Quantification in Empirical Activity: Tracing Children's Interests & Ideas

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Abstract

Changing where, when, and how objects are studied is central to lab-based science (Knorr Cetina, 1999). Science involves changing the scale of objects—particularly scales of size, time, and intensity—from what is experienced in the world. Similar to investigations conducted in science laboratories, classroom investigations involve re-representing and re-scaling entities, manipulating them, and observing effects in new locations and timescales. However, this aspect of investigation is under-studied and under-utilized as a resource for learning. We argue that, from elementary school, children can experience quantification, or identifying, developing, and working with variables, as consequential and can take up differences in representation and scale in empirical investigations as opportunities for sense-making and conceptual progress. We describe two instantiations of an investigation into heating and cooling, showing that seven- and eight-year-olds students oriented to gaps and ambiguities related to temperature, and that the re-design supported children and teachers to take up temperature for productive sense-making and conceptual progress. We examine opportunities for quantification across the heating and cooling investigation and a second investigation into landforms. This work has implications for supporting quantification in science activity in the early grades and using empirical investigations as opportunities for sense-making.

Keywords: empirical investigation; quantification; scale; elementary school science

1 Introduction

Ms. Olsen's[1] class of seven- and eight-year-olds is using straws and squirter bottles to explore how wind and water can move earth materials and thus shape land. After working in small groups, they share and discuss findings. They reason that their different claims could be due to groups blowing with different amounts of strength and from different directions. When Ms. Olsen asks students to develop a shared investigation, a student proposes they blow three times on each material. Adam raises his hand to disagree.

1. Adam: I think it should be like—we shouldn't just all blow at the same time, 'cause in real life, in real nature, does the wind like blow like three times and then wait for 10 minutes and then three times again? No.
2. Ms. Olsen: Okay.
3. Adam: So we're trying to prove that wind can move it, not I mean...
4. Ms. Olsen: That wind can move it.
5. Adam: Yeah, it should be like real nature.
6. Ms. Olsen: Okay.
7. Adam: If, I mean, wind does not blow just by that time, whatever.
8. Ms. Olsen: I understand, but we're trying to prove which [earth material] is going to move the easiest, so do you think controlling the amount of times then to prove it?
9. Adam: Well—
10. Ms. Olsen: I like that you're thinking of the real world, and you're absolutely right that our wind is very (*gesturing all over*), is it always blowing one direction too?
11. Adam: No, it's like... (*gesturing in many directions.*)

[1] All teachers' and children's names are pseudonyms.

12. Ms. Olsen: So, what were you thinking with that, then?
13. Adam: I'm thinking if these materials were outside, like also, we wouldn't want to put them in a Petri dish, it's like a wall preventing them from coming out?
14. Ms. Olsen: So we should definitely agree we should take them out. Is blowing three times each okay, to control our experiment?

Like laboratory investigations, classroom investigations involve representing and scaling entities, manipulating them, and observing effects in new locations and timescales. Here, Adam seeks to make sense of changes in representation and scale. He focuses on differences between how the wind blows materials outside and the conditions used inside, namely the idea that wind could “blow three times and stop,” the sense that wind is everywhere, and the idea that the classroom materials are placed in a dish with a “wall.” He sees these differences as consequential for the questions that the class is trying to answer, which, fundamentally, are about “real nature.” In contrast, Adam’s teacher focuses on the “amount of times” of blowing as something to control to “prove” which of three earth materials moves the easiest with wind. She has extracted from the world a cause (wind), considered it as a set of attributes that can be manipulated (number of times blowing, number of people blowing), and decided to use a re-represented version of the phenomenon, where both the force of wind and the materials are scaled down. Essentially, Ms. Olsen uses “wind” to refer to a directed force, and thus a variable that can be controlled, while Adam grapples with what it means to use classroom materials to make a claim about a complex phenomenon.

This excerpt illustrates consistent challenges and opportunities in classroom science investigations. The classroom investigation is a site of identifying, representing, manipulating, and scaling aspects of the world, in the process recasting these aspects as variables. However, this work is under-studied and under-utilized as a resource for learning. More generally, quantification, or identifying, developing, and working with variables and numerical values, is not well-supported in early grades science (Jin et al., 2019; Lehrer & Schauble, 2012). We argue that, from elementary school, children can experience quantification as consequential and can take up differences in representation and scale as opportunities for sense-making and conceptual progress.

To ground our argument, we examine the role of mathematizing in scientific activity and in classroom investigations. We introduce a conjecture emerging from our work, namely, that the changes in scale and representation involved in empirical investigation can function as a resource for children to engage in quantification. We describe how this conjecture emerged from and supported redesign in an investigation of heating and cooling, then summarize opportunities for quantification in both the heating and cooling investigation and the landforms investigation above.

2 Literature Review

Our examination of quantification and scale is guided by the perspective that students should experience their activity as meaningful and purposeful (Berland et al., 2016; Schwarz et al., 2017) and that grappling with some of the complexity that scientists engage with supports students' work (Manz, 2015b; Engle, 2011). This perspective entails neither copying science labs nor adopting discovery approaches where students figure everything out (Abd-El-Khalick, 2008; Furtak et al., 2012; Hmelo-Silver et al., 2007). Instead, we center the idea that the practices and ways of thinking used by scientists are taken up because they are helpful for sense-making and for generating consensus (Gouvea & Passmore, 2017; Russ, 2014). Therefore, we seek to understand how to develop learning environments that establish a need within the local classroom community for scientific forms of thinking, talking, and representing (Berland et al., 2016; Engle, 2011; Manz, 2015a).

Specifically, we take the perspective that children's science activity should invite and support sense-making, "the proactive engagement in understanding the world by generating, using, and extending scientific knowledge within communities… actively trying to figure out how the world works…or how to create or alter things to achieve design goals" (Schwarz et al., 2017, p. 6). We draw from Odden and Russ's (2017) conceptualization of sense-making as a process that can occur at different timescales and across individuals and communities, one that involves (1) a gap or ambiguity, (2) iteratively proposing, connecting, and refining understandings, and (3) seeking coherence across ideas and experiences.

Therefore, we seek to understand the gaps and ambiguities that children perceive in their work, that is what they take as uncertain or surprising. We orient to moments like the vignette above because, from Adam's perspective, there is something worth making sense of. We seek to understand how the issues and ideas that children bring up in these moments make contact with questions, forms of sense-making, and disciplinary ideas that are targets of instruction. Further, we use analysis to re-design learning experiences to elicit gaps, ambiguities, and disagreements that offer and support opportunities for sense-making.

2.1 Quantification in Scientific Activity

We take a broad view of quantification (or mathematization),[2] drawing from Jin et al.'s (2019) learning progression, descriptions of scientists' activity (Chang, 2004; Gooding, 1990; Kline, 1980), and classroom-based research on the intertwined development of attributes, measures, and understandings (Lehrer & Schauble, 2015; Sarama et al., 2021). Further, we

[2] Following Jin et al., 2019, we use these terms interchangeably

conceptualize scientific work as a modeling enterprise, one that involves constructing and revising systems of models (e.g., equations, conceptual models of entities and relations, simulations, statistical models, and empirical models such as experiments) (Giere, 1999; Nersessian, 2012; Rouse, 2015). This body of work helps us understand quantification as an activity central to scientific modeling, including to empirical activity (developing and refining instruments, experiments, and data).

Jin et al. describe quantification as a process that involves abstracting relevant measurable variables from phenomena and observations, investigating mathematical relationships among variables, and conceptualizing scientific ideas that explain mathematical relationships. Their historical accounts of quantification show that identifying and abstracting attributes as measurable quantities is central to the development of new understandings and theories. They chart developments in attributes used to describe motion, from Aristotle's understanding of "quicker" as the body that traverses a space in less time to the development of measurable and interacting attributes such as displacement, time, velocity, speed, and acceleration. These distinct, quantifiable attributes then supported the development of relational descriptions and, in turn, articulated understandings of force and motion.

The work to develop variables and mathematical descriptions is both empirically grounded and conceptually powerful. Gooding (1990) describes the ways that scientists working at the frontiers of knowledge seek "to bring perceptual order to phenomenal chaos" (p. 23). Scientists encountering new phenomena exchange construals that are situational, practical, and concrete. Through negotiating and refining construals, they develop shared ways of thinking and seeing and, eventually, shared variables. Chang's (2004) account of the historical development of measures of heat and temperature describes how fundamental understandings co-developed with measures and methods for quantifying temperature. Temperature was initially observed as bodily sensation. The development of the thermoscope, a device using the height of liquid in a bulb to signify changes in temperature, allowed scientists to compare different temperatures. Work to refine this device and establish fixed points (freezing, boiling) was entwined with advances in understanding molecular models that account for phase change, as well as the very notion of what heat is and whether cold is anything other than the absence of heat.

For scientists, then, empirical investigation both requires and supports quantification. Knorr Cetina (1999) argues that science laboratories "recast objects of investigation by inserting them into new temporal and territorial regimes" (p. 43). Rather than working with objects *as they occur in nature*, scientists substitute partial and transformed objects. Rather than dealing with them *where they are,* they bring objects to labs to manipulate. And rather than dealing with them *when they are,* they develop methods to increase their frequency and subject them to continuous study. Nersessian (2012) describes the ways that scientists' tentative ideas and construals are refined through work with scientific artifacts, such as in vitro models, instruments, and procedures. Scientific understandings embodied in models initially serve as placeholder concepts that articulate rudimentary ideas for the purpose of posing and representing questions. Concepts are developed through their representation in models and the subsequent refinement of those models. As scientists embody ideas in models and instruments, they make guesses about what aspects of a situation are relevant, develop attributes, determine how to re-represent those attributes, construct measures and quantities to work with patterns, and determine to what extent the values and relationships under study help articulate the target phenomenon (Nersessian, 2012; Rouse, 2015).

These accounts point to mechanisms by which quantification and scientific understanding co-develop. Mathematization involves thinking with and across several representational systems: an experienced complex world, a re-representation in the form of an empirical model or instrument, and an abstraction in the form of a mathematical model (Hesse, 1966; Kline, 1980; Nersessian, 2008). Representational formats are selective or abstractive of varying features and provide different possibilities for inference. Therefore, moving between representations is a site for making decisions about important entities and relations (Nersessian, 2008, 2012). Because models are exemplifications and approximations—and because they bring their own materiality and conceptual baggage—they do not fit or behave perfectly, leading to new implications, gaps, or contradictions and cuing evaluative and problem-solving practices. Seeing quantification in this way highlights the role of thinking-with-difference in the modeling enterprise, pointing us toward the conceptual power of bringing multiple representations into contact and exploring the import of their differences, a point we return to throughout the paper.

2.2 Mathematizing and Quantification in Classroom Science

Within the context of children's schooling, quantification occurs across the traditional domains of mathematics and science instruction. Aspects of quantification have been shown both to be challenging and to provide meaningful opportunities to integrate mathematical and scientific skills and understandings. However, ideas related to quantification and scale are still not developed systematically, particularly across early years of schooling (Jin et al., 2019; Osborne et al., 2018). Specifically, there has been little guidance about how to develop the quantitative underpinnings of science ideas or how to support children to grapple with mathematizing in their investigative work.

Research on quantification is more commonly conducted with older students (middle school through undergraduate). When students engage in scientific work that requires mathematization, they often move quickly from a problem situation to mathematical processing, without first developing a conceptual understanding that supports choosing appropriate variables and without continuing to move back and forth between using variables and reasoning about their meaning (Kuo et al., 2013; Tuminaro & Redish, 2007). When presented with mathematical relationships (e.g., graphs of motion), students may have trouble linking the variables and relationships represented to the concepts and relationships they depict (Leinhardt et al., 1990; Planinic et al., 2012). Jin et al. developed the following progression: (Level 1) students describe a phenomenon holistically, (L2) identify attributes, (L3) analyze phenomena in terms of measurable qualities (variables), and (L4) understand and use for reasoning the complex relationships among variables. One implication of this set of work is that connecting mathematical and conceptual meanings occurs (or can be short-circuited) throughout the process of understanding phenomena and solving problems. A second is that learning environments should attend to and support quantification as conceptual work (rather than primarily as a skill with variables or calculation that can be routinely applied across contexts).

While recent standards in the United States (Achieve, 2013; NRC, 2012) establish “Using mathematics and computational thinking” as one of eight scientific and engineering practices and “Scale, proportion, and quantity” as a concept that cuts across scientific domains and phenomena, these understandings receive less attention, and are not yet systematically developed, across the early years of schooling (Jin et al., 2019; Osborne et al., 2018). Concepts related to scale, proportion, and quantity are absent in the K-2 standards. They are first represented substantially in the fifth-grade standards, where students are expected to consider molecular models of matter and models of the solar system, exploring the ideas that "natural objects exist from the very small to the immensely large" and that "standard units are used to measure and describe physical quantities such as weight, time, temperature, and volume" (Osborne et al., 2018). Earlier standards *implicitly take up* quantification, for example, by indicating that children's models should represent amounts, relationships, and relative scales or that children should take measurements in investigations (Jin et al., 2019). Each of these performances relies on seeing and extracting variables that can be compared, measured, and controlled – steps that are accomplishments for scientists. However, little attention is typically paid to how to support these steps or what about them might be difficult.

Research on measurement in mathematics education suggests that quantification in the early grades, as in later years, both requires careful attention and can support conceptual understanding. Substantial research on the development of children's understanding of linear measure suggests that children need time and instructional support to quantify length and other attributes. Young children are intuitively sensitive to quantity and comparison, but don't recognize specific attributes, conceptualizing them generally, such as "small" and "big." They can be supported to discriminate and name attributes through comparing and aligning objects along a particular dimension (Sarama et al., 2021). They can be further supported to understand units of measure, iteration, and ways of developing informative comparisons (Cobb et al., 2001; Sarama et al., 2021). Early introduction of standard tools and procedures (e.g., rulers) without the development of conceptual understandings of attributes and measures can lead to rote use. In contrast, science activity can serve as a meaningful context to establish a need for and deepen ideas of attributes and measures of size. For example, Lehrer et al. (2002) reported on a first-grade class's scientific inquiry into whether bigger pumpkins produced more seeds. The class had to first agree on what they meant by bigger, thus differentiating bigness into attributes such as height and girth (Level 1 in Jin et al.'s progression). Then, they needed to decide how to compare one attribute, girth, developing a sense of measure (Levels 2 & 3).

Marianne Wiser, Carol Smith, and colleagues (Smith et al., 2006; Wiser et al., 2012) developed a learning progression describing understandings of the atomic-molecular theory of matter. They posit that identifying, distinguishing, understanding how to measure, and relating attributes (e.g., weight, volume, density) are central to rendering the atomic-molecular theory of matter intelligible and useful (Smith et al., 2006). The learning progression builds from the idea that objects have properties (weight, length, area, volume) that can be described, compared, and measured, to understanding weight as an additive, measurable property that is a function of both volume and material. These ideas are central to understanding transformations of matter that conserve weight, such as those described in the fifth-grade standards. However, in the standards and in typical curricular treatments, the qualitative aspects of Wiser and Smith's work have been made central while the development of the quantitative underpinnings are largely invisible. Second-grade standards related to matter (2-PS1.A) suggest that children should classify matter by observable processes, understand that matter can be solid or liquid depending on temperature, and consider how matter can be broken into pieces and rearranged. However, they do not address how ideas about measurable properties, temperature, or size might be developed or how this work might support the already quantified understandings fifth-grade students are meant to explore.

Finally, the relationship between quantification and empirical work in the early grades has received little attention (See Lehrer & Schauble, 2012, as an exception). Just as Knorr Cetina describes, classroom investigations rely on changing how objects are experienced—re-

representing objects, scaling them up or down, and changing the timescales at which processes occur. Students might place rocks in a jar with water and shake them to understand processes of erosion (Schauble et al., 1995) or melt ice-cubes to understand changes in sea-level due to climate change (Karpudewan et al., 2015). Children are often expected to collect and use numerical data. However, they are rarely allowed to grapple with how to represent objects in investigations, what to measure, or how to make sense of the consequences of representation for the conclusions they can draw (Chinn & Malhotra, 2002; Manz et al., 2020). Previous work has demonstrated that students may not understand the assumptions that underly the choices often implicit in classroom investigations (Schauble et al., 1995). In turn, they may not see or conclude what they are meant to, leading teachers to step in and tell them what they were supposed to observe and what it means (Lynch & Macbeth, 1998; Manz et al., 2020).

3 Scaling in Investigations as a Resource for Quantification

Based on the description of quantification developed so far, we see Adam's question as a moment of mathematization as he compares a complex phenomenon experienced outside (the wind) with an empirical representation (blowing through a straw) and a proposed manipulation located in the represented situation (the number of times students blow). The need to relate and problem-solve is situated in the decisions students are making about how to use the straws and the presence of a complex phenomenon they are seeking to explain. Adam is working at the frontier of his understanding as he constructs an explanation about how the empirical investigation does and does not feel satisfying as a tool for helping him understand wind. This work involves seeing differences in situations (the imagined wind outside and the materials and choices used in the investigation) and making proposals about which differences matter and why. However, sense-making of the kind Adam initiates is rarely supported in elementary learning environments. What, then, would it take for science investigations to function as a context where children develop and made sense of variables, in turn supporting deeper understandings of the concepts underlying those variables?

The conjecture that has emerged in our work across several investigations with elementary students, and that we develop further in this paper, is that the changes in scale and representation entailed in empirical investigations can function as a resource for children to engage in quantification, that is, in identifying, developing, and working with variables as quantities. This conjecture is supported by research showing that, in environments that support investigation construction in conceptually rich domains, children experience uncertainty about how their investigations represent and allow them to understand complex phenomena (Manz, 2015b, 2018; Metz, 2004, 2011). Like Adam, they are aware that representational choices affect what they can generalize from their investigation. Further, a few studies have shown that grappling with representing, defining, and measuring attributes can be accessible to children and can support new understandings (Ford, 2005; Lehrer & Schauble, 2012; Manz, 2015b; Warren et al., 2001). For example, Ford showed how sixth graders (ages 11-12) developed understandings of slope, distance, and velocity as they grappled with whether a better way to test if steepness affected speed was to vary the length of planks while holding height constant or vary the height to which planks of the same lengths were raised. Lehrer and Schauble (2012) described how third-grade students (ages 8-9) recognized the need to develop a definition and measure of “wet” and “dry” as they sought to understand whether differences in plant growth in a backyard were due to differences in “wetness.” Further, students’ conversation supported progress in thinking about where wetness mattered, what function it served, and how to conceptualize the in-between attribute, “moist.”

In this paper, we present evidence from two investigations of the ways that young children (in both cases, second-grade students, or 7-8 year-olds) notice changes in scale and pursue opportunities for quantification in empirical activity. This work takes Jin et al.'s learning progression for quantification, as well as related work on measurement in the younger grades, as a starting point. It provides a fine-grained analysis of when and how children engage in this work and see it as useful for their sense-making. Within the two investigations, we explore the questions,

1. When and how do students bring up questions and concerns related to quantification and see these as important?
2. How does quantification support students' sensemaking and conceptual progress?

4 Methods

We draw from methods of design-based research and co-design (Cobb et al., 2003; Penuel et al., 2007; Severance et al., 2016) to work with teachers to develop, implement, and analyze investigations in iterative cycles. The work is conducted as part of a larger project to develop principles and practices for redesigning elementary school science investigations to incorporate uncertainty scientists experience in representing phenomena, developing data models, and constructing explanations (Manz et al., 2020). The primary investigation described here (the heating and cooling investigation) was conducted as part of a multi-year partnership with a small, urban district. The description of the methods focuses on this investigation, describing how researchers observed and analyzed implementation of the district design, worked with teachers to re-design the investigation, then collected and analyzed data on the re-design. The secondary investigation (the landforms investigation) was conducted in a suburban district and used similar methods to those described here, but engaged teachers only in an initial analysis and re-design (without collecting data on the initial district design).

4.1 Participants and District Context

The district within which the investigation was conducted is a small, urban school district in the Northeastern United States. The student population is racially, linguistically, and economically diverse. District-wide, the student population is 40% Hispanic, 37% White, 10% African American, and 10% White. About half of students speak a language other than English at home and half are identified as “economically disadvantaged.” Further, the city has undergone significant gentrification in the past 10 years, so that classes typically include children with highly educated parents as well as children from historically minoritized communities. We report these data not to reinforce reductive characterizations of students’ assets and needs (Gutiérrez & Rogoff, 2003) but instead to form a picture of the heterogeneity in children’s backgrounds (Rosebery et al., 2010), and because they were important aspects of teachers’ design considerations. For example, design group discussions often focused on how to make sure all children felt their ideas and experiences were valued, what to do when children introduced particular vocabulary (e.g., evaporation, molecules), and how to support children newer to English to share their ideas with the class.

The nine teachers from whom the data is drawn vary in their cultural and racial backgrounds, proficiency in languages other than English (one teacher spoke both Spanish and Portuguese with her students while several were monolingual English speakers), and years of teaching experience (ranging from 3 to more than 30 years). Five teachers participated in the design group to examine data and re-design the investigation: two of those teachers taught the redesigned heating and cooling investigation and had data collected in their classrooms.

4.2 Investigation Context & Initial Design of the Heating and Cooling Investigation

In the district-designed unit, students explore properties of matter and consider reversible and irreversible changes as they seek to explain the phenomenon of baking a cake. Teachers first invite students' ideas about how ingredients change into a cake, surfacing questions about the role of heat in changing batter from a gooey liquid to a crumby solid. Midway through the unit, teachers introduce the idea of heating materials to explore how heat changed the ingredients in the cake. They use a hot plate, placing materials (butter, chocolate chips, egg, rock) in Pyrex or metal containers which are then placed in larger pans filled with water to disperse heat. Table 1 shows the design of the unit, and the heating and cooling investigation within the unit, as initially instantiated and then as redesigned.

Table 1

Overview of Design for the Cake Unit

Lessons	Initially Instantiated in District	Redesigned
1-3	Students examine the ingredients of a cake, mix them and observe the batter, and observe a baked cake. They develop explanations and questions.	Same
4-8	Students examine properties of materials, mix materials, and explore how materials (sugar dissolved in water) can be so small you can't see them.	Same with respect to temperature; some changes to support children's discussion of materials that are ambiguously solid or liquid (yarn, sugar).
9	Students discuss heat and find ways to use heat in the classroom to melt ice.	Same, with additional discussion of the oven heat.
10	Students examine the re-frozen ice, discussing how it is similar and different from the liquid water.	Rather then re-freezing ice, students discuss what made the ice change from solid to liquid and return to discussing the cake – Why did heat make ice change from a solid to a liquid, but the heat in the oven changed the cake batter from a liquid to a solid? The teacher introduces a thermometer and marks the freezer, room, body heat, and oven temperatures.
11	Students observe butter, chocolate chips, a rock, and an egg being heated and record their observations.	Students make predictions and observe ice, butter, chocolate chips, and a rock being heated. Students make claims about what happens when materials are heated. Students observe the egg being heated on a separate day, then make claims.
12a	Students examine heated butter, chocolate chips, rock, and egg from Lesson 11 that have been left overnight at room temperature. They discuss whether the materials "changed back to how they were."	Students examine heater butter, chocolate chips, ice, and egg from Lesson 11. They describe what happened when "heat was taken away." Students work in small groups to make claims about what happens when heat is added to their material and whether, when heat is taken away (1) the material changes back and (2) whether we could do anything to change it back.
12b	N/A	Students discuss how their investigation helps them understand what happens to the batter to change it into the cake.
13	Students read a text on reversible and irreversible changes due to heating and cooling and make connections to the cake and their investigation.	Same

The investigation addresses NGSS physical science standard 2-PS1-4, “construct an argument with evidence that some changes caused by heating or cooling can be reversed and some cannot.” Versions of this investigation—present in curricula and teacher-designed materials across the United States—typically rely on applications of heat that are doable and safe within the classroom context (e.g., placing objects in warm water, using the heat from students’ hands, placing objects on a hot plate or in a microwave). They use materials such as ice cubes, wax, or butter that will change within the range of temperatures used. These investigations generally focus on demonstrating that heat *can* change materials. There is often little attention given to the varying temperatures needed for different materials to change phase or the idea that many materials change only outside of the range of temperatures tested. When it is necessary to use different temperatures to provoke change, this is typically accomplished without much explanation. For example, students might melt ice with their hands, then place it in the freezer to see that it “changes back” (becomes solid again); examine wax and butter placed in a bag in warm water; and then watch a video of an egg cooking or talk about eggs to establish that even after cooling, a cooked egg will remain solid and opaque.

The ideas that are accessible to students within these contexts differ considerably from how heat operates in the world. Materials change phase at different temperatures, depending on the intermolecular forces acting on the molecules and atoms of the substance. The melting or freezing point of granite is around 1200 °C, while that of water is 0 °C. An egg solidifies between 65 and 73 °C—a *chemical*, rather than physical, change that occurs as the bonds in amino acids are changed and the proteins fold in new ways. Further, an object changes phase at particular *internal,* rather than external, temperatures through processes of heat exchanged between objects and their surroundings that occur over time and depend on differences in temperature, properties of materials, and amount of substance.

4.3 Initial Data Collection and Analysis

We first sought to understand the questions and ideas that children brought to their work with the investigation (Cobb et al., 2003; Grotzer et al., 2017). To this end, the lead author observed, video-taped, and interviewed six teachers using district-developed materials, focusing on the heating and cooling investigation (Table 1, Lessons 11-12). We collected data from one to five 30-45 minute class periods per teacher. A video camera and audio recorder were used to capture whole-class discussion and a sample of small group work. Student work, class artifacts, and field notes were collected. Further, the researcher, as a partner in the initial district implementation, asked probing questions and sometimes made suggestions during classroom work. Finally, the researcher conducted post-lesson interviews, asking teachers about how they perceived students' questions and opportunities for sense-making in the investigation.

Researchers, including both authors, then used an iterative process of reviewing field notes, videos, and artifacts to consider what teachers and children took as uncertain (that is, as a gap, ambiguity, or source of disagreement) and/or as a focus for sense-making. Based on project constructs and pilot studies (Manz, 2018), we considered how teachers and children recognized, took up, and made sense of uncertainties as they engaged with the cake phenomenon, conducted the empirical investigation, observed and developed data, made claims about what happened to materials as they were heated and cooled, and developed explanations. As indicators of teachers and students recognizing and making sense of uncertainty, we looked for where they (1) asked questions, (2) articulated a gap or problem, (3) disagreed, (4) discussed decisions as having import for what they could figure out, or (5) treated something as a focus for justification and explanation (Chen, 2020; Engle, 2011; Manz, 2018; Odden & Russ, 2018).

We iteratively developed and refined codes for *what* teachers and children recognized as uncertain and made sense of; for example, the roles of temperature and time, whether materials were liquid or solid, and whether materials “changed back”. We then exhaustively examined the data for moments when teachers or students brought up each code. To understand what students found interesting and meaningful, we attended to aspects of the investigation *children* introduced into small group or whole group conversation. We also took as an indication of children’s interest and sense of purpose moments that, regardless of who introduced a question or idea, resulted in increased participation, changes in affect (e.g., excitement, impassioned disagreement), and engagement with others’ ideas (Engle & Conant, 2002). We developed an “uncertainty log” for each classroom (Table 2), moving through the data to document when and how the uncertain aspect of the investigation (e.g., temperature, changing back) occurred. We documented when and how the uncertainty was introduced (e.g., prompted by teachers, brought up by students) and what happened as a result (e.g., teacher evaluated for correctness, initiated discussion). We used interview transcripts to examine how teachers made sense of these aspects of the investigation and the instructional moments in which they surfaced. Rather than using codes to reduce complex interactions to new data, we used coded data to point us back to topics and moments, supporting iterative analysis (Hammer & Berland, 2014).

Table 2

Excerpt From Uncertainty Log, Initial Design

Aspect of investigation that might be uncertain	Evidence uncertainty was recognized and/or made public	Response
What temperature are we using to test? (heating)	Day1_V3_08:43 Whole group observation of heating butter. **Teacher asks** What could we do to make it melt faster?	**Students reply** light it on fire, turn up the heat on the burner, and stir it.
	Day1_V4_00:00 Whole group conversation about whether it is possible to melt the rocks – **St. introduces** time and temperature as variables: An oven is hotter than the burner. It might melt the rock in 10 hours while maybe a burner could melt a rock in 20 hours.	**Tr. probes** for more ideas. One **student introduces** size as possibly important At 4:50, **Tr. asks** if it will be possible to melt the rock with how they are using heat in the classroom.
What temperature are we using to test? (cooling)	Day3_V3 00:03 Whole group. **Tr. asks** whether the chocolate chips which are melted will go back to being like the chips before.	**St. responds** they will if they put them in the freezer (other than egg). **Tr. probes. St. adds** that will not go back if you don't put it in the freezer (they will stay hot from the room). **Students** continue to bring up the freezer. A student brings up time (10 minutes in the freezer). **Tr. asks** —should we put them in the freezer? Asks for ideas from students. Researcher proposes leaving some at room temp to cool and putting some in the freezer.
	Day3_V3 11:42 **St. predicts** that nothing will happen to the materials left out in the room.	Discussion just between lead researcher and student. **St.** is using the word *freeze* for becoming a solid and differentiating between liquid/solid and shape.
	Day4_V1 **St. brings up** that butter had some liquid in it.	**Tr. guides** students to discuss that it was heat from their hands that turned it into a liquid.
	Day4_V1 11:45 **St. brings up** butter as soft when taken out of fridge.	**Tr.** accepts and moves on.

4.4 Redesign

Based on the analysis, the research team developed a list of potential sources of uncertainty and foci for sense-making in the investigation. These included: what temperatures were used in the investigation, how much time materials needed to heat and cool, what materials were investigated, what to use as evidence of phase change, what to claim about how materials changed when heated, and what it meant for a material to "change back." We then engaged with teachers in a protocol developed to facilitate co-design. We examined standards to brainstorm what might be non-obvious to students; did the investigation together; examined video of children making sense of gaps, ambiguities, and disagreements; and made re-design decisions. Throughout this process, we developed and refined answers to three questions: (1) What is uncertain from children's perspective? (2) How does this relate to the conceptual understandings we hope to develop through the investigation? (3) How does this relate to the phenomenon under study? We prioritized redesign efforts around aspects of the investigation that were uncertain from children's point of view, were made visible in the relationship between the investigation and phenomenon, and were centrally related to the conceptual work we sought to support.

Temperature emerged as one aspect of the investigation that students found uncertain, was made visible in the phenomenon, and that we thought could be productive if better supported. We therefore redesigned the investigation to invite and support thinking about temperature (as well as characteristics of phases and what it means for a material to change back; Table 1, "Redesigned"). Teachers introduced a thermometer representation, then marked and assigned values to various temperatures (body, room, oven, freezer) as they were conducting the unit. We conjectured that the rock not changing phase was a fruitful context for temperature. We therefore made space for students to make and explore claims about whether a rock changes with heat. In addition, we investigated ice as a material that changes liquid/solid phases at lower temperatures than the other materials studied and more explicitly prompted students to make claims both about what did happen in the investigation and what could happen. Finally, we decided to build from our noticing that children reasoned about differences in heat in the oven, on the hot plate, and in their life experiences. We therefore supported students to compare how materials changed when baked in the cake and on the hot plate and listened for, then supported, children reasoning about other experiences they had with temperature changing materials.

4.5 Implementation, Data Collection, and Initial Analysis of Redesigned Investigation

Three teachers, two part of the design team and one new to the work, implemented the redesigned lessons. We collected data (video, audio, artifacts) from the initial introduction of the cake phenomenon (Lessons 1-3) then from the heating and cooling investigation, beginning with ice (Lessons 9-12). Data collection ranged from eight to fifteen 30-45-minute class periods per teacher and mirrored processes described in the first instantiation. Instead of interviews, we collected records of teachers' reflections on the lessons in ongoing meetings and a subsequent summer institute. We then engaged in an uncertainty logging process identical to that described above, but focusing on the forms of uncertainty centered in the redesigned investigation. In this stage, we again shared initial findings and video with the teacher design group and, together, considered how our redesign supported student sense-making and conceptual progress.

4.6 Close Analysis of Temperature

For this paper, we focused our analytic work on temperature. We sought to make sense of how and why children brought up temperature (Research Question 1) and how their work supported opportunities for sense-making and conceptual progress (RQ 2). We first developed categories for the reasons students brought up temperature (e.g., to make sense of why chocolate chips melt on the hotplate but not in the cake), the forms of sense-making that occurred, and the aspects of conceptual understanding we saw developing. We moved through the data for each teacher, applying and refining categories and developing, then refining, themes. In this stage of analysis, the role of difference (described further in the findings) and opportunities for quantification became visible. We then engaged in cycles of making and refining conjectures about how quantification was at play, when children were perceiving differences and opening up opportunities for quantification, and how their work supported sense-making and conceptual progress, working closely with episodes and noting confirming and disconfirming evidence.

5 Findings

Across investigations, students quantified concepts such as force and temperature as they sought to make sense of differences in results and of differences between the investigation and other experiences of the phenomenon. We began to name and track these as *consequential differences;* that is, students experiencing and seeking to explain results that differed from each other or from their expectations, often in the context of a claim or generalization they were seeking to make. Specifically, as students recognized differences, they identified a factor (e.g., temperature) as differing in ways that were consequential for results, explored and differentiated the factor, and began to make comparisons including, when supported, numerical comparisons.

To illustrate these findings, we describe how children introduced temperature across the two instantiations of the heating and cooling investigation. We next examine episodes from one classroom, showing how the redesigned investigation provided a context for students to note temperature as a factor across situations; differentiate temperature into source, material, and effect; engage in explanatory practices related to the movement of heat through materials; and reason about ranges of temperature outside the investigation. We then summarize how these findings and those from the landforms investigation help us understand the shifts in scale and representation present in empirical work as resources for children to engage in quantification and make conceptual progress.

5.1 How Students Brought Up Temperature and Saw It as Important

Students introduced ideas related to temperature across the two instantiations of the investigation (Table 3). At the beginning of the unit, they often reasoned that heat (an increase in temperature of the oven compared to the room) was important for changing the batter into a solid. Students imagined the heat pushing the batter up from underneath, producing bubbles that cause the batter to expand, drying things out, and “hardening” the batter. They rarely brought up numerical values, engaged in explicit comparison of temperatures, or considered how temperatures might affect different materials. As the investigation progressed, students often introduced temperature to make sense differences such as materials not changing as expected, materials that did not change within the range of temperatures used in the investigation but that could change outside of that range, and differences in temperatures and results in the investigation as compared to the cake. In the first instantiation, which was not designed to support these questions, teachers who had not anticipated students’ sense-making or questions around temperature were often put in the position of correcting children or using structured sequences of talk to help students see what they were supposed to.

Table 3

How Students Introduced Temperature During the Heating & Cooling Investigation

Investigation Stage	When Students Brought Up Temperature	Yr 1	Yr 2	Example
Exploring the phenomenon	Making sense of the change from liquid batter to solid cake in the oven	X	X	Student: “Maybe the heat was forcing air into it and the air was trying to get out towards the ceiling *(points up)*.” Student: “I think the heat was more [powerful] than the batter because heat can [turn] it into a solid.”
Making predictions and observing materials as they are heated	Making predictions about the rock	X	X	Student: “The rock wouldn’t melt because it’s a solid and the heater probably doesn't have enough power, enough pressure to melt the rock.”
	Materials not changing as expected	X	X	When a student notes that chocolate chips did not melt as much as the butter, another student asks if they did not heat them for as much time.
Making sense/claims about what happens to different materials from heating	Materials not changing as expected	X	X	When the chocolate chips do not become a clear liquid, students argue that the “amount of heat matters.” They draw on past experiences of chocolate melting at different temperatures (e.g., “it’s really easy for chocolate to melt outside in the sunshine”).
	Making sense of the rock as something that did not change	—	X	Student: “[A rock] *could* be a liquid if it was like extremely hot heat but 350º isn’t really enough to melt a rock. It needs like the lava heat or magma.”
Making predictions about and observing materials as they cooled to room temperature	Considering “removing heat” as leaving objects at room temperature vs. placing them in the freezer	X	X	Student: “[The chocolate chips] turned solid.... because we left it overnight, and overnight, it um, sometimes at night, it gets chilling, freezing cold.”
	When materials begin to change (melt again) from the heat of students’ hands	X	—	A student notices that the cooled butter starts to melt again when touched because “the heat from people’s bodies, their body temperature was too warm for the butter and made the butter start to collapse.”
Making claims about whether a material did or could “change back”	When objects do not change back at room temperature	—	X	A student explains that liquid water did not “change back” to ice after being left out in the classroom overnight because “it's 65 degrees at night and water freezes at 32 degrees.”
Developing explanations about the cake	Considering how a material changes in the cake and investigation	—	X	Student: “The heat from the oven did not reach the chocolate chips.”

Across both instantiations, students introduced temperature as they made predictions, observed materials on the hot plate, and made claims about how materials change when heated. In several classes, the egg did not cook before the end of the allotted time, leaving it runny. Students often suggested that their teachers increase the amount of heat or heat the egg for longer. The chocolate chips also melt in ambiguous ways; they keep their shape until stirred and are then thick and gooey—this was another context where students routinely began to talk about temperature. In these instances, students engaged in problem solving to explain that when materials don't change they might need more heat, thinking about more time or comparing the relative heats of two heat sources, and suggesting changing the heat source to add more heat.

The rock also cued thinking about temperature. Students often introduced temperature as they made predictions about whether the rock would change, predicting that it would "change temperature... but not state." However, in the first instantiation, we did not see teachers moving beyond *what* happened to the rock to *what could happen* or what students *could conclude.* Our analysis suggested that teachers weren't sure which question to ask to elicit sensemaking. For example, while discussing the heated rock in Ms. Mark's classroom, students contributed broad observations about how it had changed (e.g., it was darker, shinier, or hotter), leaving Ms. Mark to correct them, telling them it had not really changed, and wondering later how she might have better supported discussion, posing alternative questions she might ask, such as "Could it have changed if I kept the rock in the water all night? Would that have been possible? Would the butter be any different?.... Is there way we could change it even more?"

In the redesign, we wanted to support students to consider and argue about why the rock did not change and put ideas about temperature to use in their sense-making. The co-design team decided to focus observations more clearly on changes in phase (rather than open-ended noticing) and then invite children to make and evaluate claims and whether heat can change materials —for example, asking, “Do we think that heat can turn a rock into a liquid?” or encouraging students to agree or disagree with the claim, “Heat *cannot* change rock into a liquid. I know because the rock just got hotter but did not get soft or pour.” In response to these design shifts, students reasoned a rock would need “really high heat to melt” due to its hardness and texture. They often spontaneously hedged about the conclusions that could be drawn from the investigation, noting that the rock *could* change even if it did not in these circumstances. In these discussions, as we explore in the next section, students called on consequential differences between the investigation (materials on a hot plate) and their understandings of higher temperatures.

Across the two years, students also introduced or grappled with temperature as they sought to understand what happened when materials were removed from heat. They often expressed uncertainty about whether materials would return to solid state at room temperature, sometimes suggesting putting them in the freezer or reasoning about the cold temperatures needed to “freeze materials.” They appeared to consider heating and cooling as separate processes or “freezing” as a process of becoming a solid associated with a freezer. In response, the co-design team decided to use the limitations of the investigation to support reasoning, a strategy we have used in other contexts (Manz, 2015b). First, building from the rock, we incorporated directions for students to make two claims, “Did it change back” and “Could it change back?” for each material cooled. As we describe below, this focus appeared to support students to begin to consider temperatures along a scale that both extended above and below room temperature and to reason about the different temperatures needed for phase change.

Finally, students made sense of heat in the context of the cake. In the second year, we more explicitly prompted students to consider similarities and differences between the cake and hotplate contexts and asked students to think about how the investigation helped them understand why the batter changed into a cake. Likely due both to the addition of targeted questions and the use of a thermometer, students introduced temperature as they brought up and made sense of differences between the cake phenomenon and the investigation. As we explore further in the next section, students took up these opportunities to point out differences in the temperature between the oven and the hot plate and to argue that materials (specifically, the chocolate chips) might behave differently in the cake because the heat was blocked from getting to them by other materials (e.g., the batter).

5.2 A Closer Look at Student Sensemaking about Temperature

Here, we focus on episodes from one classroom in the second instantiation of the heating and cooling investigation, where children had the most sustained opportunities to discuss temperature. We show how making space for children to recognize and engage in thinking about consequential differences centered temperature as something to see across situations and engage with for explanation. Specifically, we examine three areas of the investigation where consequential differences emerged: reasoning about ambiguous changes in chocolate chips, the rock as a material that did not change phases in the investigation, and ice not changing back to a solid at room temperature. In each of these cases, differences related to temperature established a gap or ambiguity that children took up, in turn, iteratively proposing and refining connections among ideas, past experiences, and empirical results. Through their sense-making, they engaged in quantification by differentiating, explaining, comparing, and using numerical values for temperature.

5.2.1 Chocolate Chips as an Ambiguous Material

As Ms. Lacy's class initially experienced the batter turning into the cake, they were struck by the fact that the chocolate chips did not mix into the batter and, instead, sank to the bottom of the cake. The chocolate chips behave in ambiguous ways. As they melt, they tend to keep their shape until stirred. When the cake is cooled, they appear not to have changed much from their initial shape and state, unlike the batter. This ambiguity was a context where children surfaced and explored thinking about heat and temperature. These discussions, in turn, provided opportunities for children to consider the role of amount of heat in melting substances and explore temperature moving through materials.

As Ms. Lacy brought out the chocolate chips, children agreed that they were a solid. Several students predicted that the chips would melt, drawing on their experiences with chocolate melting in their hands and pockets. When placed in small tins in hot water on the hot plate, the chocolate chips became shiny but kept their shape. Children observed in small groups, noting the shininess, asking, "is it hot?" and arguing, "it hasn't melted yet." They then decided to stir with a popsicle stick. When the shape changed, some students exclaimed, "it's a liquid!" while others were less convinced. This experience generated considerable excitement, with children calling the teacher and each other over and continuing to talk as the teacher directed them to start writing.

Next, Ms. Lacy supported students to make claims (Figure 1). They readily claimed that heat changes butter and ice into liquids. As they moved to make claims about the chocolate chips, students began to discuss ambiguities, highlighting that the chocolate chips became shiny and gooey but kept their shape until they were stirred. One student began to compare the chocolate chips to ice cream melting.

1. Sarah: When I have ice cream, it’s a solid, but when you leave it out, it starts melting. So it turns into half-solid, half-liquid because it’s melting but still solid. If you keep leaving it there, it will turn into ice cream soup.
2. Ms. Lacy: Why is that happening?
3. Sarah: Because of the sun. But if you are in the fridge or a cold place, it’s just going to get colder.
4. Ms. Lacy: So, what about the chocolate chips?
5. Sarah: Chocolate chips, they don’t melt as easy as ice cream because ice cream is made out of milk, and milk is a liquid. But it’s really easy for chocolate to melt outside in the sunshine. If it’s in the fridge or freezer, it’s going to get colder.

Here, Sarah described the *process* of melting and stretched this process out over time. In her comparison of the “easiness” of ice cream and chocolate chips melting, she seemed to draw on ideas about temperature, comparing contexts like “leav[ing] something out,” in the freezer, a fridge, a “cold place,” and “in the sun.” She made connections to the different substances that chocolate and ice cream are composed of, arguing that “milk is a liquid.” She seemed to use room temperature as an implicit norm, in that milk is a liquid and chocolate is a solid at room temperature. Here, Sarah oriented to the idea of “heat changing a material into a liquid” as interesting, saw it at play across several situations, and differentiated it into related attributes such as time, temperature (embedded in different contexts), and qualities of the material.

Fig. 1 Classroom claims and evidence about how heat changes materials

Students next debated whether the heated chocolate chips were solid, liquid, or both, focusing on their texture and the idea that they keep their shape until stirred. Then Terrell brought the conversation back to the process of melting.

1. Terrell: I think that it would be important to actually stay its shape. Because if it was a little shiny, then you wouldn't know if you could make it melt...So, if there's just a normal chocolate chip, then if you put it in something hot, it would melt, but if it was like a little shiny it won't, but it will kind of.
2. Ms. Lacy: Oh, so do you think there are different stages of what happens with the chocolate when heat is added? I want to make sure I understand correctly. You're bringing up that it got shiny. Are you thinking if it's only a little, little bit shiny, it wouldn't really have changed state? But if it's really shiny that's when you can stir it? So there's a change that's happening over time?
3. Terrell: Yeah.
4. Ms. Lacy: And do you think the amount of heat matters?
5. Terrell: It doesn't really matter because you won't really know how much heat got on it. You wouldn't really know how much the heat was there because you wouldn't know unless you set a timer, for like ten minutes. Then you would know.
6. Ms. Lacy: So, Terrell, do you think it is fair if I write it changed level of shininess over time? … So you think the amount of heat does matter? *(Writes question on a post-it note.)*
7. Sarah: So if it was at 20, it wouldn't get shiny. If it was like 20 degrees, you couldn't mix it. But with heat it was really hot, so it was able to mix.
8. Ms. Lacy: I just also wrote a question: "Does the amount of heat matter?"

9. Amber: I know the amount of heat matters because I melted chocolate in a pot on the stove, and it cooked a little bit, it took a couple minutes, but it got really hot and melted into a liquid. I knew it was a liquid because it poured and it didn't keep its shape, it had no shape. We poured it into a cup and containers. I know that because that would happen…I know that the stove is hotter than the hot plate because there's actually fire in the stove and the hot plate doesn't have fire.

Here, students constructed melting as a process, one that involves materials both getting hot and undergoing changes in observable properties. They drew on their experiences with heating materials and materials melting, introducing pots on stoves, "putting it in something hot," and "20 degrees" as they made sense of temperatures that do and do not change different materials. Ms. Lacy supported students to make comparisons and to think about heat or temperature as quantifiable and comparable. Her question, "Does the amount of heat matter?" opened up space for children to extend these connections. Terrell (Line 5) was unsure of heat as something that could be measured but made connections to measuring time. Sarah and Amber began to talk about temperature or "amount of heat" as quantifiable. Sarah posed an example of a temperature (20 degrees) that would not change the chocolate chips, while Amber made the argument that she had previously placed chocolate chips on a surface hotter than the hot plate, using comparison rather than a quantified temperature.

A few days later, the class considered how what they had learned in the investigation helped them understand the cake baking in the oven. They returned to the discussion of whether or not the chocolate chips in the batter had melted.

1. Aminah: I think that the chocolate chips were solid, but it was melted. It was a little solid because the cake was blocking it, because the cake was in front and the chocolate chips in

the bottom, because it just melted a little bit and it became a little more solid. Then when you bite it just one bite it’s turned into liquid…And probably your mouth was really hot that time because your body temperature was 98 percent degrees inside your body.

2. Dr. Eve: So Aminah, you’re using what you know about how chocolate chips act like when we heated them on the stove, and now you’re making connections to what you think was happening in the cake? And you think maybe they melted most of the way but not all of the way because they were in the middle of something or the bottom of something like the cake?
3. Sarah: I thought—that was like what I was thinking.
4. Dr. Eve: Sarah, you were thinking something similar?
5. Sarah: Like the chocolate chips did the same thing on the inside of the cake.
6. Dr. Eve: Hmmm. Anyone else have connections between what we did on the stove and how that was helping you explain what was happening in the cake? Let’s see some of the other connections we have.
7. Ms. Lacy: Amber?
8. Amber: I thought that the chocolate chips—I think that the chocolate chips just didn’t do anything unless we stirred them when we put them on a stove—well, the hot plate, but and I think that’s why in the cake it hadn’t melted because we didn’t do anything to touch it while it was melting and that’s why they got heavy and fell to the bottom.

Aminah made sense of the apparent lack of change of the chocolate chips in the cake batter, as compared to other ingredients and other materials that melt when heated. She drew on specific temperatures, describing the experience of causing a chocolate chip to melt by taking a bite of the cake, noting that her body temperature is 98 degrees, and connecting to an earlier conversation about body heat melting chocolate. She also proposed that the batter was "blocking it," likely referring to the heat or temperature of the oven. To make sense of why the chocolate chips did not appear to change, Amber (Line 8) drew on the discussion of the hotplate results—that the chocolate only changed shape when stirred, even after heat was applied. Other children extended these ideas. Sarah noted that she experienced one of the chocolate chips in the cake beginning to melt in her hand. Further, she argued that the chips melted and then "went back to their normal shape" when the cake cooled. Mark argued that "when we ate the cake, the chips weren't melted; I think that the heat from the oven did not reach the chocolate chips" and Samuel played out how heat might move through the batter, explaining, "maybe the batter got heated up, then the next layer and the next layer. It got the chocolate, because how could the surface of the chocolate chips be a bit melted?"

Here, the ambiguity of the chocolate chips supported students to introduce and use considerations of heat and temperature, seeing these ideas as at play across different situations then identifying, differentiating, and using numerical values for sense-making. When the chocolate chips did not immediately appear to melt, students called on experiences they had at home with needing more heat or more time for things to cook or melt. When considering why the chocolate chips did not appear to change in the batter, students introduced comparisons between the temperatures of heat sources. Furthermore, they began to imagine heat moving through materials, thus differentiating between the temperature of a heat source and a material and beginning to imagine a process of change in temperature which involved space and time.

5.2.2 The Rock as a Sense-Making Context

Students generally knew that the rock would not change into a liquid within the temperatures used in the investigation. Here, the consequential differences appeared to be (1) that rocks did not change in properties that did change for other materials and (2) what rocks did in the investigation differed from what students thought they could do in temperatures beyond the bounds of the investigation. Orienting to these differences as gaps to explain appeared to provide important resources for students to continue to differentiate the material and the temperature of a source of heat, develop explanations about *how* heat changes materials, and develop and use values for temperature. Consider the conversation as Ms. Lacy introduced the rock:

1. Ms. Lacy: I'm curious what your predictions are. What do you think is going to happen when we add heat to the rock? (*Puts rocks into the tin on the hot water.*)
2. Mark: I think it's just going to change the temperature of the rock, but not change the state of what the rock is in because rocks need really high heat to melt…Rocks are really hard, and it's hard for them to melt so they're like really hard and you need lots of heat to melt a rock. Cause lava or magma can melt rock—
3. Rick: It is magma.
4. Mark: But just like heat couldn't really melt rock and I don't think it's gonna change its state.
5. Ms. Lacy: So you think—OK, can I restate one of your sentences to make sure everyone heard it… Mark also said that he thinks the temperature is going to change, but the state of the rock is not going to change.
6. Ethan: That makes sense.
7. Rick: Since coal is just rocks that has had stuff happen to it, it's not going to change it's just going to feel really hot when you touch it, cause lots of people, like if you're in the North Pole or something that is really cold…It's like in the North Pole or some place that's really cold. If you had a cave around you, they would make a fire. Then you have a

rock and just take the rock out and put it in their pocket or put it in their shirt and stay warm.

8. Ms. Lacy: So this almost sounds like, Rick, and I want to restate part of it, and you tell me if I'm saying it correctly. It almost sounds like you're saying the rock will hold onto that heat, that it will stay warm for a long time?
9. Rick: Yeah, because rocks, like with the heat, the heat would go into the rock, because rocks are—there's iron in rocks sometimes…

As students justified their predictions that the rock would not change, they brought up several important ideas. First, they introduced ideas about temperature—drawing on experiences they had with heat sources of different temperatures and beginning to order and compare them (fire, lava, hot water, later, a stove at 100 degrees). Further, they drew on what they knew about rocks and their properties to begin to explain whether and how heat might change the rock, including their hardness, what they are made of (iron), and, later in the conversation, agreeing that the rock might feel hot but not change in other properties. Rick reasoned about rocks as materials that can hold heat without changing and can, therefore, be used to keep people warm.

Four days later, as Ms. Lacy invited students to make claims about how heat changes rocks, they extended these ideas. The conversation directly followed the claims about chocolate chips described earlier and was recorded in Figure 1.

1. Ms. Lacy: What claims can we make about heat and the rock? Mark?
2. Mark: Well, it can't be a liquid. A rock can't be a liquid.
3. Ms. Lacy: So why not?
4. Mark: Well, it could be a liquid if it was like extremely hot heat, but 350° isn't really enough to melt a rock. It needs like the lava heat or magma.

5. Ms. Lacy: So, Mark, it almost sounds like your claim is that with our experiment the rock didn't change into liquid, but are you saying that with a much higher heat it could?
6. Several students: Same. I agree. Same.
7. Mark: But like not with—not even fire couldn't probably melt it…like lava.

In this section, we see students comparing and ordering numerical values. First, Mark began to claim that rocks *can't* be a liquid. When pressed, he quickly indicated that the rock *could* become a liquid, but the heat used in the investigation (350°) was not sufficient. Ms. Lacy restated this statement, explicitly distinguishing what *did* happen in the investigation (the rock did not change) from what *could happen* with a "higher heat." In response, Mark began to think about amount of heat—ordering 350°, fire (earlier brought up as something you can put a rock in), and "lava," which he likely connected to melting rock. Across classrooms, we found that, when teachers pressed for a generalized claim about what heat can do to a rock or made space for students to differentiate what *did* happen in the classroom from what *could* happen, students routinely brought up magma or lava. This finding suggests that this is a context that cues children to think about relative amounts of heat and where they have resources for this work (including the ubiquitous "the floor is lava" game and/or references from movies such as *Moana*).

Rather than stopping there, with a claim that they agreed on, students continued to seek to explain *why* the rock didn't melt.

8. Mark: And the rock is hard and it kind of reflected the heat from the hot plate…'cause it wasn't that hot when we took it off, but well, it wasn't that hot but I think it reflected the heat a little bit.
9. Ms. Lacy: Deanna, what were you thinking about?

10. Deanna: I was thinking that the rock wouldn’t melt, but since it started off it could have been a liquid and you do something strong to make it into a strong and hard solid.
11. Ms. Lacy: Can you say a little bit more about that?
12. Deanna: So that if the heat…was way more hotter, maybe it would make the rock into a liquid. But I’m pretty sure it’s not gonna do that as well, because the rock has a lot of hard stuff on it.
13. Ms. Lacy: So you’re thinking because the rock is so hard, it would need that really strong heat? (*Deanna nods.*)

    …

14. Ms. Lacy: Would it be fair to write that the heat did not change the rock but it could if it was a much higher temperature or like lava? I heard that come up a lot. Would that be fair to record? Deanna, what are you thinking?
15. Deanna: I’m thinking that the lava is really melted. If you broke a rock and turned into pieces and put it with the heat, it might melt. The heat would just make it hotter on the outside but the inside might still be cold. If you leave it out, it will turn cool after.

Students brought up the properties of the rock as important for why it did not change, including that it reflected heat or was so strong that it remained a “hard solid.” Deanna further began to reason about the size of the material, an idea that other students were interested in and continued for several turns.

In these excerpts, temperature functions as something to make sense of —to reason about and with—rather than to demonstrate the effect of. The rock, as an object that does not change to a liquid, elicited student interest in whether, how, and why the rock changes temperature and what that has to do with changes in other properties. Further, the rock served as a context where the teacher could, building from students' ideas, highlight the difference between what did happen in the investigation and what could happen, an idea that continued to be important as students turned to removing heat.

5.2.3 Ice and Water

As described earlier, the context of changing materials back elicited students' ideas and questions about "freezing," with students across most classrooms in both years initially discussing how putting materials into the freezer or cold temperatures at night would be necessary for materials that had become liquid (butter, chocolate chips) to become solid again. In the second instantiation, the co-design team included ice in the hot plate/room temperature investigation to support explicit comparison of materials solidifying at different temperatures. The ice supported students to consider and order lower values for temperature and to see *coldness* as lower temperatures, and therefore, as taking away (more) heat. Using numerical temperatures appeared sensible to students as they explained the difference between what happened to ice in the investigation and what could happen.

At the end of the investigations, children worked in small groups to show what they had learned about how heat changes one of the materials and whether that material could "change back" when heat was removed, then developed and presented posters. Ms. Lacy prompted students to focus their presentations on the question: "What happened with the heat, then what happened when we took the heat away—did it change or not change?" Aminah and Amber's poster (Figure 2) and presentation illustrated the ideas that students found interesting to wrestle with.

Fig. 2 Aminah and Amber's ice poster

1. Amber: We learned about water and ice. And we think that when we added heat, the ice melted into water.
2. Aminah: We're wondering how is water going to change back to ice when it's water. We know a lot about ice getting back to water but we are more interested to learn about water to get back to ice, it's just like the reverse of ice and water.
3. Amber: When we took away the heat, it did not change back. Our evidence is it was still water. But we think that if we take away a lot of heat, it will go back to ice. I think it did not go back to water after we took away heat. So, we think that we need to put it in the fridge to make it go back to ice.
4. Student: The freezer?
5. Amber: No—the freezer particularly.
6. Aminah: (*Turning and gesturing to the poster.)* I wrote it's easy to melt with hot water. We wrote two thermometers to show how much the freezer is. We also wrote the hot plate and the extra one is the sun because the sun has all the heat in it.

The poster and presentation demonstrated Aminah and Amber's interest in understanding the role of temperature in water becoming solid again. Their poster showed that they were developing the central idea (as listed in the standards) that changes to ice due to heating and cooling are reversible, *even though* the classroom investigation did not demonstrate this result. In the top left corner and in the bottom center, they represented a cyclical, repeating process. Further, they differentiated the process of ice changing to water (which they represented with a + and a fire symbol for heat) from that of water changing back to ice. They used a - sign and then represented liquid water again, presumably showing how within the temperatures used in the investigation (returning items to room temperature), the water was still in liquid form. However, in each cycle representation, they added another step where heat is removed again to transform the liquid water into solid water. In the center of the representation, they drew a - with three fires and wrote, “I think that if we take away a lot of heat it will go back to ice.” They also included representations of the classroom thermometer, which they recruited as they narrated their poster. Further, as students restated and responded to Amber and Aminah’s ideas, the questions of relative temperature and taking heat away emerged again as students disagreed with the statement that “to get it back to ice, we can put it in the fridge or the freezer,” leading to agreement that the freezer was necessary because of its lower temperature.

5.3 Summary and Comparison of Investigations

Across both the heating and cooling investigation and the landforms investigation that opened this paper, students’ initial talk focused on what Jin et al. consider holistic description, (Level 1). They had experiences with temperature and the force of moving wind and water, but initially did not fully identify, differentiate, or reason numerically about these factors. Yet, in each investigation, as students experienced differences in results, experienced results that differed from their experiences, or sought to generalize, they oriented to a factor (force, temperature) as important for addressing ambiguities and gaps. When supported, they engaged in quantification by identifying and explaining factors, refining their descriptions in sets of attributes, and taking up comparison, including with numerical values, for sense-making (Table 4).

The heating and cooling investigation selected scales and values of temperature (oven, hot plate, room) that differed from the temperatures that change materials in the world. The redesigned investigation made space for children to make sense of materials that changed ambiguously or did not change within the temperatures used in the investigation. As children considered ambiguous and non-changing objects, they moved from considering “hotness” and “coldness” or “freezing” to differentiating temperature (temperature of a source of heat, temperature of an object, heat moving through an object, changes over time) and to reasoning about adding and taking away heat, then adding and/or taking away more heat. Further, they treated heat and temperature as something to make sense of and with, rather than simply engaging with it as a causal factor demonstrated by the investigation (e.g., heating a material can cause it to melt).

The landforms investigation that opened this paper involved not only selecting values but re-representing the phenomenon (water and wind shaping land) within the classroom environment. In the materials provided by the district science kit, children worked with a spray bottle, straw, and petri dishes of earth materials (sand, soil, rocks). The investigation therefore involved changes in the force, amount of material, and time at which wind and water shape land. Here, consequential differences also cued considering and quantifying force. Groups using the spray bottle and straw differently (in terms of numbers of squirts and blows, angles, setup, or amounts of materials) drew different conclusions. As groups presented claims, students asked questions that were oriented to reasoning about force to understand differences in results. They also hedged as teachers asked them to draw generalized conclusions from the investigation, making sense of ways that the materials did not adequately represent the force or mechanisms by which water in particular might move earth materials. We saw similar quantification processes at work here, namely, students began to see force as at play; oriented to explaining force; differentiated it into aspects such as source, direction, magnitude, and duration; and engaged in comparison.

Table 4

Quantification Activity in the Two Investigations

Quantification Activity	Heating & Cooling	Landforms
Using different objects/entities/mechanisms for something we as "experts" know is the same *Jin et al., Level 1; Holistic Descriptions*	• Students initially talk about hotness and cold as different causes for materials changing. They treat freezing/a freezer as synonymous with making something solid.	• Students initially talk about different forms of stronger and weaker weather that can move land (rain, tornados, hurricanes, pounding waves)
Naming a concept as at play; seeing it as present across different situations; treating it as something to explain *Jin et al., Level 2; Attributes*	• Students see heat and temperature at play across different situations. • They orient to explaining what allows the rock to change temperature but not form.	• Students identify force as at play across situations. • They orient to explaining force (e.g., how the force of water pointing down could translate into lateral motion of materials).
Seeing a cause/concept as a bundle of related attributes *Jin et al., Levels 2-3; Differentiating*	• Students differentiate between temperature applied, temperature of source, temperature of object (location), and role of time. • Students reason about differences in temperature of a heat source and different parts of a material as they seek to explain non-changing materials.	• Students recognize that force has a source, direction, magnitude, and duration. They adjust and control each of these as they test using the straw or water bottle. • Students ask each other about specific choices others made when comparing findings and seeking to understand differences.
Seeing attributes (or concepts, which sometimes points back to attributes) as comparable across situations *Jin et al., Levels 2-3; Comparing*	• Students recognize limits of investigation; with more heat or time rock would melt; if more heat were taken away water would change back. • There is movement from describing heat and cold as separate causes and to talking about adding and taking away (more) heat.	• Differences in findings leads to students talking about specific attributes and comparing them. • Students hedge conclusions from the investigation and compare what did happen with what could happen, saying that with more force, materials would move or would move more.
Using quantity/numerical values to compare or vary one attribute	• Students use the thermometer as a tool to describe and order experiences such as body heat, room temperature, and oven. • Students use numerical values to discuss the conclusions that could be drawn and to extend beyond the values used in the investigation.	• Not present.

6 Discussion

At the outset of this paper, we reviewed the literature on quantification in scientists' practice and students' work. We argued (1) that quantification is central to scientific activity and scientific understanding, (2) that its role in classroom science activity is not explicitly designed for and supported, particularly for young learners, and (3) that empirical activity, namely, re-scaling and re-representing phenomena in investigations, could serve as a resource for quantification and sense-making. We then traced opportunities for quantification through children's questions about temperature in one heating and cooling investigation, showing that children noticed differences in temperature and in results for which temperature was consequential and that, when supported, they made conceptual progress through quantification. We further substantiated these findings in a landforms investigation, showing that children asked about and differentiated aspects of force as they grappled with representing wind and water shaping land.

This analysis is consistent with, and extends, Jin et al.'s (2019) learning progression for quantifying phenomena. Through recognizing and considering gaps, ambiguities, and differences, students moved from holistic descriptions and comparisons of situations (chocolate melting in their pockets, heat making cakes dry out in an oven, tornados, waves on a beach) to using specific attributes for sense-making. For force, they considered direction, amount, and time sustained. For temperature, they worked to differentiate the temperature of a heat source, the temperature of material, and the ways that temperature interacted with the material to change the phase. Further, when the heating and cooling investigation was redesigned to support this work, they began to see attributes as comparable and measurable and to employ a temperature scale. This paper contributes to Jin et al.'s progression and the larger body of work on quantification (e.g., Kuo et al., 2013) by extending it to younger ages, focusing on the fine-grained work of moving from holistic description to attributes and measures, and exploring how this work is entwined with the development of conceptual understandings. Further, it contributes to the literature on empirical investigation in classrooms and on designing for children's meaningful science activity.

6.1 Quantification in Early Years Classroom Science

Research in early mathematics education has emphasized the importance of children developing a sense of quantity and an understanding of the meaning behind numbers, calculations, data, and measures. Yet, measurement work is often relegated to stand-alone units, where children move through rulers, volume containers, thermometer, and scales. Science curricula often introduce calculations, measures, and quantities as needed to allow students to demonstrate or calculate results (NASEM, 2021). In contrast, here the development of scientific ideas and mathematical measurements for temperature were intertwined. Developing an understanding of temperature transcended identifying it as a causal factor (e.g., heating can cause phase change) and involved (1) fine-grained sense-making about sources of heat and heat moving through materials, (2) explanatory work to consider the ways that objects' properties influenced how heat moved through them and whether they changed phase more or less easily, (3) developing a sense of an ordered set of values for temperature and considering adding and taking heat away (as opposed to heating and freezing as two different processes), and (4) connecting numerical values to experiences outside the investigation and using those to reason about the investigation.

These are ideas that might, if further developed with research over longer timescales, provide the basis for more sophisticated understandings of temperature, heat, energy, and material properties of objects (Clark, 2006; Erickson & Tiberghien, 1985; Lewis & Linn, 1994). A few researchers have sought to understand and design for the long-term integration of, or productive connections between, mathematical and scientific work in children's early schooling (Lehrer & Schauble, 2012; Sarama et al., 2017; Tytler et al., 2021). This is work that is important to build on and develop design principles for (NASEM, 2021). For example, interdisciplinary research could examine the quantitative basis of central scientific ideas through both conceptual analysis and analysis of children's questions and resources in activity, then work to sequence and connect mathematics and science units or lessons to support these connections. This work could form the basis for integrated or interdisciplinary curriculum development. To date, there has been more work developing curricula that systematically connect science and literacy/language development than science and mathematics (NASEM, 2021).

6.2 Highlighting Differences in Empirical Investigations

This analysis adds to work on quantification by exploring *when* and *how* students differentiate concepts into more specific attributes and experience a need for comparing and measuring attributes. Empirical investigations are a context where variables are central, but one that is not typically a focus for quantification (Chinn & Malhotra, 2002; Duschl et al., 2021). Curriculum materials tend to engage students only in investigations that have been scaled to perform correctly and to provide children quantities to use. In contrast, in the investigations described here, students took up opportunities for quantification when grappling with consequential differences in empirical work. In the heating and cooling investigation, differences included materials not changing as expected, materials that changed differently in the cake as compared to the hot plate investigation, and materials that did not change within the range of temperatures tested but *could* change. In the landforms investigation, students experienced differences in results as they manipulated materials in ways that affected the force of wind or water and considered differences in what happened in their investigation and what might happen at scales operating in the world.

This analysis, then, provides an alternative to the common strategy of focusing investigative work on what investigations *can demonstrate,* highlighting potential of *non-demonstration* and *difference* in investigations. Most often, the conceptual target of the work of an investigation is the claim that students are meant to support with evidence, putting the investigation in the position of needing to demonstrate that idea. For example, students shake pebbles or sugar cubes to show erosion, weigh a balloon before and after adding air to show that air has weight, or walk through a garden with socks on their hands to show that seeds can stick to animals. As described earlier, investigations fundamentally involve shifts in time, scale, and location; therefore, no investigation demonstrates perfectly. In typical investigations, the teacher often does substantial work to help children understand what the investigation is meant to show, and typically positions differences (other than those resulting from controlled variation of a factor) as unimportant.

In contrast, we center the role of non-demonstration in conceptual work, highlighting children's attention to materials not changing as expected, materials that changed differently in the cake as compared to the hot plate investigation, and materials that did not change within the range of temperatures tested but *could* change, and the ways that their engagement with these aspects of the investigation supported quantification and conceptual work. In each of these cases, students were able to extend their reasoning through digging into differences and proposing explanations and iterations to the investigation. Systematically designing ways to incorporate non-demonstration and support students to engage in evaluation and iterative problem-solving practices in empirical activity could both better represent the processes scientists engage in (Chang, 2004; Gooding, 1990; Nersessian, 2012) and elicit analogical reasoning practices shown to support the development of understandings (Gentner et al., 2003). At the root of each of these descriptions is the importance of selection, abstraction, and thinking-with-difference for conceptual progress. Further, difference is central to conceptualizations of sense-making that emphasize the role of a gap or ambiguity (e.g., Odden & Russ, 2018; Phillips et al., 2017).

This paper points toward the following approaches to incorporating difference into empirical work:

- **Scale**: Empirical investigations involve shifts in scale so that scientists can get a grip on processes at scales of time and magnitude that fit their needs (Knorr Cetina, 1999). This paper demonstrates that building in opportunities for children to grapple with what they can generalize from re-scaled representations supports them to see, differentiate, and numerically compare factors. In particular, we have found that it can be helpful to offer young students a generalization that they can push back on and argue against (e.g., "I think that heat doesn't change rocks. The rock stayed a solid on the hot plate."), then support them to consider and use quantity and scale for sense-making, as Ms. Lacy did.
- **Representation**: Asking children to participate in, and evaluate, the representational aspect of empirical work also supported quantification. Adam and other children cued into aspects of force such as source, duration, and direction as they considered how to use the spray bottles and straws to represent wind and water well enough to say something about the world. In another second-grade investigation, the height from which seeds fall and the distance they travel became meaningful as children struggled to understand why

the maple seed, which they associated with traveling with wind, dropped to the ground when placed in front of the fan in a seed travel test.

- **Manipulation and iteration:** A final resource for quantification is allowing students to manipulate materials and notice how those manipulations (e.g., of force, duration, direction) lead to differences, as well as inviting iterations of investigative work. Our findings suggest that children often see a need to quantify in the face of differences in outcomes and claims. These are more likely to be available during or after conducting an investigation, as opposed to in the planning stage, when children have to rely on previous experiences or thought experiments to generate consequential differences. An area for further work is to understand how sense-making during manipulation and iteration can be more effectively supported as resources for quantification and conceptual progress.

6.3 Designing and Supporting Sense-Making Environments for Children

We conclude by reflecting briefly on our approach to understanding children's point of view of the scientific entities under investigation and the idea of learning *from* children as integral to our design process. Current work in science education emphasizes the importance of designing coherent sequences of activity in which students make conceptual progress through the practice of science, including sense-making, modeling, and investigation (NRC, 2012; Schwarz et al., 2017). In practice, this work is difficult. It can lead to students participating in practices in rote ways (Gouvea & Passmore, 2017) or not making conceptual progress (Hmelo-Silver et al., 2007). It can exacerbate inequities in whose knowledge and practice is valued or recognized (Bang et al., 2012; Warren et al., 2001).

Along with other researchers (Hamza & Wickman, 2009; Keifert & Stevens, 2019; Rosebery & Hudicourt-Barnes, 2006; Russ, 2014), we underline the importance of centering learners' perspectives as we construct descriptions of disciplinary learning and practice. This work involves developing methods that allow us to see how young people organize their experience in science classrooms and how they move between the contingencies they experience and the generalizations they are in the process of developing (Hamza & Wickman, 2009). We and our teacher colleagues have found it imperative to have an initial design cycle in which we allow for and explore children's uncertainties in their empirical activity, combing the data to understand how children experienced empirical work by focusing on their interests, questions, and disagreements. We engage in extensive discussion and debate about how to deal honestly with their questions and concerns, supporting some by providing more explicit information (as in the case of providing temperatures on a thermometer) and considering which uncertainties are most productive to amplify and use as a context for extended discussion (as in the case of the ambiguity of the chocolate chips). This work often leads us to see the science involved in investigations, and the opportunities for science practice, in new ways.

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