Addressing CT metal artifacts using photon-counting detectors and one-step spectral CT image reconstruction

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Abstract

Purpose: The constrained one-step spectral CT image reconstruction (cOSSCIR) algorithm with a nonconvex alternating direction method of multipliers optimizer is proposed for addressing computed tomography (CT) metal artifacts caused by beam hardening, noise, and photon starvation. The quantitative performance of cOSSCIR is investigated through a series of photon-counting CT simulations.

Methods: cOSSCIR directly estimates basis material maps from photon-counting data using a physics-based forward model that accounts for beam hardening. The cOSSCIR optimization framework places constraints on the basis maps, which we hypothesize will stabilize the decomposition and reduce streaks caused by noise and photon starvation. Another advantage of cOSSCIR is that the spectral data need not be registered, so that a ray can be used even if some energy window measurements are unavailable. Photon-counting CT acquisitions of a virtual pelvic phantom with low-contrast soft tissue texture and bilateral hip prostheses were simulated. Bone and water basis maps were estimated using the cOSSCIR algorithm and combined to form a virtual monoenergetic image for the evaluation of metal artifacts. The cOSSCIR images were compared to a “two-step” decomposition approach that first estimated basis sinograms using a maximum likelihood algorithm and then reconstructed basis maps using an iterative total variation constrained least-squares optimization (MLE+TVmin). Images were also compared to a nonspectral TVmin reconstruction of the total number of counts detected for each ray with and without normalized metal artifact reduction (NMAR) applied. The simulated metal density was increased to investigate the effects of increasing photon starvation. The quantitative error and standard deviation in regions of the phantom were compared across the investigated algorithms. The ability of cOSSCIR to reproduce the soft-tissue texture, while reducing metal artifacts, was quantitatively evaluated.

Results: Noiseless simulations demonstrated the convergence of the cOSSCIR and MLE+TVmin algorithms to the correct basis maps in the presence of beam-hardening effects. When noise was simulated, cOSSCIR demonstrated a quantitative error of $-1$ HU, compared to 2 HU error for the MLE+TVmin algorithm and $-154$ HU error for the nonspectral TVmin+NMAR algorithm. For the cOSSCIR algorithm, the standard deviation in the central iodine region of interest was 20 HU, compared to 299 HU for the MLE+TVmin algorithm, 41 HU for the MLE+TVmin+Mask algorithm that excluded rays through metal, and 55 HU for the nonspectral TVmin+NMAR algorithm. Increasing levels of photon starvation did not impact the bias or standard deviation of the cOSSCIR images. cOSSCIR was able to reproduce the soft-tissue texture when an appropriate regularization constraint value was selected.
1 | INTRODUCTION

Metal objects are known to cause artifacts in computed tomography (CT) images due to several effects including beam hardening, photon starvation, nonlinear partial volume effect, and scatter. Metal objects cause streak and shading artifacts as well as CT number inaccuracies and noise. These artifacts can obscure anatomical structure and impede image analysis and diagnosis.

Numerous metal artifact reduction methods have been proposed and clinically implemented to reduce the severity of metal artifacts.1–3 Existing methods have demonstrated improved image quality such as reduced standard deviation, improved CT number accuracy, and improved subjective image quality evaluation.3 However, existing methods can result in residual artifacts and CT number inaccuracies.1–7

Many different approaches have been proposed for reducing metal artifacts. One approach is to identify, remove, and replace the sinogram data corrupted by metal.8–11 These methods can introduce new artifacts and inaccuracies, as the approximate estimated sinogram values may be inconsistent. Commercially available approaches generally use these projection correction methods.12 Other approaches use iterative or optimization-based reconstruction to ignore or down-weight datapoints corrupted by metal.13–16 Acquiring multiple spectral measurements, for example, using dual kV or photon-counting CT, provides additional methods to address metal artifacts. Material decomposition methods that use the projection data can address beam-hardening effects.17–19 For all material decomposition methods, virtual monoenergetic images (VMIs) representing higher energies contain fewer metal artifacts and have been proposed as an approach to reduce the effects of metals.4,20–23 But, soft tissue contrast also decreases with increasing VMI energy. Deep learning approaches have been recently proposed for metal artifact reduction.24–27 One type of deep learning method trains convolutional neural networks (CNN) to estimate the sinogram projection data corrupted by metal,27 while an additional step of using a CNN to correct image patches with artifacts has also been proposed.24 A different approach is to use unsupervised learning to disentangle metal artifacts from the CT images.26 A recent review paper summarized that although existing metal artifact reduction methods reduce artifacts, there remains a need for additional improvement for dense and multiple metallic objects, as well as a need for corrections with improved CT number accuracy, for example, for proton therapy planning.1

This study investigates using a “one-step” direct inversion material decomposition algorithm to directly estimate basis material maps from photon-counting spectral CT data.28,29 The proposed approach uses a physics-based forward model to optimize the basis maps that best match the spectral projection data. The polyenergetic forward model used during optimization theoretically accounts for beam-hardening effects, which are a major cause of metal artifact streaks. While other one-step decomposition algorithms have been proposed,30–35 the unique aspects of our algorithm are that it models the nonlinear polyenergetic x-ray transmission forward model and performs nonconvex and nonsmooth optimization using an algorithm that is proven to converge under the assumption of restricted strong convexity (RSC).36 The estimated basis maps can then be linearly combined to form a VMI that is theoretically free of metal artifacts regardless of the selected energy.

“Two-step” material decomposition methods, that first decompose spectral projection data into basis sinograms and then reconstruct basis maps, can also address beam-hardening effects.37–40 However, we hypothesize that the one-step direct inversion method has additional advantages for addressing the causes of metal artifacts. Material decomposition is an unstable inversion process, meaning that small errors or noise in the data can cause large errors in the material decomposition estimates. In one-step approaches, all the rays that pass through an image voxel are used concurrently to estimate the basis coefficients for that voxel. We hypothesize that the one-step approach is therefore more stable and less sensitive to noisy measurements through metal. In contrast, two-step decomposition methods perform decomposition for each ray individually using only the ray’s spectral measurements. Because the inversion into basis coefficients is unstable, noisy ray measurements may cause large errors in the basis coefficients, which then introduce streaks in the image during

Conclusions: By directly inverting photon-counting CT data into basis maps using an accurate physics-based forward model and a constrained optimization algorithm, cOSSCIR avoids metal artifacts due to beam hardening, noise, and photon starvation. The cOSSCIR algorithm demonstrated improved stability and accuracy compared to a two-step method of decomposition followed by reconstruction.

KEYWORDS
metal artifacts, photon-counting CT, reconstruction
the reconstruction step. In the one-step algorithm, constraints are placed on the basis maps to further stabilize the inversion in the presence of metal.

The one-step inversion approach is also advantageous when dealing with measurements that are starved of photons due to metal. For some rays, metal may cause photon starvation in the lower-energy windows but not the high-energy windows. Two-step decomposition methods decompose each ray individually; therefore each ray requires at least two spectral measurements. Two-step decomposition is expected to become more unstable when low-energy photons are starved. In one-step decomposition, all rays and spectral measurements are used together to estimate the basis maps. Rays for which some energy-windows are photon starved still provide useful information for optimizing the basis maps. We hypothesize that ray measurements that are completely starved of photons also can be used in the optimization.

This paper adapts and investigates our previously proposed constrained One-Step Spectral CT Image Reconstruction (cOSSCIR) algorithm for metal artifact reduction. A preliminary study demonstrated potential for metal artifact reduction with cOSSCIR, but residual artifacts remained. In the current study, a new optimization approach is implemented that can address the increased nonlinearity and nonconvexity caused by metal. The proposed cOSSCIR approach is not a metal artifact correction technique; rather, the algorithm is using the measured spectral information and a physics-based forward model to reconstruct the object and is thus able to handle the presence of metal. A series of simulation studies are presented to demonstrate that the cOSSCIR algorithm can avoid metal artifacts caused by beam hardening, noise, and photon starvation and to demonstrate the benefits of one-step direct inversion approach compared to the two-step decomposition approach.

2 cOSSCIR THEORY AND ALGORITHM

This section presents the general theory and algorithmic steps of the cOSSCIR algorithm.

2.1 The data model

The continuous X-ray fluence in the spectral CT set-up is modeled as

$$I_{w,\ell} = \int S_{w,\ell}(E) \exp \left[ - \int_{\ell} \mu(E, \tilde{r}(t)) \, dt \right] \, dE,$$  \hspace{1cm} (1)

where \(I_{w,\ell}\) is the transmitted X-ray photon fluence along ray \(\ell\) in energy window \(w\); \(t\) is a parameter indicating location along \(\ell\); \(S_{w,\ell}(E)\) is the spectral response; and \(\mu(E, \tilde{r}(t))\) is the energy and spatially dependent linear X-ray attenuation map. The image reconstruction problem involves finding \(\mu\) from fluence measurements \(I\).

As is standard, the X-ray attenuation map is expanded in basis functions and in this work we use a material basis set

$$\mu(E, \tilde{r}(t)) = \sum_m \left( \frac{\mu_m(E)}{\rho_m} \right) \rho_m f_m(\tilde{r}),$$  \hspace{1cm} (2)

where \(\rho_m\) is the density of material \(m\); \(\mu_m(E)/\rho_m\) is the mass attenuation coefficient of material \(m\); and \(f_m(\tilde{r})\) is the spatial fractional density map for material \(m\).

To form the data model, we combine Equation (1) with Equation (2), normalize the spectral response, and discretize the energy and line integration

$$\delta_{w,\ell, j}(f) = N_{w,\ell} \sum_i s_{w,\ell, i} \exp \left( - \sum_{m,k} \mu_{m,j} X_{\ell, k} f_{k,m} \right),$$  \hspace{1cm} (3)

where \(\delta_{w,\ell}(f)\) is the mean transmitted photon count; \(N_{w,\ell}\) is the mean total number of incident photons along ray \(\ell\) in energy window \(w\); \(s_{w,\ell, i}\) is the is the normalized spectral projection matrix element indicating the weighting of pixel \(k\) to the line integration along ray \(\ell\); and \(f_{k,m}\) is the pixelized material map with \(k\) and \(m\) indexing pixel and expansion-material, respectively.

2.1.1 Two-step image reconstruction

For two-step image reconstruction, the model in Equation (3) is split into two equations

$$\delta_{w,\ell, j}(f) = N_{w,\ell} \sum_i s_{w,\ell, i} \exp \left( - \sum_{m} \mu_{m,j} z_{\ell, m} \right),$$

$$z_{\ell, m} = \sum_k X_{\ell, k} f_{k,m},$$  \hspace{1cm} (4)

where the \(z_m\) are the basis material map sinograms. Image reconstruction involves obtaining the basis material sinograms \(z\) from the photon count data \(c\), followed by any one of a number of standard algorithms for inverting the X-ray transform model to obtain \(f\) from \(z\). In the present work, maximum likelihood is used for processing the counts data into material sinograms as presented in Schlomka et al. For the second step, we employ total variation (TV) constrained least-squares optimization. A summary of our implementation of TV\(_{\text{min}}\) can be found in Sidky et al.
One-step image reconstruction

One-step image reconstruction directly inverts Equation (3), obtaining the basis material maps \( f \) from the measured counts data \( c \). The presented imaging model that provides \( f \) from \( c \) is specified by constrained minimization

\[
f^* = \arg \min_f D_{TPL}(c, \hat{c}(f)) \text{ such that } \text{GTV}(f) \leq \gamma,
\]

where \( D_{TPL}(c, \hat{c}(f)) \) is the shifted, negative logarithm of the TPL

\[
D_{TPL}(c, \hat{c}(f)) = \sum_{w, \ell} \left[ \hat{c}_{w, \ell}(f) - c_{w, \ell} - c_{w, \ell} \log \frac{\hat{c}_{w, \ell}(f)}{c_{w, \ell}} \right],
\]

and the generalized total variation (GTV) is defined as

\[
\text{GTV}(f) = \sum_k \sqrt{\sum_m \| (D_{fm})_k \|_2^2},
\]

\( D \) is a numerical gradient operator. The GTV regularizer is a mixed-norm that is quadratic in the summation over the basis material index and an absolute magnitude (L1-norm) summation over the pixel index. Thus the GTV regularizer still encourages sparsity over pixels in the gradient magnitude images, but not sparsity across basis material index. This is an example of a group lasso mitigates artifacts due to beam-hardening. The use of transmission Poisson likelihood (TPL) for the data discrepancy objective function is particularly important because the data noise is mainly due to quantum noise and the dynamic range of the measured quanta can vary several orders of magnitude when metal is present in the scanned subject. The data TPL objective function effectively down-weights measurements that have fewer counts, which is important for handling photon starvation.

The GTV constraint regularizes the reconstructed basis material maps.

Nonconvex optimization solver

The \( D_{TPL} \) objective function is large-scale and non-convex. Furthermore, the constraint in Equation (5) adds nonsmoothness to the optimization problem. We discuss the optimization of the smooth nonconvex objective function \( D_{TPL} \). The primal-dual algorithm of Chambolle and Pock (CPPD)\(^{44} \) has proven useful for convex nonsmooth optimization problems relevant to CT imaging\(^{45} \), and the CPPD algorithm has been adapted to the nonconvex and nonsmooth problems that arise in spectral CT. The mirrored convex/concave optimization (MOCCA) algorithm is one such derivative algorithm, where the smooth objective function is expanded in a local convex quadratic approximation.\(^{28,46} \) We have also directly applied CPPD to a linearization of the data fidelity term, where the difference between the approximate data model and the full non-linear data model is accounted for by an additive term modifying the input data.\(^{47} \) An extension of CPPD to non-linear data models developed by Valkonen\(^{48} \) has also been applied to the one-step spectral CT image reconstruction\(^{49} \).

In this work, we make use of a different convexification of the one-step spectral CT image reconstruction problem that fits into the framework of the nonconvex alternating direction method of multipliers (ADMM).\(^{36} \) The algorithm is proven to converge assuming a mathematical property of the optimization problem called RSC.\(^{50} \) The mathematical details of the algorithm and its application to a small-scale spectral CT simulation are presented in Barber and Sidky\(^{36} \).

Nonconvex ADMM

To solve the spectral CT optimization problem, we use the ADMM algorithm in a way that is closely related to CPPD. The ADMM algorithm solves the generic optimization problem

\[
\min_{x,y} f(x) + g(y) \mid Ax + By = c.
\]

For the present purposes, \( f \) and \( g \) are both split into two terms

\[
f(x) = f_c(x) + f_d(x),
\]

\[
g(x) = g_c(x) + g_d(x),
\]

where \( f_c, g_c \) are convex functions that are possibly nonsmooth and \( f_d, g_d \) are differentiable and possibly nonconvex. If \( f_d \) or \( g_d \), we assume that the RSC condition holds, so that the following ADMM update steps converge

\[
x_{t+1} = \arg \min_x \left\{ f_c(x) + \langle x, \nabla f_d(x_t) \rangle + A^T u_t + \frac{1}{2} \| Ax + By_t - c \|_2^2 + \frac{1}{2} \| x - x_t \|_{u_t}^2 \right\}
\]
\[ y_{t+1} = \arg \min_y \left\{ g_c(y) + \langle \nabla g_d(y_t) + B^T u_t \rangle + \frac{1}{2} \|Ax_{t+1} + By - c\|_2^2 + \frac{1}{2} \|y - y_t\|_{H_y}^2 \right\} \]  

(11)

where \( C \) is independent of \( f \). Accordingly, we consider minimization of

\[ L(f) = L_1(f) + L_2(f) \]  

(15)

\[ L_1(f) = \sum_{w,\ell} \hat{c}_{w,\ell}(f) \]  

(16)

\[ L_2(f) = -\sum_{w,\ell} c_{w,\ell} \log \hat{c}_{w,\ell}(f). \]  

(17)

By computing the Hessian, it can be shown that the first-term \( L_1(f) \) is convex, and possible nonconvexity can only arise from the second-term \( L_2(f) \). Thus the term \( L_2 \) must be assigned to either \( f_c \) or \( g_d \). The \( L_1 \) term can be assigned to \( g_c \). (We do not consider assigning it to \( f_c \) because that leads to a nontrivial large-scale optimization problem in computing the \( x \)-update in Equation (10).) We have experimented with various combinations and have found that the most efficient algorithm results from assigning \( L_1 \) to \( g_c \) and \( L_2 \) to \( g_d \). Assigning \( L_1 \) to \( g_c \) does yield a minimization problem for Equation (11) that cannot be solved analytically, but the problem is smooth, convex, and separable. Accordingly, it can be solved very efficiently with Newton’s method. The full computer code for the algorithm is available for a small-scale spectral CT problem shown in Barber and Sidky.

A couple of other implementation notes are important for the efficiency of the large-scale optimization of the spectral CT optimization problem as we presented in previous work. For the efficiency of the large-scale optimization of the spectral CT optimization problem as we presented in previous work, orthogonally orthogonally the material \( \mu \) energy functions improves convergence substantially. This \( \mu \)-preconditioning involves a linear transformation of \( \mu \) to \( \mu' \) such that

\[ \sum_m \mu_{m,i} f_m = \sum_m \mu'_{m,i} f'_m \]  

and \( \sum_m \mu'_{m,j} = \delta_{ij}, \)  

(18)

where \( \delta_{ij} \) is the Kronecker delta function. Another important modification is to “soften” the exponential function in the data model. The exponential function is replaced by \( \text{qexp} \)

\[ \text{qexp}(x) = \begin{cases} 
\exp(x) & x \leq 0 \\
\frac{1}{2}x^2 + x + 1 & x > 0 
\end{cases} \]  

(19)

Implementation of cOSSCIR with nonconvex ADMM

In applying either ADMM or CPPD to imaging problems, there are several choices that need to be made involving how to split up the optimization problem between the functions \( f \) and \( g \) and how to determine the iteration step parameters. The step parameter determination has been covered in our previous work (see, e.g., Sidky et al.). For the ADMM updates, we need to decide how to split up the optimization problem into the convex functions \( f_c \) and \( g_c \) and the differentiable functions \( f_d \) and \( g_d \). Our experience with applying CPPD to convex imaging problems tells us that efficient algorithms result from assigning all terms of the optimization problem to \( g \), and this would be equivalent to making the assignment to \( g_c \) in the ADMM framework. For the spectral CT optimization problem in Equation (5), the GTV constraint is convex, and it can be assigned to \( g_c \), but the objective function is differentiable and nonconvex and at least part of this function needs to be assigned to either \( f_d \) or \( g_d \).

To decide how to make the objective function assignment, we examine the \( D_{\text{TPL}} \) objective function; it can be written as two terms plus a constant

\[ D_{\text{TPL}}(c, \hat{c}(f)) = \sum_{w,\ell} \left[ \hat{c}_{w,\ell}(f) - c_{w,\ell} \log \hat{c}_{w,\ell}(f) \right] + C, \]  

(14)
which is identical to \( \exp(\cdot) \) for nonpositive physical argument values. The softer quadratic dependence for positive arguments is helpful for iterative image reconstruction where unphysical, negative attenuation values may occur at intermediate iterations. Also, because the extension is quadratic both the function \( q_{\exp} \) and its derivative are continuous.

### 3 | SIMULATION AND RECONSTRUCTION IMPLEMENTATION DETAILS

This section describes methods used to simulate photon-counting CT data as well as the specific implementation details of the cOSSCIR and comparison algorithms. These methods were common to the three simulation studies, while methods specific to each study are described in Sections 4–6.

#### 3.1 | Pelvic phantom

A virtual pelvic phantom was generated from an anonymized CT image with 512 × 512 pixels. The image was first convolved with a low-pass Gaussian kernel with a standard deviation of 2 pixels to reduce noise. Each pixel in the filtered image was classified as air, adipose tissue, water (representing soft tissue), or bone, based on CT number thresholds. The total phantom extent was 36 cm. Disks of 11-mm diameter were placed in each of the femurs to model bilateral hip prosthesis. The metal implant was modeled as titanium. Two low-contrast elements of 10-mm diameter and containing 3 mg/mL of iodine water solution were placed in the phantom at regions susceptible to metal artifacts. To evaluate the performance of the cOSSCIR algorithm with respect to preserving low contrast image texture, anatomical texture was added to the soft tissue and adipose regions of the phantom according to a power-law noise model, which has been used to model breast tissue. To generate texture, a white noise image was filtered in frequency space with a radially symmetric power-law filter \( H(f) = |f|^{-\beta/2} \). The resulting texture-only images have radially symmetric noise power spectrum (NPS) equal to:

\[
P(f) = |f|^{-\beta} \tag{20}
\]

Texture in the adipose region was generated with \( \beta = 2 \), while texture in the soft tissue region was generated with \( \beta = 2.6 \). The modeled texture is not an accurate representation of adipose or soft tissue texture, rather it is added to the phantom to evaluate algorithm performance. The generated texture image was scaled to range from 0.95 to 1.05 and then used as a multiplicative weighting of the water and adipose densities, thus adjusting the linear attenuation coefficient of each phantom pixel in the soft tissue or adipose regions. The resulting textures in the ground truth 55 keV image exhibited a standard deviation of approximately 10 HU. The pelvic phantom and the ground truth 55 keV monoenergetic image depicting the texture are displayed in Figure 1.

Linear attenuation coefficients for all phantom materials were obtained from the NIST X-COM database. A least-squares estimation was used to generate the ground truth water and bone basis coefficients for each modeled material. Ground truth basis material maps of size 512 × 512 were then created using these basis material coefficients.

#### 3.2 | Photon-counting detector model

Simulations modeled the spectral properties of a CdTe photon-counting detector that we have previously used for experimental studies (DxRay, Northridge, CA). The detector spectral response, accounting for flux-independent effects such as charge sharing and K-escape, was modeled by a previously validated analytical model. The detector was modeled with four energy windows with thresholds of 20, 45, 60, and 70 keV. Pulse pileup effects were not modeled. Figure 2 plots the spectra of photons detected by each energy window through the air and assuming a 120-kV incident spectrum and a total of \( 10^7 \) photons, demonstrating the nonideal nature of the detector spectral response.

#### 3.3 | CT Simulation

CT data were simulated using the forward model of Equation (3), assuming the ground truth phantom bone and water basis maps, and modeling the energy-window spectra shown in Figure 2. The CT acquisition was modeled as a fan beam acquisition with 512 detector pixels of 1.5-mm dimension, 50-cm source-to-isocenter distance, and 100-cm source to detector distance. The number of photons modeled per ray, \( 10^7 \) (summed over all energy windows), was selected because it provided a realistic noise standard deviation (22 HU) in a filtered backprojection reconstructed image of the pelvic phantom without metal.

#### 3.4 | Reconstruction and evaluation

All images were reconstructed onto a grid of 512 × 512 pixels of size 0.7 × 0.7 mm. The cOSSCIR algorithm reconstructed bone and water basis material maps from the simulated photon-counting CT data. For the application of metal artifact reduction, reconstruction of the basis maps is required to accurately model the x-ray physics but the desired...
output is a conventional grayscale image with reduced metal artifacts. The estimated basis material maps were therefore linearly combined to form a 55-keV VMI.

For comparison, the simulated photon-counting CT data were also reconstructed into water and bone basis material maps using a two-step material decomposition approach. The energy-window counts data were the first decomposed into bone and water basis sinograms using a maximum likelihood estimation (MLE) algorithm \(^{38}\) using the same spectral model as cOSSCIR. An unconstrained quasi-Newton optimization with analytical gradient was used to estimate the basis sinograms, with the negative Poisson log-likelihood used as the data error objective, as in cOSSCIR. An unconstrained quasi-Newton optimization with analytical gradient was used to estimate the basis sinograms, with the negative Poisson log-likelihood used as the data error objective, as in cOSSCIR. An unconstrained quasi-Newton optimization with analytical gradient was used to estimate the basis sinograms, with the negative Poisson log-likelihood used as the data error objective, as in cOSSCIR. 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There are two reasons why we used the least-squares TV constrained reconstruction approach for the comparison two-step and nonspectral reconstructions, rather than filtered backprojection. One reason is to provide a more fair comparison of the one-step and two-step approaches, where both take advantage of an optimization-based reconstruction algorithm, with similar TV-based image constraints. The second reason is that the data simulation in this study uses a discrete object and measurement model, which would cause errors in a filtered backprojection reconstruction that assumes continuous modeling.

For all studies, algorithm performance was quantitatively evaluated by comparing the bias and standard deviation in the regions of interest corresponding to the iodine contrast elements. Bias was calculated relative to the ground truth 55 keV monoenergetic image.

4 | SIMULATION STUDY 1: BEAM-HARDENING EFFECTS

4.1 | Methods

A simulation study was first performed to evaluate the ability of the cOSSCIR algorithm to recover the basis material maps in the presence of metal and polyenergetic x-ray transmission. This simulation study is designed to demonstrate that the cOSSCIR approach can remove artifacts due to beam hardening when an accurate forward model is known and in the absence of noise. This simulation constitutes an “inverse crime” study where the same forward model is used to generate and reconstruct the data. This study expands a previous preliminary study of cOSSCIR in an ideal data setting with metal. Because the optimization problem is nonconvex, this is an important simulation to demonstrate that the cOSSCIR algorithm is converging to the correct solution, despite the nonconvexity of the objective function. The data were also reconstructed with the two-step MLE+TV min reconstruction approach, for which the forward model used in material decomposition also matched the simulation model, to demonstrate that both the MLE decomposition and TV min reconstruction algorithm components are converging to the correct solution under ideal conditions.

The forward model in Equation (3) yields the mean photon count in each energy window of each ray measurement, which is noninteger. In this simulation, Poisson noise was not modeled and the noninteger counts data were input to the reconstruction algorithms so that the data were completely consistent with the forward model used in one-step and two-step reconstruction. The true phantom GTV was used as the constraint for the cOSSCIR algorithm, while the true basis map TV values were used as the constraints in the MLE+TV min reconstructions.

To demonstrate the level of potential beam-hardening artifacts for the pelvic phantom, the sinogram resulting from summing the counts across energy windows for each ray was reconstructed with the nonspectral TV min algorithm and with the nonspectral algorithm after application of the NMAR sinogram correction method (TV min+NMAR).

To quantify the ability of cOSSCIR to retain low contrast texture, noise-free simulations were repeated with 20 realizations of the power-law anatomical textures. Regions of interest (ROIs) of size 32 × 32 pixels, as shown in Figure 1 were extracted from the adipose and soft tissue regions of the ground truth and cOSSCIR reconstructed images. The two-dimensional (2D) NPS was estimated for adipose and soft tissue texture from the resulting 200 adipose texture ROIs and 120 soft tissue texture ROIs using the equation

\[
NPS(u, v) = \frac{\Delta x \Delta y}{K 32^2} \sum_{k=1}^{K} |FT[I_k(x, y) - \hat{I}_k]|^2, \tag{21}
\]

where \(u, v\) are the spatial frequency space coordinates, \(\Delta x\) and \(\Delta y\) are the pixel dimensions, \(K\) is the total number of extracted ROIs, \(FT\) represents the 2D Fourier transform, \(I_k(x, y)\) is the \(k\)th extracted ROI, and \(\hat{I}_k\) is the mean value in the \(k\)th ROI. The 2D NPS was averaged along the radial dimension to estimate the one-dimensional (1D) radial NPS. The 1D radial NPS curves of the cOSSCIR reconstructions at different GTV constraint values were compared to the radial NPS estimated from the ground truth images. The NPS curves were also fit to the power-law function in Equation (20), with the estimated \(\beta\) values compared to the ground truth value.

4.2 | Results

To demonstrate the convergence of the cOSSCIR algorithm for the case of noiseless polyenergetic simulation, Figure 3 plots the Poisson likelihood data error across iteration number for the cOSSCIR algorithm, where the data error is normalized by the mean total number of simulated photons. To further analyze convergence of the cOSSCIR algorithm, the root mean squared error (RMSE) between the measured and predicted photon counts normalized by the total number of photons is plotted against the iteration number, as well as the RMSE of the estimated basis maps.

Figure 4 displays the ground truth pelvic phantom image at 55 keV and the virtual monoenergetic 55 keV image resulting from the cOSSCIR and MLE+TV min approaches for noiseless data. The image reconstructed by the nonspectral TV min algorithm from the sum of all photon counts (similar to a conventional acquisition) is also displayed with and without NMAR applied, demonstrating beam-hardening artifacts due to metal.
COSSCIR FOR CT METAL ARTIFACT REDUCTION

FIGURE 3 Convergence of the cOSSCIR algorithm in the absence of noise is demonstrated by (top) the Poisson likelihood data error metric, normalized by the mean total number of simulated photons, and the RMSE normalized by the total number of photons plotted against iteration number; (bottom) RMSE between estimated basis maps and ground truth phantom plotted against iteration number.

FIGURE 5 ROIs extracted from the soft tissue regions of the phantom for the ground truth 55 keV image and the 55-keV image reconstructed by cOSSCIR at a range of GTV constraint values. The GTV constraint values are labeled as the factor multiplying the true GTV value.

Image reconstructed from all counts resulted in $-209$ HU error for the iodine element between the two metal inserts and $-100$ HU error for the peripheral iodine element. NMAR reduced this error to $-154$ and $-115$ HU, respectively. Both the cOSSCIR and MLE+TV$_\text{min}$ algorithms recovered the values in the iodine elements to within 0.1 HU error, demonstrating accurate recovery of the phantom for this noiseless inverse crime case where the simulated model matched the reconstruction model. These two methods also recovered the CT number of the metal inserts to within 0.5 HU error.

To evaluate the ability of cOSSCIR to reconstruct low-contrast texture, Figures 5 and 6 display example ROIs in the soft tissue and adipose regions of images.

FIGURE 4 The ground truth 55 keV image of the pelvic phantom is displayed along with the 55-keV VMI reconstructed by the cOSSCIR and two-step MLE+TV$_\text{min}$ methods for the case of noiseless data. The image resulting from summing all counts in each ray, and reconstructing with the nonspectral TV$_\text{min}$ algorithm is also displayed to demonstrate beam-hardening artifacts due to metal, which are reduced using the TV$_\text{min}$+NMAR method. All images are displayed at a window of $-250$ to 250 HU; therefore, the metal is not visible. Instead, the metal regions are identified by the circle contours in the ground truth image.
COSSCIR FOR CT METAL ARTIFACT REDUCTION

FIGURE 6  ROIs extracted from the adipose regions of the phantom for the ground truth 55 keV image and the 55-keV image reconstructed by cOSSCIR at a range of GTV constraint values. The GTV constraint values are labeled as the factor multiplying the true GTV value.

FIGURE 7  The radial NPS of the ground truth 55 keV image and the 55-keV images reconstructed by cOSSCIR at a range of GTV constraint values for both the soft tissue and adipose textures. The images demonstrate that the anatomical texture can be accurately reconstructed by cOSSCIR when using an appropriate GTV constraint value.

reconstructed by cOSSCIR for a range of GTV constraint values. The images demonstrate that the anatomical texture can be accurately reconstructed by cOSSCIR when using an appropriate GTV constraint value. Lower GTV constraints blur the texture, while higher GTV constraints produce higher frequency textures. Figure 7 plots the radial NPS of the ground truth image and the cOSSCIR reconstructions at a range of GTV constraints for both the soft tissue and adipose textures. The plots demonstrate that cOSSCIR reconstructions with GTV constraints equal to or greater than the true constraint value accurately reconstruct the low-frequency portion of the NPS, with higher constraint values introducing more power at higher frequencies, which leads to the higher-frequency textures seen in Figures 5 and 6. Setting the GTV constraint to 0.75 times the true GTV value reduced the power at lower frequencies, causing the blurring seen in Figures 5 and 6. Compared to the true water texture $\beta$ value of 2.6, the $\beta$ values estimated from the nonzero frequencies of the radial NPS curves for the soft tissue texture were 3.5, 2.7, 2.6, and 2.6 for cOSSCIR reconstructions with GTV constraints equal to 0.75, 1.0, 1.25, and 1.5 times the true GTV constraint, respectively. For the adipose texture generated with $\beta = 2.0$, the estimated $\beta$ values were 2.4, 2.0, 1.8, and 1.6 for cOSSCIR reconstructions with GTV constraints equal to 0.75, 1.0, 1.25, and 1.5 times the true GTV constraint, respectively. Overall, the results demonstrate that the anatomical texture can be accurately reconstructed by cOSSCIR when using an appropriate GTV constraint value.

SIMULATION STUDY 2: BEAM-HARDENING EFFECTS AND NOISE

5.1  Methods

The next simulation study modeled Poisson noise in the photon-counting measurements. The number of simulated photons ($10^7$ per ray) provided sufficient statistics such that all energy-window ray measurements registered at least one count. This study evaluated the performance of the cOSSCIR, MLE+$TV_{\text{min}}$, MLE+$TV_{\text{min}}$+Mask, nonspectral $TV_{\text{min}}$, and nonspectral $TV_{\text{min}}$+NMAR methods in the presence of metal, beam-hardening effects, and noise.

As mentioned previously, the first step of the two-step approach, estimating basis map sinograms from the energy-window counts measurements, can be unstable. In this study, the true basis map sinogram values were used as the initial guess for the maximum likelihood basis sinogram estimation algorithm, so that the decomposition was only susceptible to noise instability rather than additional instability due to the initial guess. This approach provides an optimistic evaluation of the MLE+$TV_{\text{min}}$ approach.

To evaluate the ability of cOSSCIR to reproduce low-contrast texture in the presence of noise, simulations were performed for 20 texture phantom realizations followed by NPS analysis, as described in Section 4. Because these simulations include noise, the resulting NPS curves represent both the anatomical texture and noise. To isolate the NPS of the texture, simulations were repeated with a single texture realization and 20 noise...
realizations. The images with different noise realizations reconstructed by cOSSCIR were each subtracted from the first reconstructed noise realization, generating noise-only images. The radial NPS of the noise-only images was estimated using Equation (21) and then divided by two to correct for the increase in noise due to subtraction. The radial NPS curves quantifying noise power were subtracted from the NPS curves quantifying the power spectrum due to noise and texture, resulting in estimated NPS curves of the anatomical textures.

5.2 Results

Figure 8 displays images reconstructed by cOSSCIR, MLE+TV$_{\text{min}}$, and TV$_{\text{min}}$ across a range of TV parameter settings. Both the cOSSCIR and MLE+TV$_{\text{min}}$ are qualitatively free of beam-hardening artifacts. However, the MLE+TV$_{\text{min}}$ images demonstrate prominent noise streaks in rays that pass through metal, which are considerably reduced in the cOSSCIR reconstructions, even at high levels of the GTV constraint. The noise streaks in the MLE+TV$_{\text{min}}$ images are due to the instability of the material decomposition inversion, which causes the noise in the measurements through metal to be amplified during the decomposition. The image reconstructed by the nonspectral TV$_{\text{min}}$ algorithm demonstrates beam-hardening streaks and artifacts. For all algorithms, increasing the TV regularization constraint increased the noise level.

Figure 9 plots the bias in the peripheral iodine ROI against the standard deviation in the ROI for the different studied TV constraint levels, where increasing the constraint increases the standard deviation. The cOSSCIR algorithm resulted in the lowest bias at all constraint settings and showed limited reduction in bias with increasing constraint. The MLE+TV$_{\text{min}}$ algorithm required high standard deviation to obtain low bias. For all algorithms, the optimal constraint was selected as the lowest
constraint value that provided a bias within 2.5 HU of the minimum bias across all investigated constraint levels.

Figure 10 compares the images resulting from the studied reconstruction algorithms, each at their optimal TV constraint level. The optimal constraint multiplication factors were 1.0, 2.25, 1.0, 1.0, and 1.0 for the cOSSCIR, MLE+TV\textsubscript{min}, MLE+TV\textsubscript{min}+Mask, nonspectral TV\textsubscript{min}, and nonspectral TV\textsubscript{min}+NMAR algorithms, respectively. Excluding, or masking, the rays through metal when reconstructing the basis maps after material decomposition (MLE+TV\textsubscript{min}+Mask) reduced noise streak artifacts caused by metal. Performing NMAR sinogram correction for the nonspectral data reduced the dark shading artifacts and reduced bias in the central ROI; however, a bias >100 HU remained.

The distribution of CT numbers in the central and peripheral iodine ROIs is presented in Figure 11, for the optimally regularized images shown in Figure 10. The ground truth value is also plotted. Table 1 presents the bias and standard deviation in the central and peripheral iodine ROIs. Metal artifacts caused high bias and standard deviation in the iodine ROIs for the nonspectral TV\textsubscript{min} algorithm, for example, −208 HU bias and 90 HU standard deviation for the central ROI, reduced to −154 HU bias and 55 HU standard deviation when NMAR was applied. The MLE+TV\textsubscript{min} algorithm demonstrated 2 HU bias for central ROI, but a high standard deviation of 299 HU in central ROI due to noise amplification. The MLE+TV\textsubscript{min} standard deviation was reduced to 41 HU by excluding the rays that pass through metal. The cOSSCIR algorithm resulted in the lowest bias and deviation of the values in the iodine ROIs, for example, −1 HU bias and 20 HU standard deviation in the central ROI.

Figure 12 plots the radial NPS of the anatomical texture for images reconstructed by cOSSCIR at a range of GTV constraint values. The NPS due to texture was isolated from the NPS due to noise as described in Section 5. When reconstructing from noisy data, the plots in Figure 12 suggest that setting the GTV constraint equal to the true image GTV may blur the low contrast texture. When the NPS for frequencies greater than zero was fit to a power law function, setting the GTV constraint to the true image GTV resulted in $\beta = 3.2$ (soft tissue) and $\beta = 2.3$ (adipose), compared to true values of 2.6 and 2.0, respectively, further demonstrating blurring of the texture. However, setting the GTV constraint to 1.25...
FIGURE 11  Box plots comparing the CT numbers in the central and peripheral iodine ROIs of the images compared in Figure 10, for simulations with noise but without photon starvation. The solid horizontal line represents the ground truth value.

FIGURE 12  The NPS due to anatomical texture in the (top) soft tissue and (bottom) adipose regions of the phantom for cOSSCIR images reconstructed from noisy data at a range of GTV constraint values. The NPS of the ground truth 55 keV images is also plotted for reference. The NPS due to texture was estimated by subtracting the estimated NPS due to noise from the NPS estimated from images reconstructed with texture and noise, as described in Section 5.

times the true value recovered accurate $\beta$ values of 2.6 (soft tissue) and 2.0 (adipose) in the cOSSCIR reconstructions. According to the bias and standard deviation trade-off curves for cOSSCIR shown in Figure 9, increasing the GTV constraint to 1.25× reduces the bias in iodine values but increases the noise standard deviation from 20 HU to 30 HU. Therefore, accurately depicting the phantom texture in cOSSCIR reconstructed images involves a trade-off with image noise, as is generally the case for regularized reconstruction algorithms.

6 SIMULATION STUDY 3: BEAM-HARDENING EFFECTS, NOISE, AND PHOTON STARVATION

6.1 Methods

To explore the effects of photon starvation, the density of the metal implants was increased to reduce transmission. The metal density was first increased by a factor of 1.5. This caused photon starvation for 0.58% of rays in the lowest energy window and 0.14% of rays in the second lowest energy window, but no photon starvation in the two higher energy windows. For this level of photon starvation, both cOSSCIR and the two-step decomposition approach theoretically have sufficient information to perform material decomposition.

The metal density was then increased by a factor of 4, which caused 53% of rays to be starved of photons in the lowest energy window, 30% starvation in the second energy window, 6.5% starvation in the third energy window, and 0.61% starvation in the highest energy window. For this case, the cOSSCIR method is still able to execute with all rays, as all rays are used jointly to estimate the basis maps. To evaluate whether the ray measurements with zero counts are providing information useful to the reconstruction, cOSSCIR reconstruction
was repeated with photon-starved rays excluded from the reconstruction.

The rays with complete photon starvation are unusable by both the two-step MLE+TV$_{\text{min}}$ method and nonspectral TV$_{\text{min}}$ methods. Rays with only one nonzero energy window count measurement are also unusable by the MLE+TV$_{\text{min}}$ algorithm. These reconstruction methods require either a correction method to estimate the starved rays or for the photon-starved rays to be excluded from the reconstruction. In this study, reconstruction for the MLE+TV$_{\text{min}}$ and nonspectral TV$_{\text{min}}$ algorithms was performed by excluding photon-starved rays, while MLE+TV$_{\text{min}}$+Mask excluded all rays through metal and TV$_{\text{min}}$+NMAR estimated corrected sinogram for rays through metal.

6.2 Results

Figure 13 compares the reconstructed images for simulations with the titanium density increased by a factor of 1.5, causing 0.58% and 0.14% photon starvation in the lowest two energy windows, respectively. Each image is presented at the optimally determined constraint values, determined by multiplying the true phantom constraint values by multiplication factors of 1.0, 3.5, 1.0, 1.0, and 1.0 for the cOSSCIR, MLE+TV$_{\text{min}}$, MLE+TV$_{\text{min}}$+Mask, nonspectral TV$_{\text{min}}$, and nonspectral TV$_{\text{min}}$+NMAR algorithms, respectively.

The distribution of CT numbers in the central and peripheral iodine ROIs are presented in Figure 14, for the optimally regularized images shown in Figure 13 for the case of a 1.5 factor increase in titanium density. The ground truth value is also plotted. Table 2 presents the bias and standard deviation in the central and peripheral iodine ROIs. The cOSSCIR algorithm demonstrated −1 HU bias and 17 HU standard deviation in the central ROI, compared to −240 HU bias and 113 HU standard deviation for the nonspectral TV$_{\text{min}}$ algorithm reduced to −154 bias and 64 HU standard deviation by NMAR. The MLE+TV$_{\text{min}}$ algorithm demonstrated 110 HU bias and 1540 HU standard deviation for the central ROI when all rays were included and −12 HU bias and 43 HU standard deviation when excluding rays through metal.

Images resulting from increasing the titanium density by a factor of 4.0 are compared in Figure 15. The percentage of photon-starved rays in this simulation was 53%, 30%, 6.5%, and 0.61% starvation in the lowest to highest energy windows, respectively. The constraint multiplication factors for the images presented in Figure 15 were 1.0 for all reconstruction algorithms.

The distribution of CT numbers in the central and peripheral iodine ROIs are presented in Figure 16, for the optimally regularized images shown in Figure 15 for the case of a 4.0 factor increase in titanium density. Table 3 presents the bias and standard deviation in the central and peripheral iodine ROIs. The nonspectral TV$_{\text{min}}$ algorithm, which in this case excluded rays with zero-detected counts, resulted in −228 HU bias and 1632 HU standard deviation for the central ROI, which was reduced to −157 HU bias and 154 HU standard deviation by NMAR. The MLE+TV$_{\text{min}}$ algorithm, which excluded rays that did not count photons in the three lowest energy windows, demonstrated −273 HU bias and 166 HU standard deviation for the central ROI, compared to 1 HU bias and 16 HU standard deviation for the cOSSCIR algorithm. Excluding the rays through metal reduced the bias and standard deviation of the MLE+TV$_{\text{min}}$ algorithm to −18 and 78 HU standard deviations.
**FIGURE 14** Box plots comparing the CT numbers in the central and peripheral iodine ROIs of the images compared in Figure 13 for data with titanium density increased by a factor of 1.5. The solid horizontal line represents the ground truth value.

**TABLE 2** Bias and noise in iodine ROIs for the case of simulated polyenergetic transmission with noise and with the titanium density increased by a factor of 1.5, causing 0.58% and 0.14% photon starvation in the lowest two energy windows, respectively

<table>
<thead>
<tr>
<th></th>
<th>cOSSCIR</th>
<th>MLE+TV&lt;sub&gt;min&lt;/sub&gt;</th>
<th>MLE+TV&lt;sub&gt;min&lt;/sub&gt;+Mask</th>
<th>TV&lt;sub&gt;min&lt;/sub&gt;</th>
<th>TV&lt;sub&gt;min&lt;/sub&gt;+NMAR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Central ROI</strong></td>
<td>Bias (HU)</td>
<td>-1</td>
<td>110</td>
<td>-12</td>
<td>-240</td>
</tr>
<tr>
<td></td>
<td>St Dev (HU)</td>
<td>17</td>
<td>1540</td>
<td>43</td>
<td>113</td>
</tr>
<tr>
<td><strong>Peripheral ROI</strong></td>
<td>Bias (HU)</td>
<td>-3</td>
<td>-25</td>
<td>-17</td>
<td>-100</td>
</tr>
<tr>
<td></td>
<td>St Dev (HU)</td>
<td>19</td>
<td>30</td>
<td>35</td>
<td>77</td>
</tr>
</tbody>
</table>

Note: Negative bias signifies a lower CT number than the ground truth value. Results represent images reconstructed using the optimally determined constraint parameters for each algorithm.

Abbreviations: cOSSCIR, constrained one-step spectral CT image reconstruction; MLE, maximum likelihood estimation; NMAR, normalized metal artifact reduction; ROI, Regions of interest; St Dev, standard deviation.

**FIGURE 15** The 55-keV monoenergetic images resulting from the studied reconstruction algorithms for data with the titanium density increased by a factor of 4 causing 0.6% of rays to detect zero counts in all energy windows, with higher percentages of photon starvation in the lower energy windows. The image reconstructed by MLE+TV<sub>min</sub> excluded rays where all or all but one energy window detected zero counts, while the image reconstructed by nonspectral TV<sub>min</sub> excluded all rays with zero-detected counts, which is signified with the label "counts>0" in the figure. Images reconstructed by MLE+TV<sub>min</sub>+Mask excluded all rays through metal. Images reconstructed by nonspectral TV<sub>min</sub>+NMAR replaced the rays corrupted by metal using the NMAR technique. All images are displayed at a window of -250 to 250 HU. The metal regions are identified by the circle contours in the ground truth image.
The cOSSCIR reconstruction was repeated with all energy window measurements that detect zero photons excluded from the reconstruction, to investigate whether the photon-starved measurements are detrimental, neutral, or helpful to the cOSSCIR reconstruction. There was no negligible change in bias or standard deviation when excluding photon-starved measurements from the cOSSCIR reconstruction.

7 | DISCUSSION

The presented studies demonstrate that metal artifacts due to beam hardening, noise, and photon starvation can be avoided by acquiring spectral projection measurements and reconstructing with the cOSSCIR algorithm using an accurate physics-based forward model. Quantitative errors in the iodine ROIs in the cOSSCIR images were less than 4 HU in all studied cases, with a standard deviation less than or equal to 20 HU. In comparison, quantitative errors were as high as 157 HU with the nonspectral iterative reconstruction with NMAR, with a standard deviation as high as 154 HU.

The results demonstrate that the direct inversion into basis maps performed by cOSSCIR is more stable than the two-step process of decomposing each ray individually using an MLE algorithm followed by reconstruction. The instability of the projection-domain decomposition amplified noise in the rays through metal, leading to large noise streaks for the two-step method. Because of this instability, excluding the rays through metal improved the two-step method performance. By excluding rays through metal, the quantitative error for the MLE+TV_{min}+Mask method ranged from...
3 to 22 HU, with a standard deviation as high as 78 HU. The studied two-step method used regularization when reconstructing the basis maps from the basis sinograms, but regularization was not used when performing projection domain decomposition. The MLE decomposition method used in the two-step method in this study is known to meet the Cramer Rao lower bound on variance. However, algorithms that use regularization techniques or that take advantage of the correlations between energy windows or between the two basis sinograms during reconstruction may improve stability in the presence of metal for the two-step approach.40,56,59

A unique aspect of the cOSSCIR approach compared to other direct inversion algorithms is that convex constraints can be placed on the basis maps, which stabilizes the inversion. Figure 8 demonstrates that the cOSSCIR algorithm resulted in less noise streaks than the two-step approach even at high TV constraint values. Because cOSSCIR uses all rays that pass through a voxel to estimate the basis coefficients for that voxel, cOSSCIR is less sensitive to noisy rays through metal. Another advantage of direct inversion reconstruction methods, as demonstrated in this study, is that rays that are starved of photons in the some energy windows can still be used in the reconstruction.

cOSSCIR performed robustly as metal density and photon starvation increased. The mean value in the central ROI was 185 ± 20 HU at the low-density level compared to 185 ± 16 HU at the high-density level, where the ground truth value was 185 HU. In comparison, the MLE+TVmin+Mask algorithm resulted in mean values of 163 ± 41 HU at the low metal density and 168 ± 78 HU at the high density, demonstrating increasing noise streaks with metal density. Excluding the rays through metal did not improve or degrade the performance of cOSSCIR, which is likely because the TPL data error downweights the effect of noise, lower constraint values may be desirable to reduce noise. This trade-off between accurately reconstructing anatomical texture and reducing noise standard deviation can be controlled by changing the GTV constraint parameter.

This study demonstrated that the cOSSCIR algorithm can accurately reconstruct low-contrast texture when using an appropriate GTV constraint value. In the presence of noise, lower constraint values may be desirable to reduce noise. This trade-off between accurately reconstructing anatomical texture and reducing noise standard deviation can be controlled by changing the GTV constraint parameter.

This study addressed three major causes of metal artifacts in CT: beam hardening, noise, and photon starvation. Scatter and nonlinear partial volume effects are additional causes of metal artifacts that were not evaluated in this study but are the subject of ongoing work.60 One limitation of this study is that the metal object was a simple disk, whereas more irregularly shaped metal objects can introduce more severe artifacts.1 Another limitation is that this study only considered the 2D fan-beam geometry, although the cOSSCIR optimization strategy could be extended to three dimensions by changing the scan configuration from 2D circular fan-beam to three-dimensional (3D) circular cone-beam and switching from a 2D gradient to a 3D gradient in the expression for GTV. In this current study, the detector spectral model was assumed to be known exactly. In practice, the physical effects that occur during detection, including flux-independent degradations and pileup effects, are complex and challenging to accurately model. We have been working on improving this modeling to incorporate in cOSSCIR.29,61,82 An accurate forward model is expected to cause artifacts that would likely increase in regions near metal. The results of the current study demonstrate that cOSSCIR is effective when corrupted rays, in this case by photon starvation, are excluded. Masking rays corrupted by metal in the cOSSCIR algorithm may be a useful approach in the presence of scatter and nonideal detector effects that are difficult to model. Experimental studies are needed to evaluate cOSSCIR under these complex realistic conditions, and preliminary studies are underway.63 Because this paper demonstrates that cOSSCIR addresses metal artifacts due to beam hardening, noise, and photon starvation, future experimental studies will know that residual artifacts are due to other effects such as inaccurate detector modeling or scatter.

8 | CONCLUSIONS

The cOSSCIR algorithm was able to reconstruct the simulated pelvic phantom accurately in the presence of bilateral metal implants, with quantitative errors below 5 HU, while preserving low contrast anatomical texture. The proposed direct inversion cOSSCIR algorithm with TV constraint demonstrated improved stability and accuracy compared to a two-step method of decomposition followed by reconstruction. The results of this simulation study demonstrate the potential of photon-counting CT with cOSSCIR reconstruction to avoid metal artifacts due to beam hardening, noise, and photon starvation when using an accurate physics-based forward model.

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CONFLICTS OF INTEREST

TGS receives research funding from GE for projects unrelated to this study. XP serves as the Editor-in-Chief for the journal MEDICAL PHYSICS.
of *IEEE Transaction on Biomedical Engineering* and is a shareholder of Clarix Imaging Co. and XPIM LLC.

**APPENDIX**

The reason why the ADMM framework is used is that the potentials $f$ and $g$ appear in a symmetric way, and the linearization of $f_d$ and $g_d$ is natural. In the CPPD algorithm, the function $g$ is dualized in the CPPD updates, and, consequently, the linearization of $g_d$ is more convoluted.

The ADMM algorithm can be related exactly to the CPPD algorithm, which is important for translating experience in developing efficient CPPD instances, for imaging problems, to creating an efficient ADMM instance for the present spectral CT problem. The generic optimization for CPPD is

$$\min_x f(x) + g(Ax), \quad \text{(A1)}$$

and the CPPD update steps are

$$x_{t+1} = \arg \min_x \left\{ f(x) + \frac{1}{2} \| x_t - TA^\top \lambda_t - x \|^2_{T^{-1}} \right\}, \quad \text{(A2)}$$

$$\lambda_{t+1} = \arg \min_{\lambda} \left\{ g^*(\lambda) + \frac{1}{2} \| \lambda_t + \Sigma A \tilde{x} - \lambda \|^2_{\Sigma^{-1}} \right\}, \quad \text{(A4)}$$

where $g^*$ is the Legendre transform (or convex conjugate) of $g$, and $\lambda$ is the dual variable. The following steps relate ADMM and CPPD: (1) The matrix $B$ from the ADMM updates is set to the negative identity, $B = -I$, and the vector $c$ is set to zero, $c = 0$; (2) the order of the $y$-update and $u$-update in Equations (11) and (12), respectively, is interchanged; (3) the dual variable from CPPD is related to the ADMM variables by

$$\lambda_t = u_t + \Sigma (Ax_t - y_t); \quad \text{(A5)}$$

(4) the norm metrics are assigned

$$H_g = 0,$$

$$H_f = T^{-1} - A^\top \Sigma A, \quad \text{(A6)}$$

where $\Sigma$ and $T$ are the CPPD step size matrices. Note that the condition that $H_f$ is PSD imposes a constraint on the $\Sigma$ and $T$ matrices.

**REFERENCES**


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