A REDUCED-ADJOINT VARIATIONAL DATA ASSIMILATION FOR ESTIMATING SOIL MOISTURE PROFILE FROM SURFACE SOIL MOISTURE OBSERVATIONS

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ABSTRACT

Soil moisture plays an important role in the global water cycle and has an important impact on weather and climate, energy fluxes at the land surface, hence, the accurate estimation of this state variable is important. In this work, the potential of using surface soil moisture measurements (e.g. satellite soil moisture) to retrieve initial soil moisture profile will be explored in a synthetic study, using a reduced-adjoint variational data assimilation (hereafter RA-VDA) and a nonlinear 1D-soil water model (HYDRUS-1D). The proposed RA-VDA applies the Proper Orthogonal Decomposition (POD) technique to approximate the adjoint model in the reduced space. The accuracy and performance of the proposed RA-VDA method is illustrated with different synthetic experiments in a nonlinear physical model.

Index Terms— Variational Data Assimilation, Soil moisture, Richards equation, HYDRUS-1D, Proper Orthogonal Decomposition

1. INTRODUCTION

Soil moisture is an important state variable that plays a critical role in flood and landslide prediction, irrigation management, agricultural studies and land-atmosphere interaction studies [1], hence, measurement and simulation of soil moisture profile is of particular importance in many fields of engineering and science. Despite its importance, accurate measurement and estimation of soil moisture profile across different spatial scales is still a challenge. Satellite-based soil moisture observations can be obtained with well-defined spatial and temporal resolutions. However, they only provide surface soil moisture and not the soil moisture profile. Hydrological and land surface models are widely used to simulate soil moisture profile over space and time but there are several sources of uncertainty (i.e., unknown initial soil moisture profile, model structural error, etc.) that can cause large errors in the simulated soil moisture profile. As a result, the best approach to estimate the soil moisture profile is potentially the method of combining remote sensing data with hydrologic models (Data assimilation). Variational data assimilation (VDA) is a well-known method for estimation of the unknown parameters of a physical system given observations, by minimizing a cost function measuring the model-data misfit. VDA requires the computation of the gradient of the cost function through the adjoint technique. Computation of the adjoint is complex and extremely intensive for a nonlinear system with a large number of parameters. One way to overcome this complexity is to reduce the order of the problem. In this study, the Proper Orthogonal Decomposition (POD) technique is applied to VDA to reduce the order of the problem by computing the adjoint of the nonlinear system in the reduced space. The main objectives of this paper are to analyze the feasibility of the reduced-adjoint variational data assimilation (RA-VDA) approach to retrieve the initial soil moisture profile from assimilation of surface soil moisture into a physical model such as HYDRUS and thus improving the estimation/ prediction of soil moisture profile.

2. METHODOLOGY

2.1. HYDRUS-1D model

HYDRUS-1D is a software package developed by Simunek, Van Genuchten, and Sejna 2005 [2] to simulate water, heat and solute movement in one-dimensional variably saturated porous media. It is a physically based model that numerically solves the one-dimensional Richards equation, using Galerkin-type linear finite element schemes. In this study, HYDRUS as the forward model is applied to simulate soil moisture in an unsaturated root zone.

2.2. Estimation method

The method for estimating initial soil moisture profile is formulated based on the RA-VDA framework proposed by the authors in their earlier studies [3]. The objective of VDA approaches is to find a set of parameters with the smallest misfit to the observation for the original model defined as:

$$\theta\left(t_{i+1}\right)=F_i\theta\left(t_i\right), i=1,2,\ldots,m-1. \tag{1}$$

Where $\theta\left(t_{i+1}\right)\in R^n$ is a model state vector, $F_i : R^n \to R^n$ is a nonlinear dynamic operator. Suppose that $Y\left(t_i\right)\in R^q$ is the imperfect observations of a dynamical system, that are related to the model state at time t_i through the observation operator $H_i : R^n \to R^q$ as

$$Y(t_i) = H(\theta(t_i)) + \eta(t_i), \tag{2}$$

Operator H maps the model fields on observation space, $\eta(t_i)$ models' imperfection in the observations (such as measurement errors) and is assumed to be a white Gaussian observation noise process with zero mean and covariance matrix R_i. The cost function in VDA framework is given by

$$J(\theta_0) = \frac{1}{2}(\theta_0 - \theta^b)^T B_0^{-1} (\theta_0 - \theta^b) + \frac{1}{2}\sum_{i=0}^{m} \left(Y(t_i) - H(\theta(t_i))\right)^T R_i^{-1} \left(Y(t_i) - H(\theta(t_i))\right) \tag{3}$$

Where θ_b is a prior estimate of θ_0, assumed uncorrelated with covariance matrix B_0. The second term is the misfit between real observations and model estimated observations.

To reduce the order of the problem and compute the adjoint in the reduced space, Proper Orthogonal Decomposition (POD) method is used. To do this, a set of s snapshots of a physical model are collected from the numerical simulation of a forward model run. The elements of a snapshots should be able to describe the behavior of the system for unknown parameters [4]. To create POD modes matrix E which consists of centered snapshots is formed. The POD modes p_i are computed by

$$p_i = EZ_i / \sqrt{\lambda_i} \tag{4}$$

Where Z_i are eigenvectors and λ_i are eigenvalues of covariance of matrix E.

Using the method provided in [3], [5] the reduced dynamical model is constructed.

2.3. Reduced-Adjoint VDA (RA-VDA)

The goal of this approach is to use POD method to calculate the approximate adjoint model in the reduced space. In this approach, the original model is retained and only the adjoint is approximated by the POD method. The cost function here is the same as the equation 3. More details on the formula can be found in [6].

3. MATERIAL AND EXPERIMENT SETUP

With the aid of the forward model run (Hydrus 1-D), a synthetic data set will be designed, and observations will be generated (hereafter truth). The virtual surface soil moisture (observations) will be then assimilated to estimate the soil moisture profile by means of a variational data assimilation method. Forcing variables and meteorological data for forward model are required to characterize the atmospheric boundary condition and calculate the surface flux; i.e. precipitation, air temperature, wind speed, relative humidity, and net radiation were taken from FIFE experiment near Manhattan, Kansas [7]. The soil type falls in the texture classes of silty clay loam. The hydraulic parameters are obtained using the look-up table proposed by [8]. It is assumed that there is no vegetation cover, so LAI is zero. The flow domain of soil profile is 150 cm. To parametrize the soil hydraulic properties, van Genuchten-Mualem model [9], [10] is used.

Two cases in terms of type of initial soil moisture profile were analyzed under ideal condition. The ideal condition here means the same physical model, the same meteorological forcing and inputs, and the same observing system for both the synthetic truth and the assimilation. Doing so, we can test whether the assimilation algorithm is working properly.

4. RESULTS AND DISCUSSION

The results are obtained by assimilating surface soil moisture into HYDRUS-1D to demonstrate the efficiency of RA-VDA technique using a developed land surface model, as the forward model. The initial guess is assumed uniform soil moisture profile with the value of equal to the observed soil moisture at initial day.

Using the forward model setup explained in section 3, a basis consisting of only two POD modes is formed. Figure 1 demonstrates the cumulative variance of the data corresponds to the number of POD modes. As can be seen, by using two POD modes for each case, more than 95% of the total variance of the data would be captured.

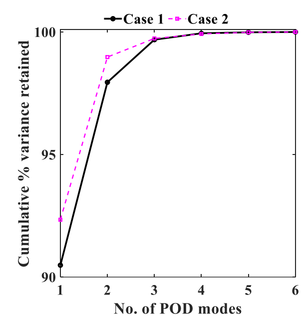

Figure 1- cumulative variance plot

The performance of the RA-VDA using HYDRUS as the forward model to retrieve the initial soil moisture profile are shown in Figure 2. As we can see in both figures, comparing the initial guess, the estimated soil moisture profiles are very close to the truth, although the only observation we had was for the surface soil moisture.

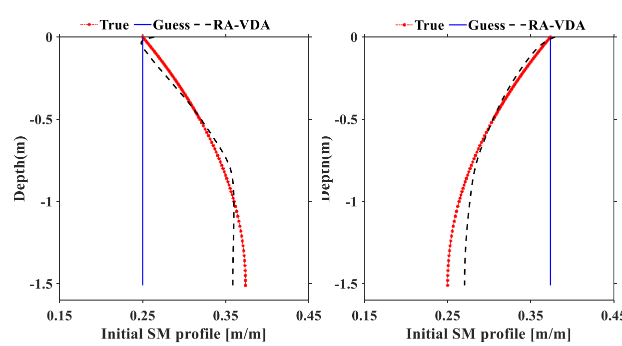

Figure 2- Estimated initial soil moisture profile, Case 1 (Left) and Case 2 (Right)

The time series of the soil moisture at the surface for observation, open loop and RA-VDA estimation is also demonstrated in Figure 3.

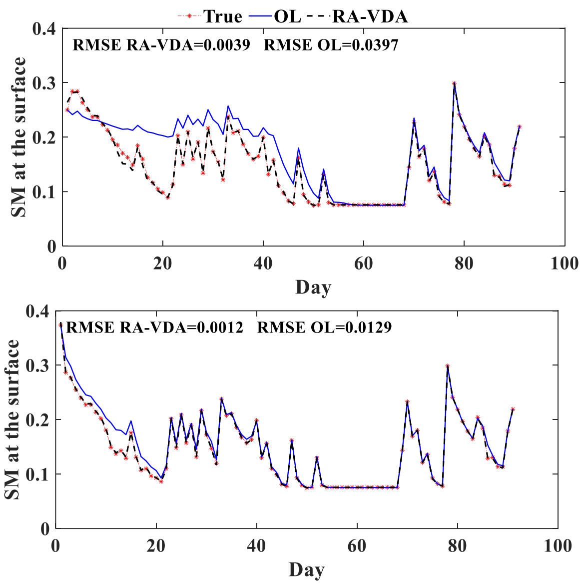

Figure 3- Surface soil moisture, Case 1 (upper figure) and Case 2 (lower figure)

As can be seen, a comparison of RMSE values for both RA-VDA and open-loop shows that the RA-VDA framework clearly improves soil moisture simulation over time.

5. CONCLUSION

To demonstrate the efficiency of the reduced-adjoint VDA framework to estimate initial soil moisture profile using a developed model such as HYDRUS-1D, a synthetic study was conducted. The preliminary results showed the success of the proposed reduced-adjoint VDA method in estimating the initial soil moisture profile from surface soil moisture observation with reasonable accuracy and its potential of being extended and used in a large-scale application using remote sensing satellite data as the observation.

11. REFERENCES

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