

Sparse System Identification of Leptin Dynamics in Women with Obesity

Md. Raful Amin ^{1,*}, Divesh Deepak Pednekar ¹, Hamid Fekri Azgomi ¹, Herman van Wietmarschen ², Kirstin Aschbacher ³, and Rose T. Faghah ^{1,*}

¹ Department of Electrical and Computer Engineering, University of Houston, USA

² Department of Nutrition and Health, Louis Bolk Instituut, Netherlands

³ Department of Psychiatry, Weill Institute for Neurosciences, University of California, San Francisco, USA

Correspondence*:

Rose T. Faghah
rtfaghah@uh.edu

2 ABSTRACT

3 The prevalence of obesity is increasing around the world at an alarming rate. The interplay of the
4 hormone leptin with the hypothalamus-pituitary-adrenal axis plays an important role in regulating
5 energy balance, thereby contributing to obesity. This study presents a mathematical model,
6 which describes hormonal behavior leading to an energy abnormal equilibrium that contributes to
7 obesity. To this end, we analyze the behavior of two neuroendocrine hormones, leptin and cortisol,
8 in a cohort of women with obesity, with simplified minimal state-space modeling. Using a system
9 theoretic approach, coordinate descent method, and sparse recovery, we deconvolved the serum
10 leptin-cortisol levels. Accordingly, we estimate the secretion patterns, timings, amplitudes, number
11 of underlying pulses, infusion, and clearance rates of hormones in eighteen premenopausal
12 women with obesity. Our results show that minimal state-space model was able to successfully
13 capture the leptin and cortisol sparse dynamics with the multiple correlation coefficients greater
14 than 0.83 and 0.87, respectively. Furthermore, the Granger causality test demonstrated a negative
15 prospective predictive relationship between leptin and cortisol, 14 of 18 women. These results
16 indicate that increases in cortisol are prospectively associated with reductions in leptin and
17 vice versa, suggesting a bidirectional negative inhibitory relationship. As dysregulation of leptin
18 may result in a abnormality in satiety and thereby associated to obesity, the investigation of
19 leptin-cortisol sparse dynamics may offer a better diagnostic methodology to improve better
20 treatments plans for individuals with obesity.

21 **Keywords:** leptin, cortisol, endocrinology, state-space, sparse recovery, system identification, deconvolution

1 INTRODUCTION

22 Obesity, dubbed the "Global Epidemic" by the World Health Organization, is said to cause or aggravate
23 various other health problems, worsening one's life expectancy Ng et al. (2014); Wang et al. (2020). The
24 prevalence of obesity is increasing worldwide at an alarming rate. The estimate shows that about 38% of
25 the adult American population suffer from obesity Liou and Kulik (2020). Obesity is associated with a
26 reduced life expectancy Blüher (2020). It also increases the risk of other chronic disorders such as diabetes
27 and cardiovascular disease Jastreboff et al. (2019). Available studies suggest different lifestyle interventions

28 for treating obesity such as a low-calorie diet, increased physical activity, and bariatric surgery Cardel
 29 et al. (2020). Nevertheless, many studies have pointed the determinants of failure of such approaches
 30 including adherence to lifestyle interventions Burgess et al. (2017), genetic background, adaptive changes
 31 in basal metabolic rate, hunger and satiety hormones that occur with weight loss Reinehr (2013). While
 32 there exists pathogenic approaches to treat the condition through studying its development, a salutogenic
 33 model focused on holistic wellbeing might be more effective as both a preventive and remedial measure
 34 Organization (2000); Seo et al. (2019). For example, obesity can be conceptualized as a deficiency of
 35 energy regulation - i.e., the brain may fail to respond to negative feedback hormonal signals from adipose
 36 tissue, thereby perpetuating non-homeostatic eating behaviors that drive obesity. Thus, identifying the
 37 role of neuroendocrine hormones and adipokines in facilitating brain-adipose communication and energy
 38 regulation may inform novel strategies for the treatment and prevention of obesity. In this research, we
 39 propose models to understand leptin and cortisol behavior, which, in the future, can further be generalized
 40 for other relevant hormones, such as growth hormone, insulin, and ghrelin. Leptin is a signaling hormone
 41 and adipokine that is essential to activating central nervous system (CNS) networks involved in the
 42 suppression of appetite. It regulates food intake, metabolism, energy expenditure, and body weight
 43 Mantzoros et al. (2011). Although yet to be proven, Jacquier *et al.* Jacquier et al. (2015) mathematically
 44 demonstrated how leptin resistance can be a result of persistent leptin infusion. They further pointed out
 45 that temporal alteration in some parameter values related to food intake can shift the equilibrium. The
 46 shift in equilibrium can lead to leptin resistance and obesity state. Blood leptin levels also correlate with
 47 changes in fat mass Ahima and Flier (2000). Leptin is a hormone produced and secreted primarily by
 48 adipocytes. Hence, excessive amounts of adipose tissue in the body correspond to an increase in leptin
 49 production rate, and thus, higher serum leptin levels Mantzoros et al. (2011). Leptin interacts with the
 50 hypothalamus-pituitary-adrenal-axis (HPA-axis), thereby constituting an important biological mechanism
 51 whereby experiences of psychological stress may influence or be influenced by body weight and energy
 52 regulation. Buckley and Schatzberg (2005); Adam and Epel (2007)

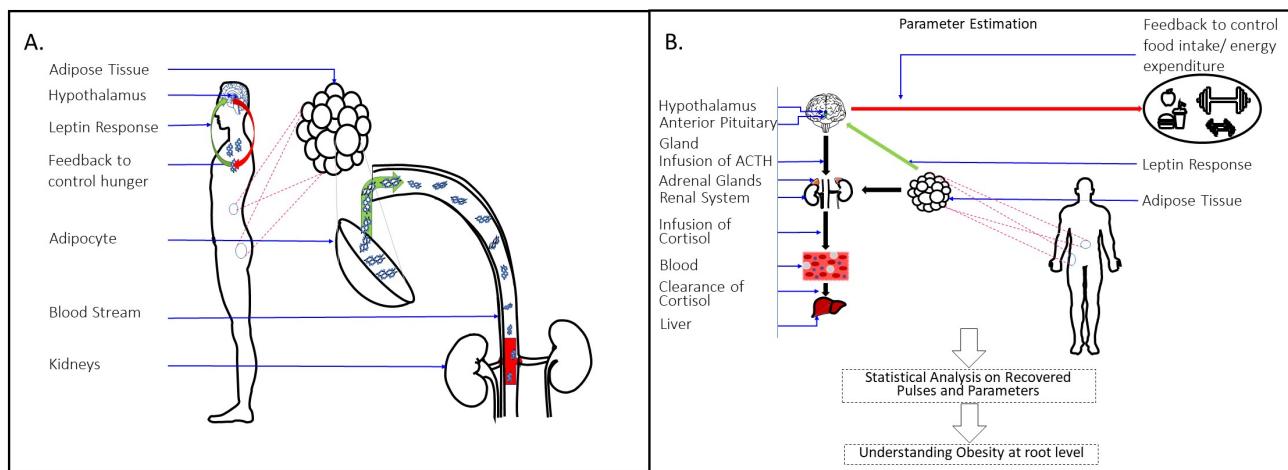


Figure 1. Leptin Regulation Model and an Overview of System-Theoretic Approach. (A) shows the leptin secretion & regulation model. Leptin is the hormone to signal the hypothalamus regarding energy storage and use. It is produced and secreted primarily by adipocytes. The renal system is responsible for clearing it. (B) shows the overall approach used in this study. A state-space model with physiological constraints is designed. A deconvolution approach is then used to estimate the secretion events and the physiological parameters. A statistical analysis is done on the estimations to understand the relationship of leptin-cortisol rhythms and dynamics in detail.

53 Another frequently studied hormone in obesity research is the glucocorticoid hormone cortisol. Björntorp
54 and Rosmond (2000) In response to different physiological demands such as blood glucose regulation,
55 inflammation suppression, physiological stress, significant HPA-axis responses are induced across all ages
56 and genders Kudielka et al. (2004). Björntorp et al. Björntorp and Rosmond (2000) compared several
57 studies and concluded that the association between obesity and cortisol is complicated. They concluded that
58 there are several factors such regulatory systems, including the HPA, gonadal, growth-hormone, leptin axes,
59 the sympathetic nervous system, the central adrenergic, serotonergic, and dopaminergic systems that are
60 related to obesity. Despite cortisol's short half-life and circadian secretion pattern, most studies use cortisol
61 measures such as serum, urine or salivary cortisol that do not reflect long-term cortisol exposure Rodriguez
62 et al. (2015). Björntorp et al. Björntorp and Rosmond (2000) suggest that it is necessary to examine the
63 cortisol secretion process to further understand the roots of obesity.

64 In an attempt to study obesity, various models based on ordinary differential equations considering
65 regulations in energy and metabolism de Graaf et al. (2009); Chow and Hall (2014); Horgan (2011);
66 Hall (2009); Aschbacher et al. (2014), as well as on the effects of different hormones such as ghrelin,
67 cholecystokinin, and leptin have been hypothesized. Traditional studies perform statistical analysis on
68 the measured serum hormone levels Kanaley et al. (2001); Antunes et al. (2008). Some models include
69 leptin, leptin resistance, and leptin receptors to understand obesity Tam et al. (2009); Jacquier et al. (2015);
70 De Gaetano et al. (2008). Other studies show the association of glucocorticoids and weight gain Udden et al.
71 (2003); Sominsky and Spencer (2014). Since obesity is associated with the polypeptide hormone, leptin
72 Buckley and Schatzberg (2005); Mantzoros et al. (2011); Jacquier et al. (2015), and the glucocorticoid
73 hormone, cortisol Myers Jr et al. (2010); Björntorp and Rosmond (2000), studying them will prove vital in
74 designing an approach to treating the condition. For example, we cannot measure leptin resistance in a living
75 human, nevertheless, investigation of the association between cortisol and leptin through mathematical
76 modeling may lead to some insights into the presence of leptin resistance.

77 Aschbacher et al. Aschbacher et al. (2014) propose a mathematical model based on the systems-
78 level understanding of the HPA-leptin axis. Incorporating leptin in the model helped them to obtain
79 three parameters: (I) inhibitory cortisol feedback signal, (II) Adrenocorticotropic hormone (ACTH)-
80 adrenal interaction, and (III) leptin-cortisol proportionality. The authors suggested that the leptin-cortisol
81 relationship may be important for understanding the neuroendocrine starvation response, and that poor
82 system control may ultimately contribute to obesity. The mathematical model presented in the research by
83 Aschbacher et al. Aschbacher et al. (2014) explains the rate of changes in cortisol concentration, ACTH,
84 and leptin dynamics. The model used in this paper is an extension of the HPA system dynamics model used
85 previously in Aschbacher et al. (2012) incorporating leptin's impact on cortisol. Though leptin regulation
86 is dependent on its relation with different hormones, understanding its behavior requires the study of leptin
87 secretion and regulation. Investigating cortisol and leptin separately, and based on their physiology, will
88 be a vital approach in interpreting how they behave concerning each other and other systems of the body.
89 Moreover, cortisol and leptin each have circadian and ultradian rhythms, which our approach may better
90 characterize. Jacquier et al. Jacquier et al. (2015) introduce a mathematical model to study leptin resistance
91 based on leptin synthesis and receptor activity. They study the changes in plasma leptin concentration, the
92 density of leptin receptors, and food intake by varying leptin infusion and food consumption. This model
93 studies leptin resistance dynamics based on many parameters such as food intake, fat mass, leptin receptors,
94 and leptin. However, it is challenging to simultaneously collect this kind of clinical data. Therefore, we
95 propose a simplified model considering only plasma leptin levels as the observation. As leptin secretion is
96 a pulsatile process, we aim to exploit this characteristic by inferring underlying impulses.

97 In this research, we propose a simplified system to understand leptin regulation by exploiting the sparse
98 nature of leptin secretion. If plasma leptin levels are sampled at a 10-minute sampling rate over 24-hours,
99 we obtain a maximum of 40 leptin secretion events out of a total of 1440 secretion events, therefore, the
100 nature being sparse. The sparse system identification of leptin dynamics over time makes it possible to
101 recover the number of leptin pulses. It further allows us to recover information about the pulse amplitudes
102 and intervals of occurrence from collected data. For an extensive understanding of leptin secretion, it is
103 necessary to unveil the pulses originating from the hypothalamus as an outcome to leptin signaling. The
104 pulses resulting in leptin regulation and secretion are the outcome of several contributing factors such
105 as effects of other hormones on leptin regulation, density of leptin receptors, and food intake. Similar to
106 leptin, cortisol secretion is a sparse process. Sparse system identification of cortisol will also lead us to
107 recover the abstraction of secretion events coming from the HPA-axis. It also provides us important insight
108 about the way it is infused by the adrenal glands and cleared by the liver. The proposed architecture has
109 been previously verified for cortisol in healthy subjects and also on patients suffering from chronic fatigue
110 and fibromyalgia syndromes Pednekar et al. (2020); Faghah et al. (2014); Pednekar et al. (2019). Here,
111 we propose to study leptin-cortisol behavior by first observing the underlying pulses and estimating the
112 physiological infusion and clearance rates in the system using two separate regulatory minimal models, one
113 each for cortisol and leptin, and then determining the relationship between them. Similar to the results of
114 previous study by Aschbacher et al. Aschbacher et al. (2014), we observe a negative relationship between
115 cortisol and leptin. This relationship is observed based on a negative correlation. To further strengthen this
116 observation, we perform the Granger causality test.

117 Figure 1-A shows a pictorial representation of the leptin regulation and secretion model. Figure 1-B
118 shows a pictorial over all of the approach used in this research. This approach yields three important
119 parameters: the timing and amplitudes of the leptin secretion events, the infusion rate of leptin by the
120 adipose tissue, and the clearance rate by the renal system. We further conduct statistical analysis on the
121 measured and estimated hormone levels. Finally, we propose that this method can be utilized to identify
122 difference between patient and matched control participants for future characterization related to leptin
123 deregulation similar to our previous approach Pednekar et al. (2020).

2 RESULTS

124 We utilize hormone assay from eighteen premenopausal women with mean body mass index (BMI) 33
125 with range 30–41 kgm^{-2} ; mean age 37.5 with range 22–51 years; at morning (at 9 AM) mean cortisol
126 level 11.02 $\mu g/dl$ and the corresponding standard deviation 3.65 $\mu g/dl$. Th distribution of average leptin
127 level for all subject is $3.3583 \pm 1.0633 \mu g/dl$. The corresponding fat distribution is $39.61 \pm 3.08 kg$ and
128 lean body mass distribution is $55.094 \pm 6.55 kg$. Total cholesterol $4.7 \pm 0.2 mmol/l$ (range 3.7–5.8 mmol/l),
129 LDL cholesterol $2.99 \pm 1.57 mmol/l$ (range 2.03–4.00 mmol/l), and HDL cholesterol $1.54 \pm 0.08 mmol/l$
130 (range 1.03–2.32 mmol/l). Homeostasis model assessment for insulin resistance (HOMA-IR) was estimated
131 to be 3.25 ± 0.48 with placebo and 2.32 ± 0.19 after treating with Bromocriptine Kok et al. (2006). The
132 statistical test between two distribution showed significant difference with $p = 0.01$.

133 2.1 Pulsatile Nature of Leptin:

134 Figure 2 shows the example deconvolution results of measured and reconstructed blood leptin levels
135 of women with obesity for both experimental and simulated data for one subject. Figure 2-A exhibits
136 the estimated amplitudes and timings of hormonal secretory events. It also presents the experimental
137 leptin data and estimates predicted by the model. The amplitude variations of the pulses are due to the

138 circadian rhythm of underlying leptin release, and the variations in the timings are because of the ultradian
 139 rhythm Simon et al. (1998). The number of recovered secretory events for all subjects were within the
 140 physiologically plausible range (i.e., 20-40 events per day). This experimental data includes 18 subjects
 141 with obesity. The black diamonds in Figure 2-A represent the measured leptin levels obtained from blood
 142 samples. By performing deconvolution, we obtain the hormonal secretory pulses, which are used to obtain
 143 the reconstructed signal. The square of the multiple correlation coefficient (R^2) is between 0.8327 and
 144 0.98603. γ_1 and γ_2 are the infusion rate of leptin by adipose tissue and the clearance rate of leptin by the
 145 renal system, respectively.

146 Moreover, we simulated the data to further validate the proposed model. We simulated 18 leptin datasets,
 147 each corresponding to an experimental profile. An example of the result is presented in Figure 2-B. The
 148 datasets are simulated from the estimated pulses of the experimental measurements shown in Figure 2-A.
 149 We added Gaussian noise with standard deviation based on interassay co-efficient of variability (σ) provided
 150 in Aschbacher et al. (2014), defined in the supplementary information. Figure 2-B shows an example of the
 151 ground truth of the sparse input, the estimated input, and the simulated leptin data. The blue stars show
 152 the estimated 24-hour leptin data, the black curve shows the estimated leptin levels, and the vertical blue
 153 lines show the amplitudes and timings of the simulated data. The vertical red lines in Figure 2-B show the
 154 amplitudes and timings of the estimated hormone secretory events. Figure 3 shows the sample distribution
 155 for infusion and clearance rates for simulated and experimental leptin levels.

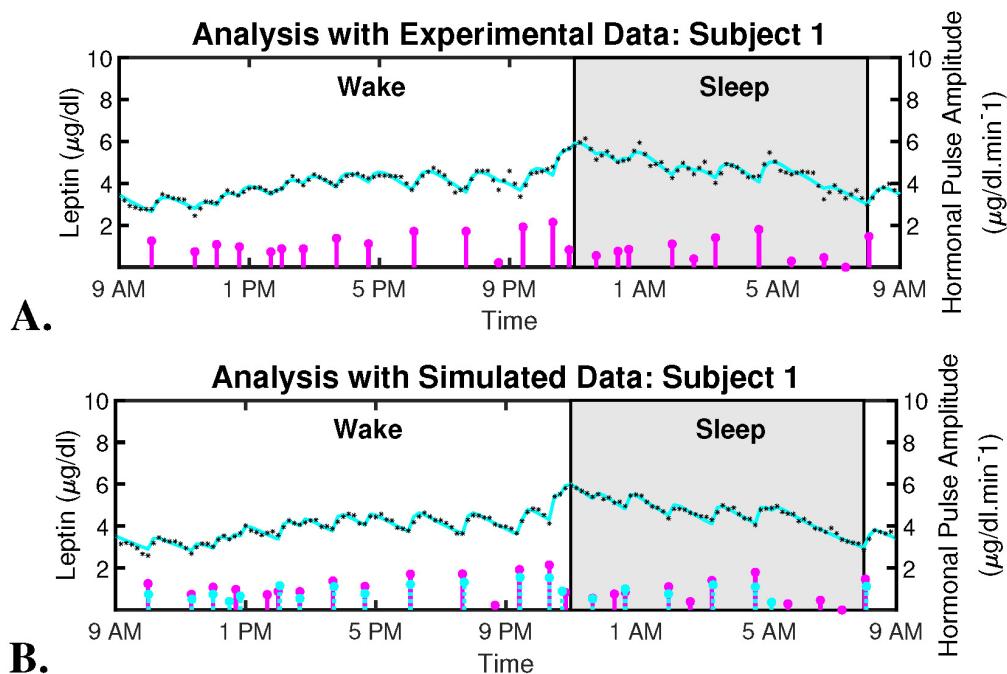


Figure 2. Deconvolved Twenty-Four Hours Leptin Levels in Patient with Obesity. (A) Sub-plot shows the measured 24-hour leptin time series (black diamonds), the reconstructed leptin levels (cyan curve), the estimated pulse timings, and amplitudes (magenta vertical lines) for experimental data. (B) Sub-plot shows the simulated 24-hour leptin time series (black diamonds), the reconstructed leptin levels (cyan curve), the simulated pulse timings and amplitudes (cyan vertical lines), the recovered pulse timings, and amplitudes (magenta vertical lines) for simulated data.

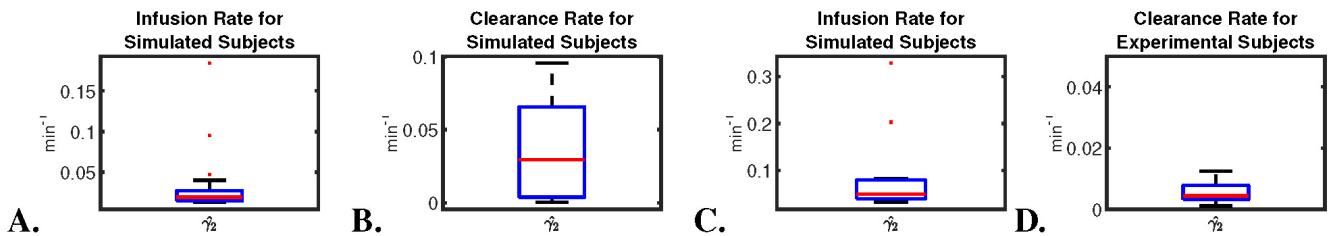


Figure 3. Box-plot of Physiological Parameters of Leptin Levels. Subplots, respectively illustrate the sample distribution of (A) the infusion rate for simulated leptin data, (B) the clearance rate for simulated leptin data, (C) the infusion rate for experimental leptin data, and (D) the clearance rate for experimental leptin data, depicting the median (red line), the lower (Q1) to upper (Q3) quartile range (blue rectangle), and 9 to 91 percentile range (black line and black dashed line).

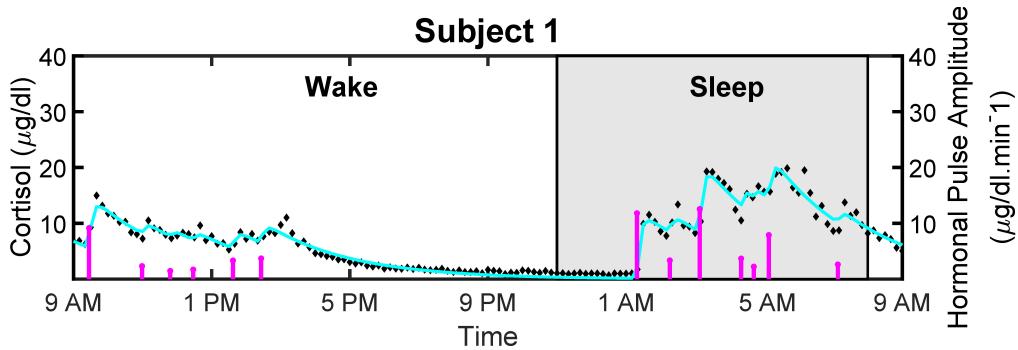


Figure 4. Deconvolved Experimental Twenty-Four Hour Cortisol Levels in Patient with Obesity. Plot shows the measured 24-hour cortisol time series (black diamonds), the reconstructed cortisol levels (cyan curve), the estimated pulse timings and amplitudes (magenta vertical lines).

156 2.2 Pulsatile Nature of Cortisol:

157 Figure 4 shows the measured and reconstructed blood cortisol levels of women with obesity. It shows the
 158 measured cortisol levels, reconstructed cortisol levels, and estimated amplitudes and timings of hormonal
 159 secretory events. The amplitude and timing variations of the pulses are due to the circadian rhythm and
 160 ultradian rhythm of the underlying cortisol release Brown et al. (2001). The black diamonds in Figure 4
 161 represent the measured cortisol levels obtained from blood samples. By performing deconvolution, we
 162 obtain the abstraction of hormonal secretory events (blue vertical lines in Figure 4), which are used to
 163 obtain the reconstructed signal (red curve in Figure 4). The square of the multiple correlation coefficient
 164 (R^2) is between 0.87557 and 0.98023. As this approach has been previously examined for cortisol data in
 165 different studies, we do not validate it on simulated data Pednekar et al. (2019); Faghah et al. (2014, 2015).

166 2.3 Leptin-cortisol antagonism

167 We seek to test how the leptin hormone samples and cortisol hormone samples are related. In this study, we
 168 look at the direct correlation between the sample regardless of other factors. Then, we also perform Granger
 169 causality analysis. This way we obtain an overall understanding of the apparent relationship between the
 170 leptin and cortisol hormone regardless of the other factors for the current dataset. Consideration of more
 171 flexible models that accounts for different factors such as age, gender, BMI, etc., and complexities related
 172 to nonlinearity is kept for future work.

173 2.3.1 Spearman Correlation

174 We obtain Spearman correlation coefficient to measure the strength of a monotonic association between
 175 two variables based on the nonparametric measure of rank correlation. Table 1 shows the Spearman
 176 correlation coefficient between measured leptin levels and measured cortisol levels for 18 subjects. It also
 177 shows correlation coefficients for estimated as well as measured leptin and cortisol levels. The outcome
 178 for both these cases is very similar and shows that the proposed model retains the properties previously
 179 known.

180 Figure 5 shows the correlation plots for leptin and cortisol levels in subjects 5 and 12 who showed the
 181 highest negative correlation for both measured and estimated cases. We plot for both the measured and
 182 estimated levels to show the similarities. Except for in subjects 3, 15, and 18, we have observed significant
 183 correlation (i.e. $p \leq 0.05$). All other correlation plots are provided in the supplementary information.
 184 This analysis shows, a negative association exists between leptin and cortisol levels across all time points
 185 in most, but not all, patients. The negative correlation on the same experimental dataset has been first
 186 identified by Aschbacher *et al.* Aschbacher et al. (2014). In this study, we see that the outcome also hold
 187 on the reconstructed data from the proposed model and sparse deconvolution.

Table 1. Comparison of Spearman Correlation. Comparing Spearman correlation coefficients between the measured serum leptin and cortisol levels, and the model estimated leptin and cortisol levels.

Subject	Avg. Cortisol ($\mu\text{g}/\text{dl}$)	Avg. Leptin ($\mu\text{g}/\text{dl}$)	Coefficient for measured levels	p-value	Coefficient for estimated levels	p-value	BMI	Age
1	7.07	4.19	-0.29	< 0.001	-0.28	< 0.001	32.7	34
2	7.27	5.19	-0.54	< 0.001	-0.57	< 0.001	40.5	33
3	6.05	4.72	-0.05	0.5732	-0.06	0.441	30.1	44
4	4.76	4.99	-0.32	< 0.001	-0.28	< 0.001	38.4	34
5	5.44	2.69	-0.76	< 0.001	-0.81	< 0.001	33.8	38
6	5.75	2.86	-0.43	< 0.001	-0.43	< 0.001	30.3	36
7	8.40	2.38	0.28	< 0.001	0.27	< 0.01	31.0	40
8	7.10	1.47	-0.45	< 0.001	-0.43	< 0.001	32.2	46
9	7.63	4.02	-0.30	< 0.001	-0.26	< 0.01	35.1	25
10	7.35	1.94	-0.20	0.0190	-0.22	< 0.01	31.8	37
11	5.14	2.95	-0.31	< 0.001	-0.30	< 0.001	31.1	22
12	5.13	3.83	-0.65	< 0.001	-0.69	< 0.001	34.3	45
13	4.32	2.96	-0.36	< 0.001	-0.37	< 0.001	35.3	38
14	5.25	3.97	-0.30	< 0.001	-0.33	< 0.001	31.4	32
15	6.38	2.20	0.05	0.5888	0.05	0.572	33.3	51
16	4.60	3.83	-0.19	0.0226	-0.20	0.019	31.6	39
17	5.98	2.67	-0.20	0.0136	-0.19	0.022	31.2	43
18	7.07	3.54	-0.16	0.0609	-0.15	0.068	32.7	38

188 2.3.2 Granger Causality

189 We perform Granger causality analysis to infer the prospective prediction between leptin and cortisol.
 190 Figure 6 shows the results from the Granger causality analysis. Fourteen out of eighteen subjects showed
 191 statistically significant causal relationship from cortisol to leptin (i.e., cortisol samples from past can predict
 192 the present leptin samples). The corresponding lags are 36.64 ± 21.68 samples. Out of these fourteen
 193 subjects, ten showed the negative causal relationship. Seven out of eighteen showed statistically significant
 194 causal relationships from leptin to cortisol and four showed statistically significant negative relationships.
 195 The corresponding lags are 10.43 ± 17.95 samples. The evidence of possible negative association is in

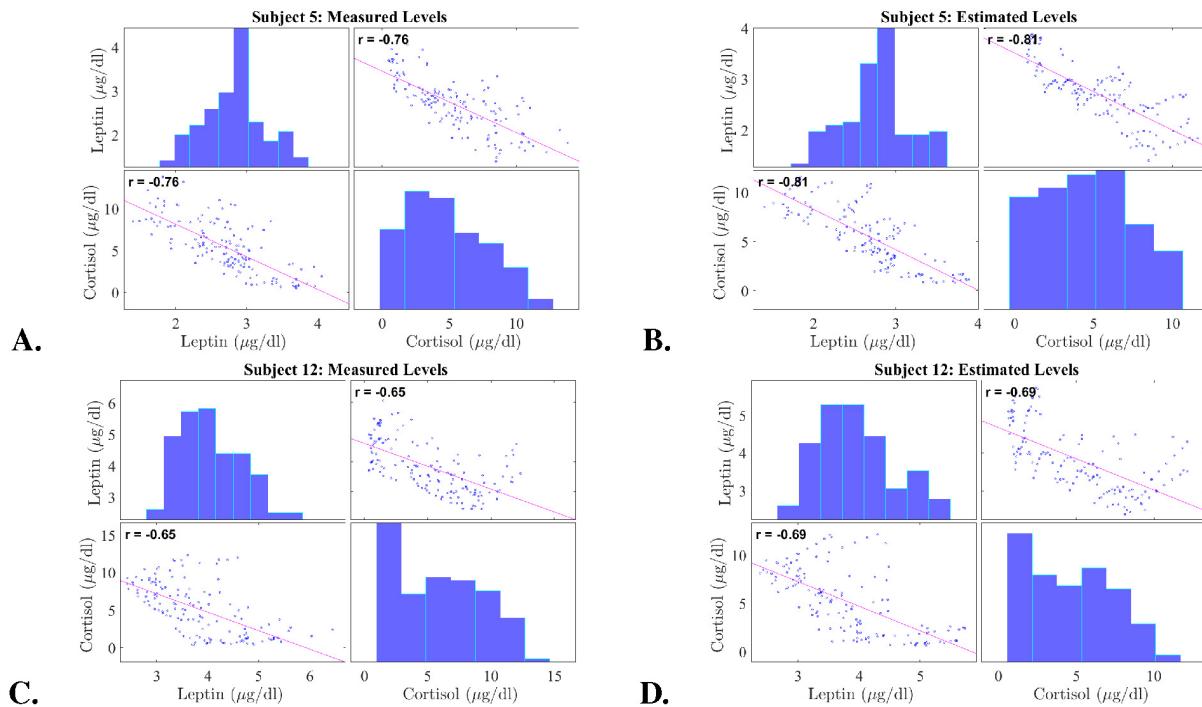


Figure 5. Correlation Plot between Leptin and Cortisol Levels in Patients with Obesity. The correlation plots are as follows: (A) measured leptin and cortisol levels for subject 5, (B) estimated leptin and cortisol levels for subject 5, (C) measured leptin and cortisol levels for subject 12, (D) estimated leptin and cortisol levels for subject 5. Each correlation plot incorporates a histogram and a scatter plot: the top left plot shows a histogram depicting leptin distributions and the bottom right plot shows a histogram depicting cortisol distributions, the bottom left plot and top right plot shows the scatter plot depicting correlation between leptin and cortisol.

196 accordance with the previous findings in previous correlation based study by Aschbacher *et al.* Aschbacher
 197 et al. (2014).

198

199 2.3.3 Secretion Characteristics Based on Between Subject Variability

200 In order to investigate the between subject variability, we perform investigation of the leptin secretion
 201 characteristics such as the l_2 -norm of the recovered leptin pulse vector (i.e. $\|\mathbf{u}_l\|_2$), infusions rates, and
 202 clearance rates against the factors such as BMI, lean body mass (LBM), HOMA-IR, and age with regression
 203 analysis. Similarly, we investigated the cortisol secretion characteristics such as the l_2 -norm of the recovered
 204 leptin pulse vector (i.e. $\|\mathbf{u}_c\|_2$), infusions rates, and clearance rates against the same factors factors. All
 205 the investigation were statistically insignificant ($p > 0$) except for the two cases. The regression analysis
 206 results between the l_2 -norm of leptin secretion (\mathbf{u}_l) vs. LBM and the l_2 -norm of leptin secretion (\mathbf{u}_l)
 207 vs. BMI showed statistically significant relationships ($p < 0.05$). Figure 7 show the scatter plot and the
 208 corresponding regression lines.

3 DISCUSSION

209 A complete model for the representation of hormonal variations must include all important intrinsic
 210 parameters such as forward and backward linkages between the hypothalamus, adipose tissue, anterior
 211 pituitary, adrenal gland, renal system, and liver as well as extrinsic parameters like stress, sleep, light, and

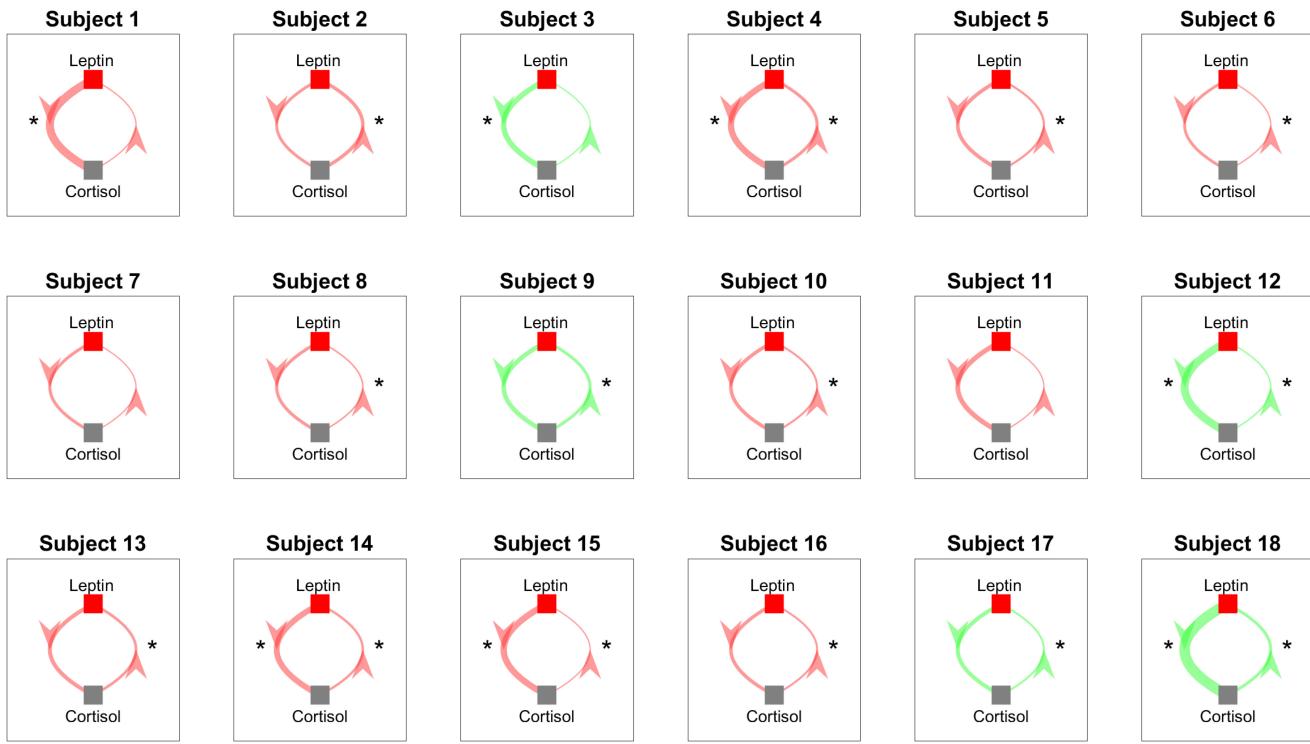


Figure 6. Granger Causality Map Estimated for Leptin and Cortisol for 18 Women with Obesity. Red color shows a negative causal influence and green color shows a positive causal influence, respectively. The direction of each arrow indicates that the variable at the start of the arrow is causing the variable at the end of the arrow. The thickness of the arrows represent the magnitude of the interaction. Starts next to the arrows denote that the corresponding prospective predictions are statistically significant with $p < 0.05$.

212 food Brown et al. (2001). It is a challenge to include all these factors while collecting human data, therefore,
 213 we propose this simplified minimal model considering the leptin measurements only. With the proposed
 214 minimal system theoretic model we formulate an optimization problem to infer unknown underlying the
 215 leptin secretion events as well as the corresponding system parameters. Furthermore, we performed the
 216 correlation analysis and the Granger causality analysis to investigate some possible directions to combine
 217 both cortisol and leptin in a single model in future. Investigation of secretion dynamics my offer a deeper
 218 understanding of the diseased conditions.

219 Understanding the secretion dynamics of leptin and cortisol in obese subjects and designing a model
 220 to understand their irregularities is difficult and challenging because the range of the number of leptin
 221 secretion events over a 24-hour period has not been accurately established. Licinio *et al.* Licinio et al. (1997)
 222 compared the leptin levels of women with obesity against healthy women and found that the concentration
 223 of independent pulsatile parameters of leptin, such as pulse duration and frequency, remained consistent
 224 between the groups, and that the excessive leptin levels in the persons with obesity were due to only an
 225 increased pulse height during secretion. The average number of leptin secretory events was found to be
 226 32.0 ± 1.5 per day Licinio et al. (1997), within a range of 29 to 39 pulses among the subjects. In another
 227 study Licinio et al. (1998), they reported the average number of leptin pulses to be 30.0 ± 1 with the range
 228 of pulses between 21 to 39 over a 24-hour sampling period. These studies give a likely average of around
 229 30 pulses per day of leptin, with a range between 20 to 40 being a possibility. We assumed there to be
 230 between 20-40 secretory events during a 24-hour period. Since there is no prior knowledge about the exact
 231 range of secretion events in women with obesity, we relax the constraints on the pulse range. We relaxed

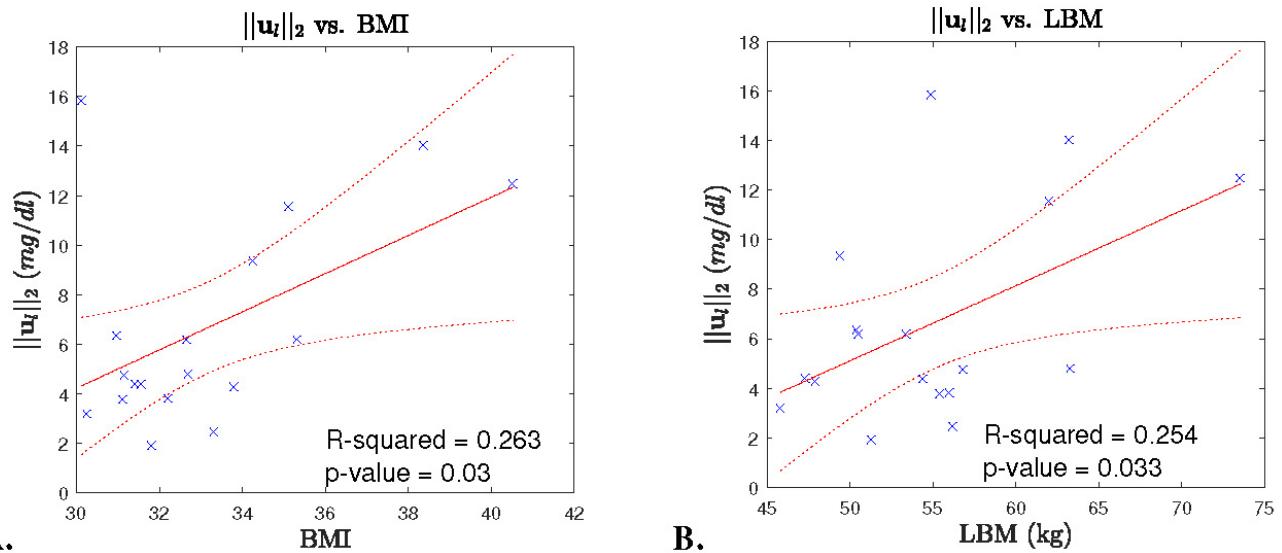


Figure 7. Leptin Secretion Characteristics vs. BMI/LBM. Each panel shows: A) the scatter plot (blue cross) of BMI vs l_2 -norm of leptin secretion \mathbf{u}_l , fitted regression line (red solid line), corresponding 95% confidence interval (dotted red curve), B) the scatter plot (blue cross) of LBM vs l_2 -norm of leptin secretion \mathbf{u}_l , fitted regression line (red solid line), corresponding 95% confidence interval (dotted red curve).

232 the upper and lower limits of this problem while taking care of the overfitting problem using Generalized
 233 Cross Validation - FOCal Under-determined System Solver (GCV-FOCUSS) to find λ . Although the upper
 234 limit was set to 50, we obtained less than or equal to 40 pulses for all patients. The proposed approach
 235 is applicable for a shorter length of hormone assay than a 24-hour assay, as long as it contains at least
 236 one secretion event. Therefore, the samples collected over an hour or more can be utilized to obtain the
 237 corresponding secretion event and the system parameters. As it can be seen in Figure 3, there is a difference
 238 in the distribution of the estimated parameters from the experimental data and the simulated data. This
 239 could be an indication of the consideration of a complex model in the optimization problem relative to the
 240 number of data-point available. Therefore proposed optimization problem might suffer from overfitting
 241 as pointed out in Ghasemi et al. (2018). As a future work of this, appropriate probabilistic priors can be
 242 applied on the system parameters to prevent such overfitting Amin and Faghah (2020).

243 Unlike leptin, the mathematical modeling of glucocorticoids such as cortisol is a more widely studied
 244 problem. Using the coordinate descent approach, we recover the underlying pulses of cortisol along with
 245 the infusion of cortisol into plasma by the adrenal glands and clearance by the liver. After recovering the
 246 underlying leptin and cortisol pulses, we check the relation between them. In this research, to find the
 247 correlation between cortisol and leptin secretory patterns in women with obesity, we keep the units for
 248 both hormones consistent. Both cortisol and leptin measurements were converted to $\mu\text{g}/\text{dL}$. As compared
 249 to cortisol measurements, the leptin measurements have lower levels, making it more challenging to
 250 deconvolve. Aschbacher et al. observed a leptin-cortisol antagonism Aschbacher et al. (2014). This
 251 research further suggests that leptin-cortisol dynamics might provide an indirect or functional biomarker
 252 of the neuroendocrine starvation response Aschbacher et al. (2014). Based on our statistical analysis, we
 253 observe a negative correlation for 14 out of 18 subjects between measured leptin and cortisol levels, as
 254 well as a negative correlation between the estimated leptin and cortisol levels. However, as no lag has been
 255 considered during the correlation analysis which may explain the inhibition action, the negative correlation
 256 in this case may suggest an existence of an underlying factor which is causing both of these hormones to
 257 change in opposite direction. Therefore, the proposed simplified minimal model to study cortisol and leptin,

258 and then finding the relationship between them, retains the previous properties. Consequently, exploiting
259 the sparse and pulsatile nature of these hormones provides us with a more coherent insight.

260 As the distributions of the measured and the estimated samples are not Gaussian, we consider the
261 rank-based Spearman correlation measure. A negative relationship has been observed between leptin and
262 cortisol based on Spearman correlation for most of the subjects. The negative correlation between the
263 reconstructed leptin and cortisol data also demonstrate that the proposed model and the corresponding
264 sparse deconvolution also preserves the negative association. To further verify the interdependence of
265 leptin and cortisol dynamics, we investigate the causal relationship between them. We perform the Granger
266 causality analysis on leptin and cortisol to obtain the directional relationship. We observe a negative
267 relationship, i.e., leptin-cortisol antagonism in 14 out of 18 subjects. leptin inhibits the HPA axis, which,
268 via neuroendocrine hormones like ACTH, results in a reduction in cortisol Bornstein et al. (1997); Pralong
269 et al. (1998). Cortisol increases or stimulates production of leptin in adipose tissue Pralong et al. (1998). It
270 is possible that this relationship may help explain behavioral contributors to obesity such as "stress-eating."
271 In the remaining 4 subjects, we do not observe an antagonism. It is possible that there exists a potential
272 leptin resistance Enriori et al. (2006); Engin (2017); González Izquierdo et al. (2019); Gruzdeva et al.
273 (2019) in these subjects. Leptin resistance is a condition in which, even though the body has enough high
274 leptin levels, receptors are desensitized leading to a "low signal." Consequently, the brain may respond
275 as if it were starving leading to a decrease in postprandial satiety. It is also possible that, due to a faulty
276 leptin signalling, the brain is unable to realize the leptin levels in the body and produce high leptin levels
277 during high cortisol levels. On the other hand, persistent higher concentration of leptin level plasma can
278 also lead to leptin resistance Enriori et al. (2006); Izquierdo et al. (2019). Izquierdo et al. Izquierdo et al.
279 (2019), also stated that obese people have high leptin levels and that the treatment of leptin resistance is
280 still a great challenge in the treatment of obesity. Therefore, negative causation further provides a strong
281 reason to study the negative relationship between leptin-cortisol. Furthermore, for some participants, the
282 negative relationship is statistically not significant, while for other participants the relationship is positive.
283 The result shows an indication of individual differences in leptin-cortisol regulation. Moreover, there are
284 studies that did not find any relationship between cortisol and leptin in different groups such as children
285 and adolescents investigating the relation in a single morning measurement of serum cortisol Sudi et al.
286 (2000). Therefore, it would be very interesting to study these individual and group differences which
287 will be crucial to develop more personalized obesity treatment programs. In other words, if there are
288 differences in the hormone regulation dynamics in presence of obesity, these differences should be studied
289 more thoroughly to before any intervention. This study has limitations in addition to observed variability
290 including lack of measurement of insulin and ACTH, no control group without obesity, no data on the
291 participants' eating behaviors/ physical activity/metabolism, use of Granger causality test, etc. All these
292 factors should be also considered for understanding before considering any treatment program. Personalize
293 treatment programs can be based on many parameters and preferences from patients, which can include
294 possible existence of leptin resistance. That way personalized programs might benefit from taking the
295 leptin cortisol dynamics and possible leptin resistance into account. Although the results obtained from
296 Granger causality test provide some lights towards possible explanation, a conclusive understanding is still
297 elusive which require further future investigation. The challenges includes limited number of subjects and
298 observed between-subject variability. Further investigation is required with consideration of more related
299 factors with larger population.

300 The leptin regulation model used in this research may be further be useful to understand the regulation
301 dynamics in different symptoms such as leptin congenital deficiency, Alzheimer's disease, and metabolic
302 syndrome Yupanqui-Lozano et al. (2019); Maioli et al. (2015). The advantage of using a model based on

303 human physiology is that it can be used to further isolate the organs or tissues responsible for causing
304 a particular deficiency. Determining a system model for the relationship between leptin and cortisol
305 concentrations provides us with a more comprehensive understanding of the biological system's behavior.
306 This model can provide insight into the effects of the relationship of hormones on weight gain. There exist
307 several bioscience and biosystems technologies used presently to simplify laboratory procedures and ease
308 human life Salehi (2017); Ghosh et al. (2018). Similar to those available to obtain insulin levels in the
309 human body Dias and Paulo Silva Cunha (2018), there will be bioscience technologies that would be able
310 to obtain leptin levels in the human body in the near future.

311 Similar to leptin and cortisol, skin conductance responses are sparse. Wickramasuriya *et al.* presents a
312 method to relate internal arousal state to the changes in skin conductance responses Wickramasuriya et al.
313 (2019). Wickramasuriya *et al.* employ a deconvolution algorithm along with the Bayesian estimation to
314 estimate the hidden arousal state. Azgomi *et al.* use control technique methods to close the loop and regulate
315 the estimated arousal state within a desired range Azgomi et al. (2019). Furthermore, Wickramasuriya
316 *et al.* utilize the cortisol observations to track energy regulation in Cushing's patients Wickramasuriya
317 and Faghah (2019). Azgomi *et al.* employ the model presented in Wickramasuriya and Faghah (2019) to
318 design a control system for closing the loop and regulating energy state in patients with hypercortisolism
319 Azgomi and Faghah (2019). Similarly, by deconvolving leptin levels and obtaining the hormonal pulses,
320 one may develop a framework to obtain hunger states and regulate it. In another study, researchers in
321 Wickramasuriya and Faghah (2019) have revealed the fact that cortisol secretion in healthy humans has a
322 circadian nature Wickramasuriya and Faghah (2019). They derive multi-day data and observe differences
323 between healthy humans and patients. A similar study can be done for leptin since leptin secretion is also
324 influenced by the circadian rhythm. Similarly, another possible outcome of this study is to understand the
325 leptin regulation behavior in women with obesity against their matched healthy subjects. Pednekar *et al.*
326 Pednekar et al. (2020) uses deconvolution to obtain the underlying hormonal pulses from cortisol data
327 for fibromyalgia, chronic fatigue syndrome, and their matched healthy subjects. The healthy subjects are
328 matched to the patients based on several factors such as age, sex, and weight. The comparison between the
329 parameters and pulses obtained from the patients against their matched healthy subjects showed significant
330 differences in clearance rates and hormonal pulses. A similar study can be done in the future, using leptin
331 data in women with obesity. As another future direction, the model could be further extended to include
332 ACTH and study the differences in hormonal secretory events in women with obesity with respect to
333 healthy subjects Faghah et al. (2015). These hormonal secretory events are an abstraction of their pulsatile
334 nature and provide a good estimation of the pulses originating in the CNS. A thorough investigation of
335 these events could further help us to understand the role of the CNS as well as kidneys and liver in causing
336 obesity.

337 In this study, we have proposed a state-space model for describing sparse leptin hormone dynamics.
338 Furthermore, we have proposed a coordinate descent approach for the sparse deconvolution of the sampled
339 leptin data to obtain the pulsatile secretion events. The deconvolution framework also provides estimates of
340 the state-space model parameters. The successful modeling of leptin is the first step to a more comprehensive
341 understanding of dynamics and its interaction with cortisol. Our analysis with Spearman correlation and
342 Granger causality has provided some evidence of negative relation between leptin and cortisol. However,
343 some variability has also been observed which prevents us from concluding a concrete explanation. A more
344 thorough investigation is required as the study has limitation such as lack of including insulin and ACTH
345 measurement, no control group without obesity, no data on the participants' eating behaviors/physical
346 activity/metabolism, use of Granger causality test that only consider two factors, a small number of
347 participants, low number of samples, etc.

4 METHODS

348 4.1 Experimental Dataset

349 In this research, we utilize blood sample data collected from eighteen premenopausal women with
350 obesity for our analysis of leptin and cortisol pulse recovery Kok et al. (2008). The participants have
351 given informed consent for participation in the study. They collected data for 18 premenopausal women
352 with obesity over a 24-hour period with 10-minute sampling frequency. All participants were required to
353 have regular menstrual cycles. All studies were done in the early follicular phase of the menstrual cycle.
354 The subjects were within the age group of 22-51 and with BMI between 30-41 kg /m². Subjects were
355 admitted to the research center at 7:00 after an overnight fast. After the subjects rested for 45 min, indirect
356 calorimetry was performed using a ventilated hood for 30 min. Thereafter, a cannula for blood sampling
357 was inserted into an antecubital vein, which was attached to a three-way stopcock and kept patent by a
358 continuous 0.9% NaCl and heparin (1 U/ml) infusion (500 ml/24 h). Each subject followed a consistent
359 schedule with breakfast, lunch, and dinner provided, and had an 8.5 hour sleeping period between 23:00
360 and 7:30. The blood samples were collected without waking the subjects during this period. Mean fasting
361 glucose concentration was 5.0 ± 0.1 mmol/l (range 4.2– 6.3 mmol/l), insulin 15.3 ± 1.7 mU/l (range 7–28
362 mU/l), glycosylated hemoglobin (HbA1C) $4.7 \pm 0.1\%$ (range 3.9 –5.3%). The above study was approved
363 by the Medical Ethics Committee of Leiden University Aschbacher et al. (2014). Subjects were required
364 to be free of chronic disease, and exclusion criteria were fixed, such as shift work, depression, alcohol
365 abuse, and oral contraceptives. The blood samples were assayed every 10 minutes for cortisol and every 20
366 minutes for leptin and intermediate points were interpolated. Plasma leptin concentrations were estimated
367 by radioimmunoassay with a detection limit of 0.5 ng/liter, and interassay coefficient of variation of 3.6%
368 to 6.8%. Plasma cortisol was measured with a radioimmunoassay with a detection limit is 25 nmol⁻¹ and
369 the intra-assay coefficient of variation ranges between 2% and 4%. A detailed description of the experiment
370 is provided in Aschbacher et al. (2014); Kok et al. (2008). In this research, we analyze the leptin and
371 cortisol abstraction pulses for 18 subjects obtained from this dataset.

372 4.2 Leptin Model Formulation

The state-space model introduced in this research considers the first-order differential system of equations for leptin synthesis in the adipose tissue and clearance by the renal system. In a 24-hour period, the blood samples were collected at a 10-minute interval, i.e., 144samples in a day. However, leptin was assayed every 20 min and concentrations was obtained for every 10 min by interpolation. If we assume that leptin secretion can happen in every minutes then there are 1440 possible events for 24-h duration. In a discrete space of 1440 events, we observe between 20 to 40 pulses making it sparse in nature. In this model, we intend to utilize the sparse nature of the hormonal secretory events along with other physiological constraints in a state-space model to estimate the amplitude and frequency of hormonal secretory events. The rate of change of leptin concentration in the adipose tissue is equal to the difference between leptin synthesis rate and the leptin infusion rate from adipose tissue into the blood. Similarly, the rate of change of leptin concentration in the serum is equal to the difference between the leptin infusion rate from adipose tissue into the blood and the leptin clearance rate by the renal system Wolf et al. (2002). The leptin secretion

dynamics are represented as follows:

$$\frac{dx_1(t)}{dt} = -\gamma_1 x_1(t) + u_l(t) \quad (\text{Adipose Tissue}) \quad (1)$$

$$\frac{dx_2(t)}{dt} = \gamma_1 x_1(t) - \gamma_2 x_2(t) \quad (\text{Plasma}) \quad (2)$$

where $x_1(t)$ and $x_2(t)$ represent the effective leptin concentrations inside adipose tissue and leptin concentration in the blood serum, respectively. The model parameters γ_1 and γ_2 represent the infusion rate of leptin by the adipose tissue and the clearance rate of leptin by the kidneys, respectively. Input $u_l(t)$ represents the abstraction of effective pulses mainly produced by the adipose tissue. $u_l(t)$ can be modeled as a summation of delta functions with $u_l(t) = \sum_{i=1}^N q_i \delta(t - \tau_i)$ where q_i is the magnitude of the pulse initiated at time τ_i . If no pulse occurs at τ_i , q_i will equal zero. The pulses are assumed to occur at integer minutes, i.e., in a 24-hour period, there are 1440 distinct locations ($N = 1440$). The blood samples are collected every 10 minutes for M samples ($M = 144$). Blood samples were collected beginning at y_{l_0} and then assayed for leptin. Let $y_{t_{10}}, y_{t_{20}}, \dots, y_{t_{10M}}, t_k : k = 10, 20, \dots, 10M$. Furthermore, $y_{l_0} = y_{t_{10}}$.

$$y_{l_{t_k}} = x_2(t_k) + \nu_{t_k} \quad (3)$$

where y_{t_k} and ν_{t_k} represent the observed leptin level and hormone measurement error at a time t_k , respectively. We model ν_{t_k} random variables as Gaussian random variable. Then for the system output, we assume that the observed output value at time t_k will be equal to the previous value, y_0 , multiplied by a decay term and added to a secretion value and the error term. We can represent the solution for every time point t_k with the following equation:

$$y_{l_{t_k}} = a_{t_k} y_{l_0} + \mathbf{b}_{t_k} \mathbf{u}_l + \boldsymbol{\nu}_{t_k} \quad (4)$$

373 where $\mathbf{b}_{t_k} = \left[\frac{\gamma_1}{\gamma_1 - \gamma_2} (e^{-\gamma_2 k} - e^{-\gamma_1 k}) \quad \frac{\gamma_1}{\gamma_1 - \gamma_2} (e^{-\gamma_2(k-1)} - e^{-\gamma_1(k-1)}) \quad \dots \quad \frac{\gamma_1}{\gamma_1 - \gamma_2} (e^{-\gamma_2} - e^{-\gamma_1}) \quad 0 \quad \dots \quad 0 \right]'$ and $a_{t_k} = e^{-\gamma_2 k}$. \mathbf{u}_l represents the input over the entire 24-hour sampling period, 374 with values q_i over $i = 1, \dots, 1440$. $\boldsymbol{\nu}_{t_k}$ stands for the vector representation of the random variable ν_{t_k} . We 375 then solve for a_{t_k} and \mathbf{b}_{t_k} using a forced solution approach, multiplying each side of the equation by $e^{-\gamma_2 t}$ 376 and using mathematical methods to obtain the solutions.

378 From these matrices, we can form a combined representation for the system at any time, using
379 $\mathbf{y}_l = [y_{t_{10}} \quad y_{t_{20}} \quad \dots \quad y_{t_{10M}}]'$, $\boldsymbol{\gamma} = [\gamma_1 \quad \gamma_2]'$, $\mathbf{A}_{\boldsymbol{\gamma}} = [a_{t_{10}} \quad a_{t_{20}} \quad \dots \quad a_{t_{10M}}]'$, $\mathbf{B}_{\boldsymbol{\gamma}} =$
380 $[\mathbf{b}_{t_{10}} \quad \mathbf{b}_{t_{20}} \quad \dots \quad \mathbf{b}_{t_{10M}}]'$, $\mathbf{u}_l = [q_1 \quad q_2 \quad \dots \quad q_{1440}]'$, and $\boldsymbol{\nu} = [\nu_{t_{10}} \quad \nu_{t_{20}} \quad \dots \quad \nu_{t_{10M}}]'$.
381 We then represent the system output as:

$$\mathbf{y}_l = \mathbf{A}_{\boldsymbol{\gamma}} y_{l_0} + \mathbf{B}_{\boldsymbol{\gamma}} \mathbf{u}_l + \boldsymbol{\nu}_l$$

382 where output vector \mathbf{y}_l is dependent on the initial signal value, the input pulses, and the error term. The
383 values of matrices $\mathbf{A}_{\boldsymbol{\gamma}}$ and $\mathbf{B}_{\boldsymbol{\gamma}}$ are dependent on the values of $\boldsymbol{\gamma}$ for the given subject.

384 4.3 Cortisol Model Formulation

In this research, we use the cortisol secretion model provided by Faghah *et al.* Faghah et al. (2014). It exploits the sparse nature of hormonal secretory events and other physiological constraints to estimate

the amplitude and timings of the secretory events using a state-space model. The rate at which cortisol concentration changes in the adrenal glands is equal to the difference between the rate at which it is infused in the blood and the rate at which it is secreted. The rate of change of serum cortisol concentration is equal to the difference between the rate at which it is infused by the adrenal gland into the blood and the rate at which it is cleared by the liver from the blood. The cortisol secretion dynamics are represented as follows:

$$\frac{dx_3(t)}{dt} = -\psi_1 x_3(t) + u_c(t) \quad (\text{Adrenal Glands}) \quad (5)$$

$$\frac{dx_4(t)}{dt} = \psi_1 x_3(t) - \psi_2 x_4(t) \quad (\text{Serum}) \quad (6)$$

where $x_3(t)$ and $x_4(t)$ represent the cortisol concentrations in the adrenal glands and the blood serum, respectively. The physiological model parameters ψ_1 and ψ_2 represent the infusion rate of cortisol by the adrenal glands and the cortisol clearance rate by the liver respectively. Input $u_c(t)$ represent the abstraction of pulses coming from the hypothalamus responsible for the production of cortisol. $u_c(t)$ can be modeled as a summation of delta functions with $u_c(t) = \sum_{i=1}^N p_i \delta(t - \tau_i)$. p_i is the magnitude of the pulse initiated at time τ_i . If no pulse occurs at τ_i , q_i will equal zero. The pulses are assumed to occur at integer minutes, i.e. in a 24-hour period, there are 1440 distinct locations ($N = 1440$). The blood samples are collected every 10 minutes, for M samples ($M = 144$). As explained in Section 4.2 the measurement output can be represented as:

$$y_{c_{t_k}} = x_4(t_k) + \omega_{t_k}$$

where y_{t_k} and ω_{t_k} are the observed cortisol concentration and the measurement error, respectively. The initial concentration of cortisol in adrenal glands and serum is assumed to be zero and y_{c_0} . The system is further expressed as:

$$\mathbf{y}_c = \mathbf{A}_\psi \mathbf{y}_{c_0} + \mathbf{B}_\psi \mathbf{u}_c + \boldsymbol{\omega}_c$$

385 where $\mathbf{y}_c = [y_{t_{10}} \ y_{t_{20}} \ \dots \ y_{t_{10M}}]'$, $\psi = [\psi_1 \ \psi_2]'$, $\mathbf{A}_\psi = [a_{t_{10}} \ a_{t_{20}} \ \dots \ a_{t_{10M}}]'$,
 386 $\mathbf{B}_\psi = [b_{t_{10}} \ b_{t_{20}} \ \dots \ b_{t_{10M}}]'$, $\mathbf{u}_c = [q_1 \ q_2 \ \dots \ q_N]'$, $\boldsymbol{\omega}_c = [\omega_{t_{10}} \ \omega_{t_{20}} \ \dots \ \omega_{t_{10M}}]'$,
 387 $a_{t_i} = e^{-\psi_2 i}$ and $b_{t_i} = [\frac{\psi_1}{\psi_1 - \psi_2}(e^{-\psi_2 i} - e^{-\psi_1 i}) \quad \frac{\psi_1}{\psi_1 - \psi_2}(e^{-\psi_2(i-1)} - e^{-\psi_1(i-1)}) \quad \dots \quad \frac{\psi_1}{\psi_1 - \psi_2}(e^{-\psi_2} -$
 388 $e^{-\psi_1}) \quad \underbrace{0 \ \dots \ 0}_{N-i}]'$.

389 4.3.1 Granger Causality Analysis

We utilize Granger Causality to determine the prospective prediction between leptin and cortisol. Let $y_{c_{t_k}}$ and $y_{l_{t_k}}$ are two stationary processes sampled every 10 minutes. A simple causal model for the interaction between leptin and cortisol is given as follows Granger (2001):

$$y_{l_{t_k}} = \sum_{j=1}^{m_{l_1}} \alpha_{l,j} y_{l_{t_k-10j}} + \sum_{j=1}^{m_{l_2}} \beta_{l,j} y_{c_{t_k-10j}} + \epsilon_{l_t} \quad (7)$$

$$y_{c_{t_k}} = \sum_{j=1}^{m_{c_1}} \alpha_{c,j} y_{c_{t_k-10j}} + \sum_{j=1}^{m_{c_2}} \beta_{c,j} y_{l_{t_k-10j}} + \epsilon_{c_t} \quad (8)$$

390 According to the definition of the causality, y_{ct_k} is causing y_{lt_k} provided some $\beta_{l,j}$ is not zero and y_{lt_k} is
 391 causing y_{ct_k} if some $\beta_{c,j}$ is not zero. If both $\beta_{c,j}$ and $\beta_{l,j}$ are not zero, there is exist a feedback relationship
 392 between y_{ct_k} and y_{lt_k} . Here, $\alpha_{j,j}$ and $\alpha_{c,j}$ represents the coefficients related to the self interaction. ϵ_{lt} and
 393 ϵ_{ct} represent the model errors. We model them as i.i.d Gaussian random variables.

We use the Bayesian information criteria (BIC) to find m_{l_1} and m_{c_1} considering all $\beta_{l,j} = \beta_{c,j} = 0$. In contrast, we find the m_{l_2} and m_{c_2} using BIC considering m_{l_1} and m_{c_1} numbers of nonzero $\alpha_{l,j}$ and $\alpha_{c,j}$ in (7) and (8), respectively. We first fit the models in (7) and (8) to the data with $\beta_{l,j} = \beta_{c,j} = 0$. We obtain the likelihoods as $\mathcal{L}_l(y_{lt_k})$ and $\mathcal{L}_c(y_{ct_k})$. Then, we assume $\beta_{l,j}, \beta_{c,j} \neq 0$ and fit the model to the data. We obtain the corresponding likelihood $\mathcal{L}_l(y_{lt_k}|y_{ct_k})$ and $\mathcal{L}_c(y_{ct_k}|y_{lt_k})$. Therefore, the Granger causality depicts how cortisol is causing leptin,

$$LLR_{l,c} = \text{sign}\left(\sum_{j=1}^{m_{l_2}} \beta_{l,j}\right) \times \left(-\log \frac{\mathcal{L}_l(y_{lt_k})}{\mathcal{L}_l(y_{lt_k}|y_{ct_k})}\right)$$

and how leptin is causing cortisol,

$$LLR_{c,l} = \text{sign}\left(\sum_{j=1}^{m_{c_2}} \beta_{c,j}\right) \times \left(-\log \frac{\mathcal{L}_c(y_{ct_k})}{\mathcal{L}_c(y_{ct_k}|y_{lt_k})}\right)$$

394 where negative sign denotes inhibitory, positive sign denotes excitatory, and zero means no prospective
 395 prediction. The magnitude gives the relative strength of the prospective prediction. Furthermore, we
 396 calculate the F-statistics for testing whether the prospective prediction is statistically significant or not
 397 Granger (2001).

AUTHOR CONTRIBUTIONS STATEMENT

398 RF conceived and designed the study. MA and DP performed the research, analyzed data and wrote the
 399 manuscript. RF, and MA developed the algorithms and analysis tools. All authors revised the manuscript.

ADDITIONAL INFORMATION

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558 to draw Diagrams/Figures/Structures should be avoided. They should be external callouts including
559 graphics.