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Engineering parafermions in helical Luttinger liquids

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ABSTRACT

Parafermions or Fibonacci anyons leading to universal quantum computing, require strongly interacting systems. A leading contender is the fractional quantum Hall effect, where helical channels can arise from counter-propagating chiral modes. These modes have been considered weakly interacting. However, experiments on transport in helical channels in the fractional quantum Hall effect at a 2/3 filling shows current passing through helical channels on the boundary between polarized and unpolarized quantum Hall liquids nine-fold smaller than expected. This current can increase three-fold when nuclei near the boundary are spin polarized. We develop a microscopic theory of strongly interacting helical states and show that emerging helical Luttinger liquid manifests itself as unequally populated charge, spin and neutral modes in polarized and unpolarized fractional quantum Hall liquids. We show that at strong coupling counter-propagating modes of opposite spin polarization emerge at the sample edges, providing a viable path for generating proximity topological superconductivity and parafermions. Current, calculated in strongly interacting picture is in agreement with the experimental data.

Keywords: Parafermions, topological superconductivity, fractional quantum Hall effect, Luttinger Liquid, Helical states, Spin current, Tunneling, Edge states

1. INTRODUCTION

Quest for discovery of ways to harness the full power of quantum physics for developing information technology is the frontier of present research. Traditional schemes for quantum computing are about to achieve hundreds of qubits and racing towards a milestone of a thousand qubits.[1] Successful error correction schemes have significantly improved the error rate, and promise further reduction in errors.[2] However, to be able to achieve millions of qubits for real quantum computation, further drastic improvement in error correction schemes is needed, and it is far from clear that limits on improving coherence of physical qubits would not emerge or error correction schemes would not require prohibitive amount of additional physical qubits, leading to a stumbling block on the pathway to full-scale quantum computing.

For this reasons, physicists continue to search for alternative paths towards quantum computing that would present fundamentally new ways to avoid decoherence, particularly topological quantum computing. The principal idea is a possible insensitivity of topological qubits to local fields and related decoherence. Currently, the main body of efforts in this direction is concentrated on quantum computing with Majorana fermions.[3,4] However, other excitations with non-Abelian quantum statistics[5–7] present considerable interest for development of topological quantum bits. In particular, parafermions are free from electron poisoning,[8] and allow fuller set of single-qubit operations compared to Majorana fermions.[9] Two-dimensional array of parafermions provides a path to achieve Fibonacci anyons, which permit universal quantum computation.

Parafermion non-Abelian excitations were discussed by Fendley using simple statistical physics Ising-like model. It was shown by Clarke, Alicea and Stengel[10] that commutation relations characterising parafermion excitations coincide with commutation relations for $1/m$ fractional quantum Hall effect edge states. Thus, fractional quantum Hall effect systems can serve as a breeding ground for parafermion excitations. The required

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systems must exhibit counter-propagating fractional quantum Hall edge states with opposite spin polarization coupled to an s-type superconductor. However, practical realizations suggested in[10,11] are based on systems which are currently not feasible, e.g. adjacent 2D systems with g-factors of opposite signs. In,[12,13] we proposed the path to parafermions using systems with fractional quantum Hall effect spin transitions.

Spin transition in the integer and fractional quantum Hall effects originate from competition of orbital and spin degrees of freedom of electrons in quantized magnetic fields and were observed first by varying magnetic field within the range of quantum Hall plateaus, particularly using tilted magnetic fields, that affect only Zeemann energy. As the electron density is not uniform, changing Zeemann energy leads to domains, in which electrons in the topmost filled spin resolved Landau level in the case of the integer quantum Hall effect (or composite fermion Λ-level in the case of the fractional quantum Hall effect) have a different spin polarization S_2 from the rest of the sample with polarization S_1. When the size of S_2 domains and their number increase, their boundaries form continuous network connecting contacts to the sample. The boundaries of states with different spin polarization are the domain walls. Edge states flow at the domain walls. Their continuous network allow percolation of charge between contacts via these edge states. This leads to electric current and to the disappearance of zero-resistance state of the quantum Hall plateau. In magnetic fields corresponding to such currents, the numbers of domains with spin polarization S_1 and S_2 are roughly equal. Further change in the number of S_2-domains is going to destroy the percolation path and lead to the re-entrance of the quantized Hall plateau, which will be characterized by a spin state S_2 of the prevailing phase surrounding isolated S_1-domains. With further increase in magnetic field, the quantum Hall state will ultimately have uniform polarization S_2.

Spin transitions in the integer and fractional quantum Hall effects can also be induced by gate voltage. In CdMnTe quantum wells, Zeemann splitting of Landau levels is strongly affected by spin polarization of Mn ions. For asymmetric Mn doping of quantum wells, electron wavefunction in z- direction has stronger overlap with Mn ions for one of the signs of the applied gate voltage, leading to the controlled Zeeman splitting of electrons and therefore electrostatically controlled spin transitions in the integer quantum Hall effect.[14–16]

The spin transitions in the fractional quantum Hall effect have been observed at the filling factor $\nu = \frac{2}{3}$ and other fractions.[17,18] These spin transitions can be understood by using the composite fermion (CF) picture.[?] The energy of the n-th composite fermion Λ-level is given by

$$E_{ns} = \hbar\omega^{cf}(n + \frac{1}{2}) + sg\mu_B B, \tag{1}$$

where μ_B is the Bohr magneton, B is the magnitude of magnetic field, the index $s = \pm 1$ describes the up and down spin states of the composite fermions. The orbital quantization energy $\hbar\omega^{cf}$ is proportional to the characteristic electron-electron interaction energy scale $\frac{e^2}{\lambda}$, $\lambda = \sqrt{\hbar c e B_\perp}$ is the magnetic length, and $H_\perp$ is the out of plane component of the magnetic field $\mathbf{B}$. As a result of different magnetic field dependencies of the orbital quantization and Zeeman energies, the levels $\Lambda_{n,\downarrow}$ and $\Lambda_{n+1,\uparrow}$ have to cross at some B^* denoted by a black circle in Fig.1a. In this figure, describing the filling factor $\nu = 2/3$ fractional quantum Hall effect, composite fermions fill either the ground $\Lambda_{0,-1}$ level and the same orbital level $\Lambda_{0,1}$, resulting in un-polarized $\nu = 2/3$ quantum Hall state at $B < B^*$ (opposite spin polarizations of the filled levels), or the ground $\Lambda_{0,-1}$ level and the next orbital level $\Lambda_{1,-1}$ resulting in polarized $\nu = 2/3$ fractional quantum Hall state at $B > B^*$.

Furthermore, it has been shown that electrostatic gates control electron-electron interactions, so that for the two-dimensional electron gas confined to a triangular quantum well, the orbital quantization energy

$$\hbar\omega^{cf} \propto \frac{e^2}{\sqrt{\lambda^2 + z_0^2}}, \tag{2}$$

where z_0 is the extent of the electron wavefunction in the third spatial dimension. Thus, spin-polarized and spin-unpolarized phases can be induced via gate voltage control of the orbital quantization energy. Experimentally transitions can be achieved by both tuning the effective Coulomb interaction and/or by tuning the Zeemann coupling via the in-plane component of the magnetic field.[12] Furthermore, electrostatically controlled single domain wall can be induced on the boundary of two regions with different spin state underneath two electrostatic gates with different gate voltage.

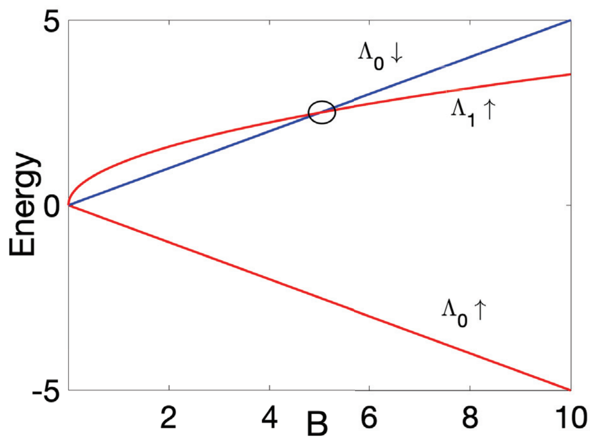

Figure 1. The schematic plot of the composite fermion Λ levels. When the in-plane magnetic field affecting Zeemann splitting of electrons as well as composite fermions, is increased, while the orbital out-of plane component of magnetic field keeps the system on the fractional quantun Hall plateau, there is a level crossing (black circle) of the $\Lambda_{0\downarrow}$ and $\Lambda_{1\uparrow}$ defining B^*, which leads to a spin transition from spin-unpolarized state to spin-polarized state.

Due to chirality of electron trajectories in magnetic field, edge states in domain walls between neighboring domains are counter-propagating. Furthermore, the topmost filled levels in these domains are characterized by opposite spin polarization. Edge states originating from these levels have opposite spin and counter-propagate at the boundary of two domains underneath the two gates, potentially leading to parafermion setting. However, e.g., for spin transition in the $\nu = 2/3$ fractional quantum Hall effect the structure of edge states is more complicated. Nevertheless, in,[19] by using effective theory,[20] it was shown that in long domain walls, there are two counter-propagating edge states with opposite spin polarization indeed. Numerical simulations on disk and torus[19] also confirmed this picture. What these considerations did not take into account, however, is coupling of the true edge states at the boundary of the sample to states propagating along the domain wall: an isolated domain wall was studied in effective theory, and both disc and torus configurations do not possess contacts of the true edge states and states propagating along the domain wall. As we shall see, analysis of the role of such contacts is required for understanding of experimental observations on the resistivity of the electrostatically induced domain wall.

In this paper we will first present experimental results on transport through the domain wall between polarized and unpolarized fractional quantum Hall states. Some of these results have been presented in.[12,13] We will then describe theoretical model for tunneling of helical Luttinger liquid modes through the domain wall proposed in,[13] explaining these experimental results. In particular, we will discuss role of the nuclear spin subsystem on transport through the domain wall.

2. EXPERIMENT

Hall bar geometry devices with multiple gates have been fabricated to study transport through helical domain walls, Fig. 2a (for heterostructure and fabrication details see[13]). Devices were separated into two regions $G1$ and $G2$; electron density n in these regions was controlled independently by electrostatic gates. Changing density within the $\nu = 2/3$ plateau in our devices makes it possible to control magnetic field B^* characterizing the spin transition by electrostatic gating, and a small peak within the $2/3$ plateau in Fig. 2b corresponds to the fractional quantum Hall spin transition between polarized (p) and unpolarized (u) regions.

Controlling of filling factors ν_1 and ν_2 under gates G1 and G2 independently divides the $2/3$ region into four quadrants uu, pp, up and pu, where the first letter corresponds to polarization of the state under G1 and the second corresponds to polarization under G2.

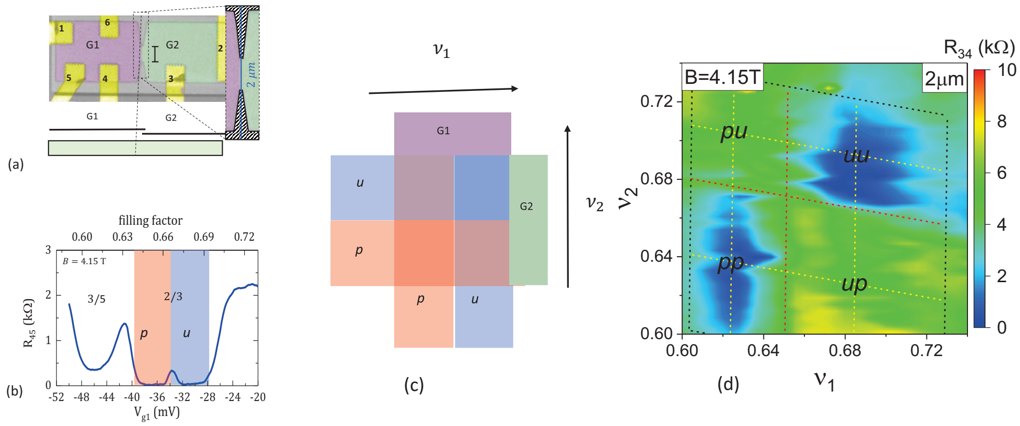

Figure 2. Formation of helical domain walls at $\nu = 2/3$. (a) A false color image of a typical device. Yellow regions show Ohmic contacts, 2D gas in green and magenta regions is controlled by gates G1 and G2. In the enlarged fragment black lines outline mesa boundary and a blue line marks a physical boundary between gates G1 and G2. (b) Magnetoresistance $R_{45} = (V_5 - V_4)/I$ of a 2D gas as a function of gate voltage (that controls filling factor ν) at a fixed $B = 4.15$ T. Small peak corresponds to the phase transition between unpolarized (u) and polarized (p) $\nu = 2/3$ fractional quantum Hall liquids. (c) A diagram of u and p phases as a function of ν_1 and ν_2 under gates $G1$ and $G2$; (d) Resistance $R_{34} = (V_4 - V_3)/I$ across a 2 μm - long gates boundary is plotted as a function of $\langle\nu\rangle = (\nu_1 + \nu_2)/2$ and $\Delta\nu = (\nu_1 - \nu_2)$. Black square outlines the $\nu = 2/3$ region, red dotted lines mark u-p transitions and yellow dotted lines cross at centers of u and p regions.

Experimentally measured R_{34} is vanishingly small in uu and pp quadrants, corresponding to unpolarized or polarized state extending over the whole device. Sizable R_{34} in up and pu quadrants indicates formation of a conducting channel along the gates boundary. Resistance measured across the boundary of u and p quantum liquids at $\nu = 2/3$ shows practically no dependence on the external magnetic field direction and the gradient of electron density, suggesting the formation of a helical domain wall.[12]

The fraction of the external current I that flows through the hDW, $i_{DW} = I_{DW}/I = 0.11$ for the $2\mu m$ domain wall. However, in a model of $\nu = 2/3$ edge states with equally populated $\nu = 1/3$ chiral channels with no scattering between them, which is clearly the case if they have opposite spin polarizations, $i_{DW} = 1$, an order of magnitude larger than the value observed experimentally.

In GaAs heterostructures, the spin transition at $\nu = 2/3$ is coexists with a dynamic nuclear spin polarization[?] due to the hyperfine coupling of electron and nuclear spins. On the quantum Hall effect plateau, the electron-nuclear spin flips are usually suppressed due to a large difference between electron and nuclear Zeemann splitting, similarly to nuclear spin relaxation in quantum dots.[21]

Near the u-p spin transition, however, spin-up and spin-down electronic states are almost degenerate, enabling flip-flop processes due to hyperfine coupling. This can lead to polarization of nuclear spins via Overhauser effect in non-equilibrium conditions, e.g. in the presence of electric current. Such pumping of nuclear spins results in effective magnetic field, contributing to Zeemann splitting of electron or composite fermions.

Polarization of nuclear spins near the domain wall after a large dc current is injected weakens possible hyperfine-assisted tunneling between channels with opposite spin polarization propagating in the domain wall region. Experiment shows almost a threefold increase in current diverted by the domain wall when nuclear spins are polarized, compared to the domain wall current in the absence of nuclear pumping.

3. THEORY

To explain significant difference of experimentally measured current diverted from the sample edge and passing through the domain wall, in[?] we calculate scattering of edge modes at a sample boundary by a helical domain wall.

Chiral edge states at a filling factor $\nu = 2/3$ are two filled Λ-levels of composite fermions with equal or opposite spin in the p and u phases, with edge modes described by densities $\Phi_{p1\uparrow}$, $\Phi_{p2\uparrow}$ and $\Phi_{u1\uparrow}$, $\Phi_{u2\downarrow}$ correspondingly. As we demonstrated in,[13] applying unitary transformations, the chiral edge can be described in terms of the separated charged and neutral modes $\varphi_{pc} = (\Phi_{p1\uparrow} + \Phi_{p2\uparrow})/3\sqrt{2}$ and $\varphi_{pn} = (\Phi_{p1\uparrow} - \Phi_{p2\uparrow})/\sqrt{2}$ for the p phase, and separated charged and spin modes $\varphi_{uc} = (\Phi_{u1\uparrow} + \Phi_{u2\downarrow})/3\sqrt{2}$ and $\varphi_{us} = (\Phi_{u1\uparrow} - \Phi_{u2\downarrow})/\sqrt{2}$ for the u phase, equivalent to the composite fermion description.

The Luttinger liquid action for $\nu = 2/3$ edge states, e.g., for the p phase reads

$$S = \frac{1}{4\pi} \int dt \int dx \left[-3\partial_x \varphi_c (\partial_t + v_c \partial_x) \varphi_c + \partial_x \varphi_n (\partial_t + v_n \partial_x) \varphi_n \right], \tag{3}$$

where indices c and n describe charge and neutral modes. The Luttinger liquid action in u phase is similar with spin modes entering instead of neural modes. The spin mode determines the spin current in the u phase.

These actions are the same as the Kane, Fisher and Polchinsky[22] Luttinger liquid action expressed in terms of the charge and neutral fields, see also Refs.,[20,23] in the absence of scattering between composite fermion modes. In the u phase such scattering is not allowed in the absence of spin-flip processes, and the u phase exhibits spin-charge separation with pure spin and charge modes moving in opposite directions.[24] Treating both phases that exhibit quantization of Hall resistance $3/2\ h/e^2$ on equal footing, we assume that scattering does not occur between different modes in the p phase. In both phases quantization of the Hall resistance and change and neutral (spin) mode separation is a consequence of the long-range Coulomb interaction.[20]

We generalized the solution for tunneling[25] of the fractional QHE modes through the point contact to consideration of tunneling through finite length domain wall between p and u phases. Point contact tunneling carried by electrons with the same spin can be described by Hamiltonian

$$\begin{aligned} \mathcal{H}_T = -\tilde{t} \cos\left(\Phi_{p1\uparrow} - \Phi_{p2\uparrow}\right) = \\ -\tilde{t} \cos\left(\tfrac{1}{\sqrt{2}} \left[3\varphi_{pc} - \varphi_{pn} - 3\varphi_{uc} + \varphi_{us} \right] \right). \end{aligned} \tag{4}$$

We demonstrated[13] that imposing strong coupling boundary conditions $\tilde{t} \to \infty$ results in the same currents flowing outside the domain wall in the models of single-junction connecting edges on opposite sample boundaries, two junctions on opposite edges, two junctions with scattering between same spin modes in between, and, finally, the domain wall of finite length. Inside the domain wall, the chiral evolution of modes is controlled by the average voltage shifts at their corresponding boundaries. In the presence of voltage V, the only incoming mode changing due to charge injection in the p phase is $\varphi_{pc}(x_1)$, where x_1 is the junction coupling edge modes in the polarized phase, edge modes in the unpolarized phase and helical modes in the domain wall. The charge mode $\varphi_{pc}(x_1)$ is characterized by an average induced current $\bar{j} = \frac{e^2 V}{3\pi\hbar}$.

When spin-flip processes are absent, it is convenient to discuss the results in terms of currents carried by Φ_{p1} (Φ_{u1}) and quasiparticle $\chi_{p2} = (\varphi_{pc} - \varphi_{pn})/\sqrt{2}$ and $\chi_{u2} = (\varphi_{pc} - \varphi_{ps})/\sqrt{2}$ modes. We have demonstrated that Φ_{p1} (Φ_{u1}) modes propagate along the edges of the 2D gas and do not enter the domain wall, while χ_{p2} and χ_{u2} modes flow along the boundaries of p or u phases, including inside the domain wall but do not tunnel. Notable features of our solution are unequal distribution of carried currents between the modes caused by strong coupling to the domain wall and the presence of spin current along the edge of the u phase for $x > x_2$, where junction x_2, similarly to x_1 couples edge modes in the polarized phase, edge modes in the unpolarized phase and helical modes in the domain wall on the edge of the sample opposite to the edge containing x_1. In Fig.3 we present the results for spin and charge currents in the unpolarized phase, and charge and neutral currents in polarized phase. The total current flowing along the hDW $I_{DW} = 1/4(I + I_{DW})$ or $i_{DW} = 1/3$.

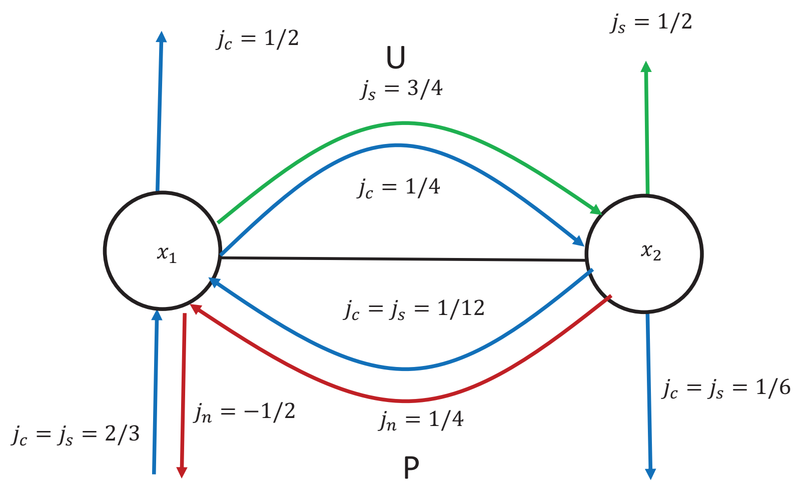

Figure 3. The current flow for current $j_c = 2/3$ injected from the polarized into the unpolarized region. Values of currents are shown in units of $\sigma_0 V$, $\sigma_0 = e^2/h$. Charge currents in the polarized and unpolarized regions are shown in blue. In the polarized region, the charge current and the spin current coincide. The neutral current in polarized region, describing the difference of currents carried by two quasiparticle modes, is shown in red. Spin currents in the unpolarized region, corresponding to the difference of currents carried by spin up and spin down modes in there, are shown in green.

It is easy to see that at each junction the charge current is conserved and the spin currents in the unpolarized liquid and the neutral currents in the polarized liquid are conserved as well. Also, spin projection of incoming electrons is equal to spin projection of outgoing electrons at junctions x_1 and x_2. These results hold for general case of the domain wall of finite length.

The case of incoming charge current from the unpolarized liquid flowing into polarized liquid, is described similarly. In both cases, for the current injected from the p phase into the u phase, and for the current injected from the u phase into the p phase, the domain wall in the absence of tunneling between modes with different spin polarization, is characterized by $i_{DW} = 1/3$, i.e. the current flowing between opposite edges of the sample is $1/3$ of the incoming current.

Thus far we assumed that tunneling conserves spin. Experiment, however, clearly demonstrates substantial role of spin-flip processes. In the vicinity of the domain wall, it is possible to match small nuclear spin splitting and electron energy splitting near the crossing of composite fermion levels.

We have modeled partial spin flip by assuming that nuclear spin flips may result in admixture of polarized and unpolarized propagating density modes inside the domain wall. For zero admixture, all results above hold. Allowing all spin-flip processes in electron tunneling and scattering, leads to the absence of current through the domain wall and all current is flowing along sample edges. Assuming probability of admixture $3/4$, allows to explain the observed value i_{dw} by our model.

In the presence of spin flips, Φ_{p1} (Φ_{u1}) modes, as in the absence of spin flips, propagate along the edges of the 2D gas and do not enter the domain wall, Fig.4, but χ_{p2} and χ_{u2} quasiparticle modes, in contrast to no spin flip case, tunnel between polarized and unpolarized regions through the domain wall, as shown by color-changing arrows.

It is important to note that even in the presence of spin flips, counter-propagating modes in unpolarized region near x_2 are characterized by spin current and constitute helical modes.

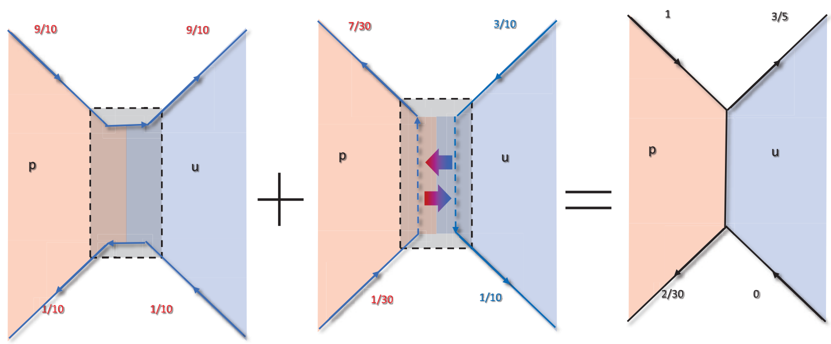

Figure 4. Visualization of currents due propagation of $\Phi_p 1$ and $\Phi_u 1$ charge modes and $\chi_p 2$ and $\chi_p 2$ quasiparticle modes in the presence of spin flips with the probability $r = 3/4$. Red (blue) mode color indicates a spin-up (spin-down) polarization. Numbers indicate the fraction of the incoming applied voltage carried by the mode. Arrows correspond to directions of currents carried by corresponding modes.

4. CONCLUSION

We presented the results of our studies of transport properties of helical states in the $\nu = 2/3$ fractional Quantum Hall regime. Experimentally we found that current carried by domain walls is substantially smaller than the prediction of the naïve non-intercting model. Luttinger liquid theory of the system reveals redistribution of currents between quasiparticle charge, spin and neutral modes, and predicts the reduction of the domain wall current compared to non-interacting model. Furthermore, inclusion of spin-flip-induced tunneling processes reconciles theory with experiment. The theory confirms emergence of spin modes, which, when coupled to an s-type superconductor, can result in the fractional topological superconductivity.

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