Quantized Floquet Topology with Temporal Noise

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Time-periodic (Floquet) drive is a powerful method to engineer quantum phases of matter, including fundamentally nonequilibrium states that are impossible in static Hamiltonian systems. One characteristic example is the anomalous Floquet insulator, which exhibits topologically quantized chiral edge states similar to a Chern insulator, yet is amenable to bulk localization. We study the response of this topological system to time-dependent noise, which breaks the topologically protecting Floquet symmetry. Surprisingly, we find that the quantized response, given by partially filling the fermionic system and measuring charge pumped per cycle, remains quantized up to finite noise amplitude. We trace this robust topology to an interplay between diffusion and Pauli blocking of edge state decay, which we expect should be robust against interactions. We determine the boundaries of the topological phase for a system with spatial disorder numerically through level statistics, and corroborate our results in the limit of vanishing disorder through an analytical Floquet superoperator approach. This approach suggests an interpretation of the state of the system as a non-Hermitian Floquet topological phase. We comment on quantization of other topological responses in the absence of Floquet symmetry and potential experimental realizations.

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Introduction.—Periodic Floquet drive is an indispensable tool in engineered quantum systems [1–6]. Recently, Floquet drive has enabled the realization of fundamentally nonequilibrium phases of matter, such as Floquet time crystals [7–14] and Floquet symmetry-protected topological states (SPTs) [15–28]. A quintessential example of Floquet SPT is the anomalous Floquet-Anderson insulator (AFAI), which has topologically protected chiral edge states similar to a Chern insulator but with a fully localizable bulk [29–34]. Topologically protected transport in the AFAI can be measured via current flowing through the system [30,35], magnetization density in a fully filled patch within the bulk [36], or quantized transport of quantum information at the edge [22,37,38].

All of these nonequilibrium states are protected by discrete time-translation symmetry of the Floquet Hamiltonian, $H(t) = H(t + T)$, where $T = 2\pi/\Omega$ is the driving period. In this Letter, we ask what happens to the AFAI upon breaking time-translation symmetry via time-dependent random noise. A similar question has been studied in the case of a Floquet SPT protected by chiral symmetry [39,40], where the authors found that edge states decay at a slow but finite rate set by diffusion. In this work, we instead find that for the two most realistic experimental protocols, namely bulk magnetization or current measurements in partially filled samples as illustrated in Fig. 1(a), the topological response remains *fully protected* over a timescale that diverges in the thermodynamic limit. We trace this topological protection back to Pauli blocking, which prevents diffusive loss of the topological edge state pumping up to approximately the Thouless time as shown in Fig. 1(b). We argue that the results should hold for many-body localization as well as Anderson localization, and comment on the potential for experimental realization.

Model.—Throughout this Letter, we study a single-particle model of the anomalous AFAI with time-dependent noise. We start from the original AFAI model [30], which involves a five-step Floquet drive. The first four steps involve hopping between sites of the two sublattices. Specifically, for step $\ell \in \{1, 2, 3, 4\}$, the Hamiltonian is

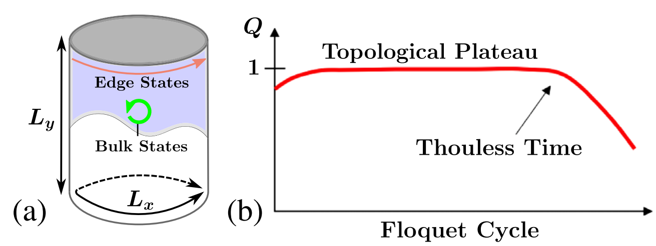

FIG. 1. Illustration of quantized nonadiabatic pumping in the presence of noise. (a) The two-dimensional system is placed on a cylinder with the top half filled and bottom half empty, and driven via a five-step Floquet drive (Fig. 2). Pumped charge Q around the cylinder per Floquet cycle is quantized without noise due to topological edge states. The bulk states are localized, undergoing cyclotronlike orbits during each Floquet cycle (green arrow). Noise is added by disordering the timings of the five-step drive. (b) For weak noise, Q goes to a topological plateau after a nonuniversal short-time transient, before decaying when the edge states start to depopulate at times of order the Thouless time.

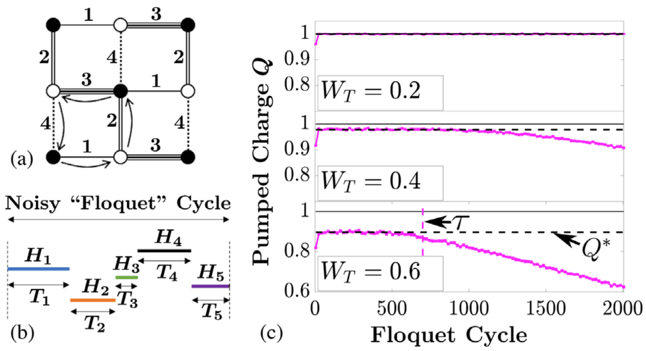

FIG. 2. Noisy AFAI model. (a) First four steps of drive protocol. Hopping occurs on bonds labeled 1 for $0 < t < T_1$, on bonds labeled 2 for $T_1 < t < T_1 + T_2$, etc. Filled (empty) circles are sites of sublattice A (B). (b) Noise is added by randomly changing the time over which the Hamiltonians are present, $T_\ell = T(1+\delta_\ell)/5$. The random noise $\delta_\ell \in [-W_T, W_T]$ is different for each "Floquet" cycle. (c) Charge pumped per Floquet cycle for 1D spatial disorder with $W = 0.2$ and $L = 100$, averaged over spatial and temporal disorder. Dashed lines show nonquantized plateau value.

$H_\ell = -J \sum_{\langle ij \rangle_\ell} c_i^\dagger c_j$, where c_j is the fermion annihilation operator on site j and $\langle ij \rangle_\ell$ indicates the bonds that are "turned on" during step ℓ, as illustrated in Fig. 2(a). During step 5, a sublattice-dependent potential of strength Δ is applied: $H_5 = \Delta \sum_j \eta_j c_j^\dagger c_j$, where $\eta_j = +1$ (-1) on the A (B) sublattice. Each Hamiltonian H_ℓ is present for time T_ℓ, which in the absence of temporal order is just $T_\ell = T/5$. The hopping Hamiltonians H_{1-4} are chosen such that, for the fine-tuned value $J = J_0 \equiv 5\Omega/4$, bulk electrons undergo a "cyclotron" orbit during each Floquet cycle and return to their original site, as illustrated in Figs. 1(a) and 2(a). A static chemical potential disorder is added throughout the cycle with Hamiltonian $H_{\text{dis}} = \sum \mu_j c_j^\dagger c_j$, where each μ_j is uniformly sampled from the interval $[-W, W]$. Units are set by $\Omega = \hbar = 1$, and we choose $\Delta = 0.4\Omega$ and $J = J_0 = 5\Omega/4$ throughout.

In this work, we modify the Hamiltonian by adding temporal disorder (noise). Explicitly, noise is introduced via random modification of the Floquet timing: $T_\ell = T(1+\delta_\ell)/5$, where $\delta_\ell \in [-W_T, W_T]$ is sampled uniformly and independently during each Floquet cycle [41]. Naively, one expects that noise will immediately destroy the Floquet topological phase, as it breaks the time periodicity [39]. Yet, as we will show, the topological response remains robust against weak noise due to special properties of the AFAI's topological response.

In our numerics, we measure topologically protected nonadiabatic charge pumping for a cylinder of $L_x = 2L$ and $L_y = L$ lattice sites [30]. As shown in Fig. 1(a), the system is initialized with one half of the cylindrical crystal filled with particles and the other half left empty. We measure the charge pumped during each cycle,

$$Q = \int_{t_0}^{t_0+\tilde{T}} dt \langle \psi(t) | J_x | \psi(t) \rangle, \tag{1}$$

where J_x is the current in the x direction, $\tilde{T} = \sum_\ell T_\ell$ is the "Floquet" period appropriately modified by noise, and t_0 is the time at the start of the cycle. In the absence of temporal disorder, Titum *et al.* [30] demonstrated quantization of Q in the presence of spatial disorder. One may think of this quantization as coming from the single filled edge state, which pumps $Q = 1$ per cycle in the topological phase, while the localized bulk states do not carry current. In the presence of temporal disorder, the bulk states no longer remain localized; we now demonstrate how this affects Q.

One-dimensional disorder.—Large two-dimensional (2D) lattices without translation symmetry are computationally challenging to simulate. Therefore, as a warmup problem in which we can address large system sizes, we begin by implementing one-dimensional (1D) spatial disorder in the y direction, meaning that for site $j = (x, y)$, μ_j only depends on the y position.

Some characteristic traces of Q vs t are shown in Fig. 2(c). For weak temporal and spatial disorder, the charge approaches a plateau value and remains nearly perfectly quantized up to more than 2000 drive cycles. As W_T is increased, the plateau value of the pumped charge is no longer quantized and the pumped charge begins to decay at late times. To quantify this behavior, we define two quantities: the plateau value of pumped charge Q^* and the decay timescale τ.

The key to understanding these quantities is their dependence on system size L, shown in Fig. 3(a). We see that the plateau value Q^* does not depend on system size, while τ increases sharply with system size. We have confirmed that this finite-size dependence of τ reflects the known fact that temporal disorder causes the particles of the system to undergo a diffusive random walk [40]. The consequence of this diffusion is that the sharp density edge separating the top and bottom half of the system spreads diffusively into a smooth position dependence of the density, until eventually the edge state starts to depopulate on a timescale of order the Thouless time, $t_D = L^2/D$ with diffusion constant D. We have confirmed that this diffusive behavior behaves independently of the initial configuration of occupied sites through magnetization calculations in Supplemental Material [42]. Since nonadiabatic charge pumping comes entirely from the edge state, the loss of edge state occupation corresponds to a loss of the signal in Q, and thus τ will be proportional to the Thouless time.

We can now draw two important conclusions about the system with one-dimensional disorder. First, the pumped charge reaches a plateau that eventually decays on a timescale $\tau \sim L^2$. Importantly, this implies that the plateau will be infinitely long-lived in the thermodynamic limit, where the topological phase is defined. Second, we learned that the plateau value Q^* loses quantization as either spatial

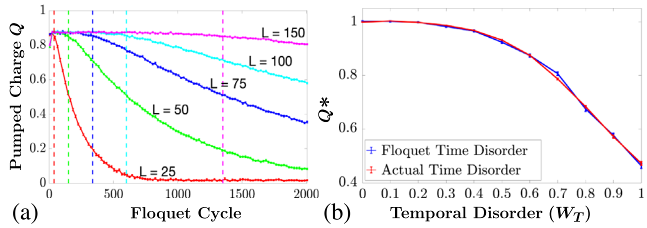

FIG. 3. Finite size effects for 1D disorder. (a) System size dependence of Q for $W = 0.5$ and $W_T = 0.6$. The dashed lines show times $\tau \sim L^2$, illustrating that the pumped charge begins to decay on a timescale of order the Thouless time, which is set by diffusion. (b) Comparison of plateau value Q^* for actual time disorder and "Floquet time disorder," in which the same random pattern of δ_ℓ is repeated indefinitely. Finite size effects have been removed by extrapolating to $L \to \infty$ using a linear fit to Q^* versus $1/L$ at large L. All data shown are averaged over spatial and temporal disorder.

or temporal disorder are added. We note that this loss of quantization with W is similar to the Floquet-Thouless energy pump [44], which is a 1D system where the spatial x direction and the associated conserved momentum k_x in our system is replaced by an adiabatic pump parameter λ. Temporal disorder had not been studied earlier, but it causes a similar smooth reduction of Q^* from its quantized value. We conjecture that this physics is, in fact, exactly captured by that of the Floquet-Thouless energy pump. Specifically, we consider the behavior of a related Floquet system created by randomly sampling the times $T_1, T_2, \ldots, T_5$ as before, but then repeating this random sequence for each Floquet cycle. Such a Floquet system will achieve a plateau value Q^* and then stay there [44], as there is no diffusion to prevent localization. We refer to this construction as "Floquet time disorder."

We compare the results of actual time disorder and Floquet time disorder in Fig. 3(b), showing that they match within error bars after extrapolation to the thermodynamic limit. Importantly, each realization of the Floquet time disorder can be analyzed in the language of the Floquet-Thouless energy pump, meaning that our nontopological response with time disorder is obtained by averaging over the nonquantized responses from the Floquet-Thouless energy pump. This explains why the response is not quantized, and provides a valuable method for defining (average) topology in this temporally disordered system.

Two-dimensional disorder.—Having understood one-dimensional disorder, we can now make predictions for the actual case of interest, namely full two-dimensional disorder, in which μ_j is chosen independently for each site. The dependence of τ on system size will be the same with 2D disorder, since the mapping to a diffusive random walk still applies. This means that the plateau value Q^* should again be infinitely long-lived in the thermodynamic limit, which we have confirmed in Supplemental Material [42]. However, a more interesting fact comes out of thinking about this plateau value. Unlike the case of 1D disorder, 2D disorder has a nontrivial topological phase (the AFAI) that survives to finite disorder, with a sharp transition from $Q = 1$ to $Q = 0$ at finite W_c [30]. Therefore, our analysis of 1D disorder implies that the nontrivial topological phase will also survive for weak Floquet time disorder, since this is a perturbative deformation of the original AFAI model. Given that time disorder and Floquet time disorder demonstrate identical plateau values for Q^* upon averaging over disorder configurations, we thus predict that the AFAI is *stable* to weak temporal noise.

This intuition is confirmed numerically in Fig. 4 using the a well-established technique introduced by Titum *et al.* [30]. Specifically, for a given realization of Floquet time disorder, we calculate the Floquet quasienergies ϵ_n^F and determine the statistics of their nearest-neighbor level spacings: $\Delta_n \equiv \epsilon_{n+1}^F - \epsilon_n^F$. We calculate the r statistic [45]:

$$r_n = \min[\Delta_n, \Delta_{n+1}] / \max[\Delta_n, \Delta_{n+1}], \qquad (2)$$

whose average over disorder and eigenstates $\langle r \rangle$ is a useful indicator of level repulsion. $\langle r \rangle$ converges to the Poisson

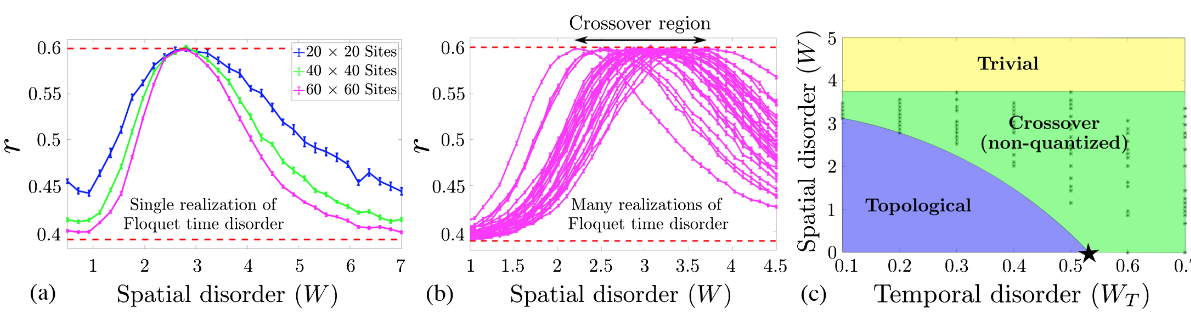

FIG. 4. Topological phase diagram for 2D disorder in the presence of temporal noise. (a) Level spacing ratio r averaged over spatial disorder for a single realization of Floquet temporal disorder with $W_T = 0.3$. A clear peak is seen at $W_c \approx 2.8$, becoming increasingly sharp with increasing system size. We identify this as the critical point. (b) Data for system size of 60×60 sites with 30 different realizations of Floquet temporal disorder, showing that different realizations lead to different values of W_c. (c) Phase diagram obtained from peaks of r, plotted as black dots. The value of $W_{T,c}$ for $W = 0$, which is indicated with an asterisk, is obtained from the gap closing of the noise-averaged Floquet superoperator (see Supplemental Material [42]).

value $r_P \approx 0.39$ for localized systems that do not display level repulsion, and to the circular unitary ensemble (CUE) value $r_C \approx 0.6$ for delocalized systems. In the present case of noninteracting particles, both the topologically nontrivial phase at low W and the topologically trivial phase at high W are localized, giving r_P. Right at the phase transition, the system delocalizes, creating a sharp peak with CUE level statistics. This peak was shown to be a sensitive indicator of the phase transition for the Floquet model [30], and we see this holds with Floquet time disorder as well [Fig. 4(a)] [46]. Therefore, for a given realization of Floquet time disorder, we can obtain the critical disorder value W_c by finding this peak.

There is one notable effect of Floquet time disorder, namely that different realizations of time disorder yield different values for this critical W_c, as seen in Fig. 4(b). In other words, Floquet time disorder does not self average. This means that there is not a sharp transition from topologically nontrivial to trivial, but rather a topologically nontrivial phase for $W < W_{c,\min}$, a topologically trivial phase for $W > W_{c,\max}$, and a crossover region in between where the response is not quantized. The full phase diagram showing these three regions is plotted in Fig. 4(c), with best estimates for the phase transition lines $W_{c,\min/\max}$.

Floquet superoperator approach.—The topological transition can be obtained directly from the noise-averaged Floquet superoperator; an approach, which unlike that of the level spacing ratio, does not involve the auxiliary system with Floquet time disorder. The evolution of the density matrix ρ of a single particle during a noisy Floquet cycle is described by a superoperator $\mathcal{U}\rho = U\rho U^\dagger$, where $U = e^{-iT_5H_5}, \ldots, e^{-iT_1H_1}$. While spatial disorder can be incorporated into this superoperator, we consider a system with no disorder. Averaging over temporal noise, this becomes a nonunitary Floquet superoperator $\mathcal{F}$, whose eigenvalues lie within the unit circle on the complex plane [39]. We analyze this superoperator in detail in Supplemental Material [42]. We find that a gap closes on the real axis at $W_{T,c} = 0.535$, which is indicated in Fig. 4(c) with an asterisk, suggesting a topological transition. Furthermore, one can define a generalized winding number for this superoperator, which ceases to be quantized for $W_T > W_{T,c}$ due to issues taking a branch cut along the real axis. This is consistent with the nonquantized crossover regime found earlier, and provides a readily generalizable, complementary perspective on our topological phase diagram. The Floquet superoperator analysis also allows us to define our topological system in the language of non-Hermitian Floquet topological SPTs [47], which should be readily extensible to other systems and symmetry classes.

Discussion.—We have shown that the two-dimensional anomalous Floquet-Anderson insulator is stable to weak temporal noise. The argument involves constructing a related Floquet system for a given noise realization and then arguing that if each such realization is topological, then their noise average, which is given by the superoperator approach, is topological as well. This argument should hold for other types of environmental noise, and therefore we expect that the AFAI is stable to a wide class of weak dissipative couplings. Correlated noise would kill this argument, hence we leave generic non-Markovian baths for future work. We have begun to explore the implementation of different types of temporal noise in Supplemental Material [42]. These responses may also be stable to quasiperiodic driving, which leads to a variety of interesting steady states in other contexts [48–50].

While we numerically studied the topological response via charge pumping in a half-filled system, our arguments indicate that a similar story should hold for other proposed experimental measurements of the anomalous Floquet insulator [36,37,51]. For instance, topologically quantized magnetization for a filled region of linear size l [51] should hold up to time $\tau \sim l^2$ and remain measurable by the same protocols. We have provided data that confirm this quantized magnetization in the supplement [42], even though the diffusive scaling of τ is not tested explicitly. This fact will be important in practical experimental realizations, as there are always finite noise sources—such as laser fluctuations or spontaneous emission into lattice lasers—that break the Floquet symmetry of the problem.

It has recently been argued that the AFAI is stable to interactions [36], and we suspect the same will be true in the presence of noise. An interesting question is how noise affects other topological invariants that have been identified in the AFAI [52], which are also theoretically measurable. Finally, we speculate that similar ideas may be used to demonstrate stability in other Floquet topological phases, such as the Floquet topological superconductor, with possible implications for robust quantum information processing and computation [53,54].

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