# Password Strength Signaling: A Counter-Intuitive Defense Against Password Cracking 

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#### Abstract

We introduce password strength signaling as a potential defense against password cracking. Recent breaches have exposed billions of user passwords to the dangerous threat of offline password cracking attacks. An offline attacker can quickly check millions (or sometimes billions/trillions) of password guesses by comparing a candidate password's hash value with a stolen hash from a breached authentication server. The attacker is limited only by the resources he is willing to invest. We explore the feasibility of applying ideas from Bayesian Persuasion to password authentication. Our key idea is to have the authentication server store a (noisy) signal about the strength of each user password for an offline attacker to find. Surprisingly, we show that the noise distribution for the signal can often be tuned so that a rational (profit-maximizing) attacker will crack fewer passwords. The signaling scheme exploits the fact that password cracking is not a zero-sum game i.e., it is possible for an attacker to increase their profit in a way that also reduces the number of cracked passwords. Thus, a well-defined signaling strategy will encourage the attacker to reduce his guessing costs by cracking fewer passwords. We use an evolutionary algorithm to compute the optimal signaling scheme for the defender. We evaluate our mechanism on several password datasets and show that it can reduce the total number of cracked passwords by up to $12 \%$ (resp. $5 \%$ ) of all users in defending against offline (resp. online) attacks. While the results of our empirical analysis are positive we stress that we view the current solution as a proof-of-concept as there are important societal concerns that would need to be considered before adopting our password strength signaling solution.


Keywords: Bayesian Persuasion, Password Authentication, Stackelberg Game.

## 1 Introduction

In the last decade, large scale data-breaches have exposed billions of user passwords to the dangerous threat of offline password cracking. An offline attacker who has obtained the salt and cryptographic hash $\left(\left(h_{u}\right.\right.$, salt $\left._{u}\right)=$
$\left(H\left(\right.\right.$ salt $\left._{u}, p w_{u}\right)$, salt $\left.\left._{u}\right)\right)$ of a user $u$ 's password $\left(p w_{u}\right)$ can attempt to crack the password by comparing this hash value with the hashes of likely password guesses i.e., by checking if $h_{u}^{\prime}=H\left(s a l t_{u}, p w^{\prime}\right)$ for each $p w^{\prime}$. The attacker can check as many guesses as he wants offline - without interacting with the authentication server. The only limit is the resources that the attacker is willing to invest in trying to crack the password. A rational password cracker $[9,12]$ will choose the number of guesses that maximizes his utility.

Password hashing serves as a last line of defense against an offline password attacker. A good password hash function $H$ should be moderately expensive to compute so that it becomes prohibitively expensive to check millions or billions of password guesses. However, we cannot make $H$ too expensive to compute as the honest authentication server needs to evaluate $H$ every time a user authenticates. In this paper, we explore a highly counter-intuitive ${ }^{1}$ defense against rational attackers which does not impact hashing costs: password strength signaling! In particular, we apply Bayesian Persuasion [30] to password authentication. Specifically, we propose to have the authentication server store a (noisy) signal $s i g_{u}$ which is correlated with the strength of the user's password.

Traditionally, an authentication server stores the tuple ( $u$, salt $_{u}, h_{u}$ ) for each user $u$ where salt $_{u}$ is a random salt value and $h_{u}=H\left(s a l t_{u}, p w_{u}\right)$ is the salted hash. We propose to have the authentication server instead store the tuple ( $u$, salt ${ }_{u}, \operatorname{sig}_{u}, h_{u}$ ), where the (noisy) signal $\operatorname{sig}_{u}$ is sampled based on the strength of the user's password $p w_{u}$. The signal $s i g_{u}$ is simply recorded for an offline attacker to find if the authentication server is breached. In fact, the authentication server never even uses $\operatorname{sig}_{u}$ when the user $u$ authenticates ${ }^{2}$. The attacker will only use the signal $s i g_{u}$ if it is beneficial - at minimum the attacker could always choose to ignore the signal.

It is natural, but incorrect, to imagine that password cracking is a zerosum game i.e., the attacker's gain is directly proportional to the defender's loss. In a zero-sum game there would be no benefit from information signaling [59] e.g., in a zero-sum game like rock-paper-scissors there is no benefit to leaking information about your action. However, we stress that password cracking is not a zero-sum game. The defender's (the sender of strength signal) utility is inversely proportional to the fraction of user passwords that are cracked. By contrast, it is possible that the attacker's utility is marginal even when he cracks a password i.e., when guessing costs offset the reward. In particular, the attacker's utility is given by the (expected) value of all of the cracked passwords minus his (expected) guessing costs. Thus, it is possible that password strength signaling would persuade the attacker to crack fewer passwords to reduce guessing costs. Indeed, we show that the signal distribution can be tuned so that a rational (profit-maximizing) attacker will crack fewer passwords.

[^0]To provide some intuition of why password strength signaling might be beneficial, we give two examples.
Example 1 Suppose that we add a signal $\operatorname{sig}_{u}=1$ to indicate that user $u$ 's password $p w_{u}$ is uncrackable (e.g., the entropy of the password is over 60-bits) and we add the signal $\operatorname{sig}_{u}=0$ otherwise. In this case, the attacker will simply choose to ignore accounts with $\operatorname{sig}_{u}=1$ to reduce his total guessing cost. However, the number of cracked user passwords stays unchanged.
Example 2 Suppose that we modify the signaling scheme above so that even when the user's password $p w_{u}$ is not deemed to be uncrackable we still signal $\operatorname{sig}_{u}=1$ with probability $\epsilon$ and $\operatorname{sig}_{u}=0$ otherwise. If the user's password is uncrackable we always signal $\operatorname{sig}_{u}=1$. Assuming that $\epsilon$ is not too large a rational attacker might still choose to ignore any account with $\operatorname{sig}_{u}=1$ i.e., the attacker's expected reward will decrease slightly, but the attacker's guessing costs will also be reduced. In this example, the fraction of cracked user passwords is reduced by up to $\epsilon$ i.e., any lucky user $u$ with $\operatorname{sig}_{u}=1$ will not have their password cracked.

In this work, we explore the following questions: Can password strength signaling be used to protect passwords against rational attackers? If so, how can we compute the optimal signaling strategy?

### 1.1 Contributions

We introduce password information signaling as a novel, counter-intuitive, defense against rational password attackers. We adapt a Stackelberg game-theoretic model of Blocki and Datta [9] to characterize the behavior of a rational password adversary and the optimal signaling strategy for an authentication server (defender). We analyze the performance of password information signaling using several large password datasets: Bfield, Brazzers, Clixsense, CSDN, Neopets, 000webhost, RockYou, Yahoo! [10, 14], and LinkedIn [8]. We analyze our mechanism both in the idealistic setting, where the defender has perfect knowledge of the user password distribution $\mathcal{P}$ and the attacker's value $v$ for each cracked password, as well as in a more realistic setting where the defender only is given approximations of $\mathcal{P}$ and $v$. In our experiments, we analyze the fraction $x_{s i g}(v)$ (resp. $\left.x_{n o-s i g}(v)\right)$ of passwords that a rational attacker would crack if the authentication server uses (resp. does not use) password information signaling. We find that the reduction in the number of cracked passwords can be substantial e.g., $x_{n o-s i g}(v)-x_{s i g}(v) \approx 8 \%$ under empirical distribution and $13 \%$ under Monte Carlo distribution. We also show that password strength signaling can be used to help deter online attacks when CAPTCHAs are used for throttling.

An additional advantage of our password strength signaling method is that it is independent of the password hashing method and requires no additional hashing work. Implementation involves some determination of which signal to attach to a certain account, but beyond that, any future authentication attempts are handled exactly as they were before i.e. the signal information is ignored.

We conclude by discussing several societal and ethical issues that would need to be addressed before password strength signaling is used. While password
strength signaling decreases the total number of compromised accounts, there may be a few users whose accounts are cracked because they were assigned an "unlucky" signal. One possible solution might be to allow users to opt-in (resp. opt-out). Another approach might try to constrain the solution space to ensure that there are no "unlucky" users.

### 1.2 Related Work

The human tendency to pick weaker passwords has been well documented e.g., [14]. Convincing users to select stronger passwords is a difficult task [16,28,33,4749]. One line of research uses password strength meters to nudge users to select strong passwords $[17,32,52]$ though a common finding is that users were not persuaded to select a stronger password $[17,52]$. Another approach is to require users to follow stringent guidelines when they create their password. However it has been shown that these methods also suffer from usability issues [3, 24, 28,50], and in some cases can even lead to users selecting weaker passwords [13, 33].

Offline password cracking attacks have been around for decades [38]. There is a large body of research on password cracking techniques. State of the art cracking methods employ methods like Probabilistic Context-Free Grammars [31, 55, 58], Markov models [19, 20, 36, 53], and neural networks [37]. Further work [35] has described methods of retrieving guessing numbers from commonly used tools like Hashcat [1] and John the Ripper [23].

Blocki and Datta [9] used a Stackelberg game to model the behavior of a rational (profit-motivated) attacker against a cost-asymmetric secure hashing (CASH) scheme. However, the CASH mechanism is not easily integrated with modern memory-hard functions. By contrast, password strength signaling does not require any changes to the password hashing algorithm.

A large body of research has focused on alternatives to text passwords. Alternatives have included one time passwords [25,34, 41], challenge-response constructions [21,29], hardware tokens [40,46], and biometrics [4,22,45]. While all of these offer possible alternatives to traditional passwords it has been noted that none of these strategies outperforms passwords in all areas [15]. Furthermore, it has been noted that despite the shortcomings of passwords they remain the dominant method of authentication even today, and research should acknowledge this fact and seek to better understand traditional password use [27].

Password strength signaling is closely related to the literature on Bayesian Persuasion. Kamenica and Gentzkow [30] first introduced the notion of Bayesian Persuasion where a person (sender) chooses a signal to reveal to a receiver in an attempt to convince the receiver to take an action that positively impacts the welfare of both parties. There are a few prior results applying Bayesian Persuasion in security contexts, e.g., patrols [18], honeypots [43], with the sender (resp. receiver) playing the roles of defender (resp. attacker). To the best of our knowledge Bayesian Persuasion has never been applied in the context of password authentication. Most prior works use linear programming to find (or approximate) the sender's optimal signaling strategy. We stress that there are several unique challenges in the context of password authentication: (1) the
action space of the receiver (attacker) is exponential in the size of (the support of) the password distribution, and (2) the sender's objective function is nonlinear.

## 2 Preliminaries

We use $\mathbb{P}$ to denote the set of all passwords that a user might select and use $\mathcal{P}$ to denote a distribution over user-selected passwords i.e., a new user will select the password $p w \in \mathbb{P}$ with probability $\operatorname{Pr}_{x \sim \mathcal{P}}[x=p w]$ - we typically write $\operatorname{Pr}[p w]$ for notational simplicity.
Password Datasets Given a set of $N$ users $\mathcal{U}=\left\{u_{1}, \ldots, u_{N}\right\}$ the corresponding password dataset $D_{u}$ is given by the multiset $D_{u}=\left\{p w_{u_{1}}, \ldots, p w_{u_{N}}\right\}$ where $p w_{u_{i}}$ denotes the password selected by user $u_{i}$. Fixing a password dataset $D$ we let $f_{i}$ denote the number of users who selected the $i$ th most popular password in the dataset. We note that that $f_{1} \geq f_{2} \geq \ldots$ and that $\sum_{i} f_{i}=N$ gives the total number $N$ of users in the original dataset.

Empirical Password Distribution Viewing our dataset $D$ as $N$ independent samples from the (unknown) distribution $\mathcal{P}$, we use $f_{i} / N$ as an empirical estimate of the probability of the $i t h$ most common password $p w_{i}$ and $D_{f}=\left(f_{1}, f_{2}, \ldots\right)$ as the corresponding frequency list. In addition, $\mathcal{D}_{e}$ is used to denoted the corresponding empirical distribution i.e., $\operatorname{Pr}_{x \sim \mathcal{D}_{e}}\left[x=p w_{i}\right]=f_{i} / N$. Because the real distribution $\mathcal{P}$ is unknown we will typically work with the empirical distribution $\mathcal{D}_{e}$. We remark that when $f_{i} \gg 1$ the empirical estimate will be close to the actual distribution i.e., $\operatorname{Pr}\left[p w_{i}\right] \approx f_{i} / N$, but when $f_{i}$ is small the empirical estimate will likely diverge from the true probability value. Thus, while the empirical distribution is useful to analyze the performance of password strength signaling when the password value $v$ is small, this analysis will be less accurate for larger values of $v$ i.e., once the rational attacker has an incentive to start cracking passwords with lower frequency.
Monte Carlo Password Distribution Following [5] we also use the Monte Carlo Password Distribution $\mathcal{D}_{m}$ to evaluate the performance of our password signaling mechanism when $v$ is large. The Monte Carlo distributions is derived by subsampling passwords from our dataset $D$, generating guessing numbers from state-of-the-art password cracking models, and fitting a distribution to the resulting guessing curve. Due to the length limits, we omit discussion and experiment results for Monte Carlo Password Distribution. See more details in the full version of this paper [6].

## 3 Strength Signaling and Password Storage

In this section, we overview our basic signaling mechanism deferring until later how to optimally tune the parameters of the mechanism to minimize the number of cracked passwords.

### 3.1 Account Creation and Signaling

When users create their accounts they provide a user name $u$ and password $p w_{u}$. First, the server runs canonical password storage procedure - randomly selecting a salt value $s a l t_{u}$ and calculating the hash value $h_{u}=H\left(s a l t_{u}, p w_{u}\right)$. Next, the server calculates the (estimated) strength $\operatorname{str}_{u} \leftarrow \operatorname{getStrength}\left(p w_{u}\right)$ of password $p w_{u}$ and samples the signal $\operatorname{sig}_{u} \stackrel{\$}{\leftarrow}$ getSignal $\left(s t_{u}\right)$. Finally, the server stores the tuple ( $u$, salt $_{u}, \operatorname{sig}_{u}, h_{u}$ ) - later if the user $u$ attempts to login with a password $p w^{\prime}$ the authentication server will accept $p w^{\prime}$ if and only if $h_{u}=H\left(s_{a l t}, p w^{\prime}\right)$. The account creation process is formally presented in Algorithm 1.

```
Algorithm 1 Signaling during Account Creation
Input: \(u, p w_{u}, L, d\)
    salt \(_{u} \stackrel{\$}{\leftarrow}\{0,1\}^{L}\)
    \(h_{u} \leftarrow H\left(s a l t_{u}, p w_{u}\right)\)
    str \(_{u} \leftarrow\) getStrength \(\left(p w_{u}\right)\)
    sig \(_{u} \stackrel{\$}{\stackrel{\$}{\leftarrow}}\) getSignal \(\left(\operatorname{str}_{u}\right)\)
    StoreRecord \(\left(u\right.\), salt \(\left._{u}, \operatorname{sig}_{u}, h_{u}\right)\)
```

A traditional password hashing solution would simply store the tuple ( $u, s a l t_{u}, h_{u}$ ) i.e., excluding the signal $\operatorname{sig}_{u}$. Our mechanism requires two additionally subroutines getStrength() and getSignal() to generate this signal. The first algorithm is deterministic. It takes the user's password $p w_{u}$ as input and outputs $s t r_{u}$ - (an estimate of) the password strength. The second randomized algorithm takes the (estimated) strength parameter $s t r_{u}$ and outputs a signal $s i g_{u}$. The whole signaling algorithm is the composition of these two subroutines i.e., $\mathcal{A}=\operatorname{getSignal}(\operatorname{get} \operatorname{Strength}(p w))$. We use $s_{i, j}$ to denote the probability of observing the signal $\operatorname{sig}_{u}=j$ given that the estimated strength level was $\operatorname{str}_{u}=i$. Thus, getSignal() can be encoded using a signaling matrix $\mathbf{S}$ of dimension $a \times b$,

$$
\left[\begin{array}{cccc}
s_{0,0} & s_{0,1} & \cdots & s_{0, b-1} \\
s_{1,0} & s_{1,1} & \cdots & s_{1, b-1} \\
\vdots & \vdots & \ddots & \vdots \\
s_{a-1,0} & s_{a-1,1} & \cdots & s_{a-1, b-1}
\end{array}\right]
$$

where $a$ is the number of strength levels that passwords can be labeled, $b$ is the number of signals the server can generate and $\mathbf{S}[i, j]=s_{i, j}$.

We remark that if $\mathbf{S}[i, 0]=1$ for all $i^{3}$ then the actual signal $\operatorname{sig}_{u}$ is uncorrelated with the password $p w_{u}$. In this case our mechanism is equivalent to the traditional (salted) password storage mechanism where getSignal() is replaced with a constant/null function. getStrength() is password strength oracle that outputs the actual/estimated strength of a password. We discuss ways that getStrength () could be implemented in full version of this paper [6].

[^1]
### 3.2 Generating Signals

We use $[a]=\{0,1, \ldots, a-1\}$ (resp. $[b]=\{0,1, \ldots, b-1\}$ ) to denote the range of getStrength() (resp. getSignal()). For example, if $[a]=\{0,1,2\}$ then 0 would correspond to weak passwords, 2 would correspond to strong passwords and 1 would correspond to medium strength passwords. To generate signal for $p w_{u}$, the server first invokes subroutine getStrength $\left(p w_{u}\right)$ to get strength level $\operatorname{str}_{u}=$ $i \in[a]$ of $p w_{u}$, then signals $\operatorname{sig}_{u}=j \in[b]$ with probability $\operatorname{Pr}\left[\operatorname{get} \operatorname{Signal}\left(p w_{u}\right)=\right.$ $\left.j \mid \operatorname{getStrength}\left(p w_{u}\right)=i\right]=\mathbf{S}[i, j]=s_{i, j}$.
Bayesian Update. An attacker who breaks into the authentication server will be able to observe the signal $\operatorname{sig}_{u}$ and $\mathbf{S}$. After observing the signal $\operatorname{sig}_{u}=y$ and $\mathbf{S}$ the attacker can perform a Bayesian update. In particular, given any password $p w \in \mathbb{P}$ with strength $i=\operatorname{get}$ Strength $(p w)$ we have

$$
\begin{align*}
\operatorname{Pr}[p w \mid y] & =\frac{\operatorname{Pr}[p w] \mathbf{S}[i, y]}{\sum_{p w^{\prime} \in \mathbb{P}} \operatorname{Pr}\left[\text { getSignal }\left(\operatorname{get} \operatorname{Strength}\left(p w^{\prime}\right)\right)\right] \cdot \operatorname{Pr}\left[p w^{\prime}\right]} \\
& =\frac{\operatorname{Pr}[p w] \mathbf{S}[i, y]}{\sum_{i^{\prime} \in[a]} \operatorname{Pr}_{p w^{\prime} \sim \mathcal{P}}\left[\operatorname{getStrength}\left(p w^{\prime}\right)=i^{\prime}\right] \cdot \mathbf{S}\left[i^{\prime}, y\right]} \tag{1}
\end{align*}
$$

If the attacker knew the original password distribution $\mathcal{P}$ then $s /$ he can update posterior distribution $\mathcal{P}_{y}$ with $\operatorname{Pr}_{x \sim \mathcal{P}_{y}}[x=p w]:=\operatorname{Pr}[p w \mid y]$. We extend our notation, let $\lambda(\pi, B ; y)=\sum_{i=1}^{B} \operatorname{Pr}\left[p w_{i}^{\pi} \mid y\right]$ where $p w_{i}^{\pi}$ is the $i$ th password in the ordering $\pi$. Intuitively, $\lambda(\pi, B ; y)$ is the conditional probability of cracking the user's password by checking the first $B$ guesses in permutation $\pi$ after observing signal $y$.

### 3.3 Delayed Signaling

In some instances, the authentication server might implement the password strength oracle getStrength() by training a (differentially private) Count-Sketch based on the user-selected passwords $p w_{u} \sim \mathcal{P}$, detailed discussion about use of count-sketch in password strength signaling can be found in full version of this paper [6]. The strength estimation will not be accurate until a larger number $N$ of users have registered. In this case, the authentication server may want to delay signaling until after the Count-Sketch has been initialized. In particular, the authentication server will store the tuple ( $u, \operatorname{salt}_{u}, \operatorname{sig}_{u}=\perp, h_{u}$ ) when users first register their accounts. After the count-sketch has been initialized, the server can update $\operatorname{sig}_{u}=$ getSignal (getStrength $\left.\left(p w_{u}\right)\right)$ upon users' next successful login.

## 4 Adversary Model

We adapt the economic model of [9] to capture the behavior of a rational attacker. We also make several assumptions: (1) there is a value $v_{u}$ for each password $p w_{u}$ that the attacker cracks; (2) the attacker is untargeted and that the value $v_{u}=v$
for each user $u \in U$; (3) by Kerckhoffs's principle, the password distribution $\mathcal{P}$ and the signaling matrix are known to the attacker.
Value/Cost Estimates One can derive a range of estimates for $v$ based on black market studies e.g., Symantec reported that passwords generally sell for $\$ 4-\$ 30[26]$ and [51] reported that Yahoo! e-mail passwords sold for $\approx \$ 1$. Similarly, we assume that the attacker pays a cost $k$ each time he evaluates the hash function $H$ to check a password guess. We remark that one can estimate $k \approx \$ 1 \times 10^{-7}$ if we use a memory-hard function ${ }^{4}$.

### 4.1 Adversary Utility: No Signaling

We first discuss how a rational adversary would behave when no signal is available (traditional hashing). We defer the discussion of how the adversary would update his strategy after observing a signal $y$ to the next section. In the nosignaling case, the attacker's strategy $(\pi, B)$ is given by an ordering $\pi$ over passwords $\mathbb{P}$ and a threshold $B$. Intuitively, this means that the attacker will check the first $B$ guesses in $\pi$ and then give up. The expected reward for the attacker is given by the simple formula $v \times \lambda(\pi, B)$, i.e., the probability that the password is cracked times the value $v$. Similarly, the expected guessing cost of the attacker is

$$
\begin{equation*}
C(k, \pi, B)=k \sum_{i=1}^{B}(1-\lambda(\pi, i-1)) \tag{2}
\end{equation*}
$$

Intuitively, $(1-\lambda(\pi, i-1))$ denotes the probability that the adversary actually has to check the $i$ th password guess at cost $k$. With probability $\lambda(\pi, i-1)$ the attacker will find the password in the first $i-1$ guesses and will not have to check the $i$ th password guess $p w_{i}^{\pi}$. Specially, we define $\lambda(\pi, 0)=0$. The adversary's expected utility is the difference of expected gain and expected cost, namely,

$$
\begin{equation*}
U_{a d v}(v, k, \pi, B)=v \cdot \lambda(\pi, B)-C(k, \pi, B) \tag{3}
\end{equation*}
$$

Sometimes we omit parameters in the parenthesis and just write $U_{a d v}$ for short when the $v, k$ and $B$ are clear from context.

### 4.2 Optimal Attacker Strategy: No Signaling

A rational adversary would choose $\left(\pi^{*}, B^{*}\right) \in \arg \max U_{a d v}(v, k, \pi, B)$. It is easy to verify that the optimal ordering $\pi^{*}$ is always to check passwords in descending order of probability. The probability that a random user's account is cracked is

$$
\begin{equation*}
P_{a d v}=\lambda\left(\pi^{*}, B^{*}\right) \tag{4}
\end{equation*}
$$

[^2]We remark that in practice $\arg \max U_{a d v}(v, k, \pi, B)$ usually returns a singleton set $\left(\pi^{*}, B^{*}\right)$. If instead the set contains multiple strategies then we break ties adversarially i.e.,

$$
P_{a d v}=\max _{\left(\pi^{*}, B^{*}\right) \in \arg \max U_{a d v}(v, k, \pi, B)} \lambda\left(\pi^{*}, B^{*}\right)
$$

## 5 Information Signaling as a Stackelberg Game

We model the interaction between the authentication server (leader) and the adversary (follower) as a two-stage Stackelberg game. In a Stackelberg game, the leader moves first and then the follower may select its action after observing the action of the leader.

In our setting the action of the defender is to commit to a signaling matrix $\mathbf{S}$ as well as the implementation of getStrength() which maps passwords to strength levels. The attacker responds by selecting a cracking strategy $(\vec{\pi}, \vec{B})=$ $\left\{\left(\pi_{0}, B_{0}\right), \ldots,\left(\pi_{b-1}, B_{b-1}\right)\right\}$. Intuitively, this strategy means that whenever the attacker observes a signal $y$ he will check the top $B_{y}$ guesses according to the ordering $\pi_{y}$.

### 5.1 Attacker Utility

If the attacker checks the top $B_{y}$ guesses according to the order $\pi_{y}$ then the attacker will crack the password with probability $\lambda\left(\pi_{y}, B_{y} ; y\right)$. Recall that $\lambda\left(\pi_{y}, B_{y} ; y\right)$ denotes the probability of the first $B_{y}$ passwords in $\pi_{y}$ according to the posterior distribution $\mathcal{P}_{y}$ obtained by applying Bayes Law after observing a signal $y$. Extrapolating from no signal case, the expected utility of adversary conditioned on observing the signal $y$ is

$$
\begin{equation*}
U_{a d v}\left(v, k, \pi_{y}, B_{y} ; \mathbf{S}, y\right)=v \cdot \lambda\left(\pi_{y}, B_{y} ; y\right)-\sum_{i=1}^{B_{y}} k \cdot\left(1-\lambda\left(\pi_{y}, i-1 ; y\right)\right) \tag{5}
\end{equation*}
$$

where $B_{y}$ and $\pi_{y}$ are now both functions of the signal $y$. Intuitively, $\left(1-\lambda\left(\pi_{y}, i-1 ; y\right)\right)$ denotes the probability that the attacker has to pay cost $k$ to make the $i$ th guess. We use $U_{a d v}^{s}(v, k,\{\mathbf{S},(\vec{\pi}, \vec{B})\})$ to denote the expected utility of the adversary with password strength signaling,

$$
\begin{equation*}
U_{a d v}^{s}(v, k,\{\mathbf{S},(\vec{\pi}, \vec{B})\})=\sum_{y \in[b]} \operatorname{Pr}[\operatorname{Sig}=y] U_{a d v}\left(v, k, \pi_{y}, B_{y} ; \mathbf{S}, y\right) \tag{6}
\end{equation*}
$$

where

$$
\operatorname{Pr}[S i g=y]=\sum_{i \in[b]} \underset{p w \sim \mathcal{P}}{ } \operatorname{Pr}[\operatorname{get} \operatorname{Strength}(p w)=i] \cdot S[i, y] .
$$

### 5.2 Optimal Attacker Strategy

Now we discuss how to find the optimal strategy $\left(\vec{\pi}^{*}, \vec{B}^{*}\right)$. Since the attacker's strategies in reponse to different signals are independent. It suffices to find $\left(\pi_{y}^{*}, B_{y}^{*}\right) \in \arg \max _{B_{y}, \pi_{y}} U_{a d v}\left(v, k, \pi_{y}, B_{y} ; y\right)$ for each signal $y$. We first remark that the adversary can obtain the optimal checking sequence $\pi_{y}^{*}$ for $p w_{u}$ associated with signal $y$ by sorting all $p w \in \mathcal{P}$ in descending order of posterior probability according to the posterior distribution $\mathcal{P}_{y}$.

Given the optimal guessing order $\pi_{y}^{*}$, the adversary can determine the optimal budget $B_{y}^{*}$ for signal $y$ such that $B_{y}^{*}=\arg \max _{B_{y}} U_{a d v}\left(v, k, \pi_{y}^{*}, B_{y} ; y\right)$. Each of the password distributions we analyze has a compact representation allowing us to apply techniques from [5] to further speed up the computation of the attacker's optimal strategy $\pi_{y}^{*}$ and $B_{y}^{*}$.

We observe that an adversary who sets $\pi_{y}=\pi$ and $B_{y}=B$ for all $y \in[b]$ is effectively ignoring the signal and is equivalent to an adversary in the no signal case. Thus,

$$
\begin{equation*}
\max _{\vec{\pi}, \vec{B}} U_{a d v}^{s}(v, k,\{\mathbf{S},(\vec{\pi}, \vec{B})\}) \geq \max _{\pi, B} U_{a d v}(v, k, \pi, B), \forall \mathbf{S} \tag{7}
\end{equation*}
$$

implying that adversary's expected utility will never decrease by adapting its strategy according to the signal.

### 5.3 Optimal Signaling Strategy

Once the function getStrength() is fixed we want to find the optimal signaling matrix $\mathbf{S}$. We begin by introducing the defender's utility function. Intuitively, the defender wants to minimize the total number of cracked passwords.

Let $P_{a d v}^{s}(v, k, \mathbf{S})$ denote the expected adversary success rate with password strength signaling when playing with his/her optimal strategy, then

$$
\begin{equation*}
P_{a d v}^{s}(v, k, \mathbf{S})=\sum_{y \in S L} \operatorname{Pr}[\operatorname{Sig}=y] \lambda\left(\pi_{y}^{*}, B_{y}^{*} ; \mathbf{S}, y\right) \tag{8}
\end{equation*}
$$

where $\left(\pi_{y}^{*}, B_{y}^{*}\right)$ is the optimal strategy of the adversary when receiving signal $y$, namely,

$$
\left(\pi_{y}^{*}, B_{y}^{*}\right)=\arg \max _{\pi_{y}, B_{y}} U_{a d v}\left(v, k, \pi_{y}, B_{y} ; \mathbf{S}, y\right)
$$

If $\arg \max _{\pi_{y}, B_{y}} U_{a d v}\left(v, k, \pi_{y}, B_{y} ; y\right)$ returns a set, we break ties adversarially.
The objective of the server is to minimize $P_{a d v}^{s}(v, k, \mathbf{S})$, therefore we define

$$
\begin{equation*}
U_{s e r}^{s}\left(v, k,\left\{\mathbf{S},\left(\vec{\pi}^{*}, \vec{B}^{*}\right)\right\}\right)=-P_{a d v}^{s}(v, k, \mathbf{S}) \tag{9}
\end{equation*}
$$

Our focus of this paper is to find the optimal signaling strategy, namely, the signaling matrix $\mathbf{S}^{*}$ such that $\mathbf{S}^{*}=\arg \min _{\mathbf{S}} P_{a d v}^{s}(v, k, \mathbf{S})$. Finding the optimal signaling matrix $\mathbf{S}^{*}$ is equivalent to solving the mixed strategy Subgame Perfect Equilibrium (SPE) of the Stackelberg game. At SPE no player has the incentive to derivate from his/her strategy. Namely,

$$
\left\{\begin{array}{l}
U_{s e r}^{s}\left(v, k,\left\{\mathbf{S}^{*},\left(\vec{\pi}^{*}, \vec{B}^{*}\right)\right\}\right) \geq U_{s e r}^{s}\left(v, k,\left\{\mathbf{S},\left(\vec{\pi}^{*}, \vec{B}^{*}\right)\right\}\right), \forall \mathbf{S}  \tag{10}\\
U_{a d v}^{s}\left(v, k,\left\{\mathbf{S}^{*},\left(\vec{\pi}^{*}, \vec{B}^{*}\right)\right\}\right) \geq U_{a d v}^{s}\left(v, k,\left\{\mathbf{S}^{*},(\vec{\pi}, \vec{B})\right\}\right), \forall(\vec{\pi}, \vec{B})
\end{array}\right.
$$

Notice that a signaling matrix of dimension $a \times b$ can be fully specified by $a(b-1)$ variables since the elements in each row sum up to 1 . Fixing $v$ and $k$, we define $f: \mathbb{R}^{a(b-1)} \rightarrow \mathbb{R}$ to be the map from $\mathbf{S}$ to $P_{a d v}^{s}(v, k, \mathbf{S})$. Then we can formulate the optimization problem as

$$
\begin{array}{ll}
\min _{\mathbf{S}} & f\left(s_{0,0}, \ldots s_{0,(b-2)}, \ldots, s_{(a-1), 0}, s_{(a-1),(b-2)}\right) \\
\text { s.t. } & 0 \leq s_{i, j} \leq 1, \forall 0 \leq i \leq a-1,0 \leq j \leq b-2,  \tag{11}\\
& \sum_{j=0}^{b-2} s_{i, j} \leq 1, \forall 0 \leq i \leq a-1
\end{array}
$$

The feasible region is a $a(b-1)$-dimensional probability simplex. Notice that in 2-D $(a=b=2)$, the second constraint would be equivalent to the first constraint. In our experiments we will treat $f$ as a black box and use derivativefree optimization methods to find good signaling matrices $\mathbf{S}^{*}$.

## 6 Experimental Design

We now describe our empirical experiments to evaluate the performance of password strength signaling. Fixing the parameters $v, k, a, b$, a password distribution $\mathcal{D}$ and the strength oracle getStrength $(\cdot)$ we define a procedure $\mathbf{S}^{*} \leftarrow$ genSigMat $(v, k, a, b, \mathcal{D})$ which uses derivate-free optimization to solve the optimization problem defined in equation (11) and find a good signaling matrix $\mathbf{S}^{*}$ of dimension $a \times b$. Similarly, given a signaling matrix $\mathbf{S}^{*}$ we define a procedure evaluate $\left(v, k, a, b, \mathbf{S}^{*}, \mathcal{D}\right)$ which returns the percentage of passwords that a rational adversary will crack given that the value of a cracked password is $v$, the cost of checking each password is $k$. To simulate settings where the defender has imperfect knowledge of the password distribution we use different distributions $\mathcal{D}_{1}$ (training) and $\mathcal{D}_{2}$ (evaluation) to generate the signaling matrix $\mathbf{S}^{*} \leftarrow \operatorname{gen} \operatorname{SigMat}\left(v, k, a, b, \mathcal{D}_{1}\right)$ and evaluate the success rate of a rational attacker evaluate $\left(v, k, a, b, \mathbf{S}^{*}, \mathcal{D}_{2}\right)$. We can also set $\mathcal{D}_{1}=\mathcal{D}_{2}$ to evaluate our mechanism under the idealized setting in which defender has perfect knowledge of the distribution.

Password Distribution We evaluate the performance of our information signaling mechanism using 9 password datasets: Bfield ( 0.54 million), Brazzers ( $N=0.93$ million), Clixsense ( 2.2 million), CSDN ( 6.4 million), LinkedIn ( 174 million), Neopets ( 68.3 million), RockYou ( 32.6 million), 000webhost ( 153 million) and Yahoo! ( 69.3 million). The Yahoo! frequency corpus ( $N \approx 7 \times 10^{7}$ ) was collected and released with permission from Yahoo! using differential pri-
vacy [10] and other privacy-preserving measures [14]. All the other datasets come from server breaches.

Differentially Private Count-Sketch. When using the empirical distribution $\mathcal{D}_{e}$ for evaluation we evaluate the performance of an imperfect knowledge defender who trains a differentially private Count-Mean-Min-Sketch. As users register their accounts, the server can feed passwords into a Count-Mean-MinSketch initialized with Laplace noise to ensure differential privacy (we briefly introduce count sketch and discuss the use of it to guarantee differential privacy in the full version of this paper [6]). After the Count-Sketch has been trained, the server can query the sketch about the estimated frequency for new users' passwords. Thus we can obtain a differentially private password frequency list $D^{d p}$.

When working with empirical distributions in an imperfect knowledge setting we split the original dataset $D$ in half to obtain $D_{1}$ and $D_{2}$. Our noise-initialized Count-Mean-Min-Sketch is trained with $D_{1}$. We then use this count sketch along with $D_{2}$ to extract a noisy distribution $\mathcal{D}_{\text {train }}$. In particular, for every $p w \in D_{2}$ we query the the count sketch to get $\tilde{f}_{p w}$, a noisy estimate of the frequency of $p w$ in $D_{2}$ and set $\operatorname{Pr}_{\mathcal{D}_{\text {train }}}[p w] \doteq \frac{\tilde{f}_{p w}}{\sum_{w \in D_{2}} \tilde{f}_{w}}$. We also use the Count-MeanMin Sketch as a frequency oracle in our implementation of getStrength(). $\mathcal{D}_{\text {train }}$ is used to derive frequency thresholds for getStrength() and to generate the signaling matrix $\mathbf{S}^{*} \leftarrow \operatorname{genSigMat}\left(v, k, a, b, \mathcal{D}_{\text {train }}\right)$. Finally we evaluate results on the original empirical distribution $\mathcal{D}_{e}$ for the original dataset $D$ i.e., $P_{a d v}^{s} \leftarrow$ evaluate $\left(v, k, a, b, \mathbf{S}^{*}, \mathcal{D}_{e}\right)$.

Derivative-Free Optimization. Given a value $v$ and hash cost $k$ we want to find a signaling matrix which optimizes the defenders utility. Recall that this is equivalent to minimizing the function $f(\mathbf{S})=$ evaluate $(v, k, a, b, \mathbf{S}, \mathcal{D})$ subject to the constraints that $\mathbf{S}$ is a valid signaling matrix.

In experiment we will treat $f$ as a black box and use BITmask Evolution OPTimization [54] (BITEOPT) with $10^{4}$ iterations to generate signaling matrix $\mathbf{S}^{*}$ for each different $v / C_{\max }$ ratio, where $C_{\max }$ is server's maximum authentication cost satisfying $k \leq C_{\text {max }}$.

## 7 Empirical Analysis

We describe the results of our experiments. In the first batch of experiments, we evaluate the performance of password strength signaling against an offline and an online attacker where the ratio $v / C_{\max }$ is typically much smaller.

### 7.1 Password Strength Signaling against Offline Attacks

We consider four scenarios using the empirical/Monte Carlo distribution in a setting where the defender has perfect/imperfect knowledge of the distribution.

Empirical Distribution From each password dataset we derived an empirical distribution $\mathcal{D}_{e}$ and set $\mathcal{D}_{\text {eval }}=\mathcal{D}_{e}$. In the perfect knowledge setting we also set $\mathcal{D}_{\text {train }}=\mathcal{D}_{e}$ while in the imperfect knowledge setting we used a Count-MinMean Sketch to derive $\mathcal{D}_{\text {train }}$ (see details in the previous section).

We fix dimension of signaling matrix to be 11 by 3 (the server issues 3 signals for 11 password strength levels) and compute attacker's success rate for different value-to-cost ratios $v / C_{\max } \in\left\{i \times 10^{j}: 1 \leq i \leq 9,3 \leq j \leq 7\right\} \cup\left\{(i+0.5) \times 10^{j}\right.$ : $1 \leq i \leq 9,6 \leq j \leq 7\}$. In particular, for each value-to-cost ratio $v / C_{\max }$ we run $\mathbf{S}^{*} \leftarrow \operatorname{genSigMat}\left(v, k, a, b, \mathcal{D}_{e}\right)$ to generate a signaling matrix and then run evaluate $\left(v, k, a, b, \mathbf{S}^{*}, \mathcal{D}_{e}\right)$ to get the attacker's success rate. The same experiment is repeated for all 9 password datasets. We plot the attacker's success rate vs. $v / C_{\max }$ in Fig. 1. Due to space limitations Fig. 1 only shows results for 2 datasets - additional plots can be found in full version of this paper [6].

We follow the approach of [5], highlighting the uncertain regions of the plot where the cumulative density function of the empirical distribution might diverge from the real distribution. In particular, the red (resp. yellow) region indicates $E>0.1$ (resp. $E>0.01$ ) where $E$ can be interpreted as an upper bound on the difference between the two CDFs.


Fig. 1. Adversary Success Rate vs $v / C_{\max }$ for Empirical Distributions the red (resp. yellow) shaded areas denote unconfident regions where the the empirical distribution might diverge from the real distribution $E \geq 0.1$ (resp. $E \geq 0.01$ ).

Fig. 1 demonstrates that information signaling reduces the fraction of cracked passwords. The mechanism performs best when the defender has perfect knowledge of the distribution (blue curve), but even with imperfect knowledge, there is still a large advantage. For example, for the Neopets dataset when $v / C_{\max }=$ $5 \times 10^{6}$ the percentage of cracked passwords is reduced from $44.6 \%$ to $36.9 \%$ (resp. 39.1\%) when the defender has perfect (resp. imperfect) knowledge of the password distribution. Similar results hold for other datasets. The green curve (signaling with imperfect knowledge) curve generally lies in between the black curve (no signaling) and the blue curve (signaling with perfect knowledge), but sometimes has an adverse effect when $v / C_{\max }$ is large. This is because the noisy
distribution will be less accurate for stronger passwords that were sampled only once.

We also use guessing numbers generated by state-of-the-art password cracking models (neural network, Markov model, PCFG) to fit a distribution, which we call Monte Carlo distribution. Monte Carlo distributions are useful in evaluating the performance of strength signaling when $v / C_{\max }$ is large. Experiments show that the reduction in the percentage of cracked passwords is up to $12 \%$ for Neopets, results can be found in the full version of this paper [6].

Which accounts are cracked? As Fig 1 demonstrates password strength signaling can substantially reduce the overall fraction of cracked passwords i.e., many previously cracked passwords are now protected. It is natural to ask whether there are any unlucky users $u$ whose password is cracked after signaling even though their account was safe before signaling. Let $X_{u}$ (resp. $L_{u}$ ) denote the event that user $u$ is unlucky (resp. lucky) i.e., a rational attacker would originally not crack $p w_{u}$, but after password strength signaling the account is cracked. We measure $E\left[X_{u}\right]$ and $E\left[L_{u}\right]$ (See Fig. 2) for various $v / C_{\max }$ values under each dataset. Generally, we find that the fraction of unlucky users $E\left[X_{u}\right]$ is small in most cases e.g. $\leq 0.04$. For example, when $v / k=2 \times 10^{7}$ we have that $E\left[X_{u}\right] \approx 0.09 \%$ and $E\left[L_{u}\right] \approx 5 \%$ for Neopets. In all instances the net advantage $E\left[L_{u}\right]-E\left[X_{u}\right]$ remains positive.

(a) Yahoo!, Neopets

Fig. 2. Proportion of Unlucky Users for Various Datasets ( $E\left[X_{u}\right]$ )

We remark that the reduction in cracked passwords does not necessarily come from persuading the attacker to crack weak passwords, but rather through the attacker shifting his attention towards certain signals. A utility-maximizing attacker will be interested in passwords whose signals suggest the attacker will not need to spend as much effort to crack them. However, because of the noisy nature of the signaling scheme, this is only similar to, but not quite the same, as attacking only the weakest passwords in a set. Some weak passwords may be "saved" when they are signaled as being in a higher strength category than their
true strength merits. By contrast, without signaling we expect that a rational attacker will crack all of the weak passwords.
Robustness We also evaluated the robustness of the signaling matrix when the defender's estimate of the ratio $v / C_{\max }$ is inaccurate. In particular, for each dataset we generated the signaling matrix $\mathbf{S}\left(10^{5}\right)$ (resp. $\mathbf{S}\left(10^{6}\right)$ ) which was optimized with respect to the ratio $v / C_{\max }=10^{5}$ (resp. $v / C_{\max }=10^{6}$ ) and evaluated the performance of both signaling matrices against an attacker with different $v / C_{\max }$ ratios. We find that password signaling is tolerant even if our estimate of $v / k$ is off by a small multiplicative constant factor e.g., 2. For example, in Fig. 1b the signaling matrix $\mathbf{S}\left(10^{6}\right)$ outperforms the no-signaling case even when the real $v / C_{\max }$ ratio is as large as $2 \times 10^{6}$. In the "downhill" direction, even if the estimation of $v / k$ deviates from its true value up to $5 \times 10^{5}$ at anchor point $10^{6}$ it is still advantageous for the server to deploy password signaling.

### 7.2 Password Strength Signaling against Online Attacks

We can extend the experiment from password signaling with perfect knowledge to an online attack scenario. One common way to throttle online attackers is to require the attacker to solve a CAPTCHA challenge [56], or provide some other proof of work (PoW), after each incorrect login attempt [42]. One advantage of this approach is that a malicious attacker cannot lockout an honest user by repeatedly submitting incorrect passwords [2]. However, the solution also allows an attacker to continue trying to crack the password as long as $\mathrm{s} / \mathrm{he}$ is willing to continue paying the cost to solve the CAPTCHA/PoW challenges. Thus, password strength signaling could be a useful tool to mitigate the risk of online attacks.

When modeling a rational online password we will assume that $v / C_{\max } \leq 10^{5}$ since the cost to pay a human to solve a CAPTCHA challenge (e.g., $\$ 10^{-3}$ to $10^{2}$ [39]) is typically much larger than the cost to evaluate a memory-hard cryptographic hash function (e.g., $\$ 10^{-7}$ ). Since $v / C_{\max } \leq 10^{5}$ we use the empirical distribution to evaluate the performance of signaling against an online attacker. In the previous subsection, we found that the uncertain regions of the curve started when $v / C_{\max } \gg 10^{5}$ so the empirical distribution is guaranteed to closely match the real one.

Since an online attacker will be primarily focused on the most common passwords (e.g., top $10^{3}$ to $10^{4}$ ) we modify getStrength() accordingly. We consider two modifications of getStrength() which split passwords in the top $10^{3}$ (resp. $10^{4}$ ) passwords into 11 strength levels. By contrast, our prior implementation of getStrength () would have placed most of the top $10^{3}$ passwords in the bottom two strength levels. As before we fix the signaling matrix dimension to be $11 \times 3$. Our results are shown in Fig. 3. Plots for other datasets can be found in the full version of this paper [6].

Our results demonstrate that password strength signaling can be an effective defense against online attackers as well. For example, in Fig. 3a, when $v / C_{\max }=$
$9 \times 10^{4}$, our mechanism reduces the fraction of cracked passwords from $12.7 \%$ to just $9.6 \%$. Similarly, observations hold true for other datasets. We observe that the red curve (partitioning the top $10^{3}$ passwords into 11 strength levels) performs better than the blue curve (partitioning the top $10^{3}$ passwords into 11 strength levels) when $v / k$ is small e.g., $v / C_{\max }<2 \times 10^{4}$ in Fig. 3a). The blue curve performs better when $v / C_{\max }$ is larger. Intuitively, this is because we want to have a fine-grained partition for the weaker (top $10^{3}$ ) passwords that the adversary might target when $v / C_{\max }$ is small.


Fig. 3. Adversary Success Rate vs $v / C_{\max }$ in Defense of Online Attacks

### 7.3 Discussion

- While password strength signaling reduced the total number of cracked passwords a few unlucky users might be harmed i.e., instead of being deterred the unlucky signal helps the rational attacker to crack a password that they would not otherwise have cracked. The usage of password signaling raises important ethical and societal questions. How would users react to such a solution knowing that they could be one of the unlucky users? One possible way to address these concerns would be to allow users to opt-in/out of password strength signaling. However, each user $u$ would need to make this decision without observing their signal. Otherwise, the decision to opt-in/out might be strongly correlated with the signal allowing the attacker to perform another Bayesian update. Another possible way to address these concerns would be to modify the objective function (equation (11)) to penalize solutions with unlucky users.
- Can we analyze the behavior of rational targeted attackers? We only consider an untargeted attacker. In some settings, an attacker might place a higher value on some passwords e.g., celebrity accounts. Can we predict how a targeted attacker would behave if the value $v_{u}$ varied from user to user? Similarly, a targeted adversary could exploit demographic and/or biographical knowledge to improve password guessing attacks e.g., see [57].


## 8 Conclusions

We introduce password strength signaling as a novel, yet counter-intuitive defense against rational password attackers. We use Stackelberg game to model the interaction between the defender and attacker, and present an algorithm for the server to optimize its signaling matrix. We ran experiments to empirically evaluate the effectiveness of password strength signaling on 9 password datasets. When testing on the empirical (resp. Monte Carlo) password distribution distribution we find that password strength signaling reduces the number of passwords that would have been cracked by up to $8 \%$ (resp. $12 \%$ ). Additionally, we find that password strength signaling can help to dissuade an online attacker by saving $5 \%$ of all user accounts. We view our positive experimental results as a proof of concept which motivates further exploration of password strength signaling.

## Acknowledgement

This work was supported the National Science Foundation under NSF CAREER Award CNS-2047272 and under NSF Award 1755708 and by Rolls-Royce through a Doctoral Fellowship.

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[^0]:    ${ }^{1}$ The propose may be less counter-intuitive to those familiar with prior work in the area of Bayesian Persuasion [30].
    ${ }^{2}$ If a user $u$ attempts to login with password $p w^{\prime}$ the authentication server will lookup salt $u_{u}$ and $h_{u}$ and accept $p w^{\prime}$ if and only if $h_{u}=H\left(s a l t_{u}, p w^{\prime}\right)$.

[^1]:    ${ }^{3}$ The index of matrix elements start from 0

[^2]:    ${ }^{4}$ The energy cost of transferring 1GB of memory between RAM and cache is approximately $0.3 J$ [44], which translates to an energy cost of $\approx \$ 3 \times 10^{-8}$ per evaluation. Similarly, if we assume that our MHF can be evaluated in 1 second $[7,11]$ then evaluating the hash function $6.3 \times 10^{7}$ times will tie up a 1 GB RAM chip for 2 years. If it costs $\$ 5$ to rent a 1 GB RAM chip for 2 years (equivalently purchase the RAM chip which lasts for 2 years for $\$ 5$ ) then the capital cost is $\approx \$ 8 \times 10^{-8}$. Thus, our total cost would be around $\$ 10^{-7}$ per password guess.

