R\chi^0, 0, -2\bar{F}_0, -i\mathcal{D}_\mu\bar{s}_0, 0, 0\right) = \{\bar{s}_0, P_R\chi^0, \bar{F}_0\}, \tag{3.6}$$
$$\bar{\lambda}P_L\lambda = \left(\bar{\lambda}P_L\lambda, -i\sqrt{2}P_L\Lambda, 2D_-^2, 0, +i\mathcal{D}_\mu(\bar{\lambda}P_L\lambda), 0, 0\right) = \{\bar{\lambda}P_L\lambda, P_L\Lambda, -D_-^2\}, \tag{3.7}$$
$$\bar{\lambda}P_R\lambda = \left(\bar{\lambda}P_R\lambda, +i\sqrt{2}P_R\Lambda, 0, 2D_+^2, -i\mathcal{D}_\mu(\bar{\lambda}P_R\lambda), 0, 0\right) = \{\bar{\lambda}P_R\lambda, P_R\Lambda, -D_+^2\}, \tag{3.8}$$
$$(V)_D = (D, \not{\mathcal{D}}\lambda, 0, 0, \mathcal{D}^b\hat{F}_{ab}, -\not{\mathcal{D}}\not{\mathcal{D}}\lambda, -\Box^C D), \tag{3.9}$$

where

$$P_L\Lambda \equiv \sqrt{2}P_L\left(-\frac{1}{2}\gamma\cdot\hat{F} + iD\right)\lambda, \qquad P_R\Lambda \equiv \sqrt{2}P_R\left(-\frac{1}{2}\gamma\cdot\hat{F} - iD\right)\lambda, \tag{3.10}$$
$$D_-^2 \equiv D^2 - \hat{F}^-\cdot\hat{F}^- - 2\bar{\lambda}P_L\not{\mathcal{D}}\lambda, \qquad D_+^2 \equiv D^2 - \hat{F}^+\cdot\hat{F}^+ - 2\bar{\lambda}P_R\not{\mathcal{D}}\lambda, \tag{3.11}$$
$$\mathcal{D}_\mu\lambda \equiv \left(\partial_\mu - \frac{3}{2}b_\mu + \frac{1}{4}w_\mu^{ab}\gamma_{ab} - \frac{3}{2}i\gamma_*\mathcal{A}_\mu\right)\lambda - \left(\frac{1}{4}\gamma^{ab}\hat{F}_{ab} + \frac{1}{2}i\gamma_* D\right)\psi_\mu \tag{3.12}$$
$$\hat{F}_{ab} \equiv F_{ab} + e_a{}^\mu e_b{}^\nu\bar{\psi}_{[\mu}\gamma_{\nu]}\lambda, \qquad F_{ab} \equiv e_a{}^\mu e_b{}^\nu(2\partial_{[\mu}A_{\nu]}), \tag{3.13}$$
$$\hat{F}_{\mu\nu}^\pm \equiv \frac{1}{2}(\hat{F}_{\mu\nu} \pm \tilde{\hat{F}}_{\mu\nu}), \qquad \tilde{\hat{F}}_{\mu\nu} \equiv -\frac{1}{2}i\epsilon_{\mu\nu\rho\sigma}\hat{F}^{\rho\sigma}. \tag{3.14}$$

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3.1.1 $w'^2, \bar{w}'^2$ composite complex multiplets: (Weyl/Chiral) weights $= (-1, 3)$ and $(-1, -3)$

We show here the components of the first superconformal *composite* complex multiplets w'^2 and $\bar{w}'^2$ with Weyl/chiral weights $(-1, 3)$ and $(-1, -3)$ respectively. These composite multiplets are defined to be

$$w'^2 \equiv \frac{\bar{\lambda} P_L \lambda}{(S_0 \bar{S}_0 e^{-K/3})^2} = \{\mathcal{C}_w, \mathcal{Z}_w, \mathcal{H}_w, \mathcal{K}_w, \mathcal{B}^w_\mu, \Lambda_w, \mathcal{D}_w\} \tag{3.15}$$

$$\bar{w}'^2 \equiv \frac{\bar{\lambda} P_R \lambda}{(S_0 \bar{S}_0 e^{-K/3})^2} = \{\mathcal{C}_{\bar{w}}, \mathcal{Z}_{\bar{w}}, \mathcal{H}_{\bar{w}}, \mathcal{K}_{\bar{w}}, \mathcal{B}^{\bar{w}}_\mu, \Lambda_{\bar{w}}, \mathcal{D}_{\bar{w}}\}. \tag{3.16}$$

where

$$\mathcal{C}_w = h \equiv \frac{\bar{\lambda} P_L \lambda}{(s_0 \bar{s}_0 e^{-K(z,\bar{z})/3})^2}, \tag{3.17}$$

$$\mathcal{Z}_w = i\sqrt{2}(-h_a \Omega^a + h_{\bar{a}} \Omega^{\bar{a}}), \tag{3.18}$$

$$\mathcal{H}_w = -2h_a F^a + h_{a\bar{b}} \bar{\Omega}^a \Omega^b, \tag{3.19}$$

$$\mathcal{K}_w = -2h_{\bar{a}} F^{\bar{a}} + h_{\bar{a}\bar{b}} \bar{\Omega}^{\bar{a}} \Omega^{\bar{b}}, \tag{3.20}$$

$$\mathcal{B}^w_\mu = ih_a \mathcal{D}_\mu X^a - ih_{\bar{a}} \mathcal{D}_\mu \bar{X}^{\bar{a}} + ih_{a\bar{b}} \bar{\Omega}^a \gamma_\mu \Omega^{\bar{b}}, \tag{3.21}$$

$$P_L \Lambda_w = -\sqrt{2} i h_{\bar{a}b} [(\not{\mathcal{D}} X^b) \Omega^{\bar{a}} - F^{\bar{a}} \Omega^b] - \frac{i}{\sqrt{2}} h_{\bar{a}\bar{b}c} \Omega^c \bar{\Omega}^{\bar{a}} \Omega^{\bar{b}}, \tag{3.22}$$

$$P_R \Lambda_w = \sqrt{2} i h_{a\bar{b}} [(\not{\mathcal{D}} \bar{X}^{\bar{b}}) \Omega^a - F^a \Omega^{\bar{b}}] + \frac{i}{\sqrt{2}} h_{ab\bar{c}} \Omega^{\bar{c}} \bar{\Omega}^a \Omega^b, \tag{3.23}$$

$$\begin{aligned} \mathcal{D}_w = {} & 2h_{a\bar{b}} \left(-\mathcal{D}_\mu X^a \mathcal{D}^\mu \bar{X}^{\bar{b}} - \frac{1}{2} \bar{\Omega}^a P_L \not{\mathcal{D}} \Omega^{\bar{b}} - \frac{1}{2} \bar{\Omega}^{\bar{b}} P_R \not{\mathcal{D}} \Omega^a + F^a F^{\bar{b}} \right) \\ & + h_{ab\bar{c}} (-\bar{\Omega}^a \Omega^b F^{\bar{c}} + \bar{\Omega}^a (\not{\mathcal{D}} X^b) \Omega^{\bar{c}}) + h_{\bar{a}\bar{b}c} (-\bar{\Omega}^{\bar{a}} \Omega^{\bar{b}} F^c + \bar{\Omega}^{\bar{a}} (\not{\mathcal{D}} \bar{X}^{\bar{b}}) \Omega^c) \\ & + \frac{1}{2} h_{ab\bar{c}\bar{d}} (\bar{\Omega}^a P_L \Omega^b)(\bar{\Omega}^{\bar{c}} P_R \Omega^{\bar{d}}). \end{aligned} \tag{3.24}$$

Notice that when finding the multiplet $\bar{w}'^2$, we can just replace h by its complex conjugate h^*.

3.1.2 $T(\bar{w}'^2), \bar{T}(w'^2)$ chiral projection multiplets: (Weyl/Chiral) weights $= (0, 0)$

The second superconformal multiplets that we need are the *composite* chiral projection multiplets $T(\bar{w}'^2)$ and $\bar{T}(w'^2)$ with Weyl/chiral weights $(0, 0)$. From their component supermultiplets defined by

$$T(\bar{w}'^2) = \left(-\frac{1}{2} \mathcal{K}_{\bar{w}}, -\frac{1}{2} \sqrt{2} i P_L (\not{\mathcal{D}} \mathcal{Z}_{\bar{w}} + \Lambda_{\bar{w}}), \frac{1}{2} (\mathcal{D}_{\bar{w}} + \Box^C \mathcal{C}_{\bar{w}} + i \mathcal{D}_a \mathcal{B}^a_{\bar{w}}) \right), \tag{3.25}$$

$$\bar{T}(w'^2) = \left(-\frac{1}{2} \mathcal{K}^*_{\bar{w}}, \frac{1}{2} \sqrt{2} i P_R (\not{\mathcal{D}} \mathcal{Z}^C_{\bar{w}} + \Lambda^C_{\bar{w}}), \frac{1}{2} (\mathcal{D}^*_{\bar{w}} + \Box^C \mathcal{C}^*_{\bar{w}} - i \mathcal{D}_a (\mathcal{B}^a_{\bar{w}})^*) \right) \tag{3.26}$$

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we find the corresponding superconformal multiplets and their complex conjugates as follows:

$$T \equiv T(\bar{w}'^2) = \{\mathcal{C}_T, \mathcal{Z}_T, \mathcal{H}_T, \mathcal{K}_T, \mathcal{B}_\mu^T, \Lambda_T, \mathcal{D}_T\}$$
$$\bar{T} \equiv \bar{T}(w'^2) = \{\mathcal{C}_{\bar{T}}, \mathcal{Z}_{\bar{T}}, \mathcal{H}_{\bar{T}}, \mathcal{K}_{\bar{T}}, \mathcal{B}_\mu^{\bar{T}}, \Lambda_{\bar{T}}, \mathcal{D}_{\bar{T}}\}, \tag{3.27}$$

whose superconformal components are given by

$$\mathcal{C}_T = -\frac{1}{2}\mathcal{K}_{\bar{w}} = h^*_{\bar{a}}F^{\bar{a}} - \frac{1}{2}h^*_{\bar{a}\bar{b}}\bar{\Omega}^{\bar{a}}\Omega^{\bar{b}} \equiv C_T \tag{3.28}$$

$$\mathcal{Z}_T = -\sqrt{2}iP_L\Big[\not{\mathcal{D}}(-h^*_a\Omega^a + h^*_{\bar{a}}\Omega^{\bar{a}}) - h^*_{\bar{a}b}[(\not{\mathcal{D}}X^b)\Omega^{\bar{a}} - F^{\bar{a}}\Omega^b] - \frac{1}{2}h^*_{\bar{a}\bar{b}c}\Omega^c\bar{\Omega}^{\bar{a}}\Omega^{\bar{b}}$$
$$+h^*_{a\bar{b}}[(\not{\mathcal{D}}\bar{X}^{\bar{b}})\Omega^a - F^a\Omega^{\bar{b}}] + \frac{1}{2}h^*_{ab\bar{c}}\Omega^{\bar{c}}\bar{\Omega}^a\Omega^b\Big] \equiv -\sqrt{2}iP_L\Omega_T, \tag{3.29}$$

$$\mathcal{H}_T = -2\Big[h^*_{a\bar{b}}\Big(-\mathcal{D}_\mu X^a\mathcal{D}^\mu\bar{X}^{\bar{b}} - \frac{1}{2}\bar{\Omega}^a P_L\not{\mathcal{D}}\Omega^{\bar{b}} - \frac{1}{2}\bar{\Omega}^{\bar{b}}P_R\not{\mathcal{D}}\Omega^a + F^aF^{\bar{b}}\Big)$$
$$+\frac{1}{2}h^*_{ab\bar{c}}(-\bar{\Omega}^a\Omega^bF^{\bar{c}} + \bar{\Omega}^a(\not{\mathcal{D}}X^b)\Omega^{\bar{c}}) + \frac{1}{2}h^*_{\bar{a}\bar{b}c}(-\bar{\Omega}^{\bar{a}}\Omega^{\bar{b}}F^c + \bar{\Omega}^{\bar{a}}(\not{\mathcal{D}}\bar{X}^{\bar{b}})\Omega^c)$$
$$+\frac{1}{4}h^*_{ab\bar{c}\bar{d}}(\bar{\Omega}^aP_L\Omega^b)(\bar{\Omega}^{\bar{c}}P_R\Omega^{\bar{d}}) + \frac{1}{2}\Box^C h^* + \frac{1}{2}i\mathcal{D}^\mu(ih^*_a\mathcal{D}_\mu X^a - ih^*_{\bar{a}}\mathcal{D}_\mu\bar{X}^{\bar{a}} + ih^*_{a\bar{b}}\bar{\Omega}^a\gamma_\mu\Omega^{\bar{b}})\Big]$$
$$\equiv -2F_T, \tag{3.30}$$

$$\mathcal{K}_T = 0, \tag{3.31}$$

$$\mathcal{B}_\mu^T = -i\mathcal{D}_\mu\mathcal{C}_T, \tag{3.32}$$

$$\Lambda_T = 0, \tag{3.33}$$

$$\mathcal{D}_T = 0, \tag{3.34}$$

where we used $a, b, c, d = 0, i(\equiv z^i), W(\equiv \bar{\lambda}P_L\lambda)$. This gives the superfield components of the chiral projection multiplet T:

$$T(\bar{w}'^2) = (C_T, P_L\Omega_T, F_T) \tag{3.35}$$

where

$$C_T = h^*_{\bar{a}}F^{\bar{a}} - \frac{1}{2}h^*_{\bar{a}\bar{b}}\bar{\Omega}^{\bar{a}}\Omega^{\bar{b}}, \tag{3.36}$$

$$P_L\Omega_T = \not{\mathcal{D}}(-h^*_a\Omega^a + h^*_{\bar{a}}\Omega^{\bar{a}}) - h^*_{\bar{a}b}[(\not{\mathcal{D}}X^b)\Omega^{\bar{a}} - F^{\bar{a}}\Omega^b] - \frac{1}{2}h^*_{\bar{a}\bar{b}c}\Omega^c\bar{\Omega}^{\bar{a}}\Omega^{\bar{b}}$$
$$+h^*_{a\bar{b}}[(\not{\mathcal{D}}\bar{X}^{\bar{b}})\Omega^a - F^a\Omega^{\bar{b}}] + \frac{1}{2}h^*_{ab\bar{c}}\Omega^{\bar{c}}\bar{\Omega}^a\Omega^b, \tag{3.37}$$

$$F_T = h^*_{a\bar{b}}\Big(-\mathcal{D}_\mu X^a\mathcal{D}^\mu\bar{X}^{\bar{b}} - \frac{1}{2}\bar{\Omega}^aP_L\not{\mathcal{D}}\Omega^{\bar{b}} - \frac{1}{2}\bar{\Omega}^{\bar{b}}P_R\not{\mathcal{D}}\Omega^a + F^aF^{\bar{b}}\Big)$$
$$+\frac{1}{2}h^*_{ab\bar{c}}(-\bar{\Omega}^a\Omega^bF^{\bar{c}} + \bar{\Omega}^a(\not{\mathcal{D}}X^b)\Omega^{\bar{c}}) + \frac{1}{2}h^*_{\bar{a}\bar{b}c}(-\bar{\Omega}^{\bar{a}}\Omega^{\bar{b}}F^c + \bar{\Omega}^{\bar{a}}(\not{\mathcal{D}}\bar{X}^{\bar{b}})\Omega^c)$$
$$+\frac{1}{4}h^*_{ab\bar{c}\bar{d}}(\bar{\Omega}^aP_L\Omega^b)(\bar{\Omega}^{\bar{c}}P_R\Omega^{\bar{d}}) + \frac{1}{2}\Box^C h^* - \frac{1}{2}\mathcal{D}^\mu(h^*_a\mathcal{D}_\mu X^a - h^*_{\bar{a}}\mathcal{D}_\mu\bar{X}^{\bar{a}} + h^*_{a\bar{b}}\bar{\Omega}^a\gamma_\mu\Omega^{\bar{b}}). \tag{3.38}$$

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Morever,

$$\bar{T}(w'^2) = \{C_T^*, P_R\Omega_T, F_T^*\} \tag{3.39}$$

where

$$C_T^* = h_a F^a - \frac{1}{2} h_{ab} \bar{\Omega}^a \Omega^b, \tag{3.40}$$

$$\begin{aligned} P_R\Omega_T = {} & \not{\mathcal{D}}(-h_a\Omega^a + h_{\bar{a}}\Omega^{\bar{a}}) - h_{\bar{a}b}[(\not{\mathcal{D}}X^b)\Omega^{\bar{a}} - F^{\bar{a}}\Omega^b] - \frac{1}{2} h_{\bar{a}\bar{b}c}\Omega^c\bar{\Omega}^{\bar{a}}\Omega^{\bar{b}} \\ & + h_{a\bar{b}}[(\not{\mathcal{D}}\bar{X}^{\bar{b}})\Omega^a - F^a\Omega^{\bar{b}}] + \frac{1}{2} h_{ab\bar{c}}\Omega^{\bar{c}}\bar{\Omega}^a\Omega^b, \end{aligned} \tag{3.41}$$

$$\begin{aligned} F_T^* = {} & h_{a\bar{b}}\left(-\mathcal{D}_\mu X^a \mathcal{D}^\mu \bar{X}^{\bar{b}} - \frac{1}{2}\bar{\Omega}^a P_L \not{\mathcal{D}}\Omega^{\bar{b}} - \frac{1}{2}\bar{\Omega}^{\bar{b}} P_R \not{\mathcal{D}}\Omega^a + F^a F^{\bar{b}}\right) \\ & + \frac{1}{2} h_{ab\bar{c}}(-\bar{\Omega}^a\Omega^b F^{\bar{c}} + \bar{\Omega}^a(\not{\mathcal{D}}X^b)\Omega^{\bar{c}}) + \frac{1}{2} h_{\bar{a}\bar{b}c}(-\bar{\Omega}^{\bar{a}}\Omega^{\bar{b}} F^c + \bar{\Omega}^{\bar{a}}(\not{\mathcal{D}}\bar{X}^{\bar{b}})\Omega^c) \\ & + \frac{1}{4} h_{ab\bar{c}\bar{d}}(\bar{\Omega}^a P_L\Omega^b)(\bar{\Omega}^{\bar{c}} P_R\Omega^{\bar{d}}) + \frac{1}{2}\Box^C h - \frac{1}{2}\mathcal{D}^\mu(h_a\mathcal{D}_\mu X^a - h_{\bar{a}}\mathcal{D}_\mu\bar{X}^{\bar{a}} + h_{a\bar{b}}\bar{\Omega}^{\bar{b}}\gamma_\mu\Omega^a). \end{aligned} \tag{3.42}$$

3.1.3 Composite real multiplet $\mathcal{R}$: (Weyl/Chiral) weights = (0, 0)

We present here a superconformal composite real multiplet $\mathcal{R}$ with Weyl/chiral weights $(0,0)$. Defining some chiral multiplets $\mathcal{X}^A \equiv \{X^A, P_L\Omega^A, F^A\}$ where $A = \{S_0, Z^i, \bar{\lambda}P_L\lambda, T(\bar{w}'^2)\}$ and their conjugates, we represent the composite one $\mathcal{R}$ as

$$\mathcal{R} \equiv (S_0\bar{S}_0 e^{-K/3})^{-3}\frac{(\bar{\lambda}P_L\lambda)(\bar{\lambda}P_R\lambda)}{T(\bar{w}'^2)\bar{T}(w'^2)} \tag{3.43}$$

whose lowest component is

$$\mathcal{C}_{\mathcal{R}} \equiv (s_0\bar{s}_0 e^{-K/3})^{-3}\frac{(\bar{\lambda}P_L\lambda)(\bar{\lambda}P_R\lambda)}{C_T C_{\bar{T}}} \equiv f(X^A, \bar{X}^{\bar{A}}) \tag{3.44}$$

where $C_T = -D_+^2\Delta^{-2}$; $C_{\bar{T}} = -D_-^2\Delta^{-2}$, and $\Delta \equiv s_0\bar{s}_0 e^{-K/3}$. Then, the superconformal multiplet of the new Fayet-Iliopoulos term can be written by using

$$\mathcal{R}\cdot(V)_D = \{\tilde{\mathcal{C}}, \tilde{\mathcal{Z}}, \tilde{\mathcal{H}}, \tilde{\mathcal{K}}, \tilde{\mathcal{B}}_\mu, \tilde{\Lambda}, \tilde{\mathcal{D}}\}, \tag{3.45}$$

whose superconformal multiplet components are as follows:

$$\tilde{\mathcal{C}} = Df, \tag{3.46}$$

$$\tilde{\mathcal{Z}} = f\not{\mathcal{D}}\lambda + Di\sqrt{2}(-f_A\Omega^A + f_{\bar{A}}\Omega^{\bar{A}}), \tag{3.47}$$

$$\tilde{\mathcal{H}} = D(-2f_A F^A + f_{AB}\bar{\Omega}^A\Omega^B) - i\sqrt{2}(-f_A\bar{\Omega}^A + f_{\bar{A}}\bar{\Omega}^{\bar{A}})P_L\not{\mathcal{D}}\lambda, \tag{3.48}$$

$$\tilde{\mathcal{K}} = D(-2f_{\bar{A}}F^{\bar{A}} + f_{\bar{A}\bar{B}}\bar{\Omega}^{\bar{A}}\Omega^{\bar{B}}) - i\sqrt{2}(-f_A\bar{\Omega}^A + f_{\bar{A}}\bar{\Omega}^{\bar{A}})P_R\not{\mathcal{D}}\lambda, \tag{3.49}$$

$$\tilde{\mathcal{B}} = (\mathcal{D}^\nu\hat{F}_{\mu\nu})f + D(if_A\mathcal{D}_\mu X^A - if_{\bar{A}}\mathcal{D}_\mu\bar{X}^{\bar{A}} + if_{A\bar{B}}\bar{\Omega}^A\gamma_\mu\Omega^{\bar{B}}), \tag{3.50}$$

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$$
\begin{aligned}
\tilde{\Lambda} = & -f\not{\mathcal{D}}\not{\mathcal{D}}\lambda + D(P_L\Lambda^f + P_R\Lambda^f) + \frac{1}{2}\Big(\gamma_*(-f_A\not{\mathcal{D}}X^A + f_{\bar{A}}\not{\mathcal{D}}\bar{X}^{\bar{A}} - f_{A\bar{B}}\bar{\Omega}^A\not{\partial}\Omega^{\bar{B}}) \\
& + P_L(-2f_{\bar{A}}F^{\bar{A}} + f_{\bar{A}\bar{B}}\bar{\Omega}^{\bar{A}}\Omega^{\bar{B}}) + P_R(-2f_AF^A + f_{AB}\bar{\Omega}^A\Omega^B) - \not{\mathcal{D}}f\Big)\not{\mathcal{D}}\lambda \\
& + \frac{1}{2}\Big(i\gamma_*\gamma^\mu\mathcal{D}^\nu\hat{F}_{\mu\nu} - \not{\mathcal{D}}D\Big)i\sqrt{2}(-f_A\Omega^A + f_{\bar{A}}\Omega^{\bar{A}}), \qquad (3.51) \\
\tilde{\mathcal{D}} = & -f\Box^C D + D\Big\{2f_{A\bar{B}}\left(-\mathcal{D}_\mu X^A\mathcal{D}^\mu\bar{X}^{\bar{B}} - \frac{1}{2}\bar{\Omega}^A P_L\not{\mathcal{D}}\Omega^{\bar{B}} - \frac{1}{2}\bar{\Omega}^{\bar{B}}P_R\not{\mathcal{D}}\Omega^A + F^AF^{\bar{B}}\right) \\
& + f_{AB\bar{C}}(-\bar{\Omega}^A\Omega^BF^{\bar{C}} + \bar{\Omega}^A(\not{\mathcal{D}}X^B)\Omega^{\bar{C}}) + f_{\bar{A}\bar{B}C}(-\bar{\Omega}^{\bar{A}}\Omega^{\bar{B}}F^C + \bar{\Omega}^{\bar{A}}(\not{\mathcal{D}}\bar{X}^{\bar{B}})\Omega^C) \\
& + \frac{1}{2}f_{AB\bar{C}\bar{D}}(\bar{\Omega}^AP_L\Omega^B)(\bar{\Omega}^{\bar{C}}P_R\Omega^{\bar{D}})\Big\} \\
& - (\mathcal{D}_\nu\hat{F}^{\mu\nu})(if_A\mathcal{D}_\mu X^A - if_{\bar{A}}\mathcal{D}_\mu\bar{X}^{\bar{A}} + if_{A\bar{B}}\bar{\Omega}^A\gamma_\mu\Omega^{\bar{B}}) \\
& + \left(\sqrt{2}if_{\bar{A}B}[(\not{\mathcal{D}}X^B)\Omega^{\bar{A}} - F^{\bar{A}}\Omega^B] + \frac{i}{\sqrt{2}}f_{\bar{A}\bar{B}C}\Omega^C\bar{\Omega}^{\bar{A}}\Omega^{\bar{B}}\right)\not{\mathcal{D}}\lambda \\
& - \left(\sqrt{2}if_{A\bar{B}}[(\not{\mathcal{D}}\bar{X}^{\bar{B}})\Omega^A - F^A\Omega^{\bar{B}}] + \frac{i}{\sqrt{2}}f_{AB\bar{C}}\Omega^{\bar{C}}\bar{\Omega}^A\Omega^B\right)\not{\mathcal{D}}\lambda \\
& - (\mathcal{D}_\mu f)(\mathcal{D}^\mu D) - \frac{1}{2}\not{\mathcal{D}}\left[i\sqrt{2}(-f_A\Omega^A + f_{\bar{A}}\Omega^{\bar{A}})\right](\not{\mathcal{D}}\lambda) + \frac{1}{2}i\sqrt{2}(-f_A\Omega^A + f_{\bar{A}}\Omega^{\bar{A}})(\not{\mathcal{D}}\not{\mathcal{D}}\lambda), \qquad (3.52)
\end{aligned}
$$

where the indices A, B, C, D run over $0, i, W, T$. The component action of the new FI term is then given by the D-term density formula

$$
\begin{aligned}
\mathcal{L}_{\rm NEW} \equiv -[\xi\mathcal{R}\cdot(V)_D]_D = & -\frac{\xi}{4}\int d^4xe\Big[\tilde{\mathcal{D}} - \frac{1}{2}\bar{\psi}\cdot\gamma i\gamma_*\tilde{\Lambda} - \frac{1}{3}\tilde{\mathcal{C}}R(\omega) \\
& + \frac{1}{6}\Big(\tilde{\mathcal{C}}\bar{\psi}_\mu\gamma^{\mu\rho\sigma} - i\tilde{\bar{\mathcal{Z}}}\gamma^{\rho\sigma}\gamma_*\Big)R'_{\rho\sigma}(Q) \\
& + \frac{1}{4}\varepsilon^{abcd}\bar{\psi}_a\gamma_b\psi_c\Big(\tilde{\mathcal{B}}_d - \frac{1}{2}\bar{\psi}_d\tilde{\mathcal{Z}}\Big)\Big] + \text{h.c.}\,. \qquad (3.53)
\end{aligned}
$$

3.1.4 Bosonic term of the new FI term

The new FI term is obtained from the term $D^2f_{W\bar{W}}F^WF^{\bar{W}}$ inside the D-term $\tilde{\mathcal{D}}$. Thus, we get

$$
\begin{aligned}
\mathcal{L}_{\rm NEW} \supset -\xi Df_{W\bar{W}}F^WF^{\bar{W}} = -\xi D\frac{(s_0\bar{s}_0e^{-K/3})^{-3}}{C_TC_T^*}F^WF^{\bar{W}} \supset -\xi D\frac{(s_0\bar{s}_0e^{-K/3})^{-3}}{(h_WF^W)(h^*_{\bar{W}}F^{\bar{W}})}F^WF^{\bar{W}} \\
= -\xi D\frac{(s_0\bar{s}_0e^{-K/3})^{-3}}{(s_0\bar{s}_0e^{-K/3})^{-4}F^WF^{\bar{W}}}F^WF^{\bar{W}} = -\xi D(s_0\bar{s}_0e^{-K/3}). \qquad (3.54)
\end{aligned}
$$

Hence, in the superconformal gauge ($s_0\bar{s}_0e^{-K/3} = M_{\rm pl}^2 = 1$), we obtain

$$
\mathcal{L}_{\text{new FI}}/e = -\xi D, \qquad (3.55)
$$

or

$$
\mathcal{L}_{\text{new FI}}/e = -M_{\rm pl}^2\xi D. \qquad (3.56)
$$

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4 Consistency check of supergravity as effective field theory through superconformal tensor calculus

This section studies the fermionic nonrenormalizable terms in the Lagrangians containing either liberated $\mathcal{N}=1$ terms or new FI terms. We will make full use of the superconformal tensor calculus to find the limits of validity of those effective field theories.

4.1 Constraining liberated $\mathcal{N}=1$ supergravity

We investigate first the nonrenormalizable fermionic terms in "liberated" supergravity, to find possible constraints on the liberated term $\mathcal{U}$. To begin with, we rewrite the coefficients

$$N_{q,p,m,k}^{(r=q+p+m+k)} = (\partial_0^q \partial_I^{(k-n)} \Upsilon^{-2}) \Upsilon^{4+2m} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}} W^{1-p_1} \bar{W}^{1-p_2},$$

as

$$\begin{aligned} N_{q,p,m,k}^{(r=q+p+m+k)} &= ((-1)^{q_1+q_2}(q_1+1)!(q_2+1)! s_0^{-(2+q_1)} (s_0^*)^{-(2+q_2)} (\partial_I^{(k-n)} e^{2K/3})) \\ &\quad \times \Upsilon^{4+2m} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}} W^{1-p_1} \bar{W}^{1-p_2} \end{aligned} \tag{4.1}$$

where we used $\Upsilon = s_0 s_0^* e^{-K/3}$.

The most powerful constraints on $\mathcal{U}$ come from the most singular fermionic terms in the Lagrangian. Such singular terms are those containing the highest power of $\tilde{\mathcal{F}}$ in the denominator. These powers are linearly proportional to m because they are due to taking derivatives of N with respect to the lowest component of the multiplet $T(\bar{w}^2)$. Therefore, we must investigate the fermionic interactions containing only derivatives with respect to the chiral projection and matter scalar indices, i.e. T and I, to obtain the terms that are coupled to $\mathcal{U}^{(n)}$ and contain the maximal inverse powers of $\tilde{\mathcal{F}}$. Such terms are those with $q=p=0$ and $k=n$.

In particular, we find by direct inspection of the fermionic interactions that the most singular terms are given by the couplings to the derivatives proportional to $N_{T\bar{T}}$, $N_{WT\bar{T}}$, $N_{W\bar{W}T\bar{T}}$ if our theory has a single chiral matter multiplet, while they are given by $N_{TT\bar{T}\bar{T}}$ for two or more chiral matter multiplets. The latter terms vanish identically for a single multiplet because of Fermi statistics. This has been investigated in detail in ref. [11]. Here we merely recall the results from ref. [11]:

- For the single chiral matter multiplet case,

$$\mathcal{L}_{F2}|_{q=p=0,k=n} \supset M_{\rm pl}^{2(n-4)} \frac{\mathcal{U}^{(n)}}{\mathcal{F}^{2(4-n)}} \mathcal{O}_F'^{(2(6-n))}. \tag{4.2}$$

- For two or more chiral matter multiplets,

$$\mathcal{L}_{F6}|_{q=p=0,k=n} \supset c' M_{\rm pl}^{2(n-6)} \frac{\mathcal{U}^{(n)}}{\mathcal{F}^{2(6-n)}} \mathcal{O}_F'^{(2(8-n))}. \tag{4.3}$$

  where $\mathcal{O}(10^{-2}) \leq c' \leq \mathcal{O}(1)$.

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From this, by calling $\Lambda_{\rm cut}$ the cutoff UV scale of liberated supergravity and using the inequality $\mathcal{C}^{(4-\delta)} \lesssim \frac{1}{\Lambda_{\rm cut}^{\delta-4}}$ ($\delta = 2(6-n)$ for $N_{\rm mat} = 1$ and $\delta = 2(8-n)$ for $N_{\rm mat} \geq 2$) we obtain

$$\mathcal{U}^{(n)} \lesssim \begin{cases} \mathcal{F}^{2(4-n)} \left(\frac{M_{\rm pl}}{\Lambda_{\rm cut}}\right)^{2(4-n)} & \text{where} \quad 0 \leq n \leq 2 \quad \text{for } N_{\rm mat} = 1, \\ \mathcal{F}^{2(6-n)} \left(\frac{M_{\rm pl}}{\Lambda_{\rm cut}}\right)^{2(6-n)} & \text{where} \quad 0 \leq n \leq 4 \quad \text{for } N_{\rm mat} \geq 2. \end{cases} \tag{4.4}$$

A conventional definition of the supersymmetry breaking scale M_S is given in terms of F-term expectation value; therefore, we can define $M_S^4 = M_{\rm pl}^4 \mathcal{F}$ and the constraints on $\mathcal{U}^{(n)}$ then become

$$\mathcal{U}^{(n)} \lesssim \begin{cases} \left(\frac{M_S}{M_{\rm pl}}\right)^{8(4-n)} \left(\frac{M_{\rm pl}}{\Lambda_{\rm cut}}\right)^{2(4-n)} & \text{where} \quad 0 \leq n \leq 2 \quad \text{for } N_{\rm mat} = 1, \\ \left(\frac{M_S}{M_{\rm pl}}\right)^{8(6-n)} \left(\frac{M_{\rm pl}}{\Lambda_{\rm cut}}\right)^{2(6-n)} & \text{where} \quad 0 \leq n \leq 4 \quad \text{for } N_{\rm mat} \geq 2. \end{cases} \tag{4.5}$$

The consequences of these constraints were studied in ref. [11].

4.2 Constraining new Fayet-Iliopoulos terms

In this section,[7] we explore the possible constraints on the new FI parameter ξ. First of all, let us find the fermionic terms coming from the new FI terms. From its Lagrangian, we observe that the most singular terms come from the derivatives of the function f, defined in eq. (3.44) and below, with respect to the multiplet T since they produce the inverse powers of the D-term 'D'. The function f was defined by

$$f = (s_0 \bar{s}_0 e^{-K/3})^{-3} \frac{(\bar{\lambda} P_L \lambda)(\bar{\lambda} P_R \lambda)}{C_T C_{\bar{T}}}, \tag{4.6}$$

where $C_T = -D_+^2 \Delta^{-2}$; $C_{\bar{T}} = -D_-^2 \Delta^{-2}$, and $\Delta \equiv s_0 \bar{s}_0 e^{-K/3}$.

Then, the most singular terms in the Lagrangians has the generic form

$$\mathcal{L}_F \supset -c(\xi D^p \cdot \partial_T^{m_1} \partial_{\bar{T}}^{m_2} f) \mathcal{O}_F^{(\delta)}, \tag{4.7}$$

where c is a dimensionless constant of order $\mathcal{O}(1)$; ξ is the dimensionless new FI constant, $\mathcal{O}_F^{(\delta)}$ is an effective field operator of dimension δ independent of D, m_1, m_2 are the order of derivative with respect to the $T, \bar{T}$, and p is a power of D where $p = 0, 1$.

The most singular terms appear in the Lagrangian as follows:

$$\begin{aligned} \mathcal{L}_F &\supset -(m_1! m_2! c) \xi M_{\rm pl}^{4m+2} D^{-2m-4+p} (\bar{\lambda} P_L \lambda)(\bar{\lambda} P_R \lambda) \mathcal{O}_F^{(\delta)} \\ &= -(m_1! m_2! c) \xi M_{\rm pl}^{4m+2} D^{-2m-4+p} \mathcal{O}_F^{(\delta+6)}, \end{aligned} \tag{4.8}$$

where $\mathcal{O}_F^{(\delta+6)} \equiv (\bar{\lambda} P_L \lambda)(\bar{\lambda} P_R \lambda) \mathcal{O}_F^{(\delta)}$ and $\delta = 4 - 2p$ so that $[\mathcal{L}_f] = 4$. Therefore, we have

$$\mathcal{L}_F \supset \xi M_{\rm pl}^{4m+2} D^{-2m-4+p} \mathcal{O}_F^{(10-2p)}, \tag{4.9}$$

where we omitted $-(m_1! m_2! c)$ since this is $\mathcal{O}(1)$.

[7] This subsection also complements the detailed derivation of the constraint on the new FI parameter ξ omitted in ref. [12].

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Next, we explore the constraints coming from the new FI term. First of all, we recall that the new FI term proposed by Antoniadis et al. can be added to the standard FI term, when the theory contains also a gauged U(1) R-symmetry $\mathrm{U}(1)_R$. Then, we have the following auxiliary D-term Lagrangian

$$\mathcal{L}_{\text{aux D}} = \frac{1}{2}D^2 - i(G_i k^i - G_{\bar{i}} k^{\bar{i}})D - \xi D \equiv \frac{1}{2}D^2 - (\xi' + \xi)D, \tag{4.10}$$

where ξ is the new FI constant, $\xi' \equiv i(G_i k^i - G_{\bar{i}} k^{\bar{i}})$. Here $k = k^i \partial_i$ is the Killing vector of a standard R-symmetry $U_R(1)$ and G is the supergravity G-function. If we restore explicit powers of the gauge coupling g, then we get

$$\mathcal{L}_{\text{aux D}} = \frac{1}{2g^2}D^2 - \xi' D - \xi D = \frac{1}{2g^2}D^2 - (\xi' + \xi)D. \tag{4.11}$$

After solving the equation of motion for D, we have

$$D = g^2 M_{\text{pl}}^2 (\xi + \xi'). \tag{4.12}$$

Plugging this result into the Lagrangian again, we obtain

$$\begin{aligned} \mathcal{L}_F &\supset \xi M_{\text{pl}}^{4m+2} (g^2 M_{\text{pl}}^2 (\xi + \xi'))^{-2m-4+p} \mathcal{O}_F'^{(10-2p)} \\ &= \xi (g^2 (\xi + \xi'))^{-2m-4+p} M_{\text{pl}}^{-6+2p} \mathcal{O}_F'^{(10-2p)}. \end{aligned} \tag{4.13}$$

This implies $\xi (g^2 (\xi + \xi'))^{-2m-4+p} M_{\text{pl}}^{-6+2p} \lesssim \frac{1}{\Lambda_{\text{cut}}^{6-2p}}$ so we arrive at

$$\xi (g^2 (\xi + \xi'))^{-2m-4+p} \lesssim \left(\frac{M_{\text{pl}}}{\Lambda_{\text{cut}}} \right)^{6-2p}. \tag{4.14}$$

This is a constraint on the new FI term constant ξ, written in terms of the UV cutoff Λ_{cut}.

5 Summary

In section 2 of this work we computed the component action of the liberated $\mathcal{N} = 1$ supergravity using the superconformal tensor calculus. In section 3, we revisited the component action of a Kähler-invariant new FI term (called "ACIK-FI term") in the superconformal formalism. In section 4, we used the superconformal tensor calculus to constrain the size of new supergravity terms that are present in liberated supergravity and in the ACIK-FI term, but absent in standard supergravity. The computations performed in sections 2, 3, and 4 spell out the results of [11, 12]. Specifically, we relate the "liberated" scalar potential $\mathcal{U}$ to the UV cutoff and we derive the constraints on the ACIK-FI term used in [12]. What makes our constraints powerful is that differently from standard supergravity, both liberated supergravity and the ACIK-FI terms introduce nonrenormalizable interactions proportional to inverse powers of the supersymmetry breaking scale M_S. This makes it impossible to send M_S to zero while keeping the UV cutoff of the theory finite. The most singular nonrenormalizable interactions in the limit $M_S \to 0$ are cumbersome, multi-fermion operators, but they can be found and studies using the superconformal tensor calculus in a systematic and economical way. Superconformal calculus techniques are general so can be applied to any supergravity theory. Our analysis is based on standard Effective Field Theory methods and it is thus complementary to swampland [18–20] constraints.

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A Some detailed calculations in liberated supergravity

A.1 Some details

$$\mathcal{D}_\mu z^I = \partial_\mu z^I - \frac{1}{\sqrt{2}}\bar{\psi}_\mu \chi^I, \tag{A.1}$$

$$\mathcal{C}_{\bar{W}} \equiv \bar{W} = \bar{\Lambda}_K P_R \Lambda_K = 2 \text{ fermions} + 4 \text{ fermions} + 6 \text{ fermions}, \tag{A.2}$$

$$P_L \Lambda_K = -\sqrt{2} i K_{\bar{I}J}[(\not{\mathcal{D}} z^J)\chi^{\bar{I}} - \bar{F}^{\bar{I}}\chi^J] - \frac{i}{\sqrt{2}} K_{\bar{I}\bar{J}K}\chi^K \bar{\chi}^{\bar{I}}\chi^J, \tag{A.3}$$

$$(\not{\mathcal{D}}\chi^{\bar{W}})_{1f} = 2\sqrt{2}i(\not{\mathcal{D}}\tilde{\mathcal{F}})_{0f}(P_L\Lambda_K)_{1f} + 2\sqrt{2}i\tilde{\mathcal{F}}(P_L \not{\mathcal{D}}\Lambda_K)_{1f}, \tag{A.4}$$

$$\not{\mathcal{D}}\Lambda_K = \gamma \cdot \mathcal{D}\Lambda_K, \tag{A.5}$$

$$\mathcal{D}_\mu \Lambda_K = \left(\partial_\mu - \frac{3}{2}b_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - \frac{3}{2}i\gamma_* \mathcal{A}_\mu\right)\Lambda_K - \left(\frac{1}{4}\gamma \cdot \hat{F}^K + \frac{1}{2}i\gamma_* \mathcal{D}_K\right)\psi_\mu, \tag{A.6}$$

$$(P_L \not{\mathcal{D}}\Lambda_K)_{1f} = \gamma^\mu \mathcal{D}_\mu|_{\psi=0}(P_R\Lambda_K)_{1f} + \frac{i}{2}\tilde{\mathcal{F}}\gamma^\mu\psi_\mu, \tag{A.7}$$

$$\begin{aligned}
(\gamma \cdot \hat{F}^K)_{0f} &= 0, \qquad \mathcal{D}_K|_{0f} = \tilde{\mathcal{F}} \\
(\hat{F}^K_{\mu\nu})_{0f} &= (2\partial_{[\mu}\mathcal{B}^K_{\nu]})_{0f} = 2i\partial_{[\mu}(K_I\partial_{\nu]}z^I - K_{\bar{I}}\partial_{\nu]}\bar{z}^{\bar{I}}) = 2i(\partial_{[\mu}K_I\partial_{\nu]}z^I - \partial_{[\mu}K_{\bar{I}}\partial_{\nu]}\bar{z}^{\bar{I}}) \\
&= 2i(K_{IJ}\partial_{[\mu}z^{(J}\partial_{\nu]}z^{I)} - K_{\bar{I}\bar{J}}\partial_{[\mu}z^{(\bar{J}}\partial_{\nu]}\bar{z}^{\bar{I})}) = 0,
\end{aligned} \tag{A.8}$$

$$\begin{aligned}
(\not{\mathcal{D}}\Upsilon)_{0f} &= (\not{\partial} - 2\gamma^\mu b_\mu - 2i\gamma^\mu A_\mu)\Upsilon \\
&= (\not{\partial}s_0)s_0^* e^{-K/3} + s_0(\not{\partial}s_0^*)e^{-K/3} + s_0 s_0^* e^{-K/3}\left(-\frac{1}{3}\not{\partial}K\right) - 2\gamma^\mu(b_\mu + iA_\mu)\Upsilon \\
&= \Upsilon\left(\frac{1}{s_0}\not{\partial}s_0 + \frac{1}{s_0^*}\not{\partial}s_0^* - \frac{1}{3}K_I\not{\partial}z^I - \frac{1}{3}K_{\bar{I}}\not{\partial}\bar{z}^{\bar{I}} - 2\gamma^\mu(b_\mu + iA_\mu)\right),
\end{aligned} \tag{A.9}$$

$$i(\not{\mathcal{B}}_\Upsilon)_{0f} = \Upsilon\left(-\frac{1}{s_0}\not{\partial}s_0 + \frac{1}{s_0^*}\not{\partial}s_0^* + \frac{1}{3}K_I\not{\partial}z^I - \frac{1}{3}K_{\bar{I}}\not{\partial}\bar{z}^{\bar{I}}\right),$$

$$\frac{(\not{\mathcal{D}}\Upsilon)_{0f} + i(\not{\mathcal{B}}_\Upsilon)_{0f}}{\Upsilon} = \frac{2}{s_0^*}\not{\partial}s_0^* - \frac{2}{3}K_{\bar{I}}\not{\partial}\bar{z}^{\bar{I}} - 2\gamma^\mu(b_\mu + iA_\mu). \tag{A.10}$$

A.2 Derivatives of N

In this appendix we give explicit formulas for the derivatives of N with respect to the multiplets $i = 0, I, W, T$. The first derivative of N is

$$N_i = \delta_i^0\partial_0 + \delta_i^I\partial_I + \delta_i^W\partial_W + \delta_i^T\partial_T, \quad N_{\bar{i}} = \delta_{\bar{i}}^{\bar{0}}\partial_{\bar{0}} + \delta_{\bar{i}}^{\bar{I}}\partial_{\bar{I}} + \delta_{\bar{i}}^{\bar{W}}\partial_{\bar{W}} + \delta_{\bar{i}}^{\bar{T}}\partial_{\bar{T}}, \tag{A.11}$$

the second derivative of N is

$$\begin{aligned}
N_{i\bar{j}} = {} & \delta_i^T\delta_{\bar{j}}^{\bar{0}}\partial_{\bar{0}}\partial_T + \delta_i^W\delta_{\bar{j}}^{\bar{0}}\partial_{\bar{0}}\partial_W + \delta_i^0\delta_{\bar{j}}^{\bar{0}}\partial_0\partial_{\bar{0}} + \delta_i^I\delta_{\bar{j}}^{\bar{0}}\partial_I\partial_{\bar{0}} + \delta_i^T\delta_{\bar{j}}^{\bar{J}}\partial_{\bar{J}}\partial_T + \delta_i^W\delta_{\bar{j}}^{\bar{J}}\partial_{\bar{J}}\partial_W \\
& + \delta_i^0\delta_{\bar{j}}^{\bar{J}}\partial_0\partial_{\bar{J}} + \delta_i^I\delta_{\bar{j}}^{\bar{J}}\partial_I\partial_{\bar{J}} + \delta_i^T\delta_{\bar{j}}^{\bar{T}}\partial_{\bar{T}}\partial_T + \delta_i^W\delta_{\bar{j}}^{\bar{T}}\partial_{\bar{T}}\partial_W + \delta_i^0\delta_{\bar{j}}^{\bar{T}}\partial_0\partial_{\bar{T}} + \delta_i^I\delta_{\bar{j}}^{\bar{T}}\partial_I\partial_{\bar{T}} \\
& + \delta_i^T\delta_{\bar{j}}^{\bar{W}}\partial_{\bar{W}}\partial_T + \delta_i^W\delta_{\bar{j}}^{\bar{W}}\partial_{\bar{W}}\partial_W + \delta_i^0\delta_{\bar{j}}^{\bar{W}}\partial_0\partial_{\bar{W}} + \delta_i^I\delta_{\bar{j}}^{\bar{W}}\partial_I\partial_{\bar{W}},
\end{aligned} \tag{A.12}$$

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the third derivative of N is

$$
\begin{aligned}
N_{ij\bar{k}} = {} & \delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{0}}\partial_{\bar{0}}\partial_J\partial_T + \delta_i^W\delta_j^J\delta_{\bar{k}}^{\bar{0}}\partial_{\bar{0}}\partial_J\partial_W + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{0}}\partial_0\partial_{\bar{0}}\partial_J + \delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{0}}\partial_I\partial_{\bar{0}}\partial_J \\
& + 2\delta_i^{(W}\delta_j^{T)}\delta_{\bar{k}}^{\bar{0}}\partial_{\bar{0}}\partial_T\partial_W + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{0}}\partial_0\partial_{\bar{0}}\partial_T + \delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{0}}\partial_{\bar{0}}\partial_T^2 + \delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{0}}\partial_I\partial_{\bar{0}}\partial_T \\
& + 2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{0}}\partial_0\partial_{\bar{0}}\partial_W + \delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{0}}\partial_I\partial_{\bar{0}}\partial_W + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{0}}\partial_0^2\partial_{\bar{0}} + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{0}}\partial_I\partial_0\partial_{\bar{0}} \\
& + \delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{K}}\partial_{\bar{K}}\partial_J\partial_T + \delta_i^W\delta_j^J\delta_{\bar{k}}^{\bar{K}}\partial_{\bar{K}}\partial_J\partial_W + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{K}}\partial_0\partial_{\bar{K}}\partial_J + \delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{K}}\partial_I\partial_{\bar{K}}\partial_J \\
& + 2\delta_i^{(W}\delta_j^{T)}\delta_{\bar{k}}^{\bar{K}}\partial_{\bar{K}}\partial_T\partial_W + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{K}}\partial_0\partial_{\bar{K}}\partial_T + \delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{K}}\partial_{\bar{K}}\partial_T^2 + \delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{K}}\partial_I\partial_{\bar{K}}\partial_T \\
& + 2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{K}}\partial_0\partial_{\bar{K}}\partial_W + \delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{K}}\partial_I\partial_{\bar{K}}\partial_W + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{K}}\partial_0^2\partial_{\bar{K}} + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{K}}\partial_I\partial_0\partial_{\bar{K}} \\
& + \delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{T}}\partial_{\bar{T}}\partial_J\partial_T + \delta_i^W\delta_j^J\delta_{\bar{k}}^{\bar{T}}\partial_{\bar{T}}\partial_J\partial_W + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{T}}\partial_0\partial_{\bar{T}}\partial_J + \delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{T}}\partial_I\partial_{\bar{T}}\partial_J \\
& + 2\delta_i^{(W}\delta_j^{T)}\delta_{\bar{k}}^{\bar{T}}\partial_{\bar{T}}\partial_T\partial_W + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{T}}\partial_0\partial_{\bar{T}}\partial_T + \delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{T}}\partial_{\bar{T}}\partial_T^2 + \delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{T}}\partial_I\partial_{\bar{T}}\partial_T \\
& + 2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{T}}\partial_0\partial_{\bar{T}}\partial_W + \delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{T}}\partial_I\partial_{\bar{T}}\partial_W + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{T}}\partial_0^2\partial_{\bar{T}} + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{T}}\partial_I\partial_0\partial_{\bar{T}} \\
& + \delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{W}}\partial_{\bar{W}}\partial_J\partial_T + \delta_i^W\delta_j^J\delta_{\bar{k}}^{\bar{W}}\partial_{\bar{W}}\partial_J\partial_W + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{W}}\partial_0\partial_{\bar{W}}\partial_J + \delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{W}}\partial_I\partial_{\bar{W}}\partial_J \\
& + 2\delta_i^{(W}\delta_j^{T)}\delta_{\bar{k}}^{\bar{W}}\partial_{\bar{W}}\partial_T\partial_W + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{W}}\partial_0\partial_{\bar{W}}\partial_T + \delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{W}}\partial_{\bar{W}}\partial_T^2 + \delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{W}}\partial_I\partial_{\bar{W}}\partial_T \\
& + 2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{W}}\partial_0\partial_{\bar{W}}\partial_W + \delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{W}}\partial_I\partial_{\bar{W}}\partial_W + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{W}}\partial_0^2\partial_{\bar{W}} + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{W}}\partial_I\partial_0\partial_{\bar{W}}, \qquad \text{(A.13)}
\end{aligned}
$$

and the fourth derivative of N is

$$
\begin{aligned}
& N_{ij\bar{k}\bar{l}} = \partial_i\partial_j\partial_{\bar{k}}\partial_{\bar{l}}N = \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} + \textcircled{5} + \textcircled{6} \qquad \text{(A.14)} \\
& \textcircled{1} = \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_0^2 + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_0^2 + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^0\partial_{\bar{0}}\partial_{\bar{K}}\partial_0^2 + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_0^2 \\
& + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_0^2 + 2\delta_i^0\delta_j^0\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_0^2 + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_0^2 + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_0^2 \\
& + 2\delta_i^0\delta_j^0\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_0^2 + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_0^2 + \delta_i^0\delta_j^0\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_0^2 + 2\delta_i^0\delta_j^0\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_0^2 \\
& + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_I\partial_0 + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_I\partial_0 + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_I\partial_0 + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_I\partial_0 \\
& + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_I\partial_0 + 2\delta_i^I\delta_j^0\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_I\partial_0 + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_I\partial_0 + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_I\partial_0 \\
& + 2\delta_i^I\delta_j^0\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_I\partial_0 + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_I\partial_0 + \delta_i^I\delta_j^0\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_I\partial_0 \\
& + 2\delta_i^I\delta_j^0\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_I\partial_0 + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_J\partial_0 + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_J\partial_0 \\
& + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_J\partial_0 + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_J\partial_0 + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_J\partial_0 \qquad \text{(A.15)} \\
& \textcircled{2} = 2\delta_i^0\delta_j^J\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_J\partial_0 + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_J\partial_0 + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_J\partial_0 \\
& + 2\delta_i^0\delta_j^J\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_J\partial_0 + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_J\partial_0 + \delta_i^0\delta_j^J\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_J\partial_0 \\
& + 2\delta_i^0\delta_j^J\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_J\partial_0 + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_T\partial_0 + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_T\partial_0 \\
& + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_T\partial_0 + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_T\partial_0 + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_T\partial_0 \\
& + 4\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_T\partial_0 + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_T\partial_0 + 2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_T\partial_0
\end{aligned}
$$

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$$+4\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_T\partial_0+2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_T\partial_0+2\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_T\partial_0$$
$$+4\delta_i^{(0}\delta_j^{T)}\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_T\partial_0+2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_W\partial_0+2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_W\partial_0$$
$$+2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_W\partial_0+2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_W\partial_0+2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_W\partial_0 \tag{A.16}$$

$$\textcircled{3}=4\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_W\partial_0+2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_W\partial_0+2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_W\partial_0$$
$$+4\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_W\partial_0+2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_W\partial_0+2\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_W\partial_0$$
$$+4\delta_i^{(0}\delta_j^{W)}\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_W\partial_0+\delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_T^2+\delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_T^2$$
$$+\delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_T^2+\delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_T^2+\delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_T^2$$
$$+2\delta_i^T\delta_j^T\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_T^2+\delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_T^2+\delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_T^2$$
$$+2\delta_i^T\delta_j^T\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_T^2+\delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_T^2+\delta_i^T\delta_j^T\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_T^2$$
$$+2\delta_i^T\delta_j^T\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_T^2+\delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_I\partial_J+\delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_I\partial_J$$
$$+\delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_I\partial_J+\delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_I\partial_J+\delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_I\partial_J \tag{A.17}$$

$$\textcircled{4}=2\delta_i^I\delta_j^J\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_I\partial_J+\delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_I\partial_J+\delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_I\partial_J$$
$$+2\delta_i^I\delta_j^J\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_I\partial_J+\delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_I\partial_J+\delta_i^I\delta_j^J\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_I\partial_J$$
$$+2\delta_i^I\delta_j^J\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_I\partial_J+\delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_I\partial_T+\delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_I\partial_T$$
$$+\delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_I\partial_T+\delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_I\partial_T+\delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_I\partial_T$$
$$+2\delta_i^I\delta_j^T\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_I\partial_T+\delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_I\partial_T+\delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_I\partial_T$$
$$+2\delta_i^I\delta_j^T\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_I\partial_T+\delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_I\partial_T+\delta_i^I\delta_j^T\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_I\partial_T$$
$$+2\delta_i^I\delta_j^T\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_I\partial_T+\delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_J\partial_T+\delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_J\partial_T$$
$$+\delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_J\partial_T+\delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_J\partial_T+\delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_J\partial_T \tag{A.18}$$

$$\textcircled{5}=2\delta_i^T\delta_j^J\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_J\partial_T+\delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_J\partial_T+\delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_J\partial_T$$
$$+2\delta_i^T\delta_j^J\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_J\partial_T+\delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_J\partial_T+\delta_i^T\delta_j^J\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_J\partial_T$$
$$+2\delta_i^T\delta_j^J\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_J\partial_T+\delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_I\partial_W+\delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_I\partial_W$$
$$+\delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_I\partial_W+\delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_I\partial_W+\delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_I\partial_W$$
$$+2\delta_i^I\delta_j^W\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{0}}\partial_{\bar{T}}\partial_I\partial_W+\delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{K}}\partial_{\bar{T}}\partial_I\partial_W+\delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{T}}\partial_I\partial_W$$
$$+2\delta_i^I\delta_j^W\delta_{\bar{k}}^{(\bar{0}}\delta_{\bar{l}}^{\bar{W})}\partial_{\bar{0}}\partial_{\bar{W}}\partial_I\partial_W+\delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{W}}\partial_{\bar{K}}\partial_{\bar{W}}\partial_I\partial_W+\delta_i^I\delta_j^W\delta_{\bar{k}}^{\bar{W}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{L}}\partial_{\bar{W}}\partial_I\partial_W$$
$$+2\delta_i^I\delta_j^W\delta_{\bar{k}}^{(\bar{W}}\delta_{\bar{l}}^{\bar{T})}\partial_{\bar{T}}\partial_{\bar{W}}\partial_I\partial_W+\delta_i^W\delta_j^J\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}^2\partial_J\partial_W+\delta_i^W\delta_j^J\delta_{\bar{k}}^{\bar{T}}\delta_{\bar{l}}^{\bar{T}}\partial_{\bar{T}}^2\partial_J\partial_W$$
$$+\delta_i^W\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{0}}\partial_{\bar{0}}\partial_{\bar{K}}\partial_J\partial_W+\delta_i^W\delta_j^J\delta_{\bar{k}}^{\bar{0}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{0}}\partial_{\bar{L}}\partial_J\partial_W+\delta_i^W\delta_j^J\delta_{\bar{k}}^{\bar{K}}\delta_{\bar{l}}^{\bar{L}}\partial_{\bar{K}}\partial_{\bar{L}}\partial_J\partial_W \tag{A.19}$$

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$$
\begin{aligned}
\textcircled{6} &= 2\delta_i^W \delta_j^J \delta_{\bar{k}}^{(\bar{0}} \delta_{\bar{l}}^{\bar{T})} \partial_{\bar{0}} \partial_{\bar{T}} \partial_J \partial_W + \delta_i^W \delta_j^J \delta_{\bar{k}}^{\bar{K}} \delta_{\bar{l}}^{\bar{T}} \partial_{\bar{K}} \partial_{\bar{T}} \partial_J \partial_W + \delta_i^W \delta_j^J \delta_{\bar{k}}^{\bar{T}} \delta_{\bar{l}}^{\bar{L}} \partial_{\bar{L}} \partial_{\bar{T}} \partial_J \partial_W \\
&+ 2\delta_i^W \delta_j^J \delta_{\bar{k}}^{(\bar{0}} \delta_{\bar{l}}^{\bar{W})} \partial_{\bar{0}} \partial_{\bar{W}} \partial_J \partial_W + \delta_i^W \delta_j^J \delta_{\bar{k}}^{\bar{K}} \delta_{\bar{l}}^{\bar{W}} \partial_{\bar{K}} \partial_{\bar{W}} \partial_J \partial_W + \delta_i^W \delta_j^J \delta_{\bar{k}}^{\bar{W}} \delta_{\bar{l}}^{\bar{L}} \partial_{\bar{L}} \partial_{\bar{W}} \partial_J \partial_W \\
&+ 2\delta_i^W \delta_j^J \delta_{\bar{k}}^{(\bar{W}} \delta_{\bar{l}}^{\bar{T})} \partial_{\bar{T}} \partial_{\bar{W}} \partial_J \partial_W + 2\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{\bar{0}} \delta_{\bar{l}}^{\bar{0}} \partial_{\bar{0}}^2 \partial_T \partial_W + 2\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{\bar{T}} \delta_{\bar{l}}^{\bar{T}} \partial_{\bar{T}}^2 \partial_T \partial_W \\
&+ 2\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{\bar{K}} \delta_{\bar{l}}^{\bar{0}} \partial_{\bar{0}} \partial_{\bar{K}} \partial_T \partial_W + 2\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{\bar{0}} \delta_{\bar{l}}^{\bar{L}} \partial_{\bar{0}} \partial_{\bar{L}} \partial_T \partial_W + 2\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{\bar{K}} \delta_{\bar{l}}^{\bar{L}} \partial_{\bar{K}} \partial_{\bar{L}} \partial_T \partial_W \\
&+ 4\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{(\bar{0}} \delta_{\bar{l}}^{\bar{T})} \partial_{\bar{0}} \partial_{\bar{T}} \partial_T \partial_W + 2\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{\bar{K}} \delta_{\bar{l}}^{\bar{T}} \partial_{\bar{K}} \partial_{\bar{T}} \partial_T \partial_W + 2\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{\bar{T}} \delta_{\bar{l}}^{\bar{L}} \partial_{\bar{L}} \partial_{\bar{T}} \partial_T \partial_W \\
&+ 4\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{(\bar{0}} \delta_{\bar{l}}^{\bar{W})} \partial_{\bar{0}} \partial_{\bar{W}} \partial_T \partial_W + 2\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{\bar{K}} \delta_{\bar{l}}^{\bar{W}} \partial_{\bar{K}} \partial_{\bar{W}} \partial_T \partial_W + 2\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{\bar{W}} \delta_{\bar{l}}^{\bar{L}} \partial_{\bar{L}} \partial_{\bar{W}} \partial_T \partial_W \\
&+ 4\delta_i^{(W} \delta_j^{T)} \delta_{\bar{k}}^{(\bar{W}} \delta_{\bar{l}}^{\bar{T})} \partial_{\bar{T}} \partial_{\bar{W}} \partial_T \partial_W \,. \qquad \text{(A.20)}
\end{aligned}
$$

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References

[1] ATLAS collaboration, *Search for supersymmetry in final states with two same-sign or three leptons and jets using 36 fb^{-1} of $\sqrt{s} = 13$ TeV pp collision data with the ATLAS detector*, *JHEP* **09** (2017) 084 [*Erratum JHEP* **08** (2019) 121] [arXiv:1706.03731] [INSPIRE].

[2] CMS collaboration, *Search for supersymmetry in proton-proton collisions at 13 TeV in final states with jets and missing transverse momentum*, *JHEP* **10** (2019) 244 [arXiv:1908.04722] [INSPIRE].

[3] D.Z. Freedman and A. Van Proeyen, *Supergravity*, Cambridge University Press, Cambridge U.K. (2012).

[4] I. Antoniadis, A. Chatrabhuti, H. Isono and R. Knoops, *The cosmological constant in Supergravity*, *Eur. Phys. J. C* **78** (2018) 718 [arXiv:1805.00852] [INSPIRE].

[5] I. Antoniadis and F. Rondeau, *New Kähler invariant Fayet-Iliopoulos terms in supergravity and cosmological applications*, *Eur. Phys. J. C* **80** (2020) 346 [arXiv:1912.08117] [INSPIRE].

[6] I. Antoniadis, A. Chatrabhuti, H. Isono and R. Knoops, *Fayet-Iliopoulos terms in supergravity and D-term inflation*, *Eur. Phys. J. C* **78** (2018) 366 [arXiv:1803.03817] [INSPIRE].

[7] N. Cribiori, F. Farakos, M. Tournoy and A. Van Proeyen, *Fayet-Iliopoulos terms in supergravity without gauged R-symmetry*, *JHEP* **04** (2018) 032 [arXiv:1712.08601] [INSPIRE].

[8] Y. Aldabergenov, S.V. Ketov and R. Knoops, *General couplings of a vector multiplet in $N = 1$ supergravity with new FI terms*, *Phys. Lett. B* **785** (2018) 284 [arXiv:1806.04290] [INSPIRE].

JHEP11(2021)075

[9] S.M. Kuzenko, *Taking a vector supermultiplet apart: Alternative Fayet-Iliopoulos-type terms*, *Phys. Lett. B* **781** (2018) 723 [arXiv:1801.04794] [INSPIRE].

[10] F. Farakos, A. Kehagias and A. Riotto, *Liberated $\mathcal{N} = 1$ supergravity*, *JHEP* **06** (2018) 011 [arXiv:1805.01877] [INSPIRE].

[11] H. Jang and M. Porrati, *Constraining Liberated Supergravity*, *Phys. Rev. D* **103** (2021) 025008 [arXiv:2010.06789] [INSPIRE].

[12] H. Jang and M. Porrati, *Inflation, gravity mediated supersymmetry breaking, and de Sitter vacua in supergravity with a Kähler-invariant Fayet-Iliopoulos term*, *Phys. Rev. D* **103** (2021) 105006 [arXiv:2102.11358] [INSPIRE].

[13] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, *Spontaneous Symmetry Breaking and Higgs Effect in Supergravity Without Cosmological Constant*, *Nucl. Phys. B* **147** (1979) 105 [INSPIRE].

[14] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, *Yang-Mills Theories with Local Supersymmetry: Lagrangian, Transformation Laws and SuperHiggs Effect*, *Nucl. Phys. B* **212** (1983) 413 [INSPIRE].

[15] S. Ferrara, R. Kallosh, A. Van Proeyen and T. Wrase, *Linear Versus Non-linear Supersymmetry, in General*, *JHEP* **04** (2016) 065 [arXiv:1603.02653] [INSPIRE].

[16] T. Kugo, R. Yokokura and K. Yoshioka, *Component versus superspace approaches to $D = 4$, $N = 1$ conformal supergravity*, *Prog. Theor. Exp. Phys.* **2016** (2016) 073B07 [arXiv:1602.04441] [INSPIRE].

[17] M. Rausch de Traubenberg and M. Valenzuela, *A Supergravity Primer: From Geometrical Principles to the Final Lagrangian*, World Scientific (2020).

[18] C. Vafa, *The String landscape and the swampland*, hep-th/0509212 [INSPIRE].

[19] T.D. Brennan, F. Carta and C. Vafa, *The String Landscape, the Swampland, and the Missing Corner*, *PoS* **TASI2017** (2017) 015 [arXiv:1711.00864] [INSPIRE].

[20] E. Palti, *The Swampland: Introduction and Review*, *Fortsch. Phys.* **67** (2019) 1900037 [arXiv:1903.06239] [INSPIRE].

JHEP11(2021)075