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Quantum Higgs inflation

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ABSTRACT

The Higgs field is an attractive candidate for the inflaton because it is an observationally confirmed fundamental scalar field. Importantly, it can be modeled by the most general renormalizable scalar potential. However, if the classical Higgs potential is used in models of inflation, it is ruled out by detailed observations of the cosmic microwave background. Here, a new application of non-adiabatic quantum dynamics to cosmological models is shown to lead to a multi-field Higgs-like potential, consistent with observations of a slightly red-tilted power spectrum and a small tensor-to-scalar ratio, without requiring non-standard ingredients. These methods naturally lead to novel effects in the beginning of inflation, circumventing common fine-tuning issues by an application of uncertainty relations to estimate the initial quantum fluctuations in the early universe. Moreover, inflation ends smoothly as a consequence of the derived multi-field interactions.

1. Introduction

One of the most attractive features of cosmic inflation is that it may be able to explain how the observed large-scale structure of the universe, captured in the distributions of microwave background radiation or galaxies, could have evolved out of tiny initial quantum fluctuations [1,2]. It presents a stunning example of how the microscopic and macroscopic realms can be bridged, potentially allowing cosmologists to test quantum mechanics through large-scale observations. However, traditional models of cosmic inflation require the presence of a certain scalar field in the early universe, the inflaton, with specific self-interactions that imply negative pressure pervading the entire early universe. Interactions that reliably imply this behavior often need to be finely tuned to achieve observationally viable models. Moreover, it is usually unclear whether ingredients required for fine-tuning can be derived from fundamental physics.

Here, we present further evidence for the general picture painted by cosmic inflation by introducing a new combination of cosmological equations with non-adiabatic methods for quantum dynamics. The novelty of this work is to capture non-trivial quantum effects in an initial phase of inflation which have generally been missed in previous studies that considered a single scalar field on an expanding background. We will show how a quantum state actively models the self-interactions of a multi-component inflaton field, and how this new potential can overcome several conceptual and phenomenological issues encountered when one tries to build inflation on properties of the Higgs field. The resulting scenario is consistent with current observations. With more precise future data, it may be used to deduce properties of the quantum state in the very early universe.

We begin with a Higgs-like field ψ with classical potential

$$V_{\rm class}(\psi) = M^4\left(1-2\frac{\psi^2}{v^2}+\frac{\psi^4}{v^4}\right) \tag{1}$$

where M and v are constants, assumed positive. When used in a quantum field theory, this potential is the most general one (up to adding a constant) that results in a renormalizable theory. It is therefore preferred in a model of the early universe because it implies physical effects independent of poorly understood high-energy phenomena. This decoupling allows inflation to be applied as a low-energy effective theory, avoiding details of quantum gravity. (Alternatively, one could consider only potentials restricted by a high-energy theory, for instance in terms of swampland conjectures [3].) However, for ψ to play the role of a phenomenologically

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viable inflaton [4], the potential must be extended in some way, for instance by including higher-order monomials in ψ or even non-polynomial functions as in Starobinsky-type inflation [5], or by introducing non-trivial interactions between ψ and space-time curvature [6,7].

Another possibility is to consider multi-field inflation, in which ψ is just one of several interacting scalar fields. Additional constants then appear in potentials and interaction terms, and the system seems to become more ambiguous as well as more distant from fundamental physics. The new mechanism presented here, by contrast, will use basic properties of quantum mechanics to turn any single-field potential, such as (1), into an equivalent multi-field system. All coupling constants between the fields can then be derived from v and M in (1) as well as fundamental constants such as $\hbar$, but they will also depend on parameters that characterize the quantum state of ψ, such as its fluctuations. The resulting multi-field inflation is therefore much more restricted than in usual cosmological model building, in which interaction terms are postulated so as to obey phenomenological constraints. As one of our main results, the new quantum-based multi-field scenario is nevertheless consistent with current observations, without excessive fine-tuning.

Heuristically, quantum mechanics always implies that a single classical variable, such as the position x of a particle or our field ψ, is described by an infinite number of quantum degrees of freedom. For instance, an initial wave function $\Psi(x)$ can be chosen to be centered at any value $x_0 = \langle\Psi|\hat{x}|\Psi\rangle$, and independently have an arbitrary variance $(\Delta x)^2 = \langle\Psi|(\hat{x}-x_0)^2|\Psi\rangle$ around x_0. Similarly, higher moments $\Delta x^n = \langle\Psi|(\hat{x}-x_0)^n|\Psi\rangle$ are independent, amounting to infinitely many quantum degrees of freedom. There are further moments involving the momentum p, or combinations of x and p. In standard quantum mechanics, all these values are captured by a wavefunction or density matrix.

As time changes, the initial wave function evolves such that, generically, the moments depend on time. They form an infinite-dimensional dynamical system with interacting degrees of freedom in addition to the classical x. It has been known for some time that there are equivalent classical-type systems that describe the same quantum dynamics, derived from multi-component effective potentials. In particular a semiclassical approximation in which only moments of second order are considered — position and momentum variances as well as their covariance — has been formulated several times independently [8–13]: With $s = \Delta x$ a single quantum degree of freedom to this order, the effective potential

$$V_{\rm eff}(x,s) = V(x) + \frac{U}{2s^2} + \frac{1}{2}V''(x)s^2 \qquad (2)$$

describes Hamiltonian dynamics equivalent to a first-order approximation in $\hbar$ of the expectation value $x \approx \langle\hat{x}\rangle$ and fluctuation s derived from a solution of the Schrödinger equation with potential $V(\hat{x})$. The constant U does not depend on time but only on the initial state. It obeys $U \geq \hbar^2/4$ as a consequence of Heisenberg's uncertainty relation. In our application to cosmology, s will be a multi-field partner to the Higgs-like ψ.

The presence of new quantum degrees of freedom in an effective potential may be unfamiliar to particle physicists and cosmologists, but it is a common implication of non-adiabatic dynamics. It is possible to derive an effective low-energy potential from (2) by minimizing $V_{\rm eff}(x,s)$ with respect to s, at fixed x. The result, $V_{\rm low-energy}(x) = V(x) + \sqrt{UV''(x)}$, is a quantum-corrected potential which contains higher derivatives of V instead of additional degrees of freedom. Also in quantum field theory, the low-energy effective potential commonly used in high-energy physics and cosmology can be seen as a leading-order derivative expansion of a multi-field potential analogous to (2) [14]. (The field theory version of $\sqrt{UV''(x)}$ quoted here is, for $U = \hbar^2/4$, equivalent to the Coleman–Weinberg potential [15] integrated over the time component of the wave number. The applicability of these methods to the relation between quantum field theory and background quantum mechanics has been demonstrated in [16].)

It is possible to extend the low-energy potential by assuming that s is not exactly at its minimum but stays close to it and oscillates slowly. Such an expansion, characterized by the slowness condition as an adiabatic one, is equivalent to the usual perturbative methods used to derive effective potentials. A multi-field potential such as (2) completely forgoes the adiabatic or derivative expansion and instead describes the dynamics of a quantum degree of freedom, s. Independent quantum degrees of freedom such as s are therefore relevant whenever the dynamics is non-adiabatic. Inflation is usually presented in a slow-roll regime, in which the inflaton changes slowly and its potential energy dominates over its kinetic energy. The potential then acts as a medium with tension, implying negative pressure. Such a regime should be well-described by adiabatic dynamics, and low-energy effective potentials should be sufficient, as often used in this context. However, as we will show in detail, a non-adiabatic precursor phase in a multi-field potential such as (2) is nevertheless important because it can help to set correct initial conditions for subsequent long-term inflation without excessive fine-tuning. A final non-adiabatic phase then helps to end inflation.

The initial stages of inflation are expected to be dominated by quantum phenomena. A leading-order semiclassical approximation as in (2) is then insufficient. The canonical formulation of second-order moments has recently been extended to higher orders, with complete parameterizations up to fourth-order moments [17,18]. Each additional moment order implies new quantum degrees of freedom, given by three values φ_1, φ_2 and φ_3 at third order and five at fourth order. The variance is the sum of the squares of these variables, $\Delta\psi^2 = \varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \cdots$, while third and fourth order moments of ψ are polynomials of degree three and four, respectively. The higher-order extension of (2) is linear in $\Delta\psi^n$ with coefficients given by the Taylor expansion of the classical potential around the expectation value, just as seen in (2) at second order.

In order to obtain a manageable system, we will assume that one of the quantum variables, called φ, is most relevant and replaces s in the mechanics example, such that $\Delta\psi^2 = \varphi^2$. For higher moments, $\Delta\psi^3 = \alpha_3$ and $\Delta\psi^4 = \alpha_4\varphi^4$ with parameters α_3 and α_4 that describe properties of the quantum state. For a Gaussian state, $\alpha_3 = 0$ and $\alpha_4 = 3$. The parameters α_3 and $\delta = \alpha_4 - 3$, which turn out to be crucial in our analysis, may therefore be considered non-Gaussianities of the background state of ψ. They allow us to go beyond a Gaussian approximation of effective potentials. Our parameterization of moments is an example of a moment closure, a common technique for coupled or partial differential equations in fields where there is experimental input [19]. It allows us to visualize the quantum dynamics through lower-dimensional potentials. Imposing the closure condition is an approximation, which may be tested with numerical means in a full setting of quantum moments. We expect that qualitative features as we will derive as a consequence of the presence of quantum degrees of freedom are described well by our approximation.

Moments enter an effective potential such as (2) by expanding the expectation value $\langle\hat{V}_{\rm class}\rangle$ of a potential operator in a state described by these moments around the expectation value $x = \langle\hat{x}\rangle$:

$$\langle V(\hat{x})\rangle = \langle V(x + (\hat{x}-x))\rangle = V(x) + \frac{1}{2}V''(x)\langle(\hat{x}-x)^2\rangle + \cdots . \qquad (3)$$

Since our classical potential is quartic, only moments up to fourth order appear in this expansion. It turns out that there is a second contribution to effective potentials, given by the term $U/(2s^2)$ in (2), which comes from the expected kinetic energy: The canonical formulation of moments can be seen to imply $\Delta p^2 = p_s^2 + U/s^2$

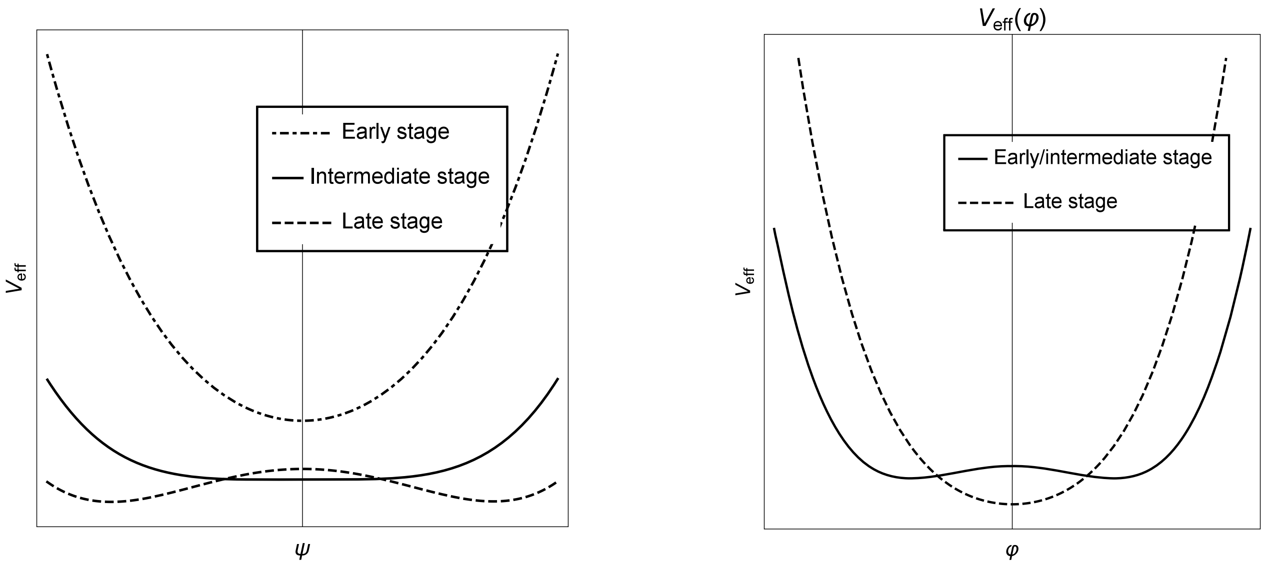

Fig. 1. Left: Build-up of a tachyonic potential for the Higgs-like field ψ. Pre-inflation stage is characterized by $\varphi > \varphi_c$, while $\varphi < \varphi_c$ signals the start of the intermediate stage. Right: Dynamical restoration of symmetry for the fluctuation field φ. The potential $V_{\rm eff}(\varphi)$ of early stages look similar to the intermediate stage.

where p_s is the momentum of s. (See [18] for a systematic derivation.) The term p_s^2 provides the usual kinetic energy of the new quantum variable, s or φ, while U/s^2 contributes to the potential.

Before we apply this method to cosmology, we have to make a small adjustment because the energy contributions of a scalar field depend on the scale factor a of expanding space. The kinetic energy of a free scalar field in an expanding universe decreases with a because of dilution, while potential energy, $V(\psi)a^3$, acts like a medium with tension and increases with a. The precise dependence on a can be derived from the canonical formulation, and leads to an additional factor of a^{-6} in the U-term in an effective potential. It is accompanied by a parameter V_0 which, by definition of a homogeneous model, determines the comoving scale over which inhomogeneities can be ignored [16]. The volume V_0a^3 should be larger than the Planck scale in order to avoid the trans-Planckian problem [20–22]. With our ansatz for moments and defining $\varphi_c^2 := \frac{1}{3}v^2$, the effective potential is

$$V_{\rm eff} = M^4\left(1 + 2\left(\frac{\varphi^2-\varphi_c^2}{\varphi_c^2}\right)\frac{\psi^2}{v^2} + \frac{4\alpha_3\,\psi}{v^4} + \frac{\psi^4}{v^4} - \frac{2}{3}\frac{\varphi^2}{\varphi_c^2} + \alpha_4\frac{\varphi^4}{v^4}\right) + \frac{U}{2a^6V_0^2\varphi^2}\,. \tag{4}$$

After a few e-folds of inflation, the last term can be ignored since a grows quickly. It is nevertheless important because it implies a repulsive potential for φ, necessitating φ to start out at large values. This alleviates the need to fine-tune the usual initial condition $\varphi > \varphi_c$ [23]. Because φ is initially large, the quartic φ-term in (4) dominates at early times, along with the U-term. Their sum has a local minimum at $\varphi = \sqrt[6]{Uv^4/(4a^6V_0^2M^4\alpha_4)}$. Using the Planckian lower bound for V_0a^3, we obtain an upper bound $\varphi_{\rm ini} < \ell_P^{-1}\sqrt[6]{Uv^4/(4\alpha_4M^4)}$ on the initial fluctuation variable. If we assume $v \sim \mathcal{O}(M_{\rm Pl})$ and $M^4 \ll M_{\rm Pl}$ as in the detailed analysis to follow, this upper bound is well beyond φ_c. (We are using units such that $M_{\rm Pl} = \hbar = c = 1$, turning M into an energy scale.)

Our potential (4), combining the dynamics of the classical field and its fluctuation, is of the hybrid-inflation type. These models typically produce a blue-shifted tilt when one starts with a large φ and small ψ, relying on a constant vacuum energy of ψ [23]. However, if instead a *waterfall* regime is responsible for a significant number of e-folds, where φ stays close to a local minimum, one may obtain a red tilt for a wide range of parameters [24,25]. In our case, as opposed to the traditional hybrid model, the inclusion of non-adiabatic dynamics implies two phase transitions and the majority of e-folds are created in between. Other variations of hybrid models [26] which produce a red tilt require additional mechanisms for stability against quantum corrections [27], making them much less natural by introducing further tunings. Our model is stable because it describes a quantization of the generic renormalizable potential (1).

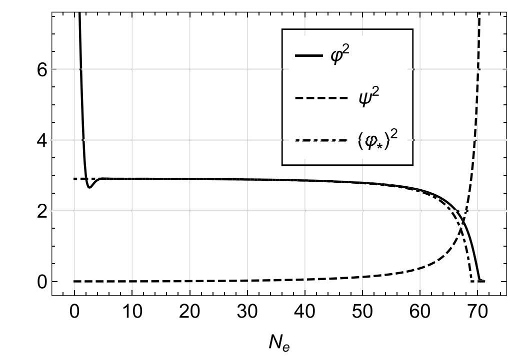

Fig. 2. The squared values of exact solutions $\varphi(N_e)$, $\psi(N_e)$ and $\varphi_*(N_e)$ are plotted using $v = 3$, $\alpha_3 = 0.05$, $\delta = 0.1$. The field φ follows its minimum closely throughout the majority of inflation, while the field ψ rolls down to its new potential minimum $\psi^2_{\rm min} = v^2$ in the end (see also Fig. 1). Non-adiabatic evolution is apparent during the start and end of inflation.

As in the original hybrid model, we start with some $\varphi > \varphi_c$, in accordance with our analysis of the last term in (4). By construction, φ describes quantum fluctuations of ψ. It should indeed be large for a highly quantum initial state, even while the expectation value ψ remains small at the local minimum of its early-time potential Fig. 1. For such a large value of φ, its early-time potential, is steep. The field quickly approaches one of its minima driven by the φ^4-term in (4).

Once φ crosses φ_c, the potential of ψ, Fig. 1, changes to its tachyonic intermediate-stage form with true minima located at non-zero ψ. In (4), reflection symmetry of ψ is broken for any non-zero α_3. As the tachyonic contribution builds up, the field starts slowly rolling away from the origin, acting as the instability required to kick-start the waterfall regime. This slow change,

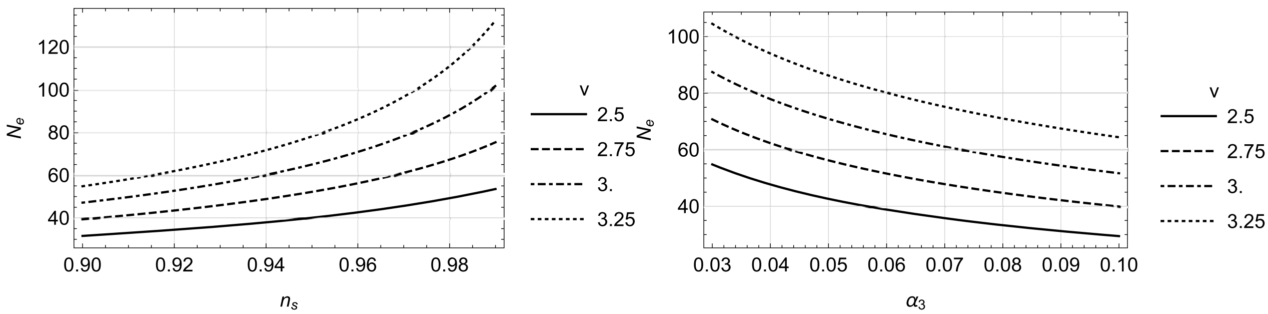

Fig. 3. Left: Number of e-folds, N_e, as a function of the spectral index n_s, using the approximation (8) with various v. Right: Number of e-folds, N_e, as a function of the non-Gaussianity parameter α_3, using (8) and assuming $n_s \approx 0.96$. Non-Gaussianity speeds up the departure from adiabatic evolution, ending inflation earlier. Here, $\delta = 0.1$. For smaller non-Gaussianity parameters δ, N_e increases because n_s, a function of the ratio $\delta/\alpha_4 \approx \delta/3$ is then closer to one.

now in an adiabatic phase, enables φ to follow its vacuum expectation value, φ_*. Eventually, $\varphi_* \to 0$, as indicated by the late-time potential, Fig. 1. This second phase transition restores reflection symmetry for φ. In summary, φ causes the traditional phase transition when it crosses φ_c, and the subsequent slow roll of ψ down its tachyonic hilltop eventually triggers a second phase transition. The adiabatic phase of slow-roll inflation takes place between the phase transitions.

For quantitative predictions, we solve the full equations

$$\ddot{\psi} + 3H\dot{\psi} = M^4 \left(-\frac{4\psi}{v^2} \left(\frac{\varphi^2 - \varphi_c^2}{\varphi_c^2} + \frac{\psi^2}{v^2} \right) - \frac{4\alpha_3}{v^4} \right) \tag{5}$$

$$\ddot{\varphi} + 3H\dot{\varphi} = M^4 \left(\frac{4\varphi}{3\varphi_c^2} \left(1 - \frac{3\psi^2}{v^2} \right) - \frac{4\alpha_4\varphi^3}{v^4} \right) \tag{6}$$

$$6H^2 = \dot{\psi}^2 + \dot{\varphi}^2 + 2V_{\rm eff}(\psi, \varphi) \tag{7}$$

using numerics. (A dot represents a derivative by proper time.) In Fig. 2, we show φ and ψ as functions of the number of e-folds, $N_e \equiv \ln(a(t)/a(0))$. Non-adiabatic dynamics is visible in both the beginning and end stages of inflation, caused by the large departure of the fluctuation field φ from its minima, $\varphi_* \equiv \pm v\sqrt{\alpha_4^{-1}(1 - 3\psi^2/v^2)}$ (early stage) or $\varphi_* = 0$ (late stage). Note that $\varphi_*^2 \approx \varphi_c^2 + \mathcal{O}(\psi^2/v^2, \delta)$. This value is large (Planckian), but it determines the local minimum of the potential where its value, of the order M^4, is sub-Planckian. The dynamics is therefore not affected by quantum gravity.

It is possible to derive analytical expressions for observables using a slow-roll approximation combined with small non-Gaussianity, δ and α_3. Since the initial ψ is small near its minimum, we may ignore the ψ^3-term in (5) to obtain the spectral index n_s at Hubble exit. Since inflation ends shortly after ψ^2 grows to $\psi^2 = v^2/3$, we have $\varphi_* = 0$. Assuming $\varphi^2 \approx \varphi_*^2$ and small non-Gaussianity,

$$n_s \approx 1 - 12\frac{\delta}{\alpha_4 v^2} \quad , \quad N_e \approx \frac{f(1-n_s, v, \alpha_3)}{1-n_s} \tag{8}$$

with a specific but lengthy function f. (For details, see [28].) In non-minimal Higgs inflation, $f(1-n_s, v, \alpha_3) \approx 2$ is constant [6] while here it increases logarithmically with growing $1-n_s$ and typically ranges from $1 \lesssim f(1-n_s, v, \alpha_3) \lesssim 5$. The second equation in (8) is plotted in Fig. 3 for $\alpha_3 = 0.05$.

Aside from the parameter v that appears in common Higgs-like or hybrid models, our observables depend on two new parameters, α_3 and δ, related to the quantum state. They describe non-Gaussianity of the background field (as opposed to perturbations) and effectively control the amount of non-adiabatic evolution due to its modulation of the shifted local φ-minima at φ_*. Using moments of a state is a standard way to parametrize non-Gaussianity in quantum information theory [29]. The dependence of the total number of e-folds on α_3 is shown in Fig. 3, using the analytical solutions. Observational requirements are consistent with a nearly Gaussian state. The different parameter values reveal that the hierarchy in our set of parameters is much more rigid than in traditional hybrid model, leaving less room for tuning and ambiguity and making our results more robust. In the traditional case there are three independent parameters, while we have only two, inherited from a single-field model accentuated by its quantum fluctuations. Importantly, having a true single-field model masquerading as a two-field one allows us to avoid fine-tuning issues known for small-field hilltop models and yet have a small tensor-to-scalar ratio r well-within observable bounds. As revealed by numerics, the small r is implied by a small slow-roll parameter ϵ of the adiabatic field (a combination of both ψ and φ responsible for the curvature perturbation).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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