

# Sparse System Identification of Leptin Dynamics in Women with Obesity

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## 2 ABSTRACT

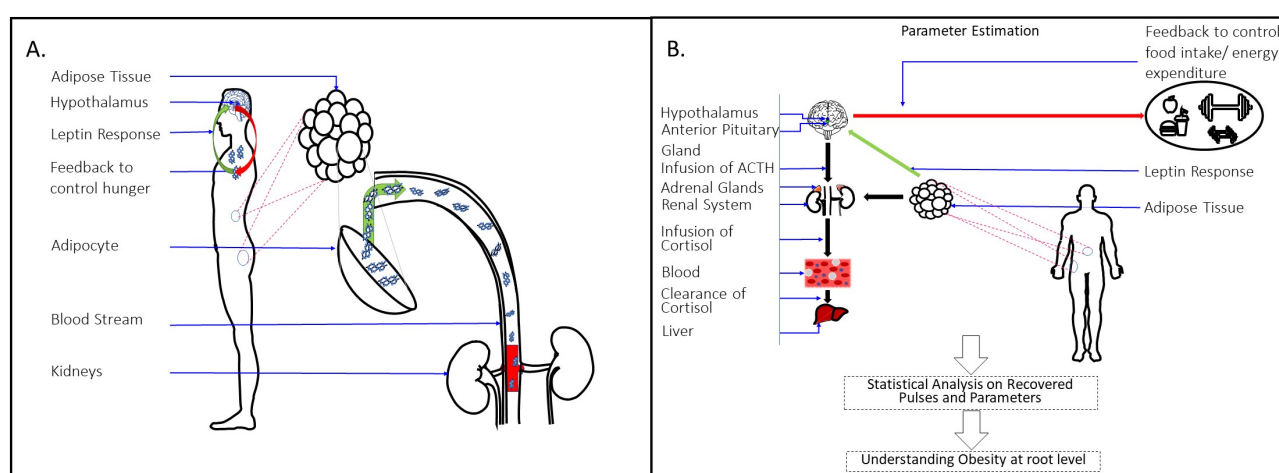
The prevalence of obesity is increasing around the world at an alarming rate. The interplay of the hormone leptin with the hypothalamus-pituitary-adrenal axis plays an important role in regulating energy balance, thereby contributing to obesity. This study presents a mathematical model, which describes hormonal behavior leading to an energy abnormal equilibrium that contributes to obesity. To this end, we analyze the behavior of two neuroendocrine hormones, leptin and cortisol, in a cohort of women with obesity, with simplified minimal state-space modeling. Using a system theoretic approach, coordinate descent method, and sparse recovery, we deconvolved the serum leptin-cortisol levels. Accordingly, we estimate the secretion patterns, timings, amplitudes, number of underlying pulses, infusion, and clearance rates of hormones in eighteen premenopausal women with obesity. Our results show that minimal state-space model was able to successfully capture the leptin and cortisol sparse dynamics with the multiple correlation coefficients greater than 0.83 and 0.87, respectively. Furthermore, the Granger causality test demonstrated a negative prospective predictive relationship between leptin and cortisol, 14 of 18 women. These results indicate that increases in cortisol are prospectively associated with reductions in leptin and vice versa, suggesting a bidirectional negative inhibitory relationship. As dysregulation of leptin may result in a abnormality in satiety and thereby associated to obesity, the investigation of leptin-cortisol sparse dynamics may offer a better diagnostic methodology to improve better treatments plans for individuals with obesity.

**Keywords:** leptin, cortisol, endocrinology, state-space, sparse recovery, system identification, deconvolution

## 1 INTRODUCTION

Obesity, dubbed the "Global Epidemic" by the World Health Organization, is said to cause or aggravate various other health problems, worsening one's life expectancy Ng et al. (2014); Wang et al. (2020). The prevalence of obesity is increasing worldwide at an alarming rate. The estimate shows that about 38% of the adult American population suffer from obesity Liou and Kulik (2020). Obesity is associated with a reduced life expectancy Blüher (2020). It also increases the risk of other chronic disorders such as diabetes and cardiovascular disease Jastreboff et al. (2019). Available studies suggest different lifestyle interventions

for treating obesity such as a low-calorie diet, increased physical activity, and bariatric surgery Cardel et al. (2020). Nevertheless, many studies have pointed the determinants of failure of such approaches including adherence to lifestyle interventions Burgess et al. (2017), genetic background, adaptive changes in basal metabolic rate, hunger and satiety hormones that occur with weight loss Reinehr (2013). While there exists pathogenic approaches to treat the condition through studying its development, a salutogenic model focused on holistic wellbeing might be more effective as both a preventive and remedial measure Organization (2000); Seo et al. (2019). For example, obesity can be conceptualized as a deficiency of energy regulation - i.e., the brain may fail to respond to negative feedback hormonal signals from adipose tissue, thereby perpetuating non-homeostatic eating behaviors that drive obesity. Thus, identifying the role of neuroendocrine hormones and adipokines in facilitating brain-adipose communication and energy regulation may inform novel strategies for the treatment and prevention of obesity. In this research, we propose models to understand leptin and cortisol behavior, which, in the future, can further be generalized for other relevant hormones, such as growth hormone, insulin, and ghrelin. Leptin is a signaling hormone and adipokine that is essential to activating central nervous system (CNS) networks involved in the suppression of appetite. It regulates food intake, metabolism, energy expenditure, and body weight Mantzoros et al. (2011). Although yet to be proven, Jacquier *et al.* Jacquier et al. (2015) mathematically demonstrated how leptin resistance can be a result of persistent leptin infusion. They further pointed out that temporal alteration in some parameter values related to food intake can shift the equilibrium. The shift in equilibrium can lead to leptin resistance and obesity state. Blood leptin levels also correlate with changes in fat mass Ahima and Flier (2000). Leptin is a hormone produced and secreted primarily by adipocytes. Hence, excessive amounts of adipose tissue in the body correspond to an increase in leptin production rate, and thus, higher serum leptin levels Mantzoros et al. (2011). Leptin interacts with the hypothalamus-pituitary-adrenal-axis (HPA-axis), thereby constituting an important biological mechanism whereby experiences of psychological stress may influence or be influenced by body weight and energy regulation. Buckley and Schatzberg (2005); Adam and Epel (2007)



**Figure 1. Leptin Regulation Model and an Overview of System-Theoretic Approach.** (A) shows the leptin secretion & regulation model. Leptin is the hormone to signal the hypothalamus regarding energy storage and use. It is produced and secreted primarily by adipocytes. The renal system is responsible for clearing it. (B) shows the overall approach used in this study. A state-space model with physiological constraints is designed. A deconvolution approach is then used to estimate the secretion events and the physiological parameters. A statistical analysis is done on the estimations to understand the relationship of leptin-cortisol rhythms and dynamics in detail.

Another frequently studied hormone in obesity research is the glucocorticoid hormone cortisol. Björntorp and Rosmond (2000) In response to different physiological demands such as blood glucose regulation, inflammation suppression, physiological stress, significant HPA-axis responses are induced across all ages and genders Kudielka et al. (2004). Björntorp *et al.* Björntorp and Rosmond (2000) compared several studies and concluded that the association between obesity and cortisol is complicated. They concluded that there are several factors such regulatory systems, including the HPA, gonadal, growth-hormone, leptin axes, the sympathetic nervous system, the central adrenergic, serotonergic, and dopaminergic systems that are related to obesity. Despite cortisol's short half-life and circadian secretion pattern, most studies use cortisol measures such as serum, urine or salivary cortisol that do not reflect long-term cortisol exposure Rodriguez et al. (2015). Björntorp *et al.* Björntorp and Rosmond (2000) suggest that it is necessary to examine the cortisol secretion process to further understand the roots of obesity.

In an attempt to study obesity, various models based on ordinary differential equations considering regulations in energy and metabolism de Graaf et al. (2009); Chow and Hall (2014); Horgan (2011); Hall (2009); Aschbacher et al. (2014), as well as on the effects of different hormones such as ghrelin, cholecystokinin, and leptin have been hypothesized. Traditional studies perform statistical analysis on the measured serum hormone levels Kanaley et al. (2001); Antunes et al. (2008). Some models include leptin, leptin resistance, and leptin receptors to understand obesity Tam et al. (2009); Jacquier et al. (2015); De Gaetano et al. (2008). Other studies show the association of glucocorticoids and weight gain Udden et al. (2003); Sominsky and Spencer (2014). Since obesity is associated with the polypeptide hormone, leptin Buckley and Schatzberg (2005); Mantzoros et al. (2011); Jacquier et al. (2015), and the glucocorticoid hormone, cortisol Myers Jr et al. (2010); Björntorp and Rosmond (2000), studying them will prove vital in designing an approach to treating the condition. For example, we cannot measure leptin resistance in a living human, nevertheless, investigation of the association between cortisol and leptin through mathematical modeling may lead to some insights into the presence of leptin resistance.

Aschbacher *et al.* Aschbacher et al. (2014) propose a mathematical model based on the systems-level understanding of the HPA-leptin axis. Incorporating leptin in the model helped them to obtain three parameters: (I) inhibitory cortisol feedback signal, (II) Adrenocorticotrophic hormone (ACTH)-adrenal interaction, and (III) leptin-cortisol proportionality. The authors suggested that the leptin-cortisol relationship may be important for understanding the neuroendocrine starvation response, and that poor system control may ultimately contribute to obesity. The mathematical model presented in the research by Aschbacher *et al.* Aschbacher et al. (2014) explains the rate of changes in cortisol concentration, ACTH, and leptin dynamics. The model used in this paper is an extension of the HPA system dynamics model used previously in Aschbacher et al. (2012) incorporating leptin's impact on cortisol. Though leptin regulation is dependent on its relation with different hormones, understanding its behavior requires the study of leptin secretion and regulation. Investigating cortisol and leptin separately, and based on their physiology, will be a vital approach in interpreting how they behave concerning each other and other systems of the body. Moreover, cortisol and leptin each have circadian and ultradian rhythms, which our approach may better characterize. Jacquier *et al.* Jacquier et al. (2015) introduce a mathematical model to study leptin resistance based on leptin synthesis and receptor activity. They study the changes in plasma leptin concentration, the density of leptin receptors, and food intake by varying leptin infusion and food consumption. This model studies leptin resistance dynamics based on many parameters such as food intake, fat mass, leptin receptors, and leptin. However, it is challenging to simultaneously collect this kind of clinical data. Therefore, we propose a simplified model considering only plasma leptin levels as the observation. As leptin secretion is a pulsatile process, we aim to exploit this characteristic by inferring underlying impulses.

In this research, we propose a simplified system to understand leptin regulation by exploiting the sparse nature of leptin secretion. If plasma leptin levels are sampled at a 10-minute sampling rate over 24-hours, we obtain a maximum of 40 leptin secretion events out of a total of 1440 secretion events, therefore, the nature being sparse. The sparse system identification of leptin dynamics over time makes it possible to recover the number of leptin pulses. It further allows us to recover information about the pulse amplitudes and intervals of occurrence from collected data. For an extensive understanding of leptin secretion, it is necessary to unveil the pulses originating from the hypothalamus as an outcome to leptin signaling. The pulses resulting in leptin regulation and secretion are the outcome of several contributing factors such as effects of other hormones on leptin regulation, density of leptin receptors, and food intake. Similar to leptin, cortisol secretion is a sparse process. Sparse system identification of cortisol will also lead us to recover the abstraction of secretion events coming from the HPA-axis. It also provides us important insight about the way it is infused by the adrenal glands and cleared by the liver. The proposed architecture has been previously verified for cortisol in healthy subjects and also on patients suffering from chronic fatigue and fibromyalgia syndromes Pednekar et al. (2020); Faghih et al. (2014); Pednekar et al. (2019). Here, we propose to study leptin-cortisol behavior by first observing the underlying pulses and estimating the physiological infusion and clearance rates in the system using two separate regulatory minimal models, one each for cortisol and leptin, and then determining the relationship between them. Similar to the results of previous study by Aschbacher et al. (2014), we observe a negative relationship between cortisol and leptin. This relationship is observed based on a negative correlation. To further strengthen this observation, we perform the Granger causality test.

Figure 1-A shows a pictorial representation of the leptin regulation and secretion model. Figure 1-B shows a pictorial over all of the approach used in this research. This approach yields three important parameters: the timing and amplitudes of the leptin secretion events, the infusion rate of leptin by the adipose tissue, and the clearance rate by the renal system. We further conduct statistical analysis on the measured and estimated hormone levels. Finally, we propose that this method can be utilized to identify difference between patient and matched control participants for future characterization related to leptin deregulation similar to our previous approach Pednekar et al. (2020).

## 2 RESULTS

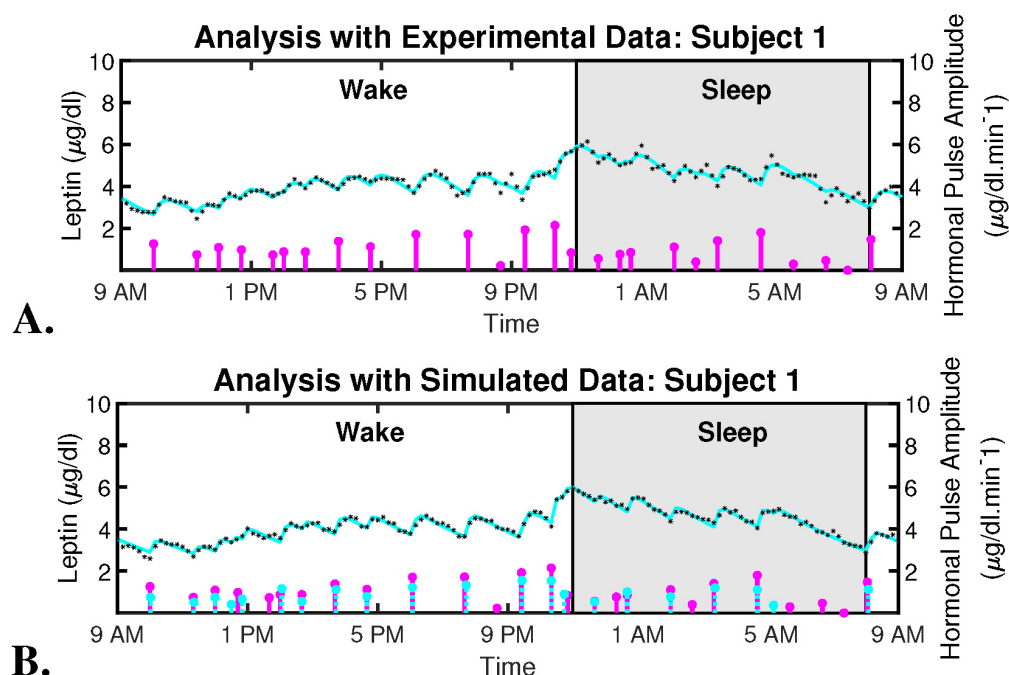
We utilize hormone assay from eighteen premenopausal women with mean body mass index (BMI) 33 with range 30–41  $kgm^{-2}$ ; mean age 37.5 with range 22–51 years; at morning (at 9 AM) mean cortisol level 11.02  $\mu g/dl$  and the corresponding standard deviation 3.65  $\mu g/dl$ . The distribution of average leptin level for all subject is  $3.3583 \pm 1.0633 \mu g/dl$ . The corresponding fat distribution is  $39.61 \pm 3.08 kg$  and lean body mass distribution is  $55.094 \pm 6.55 kg$ . Total cholesterol  $4.7 \pm 0.2 mmol/l$  (range 3.7–5.8 mmol/l), LDL cholesterol  $2.99 \pm 1.57 mmol/l$  (range 2.03– 4.00 mmol/l), and HDL cholesterol  $1.54 \pm 0.08 mmol/l$  (range 1.03–2.32 mmol/l). Homeostasis model assessment for insulin resistance (HOMA-IR) was estimated to be  $3.25 \pm 0.48$  with placebo and  $2.32 \pm 0.19$  after treating with Bromocriptine Kok et al. (2006). The statistical test between two distribution showed significant difference with  $p = 0.01$ .

### 2.1 Pulsatile Nature of Leptin:

Figure 2 shows the example deconvolution results of measured and reconstructed blood leptin levels of women with obesity for both experimental and simulated data for one subject. Figure 2-A exhibits the estimated amplitudes and timings of hormonal secretory events. It also presents the experimental leptin data and estimates predicted by the model. The amplitude variations of the pulses are due to the

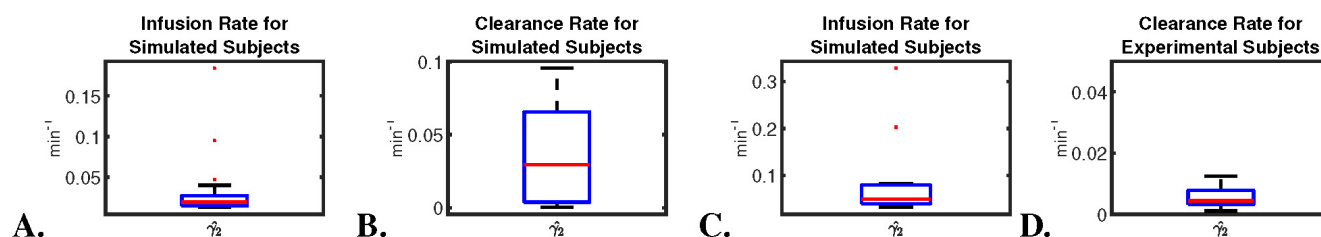
138 circadian rhythm of underlying leptin release, and the variations in the timings are because of the ultradian  
 139 rhythm Simon et al. (1998). The number of recovered secretory events for all subjects were within the  
 140 physiologically plausible range (i.e., 20-40 events per day). This experimental data includes 18 subjects  
 141 with obesity. The black diamonds in Figure 2-A represent the measured leptin levels obtained from blood  
 142 samples. By performing deconvolution, we obtain the hormonal secretory pulses, which are used to obtain  
 143 the reconstructed signal. The square of the multiple correlation coefficient ( $R^2$ ) is between 0.8327 and  
 144 0.98603.  $\gamma_1$  and  $\gamma_2$  are the infusion rate of leptin by adipose tissue and the clearance rate of leptin by the  
 145 renal system, respectively.

146 Moreover, we simulated the data to further validate the proposed model. We simulated 18 leptin datasets,  
 147 each corresponding to an experimental profile. An example of the result is presented in Figure 2-B. The  
 148 datasets are simulated from the estimated pulses of the experimental measurements shown in Figure 2-A.  
 149 We added Gaussian noise with standard deviation based on interassay co-efficient of variability ( $\sigma$ ) provided  
 150 in Aschbacher et al. (2014), defined in the supplementary information. Figure 2-B shows an example of the  
 151 ground truth of the sparse input, the estimated input, and the simulated leptin data. The blue stars show  
 152 the estimated 24-hour leptin data, the black curve shows the estimated leptin levels, and the vertical blue  
 153 lines show the amplitudes and timings of the simulated data. The vertical red lines in Figure 2-B show the  
 154 amplitudes and timings of the estimated hormone secretory events. Figure 3 shows the sample distribution  
 155 for infusion and clearance rates for simulated and experimental leptin levels.

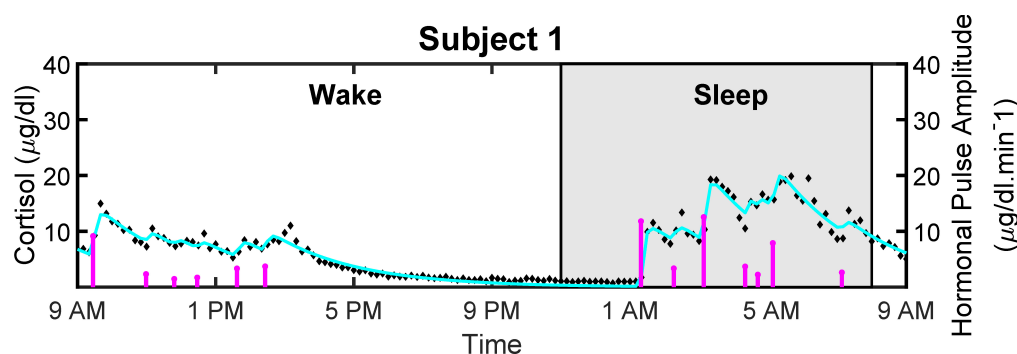


**Figure 2. Deconvolved Twenty-Four Hours Leptin Levels in Patient with Obesity.** (A) Sub-plot shows the measured 24-hour leptin time series (black diamonds), the reconstructed leptin levels (cyan curve), the estimated pulse timings, and amplitudes (magenta vertical lines) for experimental data. (B) Sub-plot shows the simulated 24-hour leptin time series (black diamonds), the reconstructed leptin levels (cyan curve), the simulated pulse timings and amplitudes (cyan vertical lines), the recovered pulse timings, and amplitudes (magenta vertical lines) for simulated data.





**Figure 3. Box-plot of Physiological Parameters of Leptin Levels.** Subplots, respectively illustrate the sample distribution of (A) the infusion rate for simulated leptin data, (B) the clearance rate for simulated leptin data, (C) the infusion rate for experimental leptin data, and (D) the clearance rate for experimental leptin data, depicting the median (red line), the lower (Q1) to upper (Q3) quartile range (blue rectangle), and 9 to 91 percentile range (black line and black dashed line).



**Figure 4. Deconvolved Experimental Twenty-Four Hour Cortisol Levels in Patient with Obesity.** Plot shows the measured 24-hour cortisol time series (black diamonds), the reconstructed cortisol levels (cyan curve), the estimated pulse timings and amplitudes (magenta vertical lines).

## 156 2.2 Pulsatile Nature of Cortisol:

157 Figure 4 shows the measured and reconstructed blood cortisol levels of women with obesity. It shows the  
 158 measured cortisol levels, reconstructed cortisol levels, and estimated amplitudes and timings of hormonal  
 159 secretory events. The amplitude and timing variations of the pulses are due to the circadian rhythm and  
 160 ultradian rhythm of the underlying cortisol release Brown et al. (2001). The black diamonds in Figure 4  
 161 represent the measured cortisol levels obtained from blood samples. By performing deconvolution, we  
 162 obtain the abstraction of hormonal secretory events (blue vertical lines in Figure 4), which are used to  
 163 obtain the reconstructed signal (red curve in Figure 4). The square of the multiple correlation coefficient  
 164 ( $R^2$ ) is between 0.87557 and 0.98023. As this approach has been previously examined for cortisol data in  
 165 different studies, we do not validate it on simulated data Pednekar et al. (2019); Faghih et al. (2014, 2015).

## 166 2.3 Leptin-cortisol antagonism

167 We seek to test how the leptin hormone samples and cortisol hormone samples are related. In this study, we  
 168 look at the direct correlation between the sample regardless of other factors. Then, we also perform Granger  
 169 causality analysis. This way we obtain an overall understanding of the apparent relationship between the  
 170 leptin and cortisol hormone regardless of the other factors for the current dataset. Consideration of more  
 171 flexible models that accounts for different factors such as age, gender, BMI, etc., and complexities related  
 172 to nonlinearity is kept for future work.

### 2.3.1 Spearman Correlation

We obtain Spearman correlation coefficient to measure the strength of a monotonic association between two variables based on the nonparametric measure of rank correlation. Table 1 shows the Spearman correlation coefficient between measured leptin levels and measured cortisol levels for 18 subjects. It also shows correlation coefficients for estimated as well as measured leptin and cortisol levels. The outcome for both these cases is very similar and shows that the proposed model retains the properties previously known.

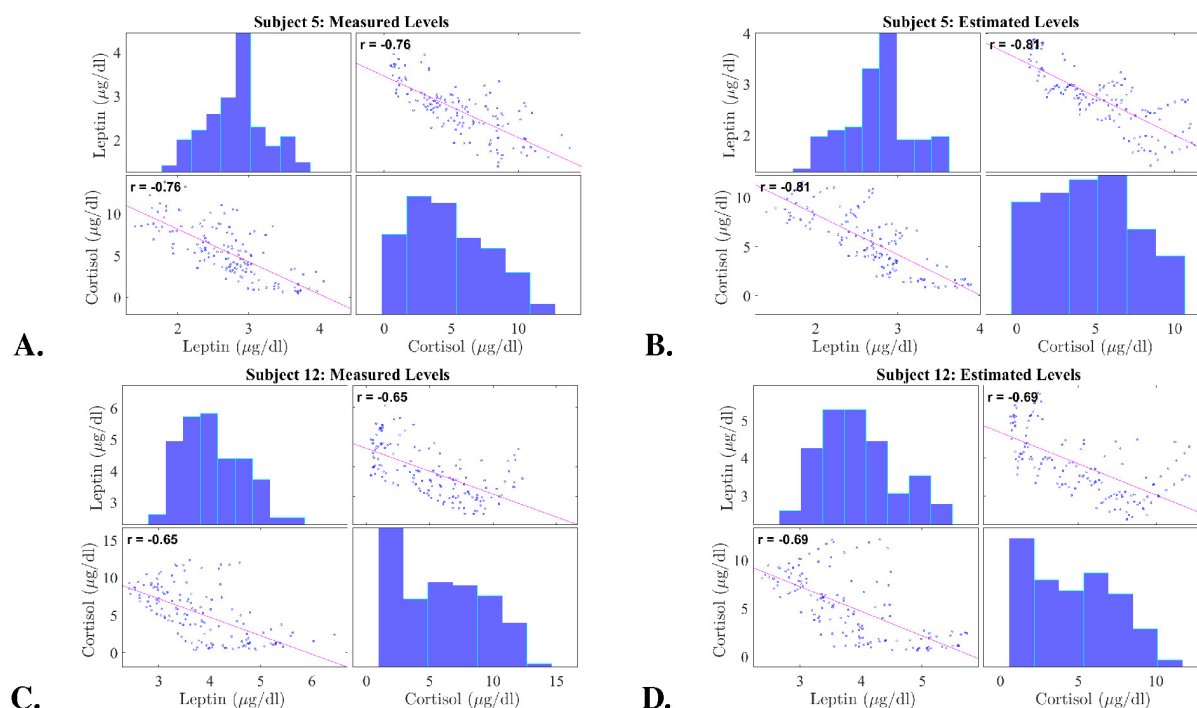
Figure 5 shows the correlation plots for leptin and cortisol levels in subjects 5 and 12 who showed the highest negative correlation for both measured and estimated cases. We plot for both the measured and estimated levels to show the similarities. Except for in subjects 3, 15, and 18, we have observed significant correlation (i.e.  $p \leq 0.05$ ). All other correlation plots are provided in the supplementary information. This analysis shows, a negative association exists between leptin and cortisol levels across all time points in most, but not all, patients. The negative correlation on the same experimental dataset has been first identified by Aschbacher *et al.* Aschbacher et al. (2014). In this study, we see that the outcome also hold on the reconstructed data from the proposed model and sparse deconvolution.

**Table 1. Comparison of Spearman Correlation.** Comparing Spearman correlation coefficients between the measured serum leptin and cortisol levels, and the model estimated leptin and cortisol levels.

Subject	Avg. Cortisol ( $\mu\text{g/dl}$ )	Avg. Leptin ( $\mu\text{g/dl}$ )	Coefficient for measured levels	p-value	Coefficient for estimated levels	p-value	BMI	Age
1	7.07	4.19	-0.29	< 0.001	-0.28	< 0.001	32.7	34
2	7.27	5.19	-0.54	< 0.001	-0.57	< 0.001	40.5	33
3	6.05	4.72	-0.05	0.5732	-0.06	0.441	30.1	44
4	4.76	4.99	-0.32	< 0.001	-0.28	< 0.001	38.4	34
5	5.44	2.69	-0.76	< 0.001	-0.81	< 0.001	33.8	38
6	5.75	2.86	-0.43	< 0.001	-0.43	< 0.001	30.3	36
7	8.40	2.38	0.28	< 0.001	0.27	< 0.01	31.0	40
8	7.10	1.47	-0.45	< 0.001	-0.43	< 0.001	32.2	46
9	7.63	4.02	-0.30	< 0.001	-0.26	< 0.01	35.1	25
10	7.35	1.94	-0.20	0.0190	-0.22	< 0.01	31.8	37
11	5.14	2.95	-0.31	< 0.001	-0.30	< 0.001	31.1	22
12	5.13	3.83	-0.65	< 0.001	-0.69	< 0.001	34.3	45
13	4.32	2.96	-0.36	< 0.001	-0.37	< 0.001	35.3	38
14	5.25	3.97	-0.30	< 0.001	-0.33	< 0.001	31.4	32
15	6.38	2.20	0.05	0.5888	0.05	0.572	33.3	51
16	4.60	3.83	-0.19	0.0226	-0.20	0.019	31.6	39
17	5.98	2.67	-0.20	0.0136	-0.19	0.022	31.2	43
18	7.07	3.54	-0.16	0.0609	-0.15	0.068	32.7	38

### 2.3.2 Granger Causality

We perform Granger causality analysis to infer the prospective prediction between leptin and cortisol. Figure 6 shows the results from the Granger causality analysis. Fourteen out of eighteen subjects showed statistically significant causal relationship from cortisol to leptin (i.e., cortisol samples from past can predict the present leptin samples). The corresponding lags are  $36.64 \pm 21.68$  samples. Out of these fourteen subjects, ten showed the negative causal relationship. Seven out of eighteen showed statistically significant causal relationships from leptin to cortisol and four showed statistically significant negative relationships. The corresponding lags are  $10.43 \pm 17.95$  samples. The evidence of possible negative association is in



**Figure 5. Correlation Plot between Leptin and Cortisol Levels in Patients with Obesity.** The correlation plots are as follows: (A) measured leptin and cortisol levels for subject 5, (B) estimated leptin and cortisol levels for subject 5, (C) measured leptin and cortisol levels for subject 12, (D) estimated leptin and cortisol levels for subject 5. Each correlation plot incorporates a histogram and a scatter plot: the top left plot shows a histogram depicting leptin distributions and the bottom right plot shows a histogram depicting cortisol distributions, the bottom left plot and top right plot shows the scatter plot depicting correlation between leptin and cortisol.

196 accordance with the previous findings in previous correlation based study by Aschbacher *et al.* Aschbacher  
197 et al. (2014).

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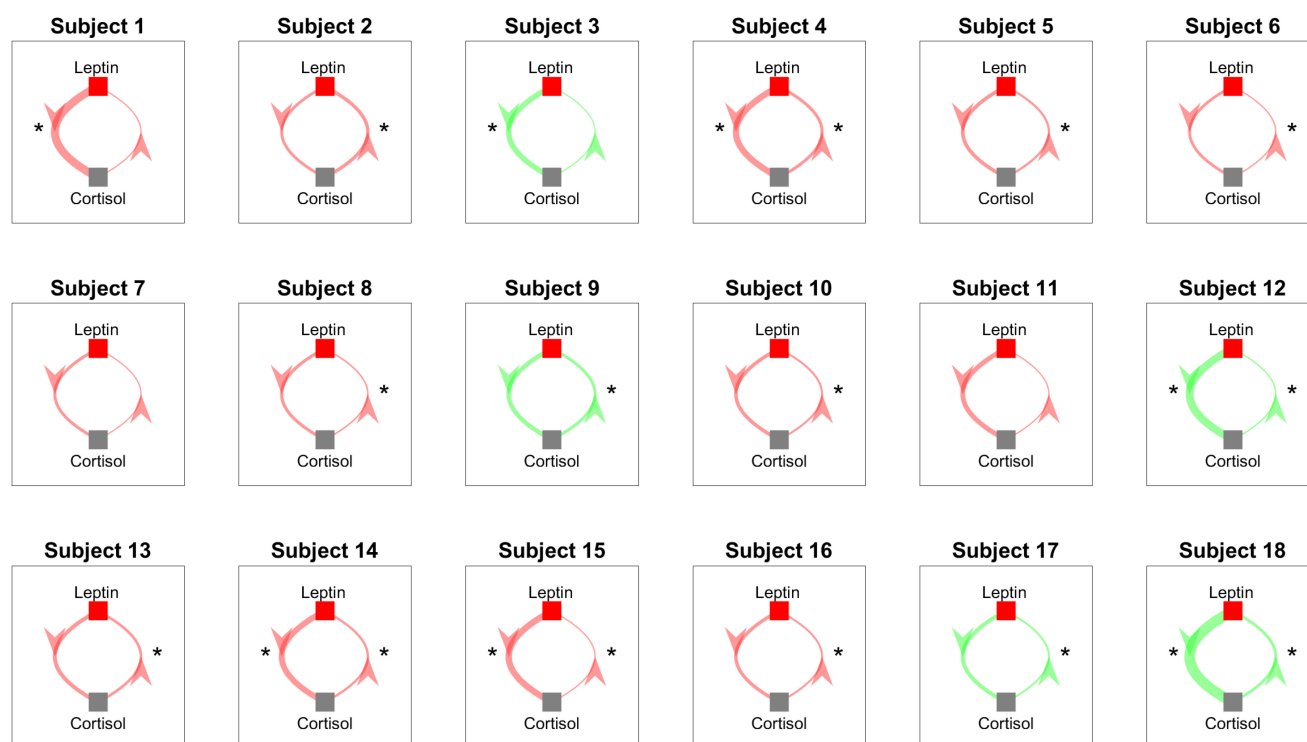
### 199 2.3.3 Secretion Characteristics Based on Between Subject Variability

200 In order to investigate the between subject variability, we perform investigation of the leptin secretion  
201 characteristics such as the  $l_2$ -norm of the recovered leptin pulse vector (i.e.  $\|\mathbf{u}_l\|_2$ ), infusions rates, and  
202 clearance rates against the factors such as BMI, lean body mass (LBM), HOMA-IR, and age with regression  
203 analysis. Similarly, we investigated the cortisol secretion characteristics such as the  $l_2$ -norm of the recovered  
204 leptin pulse vector (i.e.  $\|\mathbf{u}_c\|_2$ ), infusions rates, and clearance rates against the same factors factors. All  
205 the investigation were statistically insignificant ( $p > 0$ ) except for the two cases. The regression analysis  
206 results between the  $l_2$ -norm of leptin secretion ( $\mathbf{u}_l$ ) vs. LBM and the  $l_2$ -norm of leptin secretion ( $\mathbf{u}_l$ )  
207 vs. BMI showed statistically significant relationships ( $p < 0.05$ ). Figure 7 show the scatter plot and the  
208 corresponding regression lines.

## 3 DISCUSSION

209 A complete model for the representation of hormonal variations must include all important intrinsic  
210 parameters such as forward and backward linkages between the hypothalamus, adipose tissue, anterior  
211 pituitary, adrenal gland, renal system, and liver as well as extrinsic parameters like stress, sleep, light, and

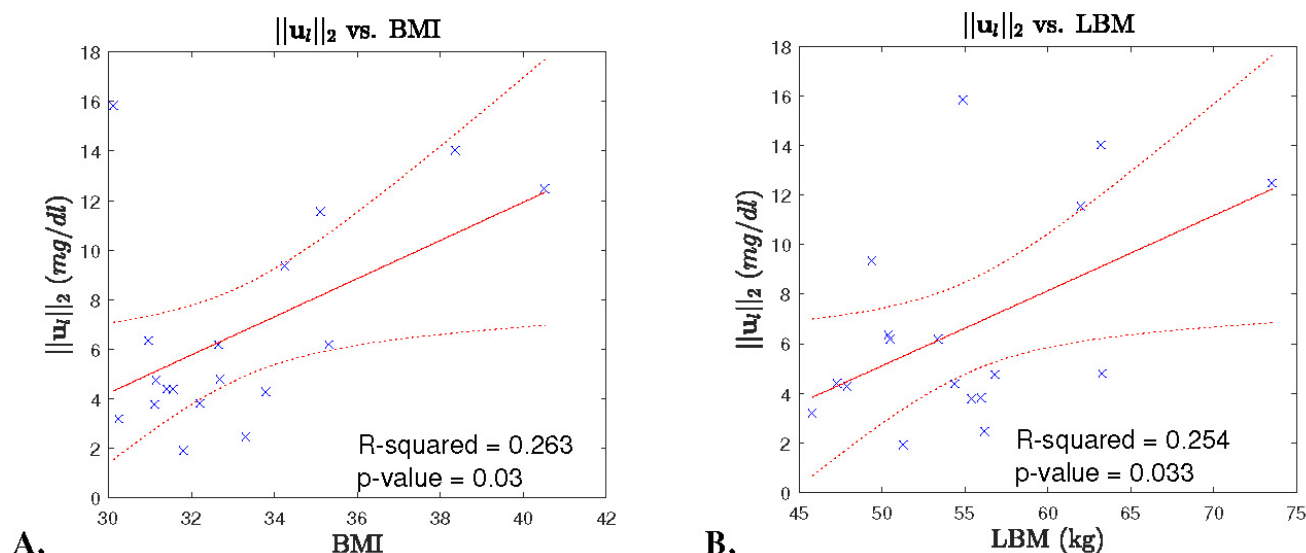




**Figure 6. Granger Causality Map Estimated for Leptin and Cortisol for 18 Women with Obesity.** Red color shows a negative causal influence and green color shows a positive causal influence, respectively. The direction of each arrow indicates that the variable at the start of the arrow is causing the variable at the end of the arrow. The thickness of the arrows represent the magnitude of the interaction. Starts next to the arrows denote that the corresponding prospective predictions are statistically significant with  $p < 0.05$ .

212 food Brown et al. (2001). It is a challenge to include all these factors while collecting human data, therefore,  
 213 we propose this simplified minimal model considering the leptin measurements only. With the proposed  
 214 minimal system theoretic model we formulate an optimization problem to infer unknown underlying the  
 215 leptin secretion events as well as the corresponding system parameters. Furthermore, we performed the  
 216 correlation analysis and the Granger causality analysis to investigate some possible directions to combine  
 217 both cortisol and leptin in a single model in future. Investigation of secretion dynamics may offer a deeper  
 218 understanding of the diseased conditions.

219 Understanding the secretion dynamics of leptin and cortisol in obese subjects and designing a model  
 220 to understand their irregularities is difficult and challenging because the range of the number of leptin  
 221 secretion events over a 24-hour period has not been accurately established. Licinio *et al.* Licinio et al. (1997)  
 222 compared the leptin levels of women with obesity against healthy women and found that the concentration  
 223 of independent pulsatile parameters of leptin, such as pulse duration and frequency, remained consistent  
 224 between the groups, and that the excessive leptin levels in the persons with obesity were due to only an  
 225 increased pulse height during secretion. The average number of leptin secretory events was found to be  
 226  $32.0 \pm 1.5$  per day Licinio et al. (1997), within a range of 29 to 39 pulses among the subjects. In another  
 227 study Licinio et al. (1998), they reported the average number of leptin pulses to be  $30.0 \pm 1$  with the range  
 228 of pulses between 21 to 39 over a 24-hour sampling period. These studies give a likely average of around  
 229 30 pulses per day of leptin, with a range between 20 to 40 being a possibility. We assumed there to be  
 230 between 20–40 secretory events during a 24-hour period. Since there is no prior knowledge about the exact  
 231 range of secretion events in women with obesity, we relax the constraints on the pulse range. We relaxed



**Figure 7. Leptin Secretion Characteristics vs. BMI/LBM.** Each panel shows: A) the scatter plot (blue cross) of BMI vs  $l_2$ -norm of leptin secretion  $u_l$ , fitted regression line (red solid line), corresponding 95% confidence interval (dotted red curve), B) the scatter plot (blue cross) of LBM vs  $l_2$ -norm of leptin secretion  $u_l$ , fitted regression line (red solid line), corresponding 95% confidence interval (dotted red curve).

the upper and lower limits of this problem while taking care of the overfitting problem using Generalized Cross Validation - FOCal Under-determined System Solver (GCV-FOCUSS) to find  $\lambda$ . Although the upper limit was set to 50, we obtained less than or equal to 40 pulses for all patients. The proposed approach is applicable for a shorter length of hormone assay than a 24-hour assay, as long as it contains at least one secretion event. Therefore, the samples collected over an hour or more can be utilized to obtain the corresponding secretion event and the system parameters. As it can be seen in Figure 3, there is a difference in the distribution of the estimated parameters from the experimental data and the simulated data. This could be an indication of the consideration of a complex model in the optimization problem relative to the number of data-point available. Therefore proposed optimization problem might suffer from overfitting as pointed out in Ghasemi et al. (2018). As a future work of this, appropriate probabilistic priors can be applied on the system parameters to prevent such overfitting Amin and Faghih (2020).

Unlike leptin, the mathematical modeling of glucocorticoids such as cortisol is a more widely studied problem. Using the coordinate descent approach, we recover the underlying pulses of cortisol along with the infusion of cortisol into plasma by the adrenal glands and clearance by the liver. After recovering the underlying leptin and cortisol pulses, we check the relation between them. In this research, to find the correlation between cortisol and leptin secretory patterns in women with obesity, we keep the units for both hormones consistent. Both cortisol and leptin measurements were converted to  $\mu\text{g}/\text{dL}$ . As compared to cortisol measurements, the leptin measurements have lower levels, making it more challenging to deconvolve. Aschbacher et al. observed a leptin-cortisol antagonism Aschbacher et al. (2014). This research further suggests that leptin-cortisol dynamics might provide an indirect or functional biomarker of the neuroendocrine starvation response Aschbacher et al. (2014). Based on our statistical analysis, we observe a negative correlation for 14 out of 18 subjects between measured leptin and cortisol levels, as well as a negative correlation between the estimated leptin and cortisol levels. However, as no lag has been considered during the correlation analysis which may explain the inhibition action, the negative correlation in this case may suggest an existence of an underlying factor which is causing both of these hormones to change in opposite direction. Therefore, the proposed simplified minimal model to study cortisol and leptin,

and then finding the relationship between them, retains the previous properties. Consequently, exploiting the sparse and pulsatile nature of these hormones provides us with a more coherent insight.

As the distributions of the measured and the estimated samples are not Gaussian, we consider the rank-based Spearman correlation measure. A negative relationship has been observed between leptin and cortisol based on Spearman correlation for most of the subjects. The negative correlation between the reconstructed leptin and cortisol data also demonstrate that the proposed model and the corresponding sparse deconvolution also preserves the negative association. To further verify the interdependence of leptin and cortisol dynamics, we investigate the causal relationship between them. We perform the Granger causality analysis on leptin and cortisol to obtain the directional relationship. We observe a negative relationship, i.e., leptin-cortisol antagonism in 14 out of 18 subjects. leptin inhibits the HPA axis, which, via neuroendocrine hormones like ACTH, results in a reduction in cortisol Bornstein et al. (1997); Pralong et al. (1998). Cortisol increases or stimulates production of leptin in adipose tissue Pralong et al. (1998). It is possible that this relationship may help explain behavioral contributors to obesity such as "stress-eating." In the remaining 4 subjects, we do not observe an antagonism. It is possible that there exists a potential leptin resistance Enriori et al. (2006); Engin (2017); González Izquierdo et al. (2019); Gruzdeva et al. (2019) in these subjects. Leptin resistance is a condition in which, even though the body has enough high leptin levels, receptors are desensitized leading to a "low signal." Consequently, the brain may respond as if it were starving leading to a decrease in postprandial satiety. It is also possible that, due to a faulty leptin signalling, the brain is unable to realize the leptin levels in the body and produce high leptin levels during high cortisol levels. On the other hand, persistent higher concentration of leptin level plasma can also lead to leptin resistance Enriori et al. (2006); Izquierdo et al. (2019). Izquierdo *et al.* Izquierdo et al. (2019), also stated that obese people have high leptin levels and that the treatment of leptin resistance is still a great challenge in the treatment of obesity. Therefore, negative causation further provides a strong reason to study the negative relationship between leptin-cortisol. Furthermore, for some participants, the negative relationship is statistically not significant, while for other participants the relationship is positive. The result shows an indication of individual differences in leptin-cortisol regulation. Moreover, there are studies that did not find any relationship between cortisol and leptin in different groups such as children and adolescents investigating the relation in a single morning measurement of serum cortisol Sudi et al. (2000). Therefore, it would be very interesting to study these individual and group differences which will be crucial to develop more personalized obesity treatment programs. In other words, if there are differences in the hormone regulation dynamics in presence of obesity, these differences should be studied more thoroughly to before any intervention. This study has limitations in addition to observed variability including lack of measurement of insulin and ACTH, no control group without obesity, no data on the participants' eating behaviors/ physical activity/metabolism, use of Granger causality test, etc. All these factors should be also considered for understanding before considering any treatment program. Personalize treatment programs can be based on many parameters and preferences from patients, which can include possible existence of leptin resistance. That way personalized programs might benefit from taking the leptin cortisol dynamics and possible leptin resistance into account. Although the results obtained from Granger causality test provide some lights towards possible explanation, a conclusive understanding is still elusive which require further future investigation. The challenges includes limited number of subjects and observed between-subject variability. Further investigation is required with consideration of more related factors with larger population.

The leptin regulation model used in this research may be further be useful to understand the regulation dynamics in different symptoms such as leptin congenital deficiency, Alzheimer's disease, and metabolic syndrome Yupanqui-Lozno et al. (2019); Maioli et al. (2015). The advantage of using a model based on

human physiology is that it can be used to further isolate the organs or tissues responsible for causing a particular deficiency. Determining a system model for the relationship between leptin and cortisol concentrations provides us with a more comprehensive understanding of the biological system's behavior. This model can provide insight into the effects of the relationship of hormones on weight gain. There exist several bioscience and biosystems technologies used presently to simplify laboratory procedures and ease human life Salehi (2017); Ghosh et al. (2018). Similar to those available to obtain insulin levels in the human body Dias and Paulo Silva Cunha (2018), there will be bioscience technologies that would be able to obtain leptin levels in the human body in the near future.

Similar to leptin and cortisol, skin conductance responses are sparse. Wickramasuriya *et al.* presents a method to relate internal arousal state to the changes in skin conductance responses Wickramasuriya et al. (2019). Wickramasuriya *et al.* employ a deconvolution algorithm along with the Bayesian estimation to estimate the hidden arousal state. Azgomi *et al.* use control technique methods to close the loop and regulate the estimated arousal state within a desired range Azgomi et al. (2019). Furthermore, Wickramasuriya *et al.* utilize the cortisol observations to track energy regulation in Cushing's patients Wickramasuriya and Faghih (2019). Azgomi *et al.* employ the model presented in Wickramasuriya and Faghih (2019) to design a control system for closing the loop and regulating energy state in patients with hypercortisolism Azgomi and Faghih (2019). Similarly, by deconvolving leptin levels and obtaining the hormonal pulses, one may develop a framework to obtain hunger states and regulate it. In another study, researchers in Wickramasuriya and Faghih (2019) have revealed the fact that cortisol secretion in healthy humans has a circadian nature Wickramasuriya and Faghih (2019). They derive multi-day data and observe differences between healthy humans and patients. A similar study can be done for leptin since leptin secretion is also influenced by the circadian rhythm. Similarly, another possible outcome of this study is to understand the leptin regulation behavior in women with obesity against their matched healthy subjects. Pednekar *et al.* Pednekar et al. (2020) uses deconvolution to obtain the underlying hormonal pulses from cortisol data for fibromyalgia, chronic fatigue syndrome, and their matched healthy subjects. The healthy subjects are matched to the patients based on several factors such as age, sex, and weight. The comparison between the parameters and pulses obtained from the patients against their matched healthy subjects showed significant differences in clearance rates and hormonal pulses. A similar study can be done in the future, using leptin data in women with obesity. As another future direction, the model could be further extended to include ACTH and study the differences in hormonal secretory events in women with obesity with respect to healthy subjects Faghih et al. (2015). These hormonal secretory events are an abstraction of their pulsatile nature and provide a good estimation of the pulses originating in the CNS. A thorough investigation of these events could further help us to understand the role of the CNS as well as kidneys and liver in causing obesity.

In this study, we have proposed a state-space model for describing sparse leptin hormone dynamics. Furthermore, we have proposed a coordinate descent approach for the sparse deconvolution of the sampled leptin data to obtain the pulsatile secretion events. The deconvolution framework also provides estimates of the state-space model parameters. The successful modeling of leptin is the first step to a more comprehensive understanding of dynamics and its interaction with cortisol. Our analysis with Spearman correlation and Granger causality has provided some evidence of negative relation between leptin and cortisol. However, some variability has also been observed which prevents us from concluding a concrete explanation. A more thorough investigation is required as the study has limitation such as lack of including insulin and ACTH measurement, no control group without obesity, no data on the participants' eating behaviors/physical activity/metabolism, use of Granger causality test that only consider two factors, a small number of participants, low number of samples, etc.

## 4 METHODS

### 4.1 Experimental Dataset

In this research, we utilize blood sample data collected from eighteen premenopausal women with obesity for our analysis of leptin and cortisol pulse recovery Kok et al. (2008). The participants have given informed consent for participation in the study. They collected data for 18 premenopausal women with obesity over a 24-hour period with 10-minute sampling frequency. All participants were required to have regular menstrual cycles. All studies were done in the early follicular phase of the menstrual cycle. The subjects were within the age group of 22-51 and with BMI between 30-41 kg /m<sup>2</sup>. Subjects were admitted to the research center at 7:00 after an overnight fast. After the subjects rested for 45 min, indirect calorimetry was performed using a ventilated hood for 30 min. Thereafter, a cannula for blood sampling was inserted into an antecubital vein, which was attached to a three-way stopcock and kept patent by a continuous 0.9% NaCl and heparin (1 U/ml) infusion (500 ml/24 h). Each subject followed a consistent schedule with breakfast, lunch, and dinner provided, and had an 8.5 hour sleeping period between 23:00 and 7:30. The blood samples were collected without waking the subjects during this period. Mean fasting glucose concentration was  $5.0 \pm 0.1$  mmol/l (range 4.2– 6.3 mmol/l), insulin  $15.3 \pm 1.7$  mU/l (range 7–28 mU/l), glycosylated hemoglobin (HbA1C)  $4.7 \pm 0.1\%$  (range 3.9 –5.3%). The above study was approved by the Medical Ethics Committee of Leiden University Aschbacher et al. (2014). Subjects were required to be free of chronic disease, and exclusion criteria were fixed, such as shift work, depression, alcohol abuse, and oral contraceptives. The blood samples were assayed every 10 minutes for cortisol and every 20 minutes for leptin and intermediate points were interpolated. Plasma leptin concentrations were estimated by radioimmunoassay with a detection limit of 0.5 ng/liter, and interassay coefficient of variation of 3.6% to 6.8%. Plasma cortisol was measured with a radioimmunoassay with a detection limit is 25 nmol<sup>-1</sup> and the intra-assay coefficient of variation ranges between 2% and 4%. A detailed description of the experiment is provided in Aschbacher et al. (2014); Kok et al. (2008). In this research, we analyze the leptin and cortisol abstraction pulses for 18 subjects obtained from this dataset.

### 4.2 Leptin Model Formulation

The state-space model introduced in this research considers the first-order differential system of equations for leptin synthesis in the adipose tissue and clearance by the renal system. In a 24-hour period, the blood samples were collected at a 10-minute interval, i.e., 144 samples in a day. However, leptin was assayed every 20 min and concentrations was obtained for every 10 min by interpolation. If we assume that leptin secretion can happen in every minutes then there are 1440 possible events for 24-h duration. In a discrete space of 1440 events, we observe between 20 to 40 pulses making it sparse in nature. In this model, we intend to utilize the sparse nature of the hormonal secretory events along with other physiological constraints in a state-space model to estimate the amplitude and frequency of hormonal secretory events. The rate of change of leptin concentration in the adipose tissue is equal to the difference between leptin synthesis rate and the leptin infusion rate from adipose tissue into the blood. Similarly, the rate of change of leptin concentration in the serum is equal to the difference between the leptin infusion rate from adipose tissue into the blood and the leptin clearance rate by the renal system Wolf et al. (2002). The leptin secretion

dynamics are represented as follows:

$$\frac{dx_1(t)}{dt} = -\gamma_1 x_1(t) + u_l(t) \quad (\text{Adipose Tissue}) \quad (1)$$

$$\frac{dx_2(t)}{dt} = \gamma_1 x_1(t) - \gamma_2 x_2(t) \quad (\text{Plasma}) \quad (2)$$

where  $x_1(t)$  and  $x_2(t)$  represent the effective leptin concentrations inside adipose tissue and leptin concentration in the blood serum, respectively. The model parameters  $\gamma_1$  and  $\gamma_2$  represent the infusion rate of leptin by the adipose tissue and the clearance rate of leptin by the kidneys, respectively. Input  $u_l(t)$  represents the abstraction of effective pulses mainly produced by the adipose tissue.  $u_l(t)$  can be modeled as a summation of delta functions with  $u_l(t) = \sum_{i=1}^N q_i \delta(t - \tau_i)$  where  $q_i$  is the magnitude of the pulse initiated at time  $\tau_i$ . If no pulse occurs at  $\tau_i$ ,  $q_i$  will equal zero. The pulses are assumed to occur at integer minutes, i.e., in a 24-hour period, there are 1440 distinct locations ( $N = 1440$ ). The blood samples are collected every 10 minutes for  $M$  samples ( $M = 144$ ). Blood samples were collected beginning at  $y_{l_0}$  and then assayed for leptin. Let  $y_{t_{10}}, y_{t_{20}}, \dots, y_{t_{10M}}, t_k : k = 10, 20, \dots, 10M$ . Furthermore,  $y_{l_0} = y_{t_{10}}$ .

$$y_{l_{t_k}} = x_2(t_k) + \nu_{t_k} \quad (3)$$

where  $y_{t_k}$  and  $\nu_{t_k}$  represent the observed leptin level and hormone measurement error at a time  $t_k$ , respectively. We model  $\nu_{t_k}$  random variables as Gaussian random variable. Then for the system output, we assume that the observed output value at time  $t_k$  will be equal to the previous value,  $y_0$ , multiplied by a decay term and added to a secretion value and the error term. We can represent the solution for every time point  $t_k$  with the following equation:

$$y_{l_{t_k}} = a_{t_k} y_{l_0} + b_{t_k} \mathbf{u}_l + \nu_{t_k} \quad (4)$$

where  $b_{t_k} = \begin{bmatrix} \frac{\gamma_1}{\gamma_1 - \gamma_2} (e^{-\gamma_2 t_k} - e^{-\gamma_1 t_k}) & \frac{\gamma_1}{\gamma_1 - \gamma_2} (e^{-\gamma_2 (k-1)} - e^{-\gamma_1 (k-1)}) & \dots & \frac{\gamma_1}{\gamma_1 - \gamma_2} (e^{-\gamma_2} - e^{-\gamma_1}) & 0 & \dots & 0 \end{bmatrix}'$  and  $a_{t_k} = e^{-\gamma_2 t_k}$ .  $\mathbf{u}_l$  represents the input over the entire 24-hour sampling period, with values  $q_i$  over  $i = 1, \dots, 1440$ .  $\nu_{t_k}$  stands for the vector representation of the random variable  $\nu_{t_k}$ . We then solve for  $a_{t_k}$  and  $b_{t_k}$  using a forced solution approach, multiplying each side of the equation by  $e^{-at}$  and using mathematical methods to obtain the solutions.

From these matrices, we can form a combined representation for the system at any time, using  $\mathbf{y}_l = [y_{t_{10}} \ y_{t_{20}} \ \dots \ y_{t_{10M}}]'$ ,  $\boldsymbol{\gamma} = [\gamma_1 \ \gamma_2]'$ ,  $\mathbf{A}_\gamma = [a_{t_{10}} \ a_{t_{20}} \ \dots \ a_{t_{10M}}]'$ ,  $\mathbf{B}_\gamma = [b_{t_{10}} \ b_{t_{20}} \ \dots \ b_{t_{10M}}]'$ ,  $\mathbf{u}_l = [q_1 \ q_2 \ \dots \ q_N]'$ , and  $\boldsymbol{\nu} = [\nu_{t_{10}} \ \nu_{t_{20}} \ \dots \ \nu_{t_{10M}}]'$ . We then represent the system output as:

$$\mathbf{y}_l = \mathbf{A}_\gamma y_{l_0} + \mathbf{B}_\gamma \mathbf{u}_l + \boldsymbol{\nu}_l$$

where output vector  $\mathbf{y}_l$  is dependent on the initial signal value, the input pulses, and the error term. The values of matrices  $\mathbf{A}_\gamma$  and  $\mathbf{B}_\gamma$  are dependent on the values of  $\boldsymbol{\gamma}$  for the given subject.

### 4.3 Cortisol Model Formulation

In this research, we use the cortisol secretion model provided by Faghih *et al.* Faghih et al. (2014). It exploits the sparse nature of hormonal secretory events and other physiological constraints to estimate



the amplitude and timings of the secretory events using a state-space model. The rate at which cortisol concentration changes in the adrenal glands is equal to the difference between the rate at which it is infused in the blood and the rate at which it is secreted. The rate of change of serum cortisol concentration is equal to the difference between the rate at which it is infused by the adrenal gland into the blood and the rate at which it is cleared by the liver from the blood. The cortisol secretion dynamics are represented as follows:

$$\frac{dx_3(t)}{dt} = -\psi_1 x_3(t) + u_c(t) \quad (\text{Adrenal Glands}) \quad (5)$$

$$\frac{dx_4(t)}{dt} = \psi_1 x_3(t) - \psi_2 x_4(t) \quad (\text{Serum}) \quad (6)$$

where  $x_3(t)$  and  $x_4(t)$  represent the cortisol concentrations in the adrenal glands and the blood serum, respectively. The physiological model parameters  $\psi_1$  and  $\psi_2$  represent the infusion rate of cortisol by the adrenal glands and the cortisol clearance rate by the liver respectively. Input  $u_c(t)$  represent the abstraction of pulses coming from the hypothalamus responsible for the production of cortisol.  $u_c(t)$  can be modeled as a summation of delta functions with  $u_c(t) = \sum_{i=1}^N p_i \delta(t - \tau_i)$ .  $p_i$  is the magnitude of the pulse initiated at time  $\tau_i$ . If no pulse occurs at  $\tau_i$ ,  $q_i$  will equal zero. The pulses are assumed to occur at integer minutes, i.e. in a 24-hour period, there are 1440 distinct locations ( $N = 1440$ ). The blood samples are collected every 10 minutes, for  $M$  samples ( $M = 144$ ). As explained in Section 4.2 the measurement output can be represented as:

$$y_{c_{t_k}} = x_4(t_k) + \omega_{t_k}$$

where  $y_{t_k}$  and  $v_{t_k}$  are the observed cortisol concentration and the measurement error, respectively. The initial concentration of cortisol in adrenal glands and serum is assumed to be zero and  $y_{c_0}$ . The system is further expressed as:

$$\mathbf{y}_c = \mathbf{A}_\psi \mathbf{y}_{c_0} + \mathbf{B}_\psi \mathbf{u}_c + \boldsymbol{\omega}_c$$

$$\begin{aligned} \text{385 where } \mathbf{y}_c &= [y_{t_{10}} \ y_{t_{20}} \ \cdots \ y_{t_{10M}}]', \ \boldsymbol{\psi} = [\psi_1 \ \psi_2]', \ \mathbf{A}_\psi = [a_{t_{10}} \ a_{t_{20}} \ \cdots \ a_{t_{10M}}]', \\ \text{386 } \mathbf{B}_\psi &= [b_{t_{10}} \ b_{t_{20}} \ \cdots \ b_{t_{10M}}]', \ \mathbf{u}_c = [q_1 \ q_2 \ \cdots \ q_N]', \ \boldsymbol{\omega}_c = [\omega_{t_{10}} \ \omega_{t_{20}} \ \cdots \ \omega_{t_{10M}}]', \\ \text{387 } a_{t_i} &= e^{-\psi_2 i} \text{ and } b_{t_i} = \left[ \frac{\psi_1}{\psi_1 - \psi_2} (e^{-\psi_2 i} - e^{-\psi_1 i}) \quad \frac{\psi_1}{\psi_1 - \psi_2} (e^{-\psi_2(i-1)} - e^{-\psi_1(i-1)}) \quad \cdots \quad \frac{\psi_1}{\psi_1 - \psi_2} (e^{-\psi_2} - \right. \\ \text{388 } &\left. e^{-\psi_1}) \quad \underbrace{0 \ \cdots \ 0}_{N-i} \right]'. \end{aligned}$$

### 389 4.3.1 Granger Causality Analysis

We utilize Granger Causality to determine the prospective prediction between leptin and cortisol. Let  $y_{c_{t_k}}$  and  $y_{l_{t_k}}$  are two stationary processes sampled every 10 minutes. A simple causal model for the interaction between leptin and cortisol is given as follows Granger (2001):

$$y_{l_{t_k}} = \sum_{j=1}^{m_{l_1}} \alpha_{l,j} y_{l_{t_k-10j}} + \sum_{j=1}^{m_{l_2}} \beta_{l,j} y_{c_{t_k-10j}} + \epsilon_{l_t} \quad (7)$$

$$y_{c_{t_k}} = \sum_{j=1}^{m_{c_1}} \alpha_{c,j} y_{c_{t_k-10j}} + \sum_{j=1}^{m_{c_2}} \beta_{c,j} y_{l_{t_k-10j}} + \epsilon_{c_t} \quad (8)$$

390 According to the definition of the causality,  $y_{c_{t_k}}$  is causing  $y_{l_{t_k}}$  provided some  $\beta_{l,j}$  is not zero and  $y_{l_{t_k}}$  is  
 391 causing  $y_{c_{t_k}}$  if some  $\beta_{c,j}$  is not zero. If both  $\beta_{c,j}$  and  $\beta_{l,j}$  are not zero, there is exist a feedback relationship  
 392 between  $y_{c_{t_k}}$  and  $y_{l_{t_k}}$ . Here,  $\alpha_{j,j}$  and  $\alpha_{c,j}$  represents the coefficients related to the self interaction.  $\epsilon_{l_t}$  and  
 393  $\epsilon_{c_t}$  represent the model errors. We model them as i.i.d Gaussian random variables.

We use the Bayesian information criteria (BIC) to find  $m_{l_1}$  and  $m_{c_1}$  considering all  $\beta_{l,j} = \beta_{c,j} = 0$ . In contrast, we find the  $m_{l_2}$  and  $m_{c_2}$  using BIC considering  $m_{l_1}$  and  $m_{c_1}$  numbers of nonzero  $\alpha_{l,j}$  and  $\alpha_{c,j}$  in (7) and (8), respectively. We first fit the models in (7) and (8) to the data with  $\beta_{l,j} = \beta_{c,j} = 0$ . We obtain the likelihoods as  $\mathcal{L}_l(y_{l_{t_k}})$  and  $\mathcal{L}_c(y_{c_{t_k}})$ . Then, we assume  $\beta_{l,j}, \beta_{c,j} \neq 0$  and fit the model to the data. We obtain the corresponding likelihood  $\mathcal{L}_l(y_{l_{t_k}}|y_{c_{t_k}})$  and  $\mathcal{L}_c(y_{c_{t_k}}|y_{l_{t_k}})$ . Therefore, the Granger causality depicts how cortisol is causing leptin,

$$LLR_{l,c} = \text{sign}\left(\sum_{j=1}^{m_{l_2}} \beta_{l,j}\right) \times \left(-\log \frac{\mathcal{L}_l(y_{l_{t_k}})}{\mathcal{L}_l(y_{l_{t_k}}|y_{c_{t_k}})}\right)$$

and how leptin is causing cortisol,

$$LLR_{c,l} = \text{sign}\left(\sum_{j=1}^{m_{c_2}} \beta_{c,j}\right) \times \left(-\log \frac{\mathcal{L}_l(y_{l_{t_k}})}{\mathcal{L}_l(y_{l_{t_k}}|y_{c_{t_k}})}\right)$$

394 where negative sign denotes inhibitory, positive sign denotes excitatory, and zero means no prospective  
 395 prediction. The magnitude gives the relative strength of the prospective prediction. Furthermore, we  
 396 calculate the F-statistics for testing whether the prospective prediction is statistically significant or not  
 397 Granger (2001).

## AUTHOR CONTRIBUTIONS STATEMENT

398 RF conceived and designed the study. MA and DP performed the research, analyzed data and wrote the  
 399 manuscript. RF, and MA developed the algorithms and analysis tools. All authors revised the manuscript.

## ADDITIONAL INFORMATION

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## REFERENCES

- 403 Adam, T. C. and Epel, E. S. (2007). Stress, eating and the reward system. *Physiology & behavior* 91,  
 404 449–458
- 405 Ahima, R. S. and Flier, J. S. (2000). Leptin. *Annual review of physiology* 62, 413–437
- 406 Amin, M. R. and Faghih, R. T. (2020). Identification of sympathetic nervous system activation from  
 407 skin conductance: A sparse decomposition approach with physiological priors. *IEEE Transactions on*  
 408 *Biomedical Engineering* 68, 1726–1736
- 409 Antunes, H., Santos, C., and Carvalho, S. (2008). Serum leptin levels in overweight children and  
 410 adolescents. *British journal of nutrition* 101, 1262–1266

- Aschbacher, K., Adam, E. K., Crofford, L. J., Kemeny, M. E., Demitrack, M. A., and Ben-Zvi, A. (2012). Linking disease symptoms and subtypes with personalized systems-based phenotypes: a proof of concept study. *Brain, behavior, and immunity* 26, 1047–1056
- Aschbacher, K., Rodriguez-Fernandez, M., van Wietmarschen, H., Tomiyama, A. J., Jain, S., Epel, E., et al. (2014). The hypothalamic–pituitary–adrenal–leptin axis and metabolic health: a systems approach to resilience, robustness and control. *Interface focus* 4, 20140020
- Azgomi, H. F. and Faghih, R. T. (2019). A wearable brain machine interface architecture for regulation of energy in hypercortisolism. In *2019 53rd Asilomar Conference on Signals, Systems, and Computers* (IEEE), 254–258
- Azgomi, H. F., Wickramasuriya, D. S., and Faghih, R. T. (2019). State-space modeling and fuzzy feedback control of cognitive stress. In *2019 41st Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)* (IEEE), 6327–6330
- Björntorp, P. and Rosmond, R. (2000). Obesity and cortisol. *Nutrition* 16, 924–936
- Blüher, M. (2020). Metabolically healthy obesity. *Endocrine reviews* 41, 405–420
- Bornstein, S. R., Uhlmann, K., Haidan, A., Ehrhart-Bornstein, M., and Scherbaum, W. A. (1997). Evidence for a novel peripheral action of leptin as a metabolic signal to the adrenal gland: leptin inhibits cortisol release directly. *Diabetes* 46, 1235–1238
- Brown, E. N., Meehan, P. M., and Dempster, A. P. (2001). A stochastic differential equation model of diurnal cortisol patterns. *American Journal of Physiology-Endocrinology And Metabolism* 280, E450–E461
- Buckley, T. M. and Schatzberg, A. F. (2005). On the interactions of the hypothalamic-pituitary-adrenal (hpa) axis and sleep: normal hpa axis activity and circadian rhythm, exemplary sleep disorders. *The Journal of Clinical Endocrinology & Metabolism* 90, 3106–3114
- Burgess, E., Hassmén, P., and Pumpa, K. L. (2017). Determinants of adherence to lifestyle intervention in adults with obesity: a systematic review. *Clinical obesity* 7, 123–135
- Cardel, M. I., Atkinson, M. A., Taveras, E. M., Holm, J.-C., and Kelly, A. S. (2020). Obesity treatment among adolescents: a review of current evidence and future directions. *JAMA pediatrics* 174, 609–617
- Chow, C. C. and Hall, K. D. (2014). Short and long-term energy intake patterns and their implications for human body weight regulation. *Physiology & behavior* 134, 60–65
- De Gaetano, A., Hardy, T., Beck, B., Abu-Raddad, E., Palumbo, P., Bue-Valleskey, J., et al. (2008). Mathematical models of diabetes progression. *American Journal of Physiology-Endocrinology and Metabolism* 295, E1462–E1479
- de Graaf, A. A., Freidig, A. P., De Roos, B., Jamshidi, N., Heinemann, M., Rullmann, J. A., et al. (2009). Nutritional systems biology modeling: from molecular mechanisms to physiology. *PLoS computational biology* 5, e1000554
- Dias, D. and Paulo Silva Cunha, J. (2018). Wearable health devices—vital sign monitoring, systems and technologies. *Sensors* 18, 2414
- Engin, A. (2017). Diet-induced obesity and the mechanism of leptin resistance. *Obesity and Lipotoxicity*, 381–397
- Enriori, P. J., Evans, A. E., Sinnayah, P., and Cowley, M. A. (2006). Leptin resistance and obesity. *Obesity* 14, 254S–258S
- Faghih, R. T., Dahleh, M. A., Adler, G. K., Klerman, E. B., and Brown, E. N. (2014). Deconvolution of serum cortisol levels by using compressed sensing. *PloS one* 9, e85204

- 454 Faghih, R. T., Dahleh, M. A., Adler, G. K., Klerman, E. B., and Brown, E. N. (2015). Quantifying  
455 pituitary-adrenal dynamics and deconvolution of concurrent cortisol and adrenocorticotrophic hormone  
456 data by compressed sensing. *IEEE Transactions on Biomedical Engineering* 62, 2379–2388
- 457 Ghasemi, Z., Lee, J. C., Kim, C.-S., Cheng, H.-M., Sung, S.-H., Chen, C.-H., et al. (2018). Estimation  
458 of cardiovascular risk predictors from non-invasively measured diametric pulse volume waveforms via  
459 multiple measurement information fusion. *Scientific reports* 8, 1–11
- 460 Ghosh, M., Sharma, N., Singh, A. K., Gera, M., Pulicherla, K. K., and Jeong, D. K. (2018). Transformation  
461 of animal genomics by next-generation sequencing technologies: a decade of challenges and their impact  
462 on genetic architecture. *Critical reviews in biotechnology* 38, 1157–1175
- 463 González Izquierdo, A., Crujeiras Martínez, A. B., and Couselo Carreira, M. (2019). Leptin, obesity, and  
464 leptin resistance: Where are we 25 years later?
- 465 Granger, C. (2001). Investigating causal relations by econometric models and cross-spectral methods. In  
466 *Essays in econometrics: collected papers of Clive WJ Granger*. 31–47
- 467 Gruzdeva, O., Borodkina, D., Uchasova, E., Dyleva, Y., and Barbarash, O. (2019). Leptin resistance:  
468 underlying mechanisms and diagnosis. *Diabetes, metabolic syndrome and obesity: targets and therapy*  
469 12, 191
- 470 Hall, K. D. (2009). Predicting metabolic adaptation, body weight change, and energy intake in humans.  
471 *American Journal of Physiology-Endocrinology and Metabolism* 298, E449–E466
- 472 Horgan, G. W. (2011). The behaviour of a neutral model of weight regulated only by body mass. *Journal*  
473 *of theoretical biology* 270, 1–6
- 474 Izquierdo, A. G., Crujeiras, A. B., Casanueva, F. F., and Carreira, M. C. (2019). Leptin, obesity, and leptin  
475 resistance: where are we 25 years later? *Nutrients* 11, 2704
- 476 Jacquier, M., Soula, H. A., and Crauste, F. (2015). A mathematical model of leptin resistance. *Mathematical*  
477 *biosciences* 267, 10–23
- 478 Jastreboff, A. M., Kotz, C. M., Kahan, S., Kelly, A. S., and Heymsfield, S. B. (2019). Obesity as a disease:  
479 the obesity society 2018 position statement. *Obesity* 27, 7–9
- 480 Kanaley, J. A., Weltman, J. Y., Pieper, K. S., Weltman, A., and Hartman, M. L. (2001). Cortisol and  
481 growth hormone responses to exercise at different times of day. *The Journal of Clinical Endocrinology*  
482 *& Metabolism* 86, 2881–2889
- 483 Kok, P., Roelfsema, F., Frolich, M., van Pelt, J., Meinders, A. E., and Pijl, H. (2008). Short-term treatment  
484 with bromocriptine improves impaired circadian growth hormone secretion in obese premenopausal  
485 women. *The Journal of Clinical Endocrinology & Metabolism* 93, 3455–3461
- 486 Kok, P., Roelfsema, F., Frolich, M., van Pelt, J., Stokkel, M. P., Meinders, A. E., et al. (2006). Activation of  
487 dopamine d2 receptors simultaneously ameliorates various metabolic features of obese women. *American*  
488 *Journal of Physiology-Endocrinology and Metabolism* 291, E1038–E1043
- 489 Kudielka, B., Buske-Kirschbaum, A., Hellhammer, D., and Kirschbaum, C. (2004). Hpa axis responses to  
490 laboratory psychosocial stress in healthy elderly adults, younger adults, and children: impact of age and  
491 gender. *Psychoneuroendocrinology* 29, 83–98
- 492 Licinio, J., Mantzoros, C., Negrão, A. B., Cizza, G., Wong, M.-L., Bongiorno, P. B., et al. (1997). Human  
493 leptin levels are pulsatile and inversely related to pituitary–adrenal function. *Nature medicine* 3, 575
- 494 Licinio, J., Negrão, A. B., Mantzoros, C., Kaklamani, V., Wong, M.-L., Bongiorno, P. B., et al. (1998). Sex  
495 differences in circulating human leptin pulse amplitude: clinical implications. *The Journal of Clinical*  
496 *Endocrinology & Metabolism* 83, 4140–4147
- 497 Liou, D. and Kulik, L. (2020). Self-efficacy and psychosocial considerations of obesity risk reduction  
498 behaviors in young adult white americans. *Plos one* 15, e0235219

- 499 Maioli, S., Lodeiro, M., Merino-Serrais, P., Falahati, F., Khan, W., Puerta, E., et al. (2015). Alterations  
500 in brain leptin signalling in spite of unchanged csf leptin levels in alzheimer's disease. *Aging cell* 14,  
501 122–129
- 502 Mantzoros, C. S., Magkos, F., Brinkoetter, M., Sienkiewicz, E., Dardeno, T. A., Kim, S.-Y., et al. (2011).  
503 Leptin in human physiology and pathophysiology. *American Journal of Physiology-Endocrinology and*  
504 *Metabolism* 301, E567–E584
- 505 Myers Jr, M. G., Leibel, R. L., Seeley, R. J., and Schwartz, M. W. (2010). Obesity and leptin resistance:  
506 distinguishing cause from effect. *Trends in Endocrinology & Metabolism* 21, 643–651
- 507 Ng, M., Fleming, T., Robinson, M., Thomson, B., Graetz, N., Margono, C., et al. (2014). Global, regional,  
508 and national prevalence of overweight and obesity in children and adults during 1980–2013: a systematic  
509 analysis for the global burden of disease study 2013. *The lancet* 384, 766–781
- 510 Organization, W. H. (2000). *Obesity: preventing and managing the global epidemic*. 894 (World Health  
511 Organization)
- 512 Pednekar, D. D., Amin, M. R., Azgomi, H. F., Aschbacher, K., Crofford, L. J., and Faghih, R. T. (2020).  
513 Characterization of cortisol dysregulation in fibromyalgia and chronic fatigue syndromes: A state-space  
514 approach. *IEEE Transactions on Biomedical Engineering* 67, 3163–3172
- 515 Pednekar, D. D., Amin, M. R., Fekri, H., Aschbacher, C. L. J., Kirstin, and Faghih, R. T. (2019). A  
516 system theoretic investigation of cortisol dysregulation in fibromyalgia patients with chronic fatigue. In  
517 *Engineering in Medicine and Biology Society (EMBC), 2019 41st Annual International Conference of*  
518 *the IEEE (IEEE)*
- 519 Pralong, F. P., Roduit, R., Waeber, G., Castillo, E., Mosimann, F., Thorens, B., et al. (1998). Leptin  
520 inhibits directly glucocorticoid secretion by normal human and rat adrenal gland. *Endocrinology* 139,  
521 4264–4268
- 522 Reinehr, T. (2013). Lifestyle intervention in childhood obesity: changes and challenges. *Nature Reviews*  
523 *Endocrinology* 9, 607–614
- 524 Rodriguez, A. C. I., Epel, E. S., White, M. L., Standen, E. C., Seckl, J. R., and Tomiyama, A. J. (2015).  
525 Hypothalamic-pituitary-adrenal axis dysregulation and cortisol activity in obesity: a systematic review.  
526 *Psychoneuroendocrinology* 62, 301–318
- 527 Salehi, S. M. (2017). Engineering cell-free protein synthesis technology for codon reassignment,  
528 biotherapeutics production using just-add-water system, and biosensing endocrine disrupting compounds
- 529 Seo, Y.-G., Lim, H., Kim, Y., Ju, Y.-S., Lee, H.-J., Jang, H. B., et al. (2019). The effect of a multidisciplinary  
530 lifestyle intervention on obesity status, body composition, physical fitness, and cardiometabolic risk  
531 markers in children and adolescents with obesity. *Nutrients* 11, 137
- 532 Simon, C., Gronfier, C., Schlienger, J., and Brandenberger, G. (1998). Circadian and ultradian variations of  
533 leptin in normal man under continuous enteral nutrition: relationship to sleep and body temperature. *The*  
534 *Journal of Clinical Endocrinology & Metabolism* 83, 1893–1899
- 535 Sominsky, L. and Spencer, S. J. (2014). Eating behavior and stress: a pathway to obesity. *Frontiers in*  
536 *psychology* 5, 434
- 537 Sudi, K., Gallistl, S., Weinhandl, G., Payer, C., Cartellieri, M., and Borkenstein, M. H. (2000). No  
538 relationship between leptin and cortisol in obese children and adolescents. *Journal of Pediatric*  
539 *Endocrinology and Metabolism* 13, 913–922
- 540 Tam, J., Fukumura, D., and Jain, R. K. (2009). A mathematical model of murine metabolic regulation by  
541 leptin: energy balance and defense of a stable body weight. *Cell metabolism* 9, 52–63

- 542 Udden, J., Björntorp, P., Arner, P., Barkeling, B., Meurling, L., and Rössner, S. (2003). Effects of  
543 glucocorticoids on leptin levels and eating behaviour in women. *Journal of internal medicine* 253,  
544 225–231
- 545 Wang, Y., Beydoun, M. A., Min, J., Xue, H., Kaminsky, L. A., and Cheskin, L. J. (2020). Has the  
546 prevalence of overweight, obesity and central obesity levelled off in the united states? trends, patterns,  
547 disparities, and future projections for the obesity epidemic. *International Journal of Epidemiology*
- 548 Wickramasuriya, D. S., Amin, M., Faghih, R. T., et al. (2019). Skin conductance as a viable alternative for  
549 closing the deep brain stimulation loop in neuropsychiatric disorders. *Frontiers in neuroscience* 13, 780
- 550 Wickramasuriya, D. S. and Faghih, R. T. (2019). A cortisol-based energy decoder for investigation of  
551 fatigue in hypercortisolism. In *2019 41st Annual International Conference of the IEEE Engineering in*  
552 *Medicine and Biology Society (EMBC)* (IEEE), 11–14
- 553 Wolf, G., Chen, S., Han, D. C., and Ziyadeh, F. N. (2002). Leptin and renal disease. *American Journal of*  
554 *Kidney Diseases* 39, 1–11
- 555 Yupanqui-Lozno, H., Bastarrachea, R. A., Yupanqui-Velazco, M. E., Alvarez-Jaramillo, M., Medina-  
556 Méndez, E., Giraldo-Peña, A. P., et al. (2019). Congenital leptin deficiency and leptin gene missense  
557 mutation found in two colombian sisters with severe obesity. *Genes* 10, 342
- 558 to draw Diagrams/Figures/Structures should be avoided. They should be external callouts including  
559 graphics.