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Multi-field inflation from single-field models

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Multi-field inflation from single-field models

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Abstract. Quantization implies independent degrees of freedom that do not appear in the classical theory, given by fluctuations, correlations, and higher moments of a state. A systematic derivation of the resulting dynamical systems is presented here in a cosmological application for near-Gaussian states of a single-field inflation model. As a consequence, single-field Higgs inflation is made viable observationally by becoming a multi-field model with a specific potential for a fluctuation field interacting with the inflaton expectation value. Crucially, non-adiabatic methods of semiclassical quantum dynamics reveal important phases that can set suitable initial conditions for slow-roll inflation (in combination with the uncertainty relation), and then end inflation after the observationally preferred number of e -folds. New parameters in the interaction potential are derived from properties of the underlying background state, demonstrating how background non-Gaussianity can affect observational features of inflation or, conversely, how observations may be used to understand the quantum state of the inflaton.

Keywords: inflation, initial conditions and eternal universe

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1 Introduction

It is a natural requirement that self-consistent inflationary models should be largely independent of the high energy quantum gravity theory, viewed in an effective field theory framework. However, an exact decoupling of scales relevant for inflation from high-energy modes can happen only if the low-energy Lagrangian consists entirely of terms that are renormalizable using Wilsonian effective actions. This condition restricts single-field models of inflation to be of chaotic type with quartic potentials.

If the inflationary action contains terms beyond mass-dimension four, then the theory is liable to be affected by as yet unknown high-energy physics. In fact, one even has to rely on ultraviolet physics in order to derive a suitable higher-order form of the potential. In common single-field inflation, this problem can rarely be avoided as the models preferred by observations [1] depend crucially on non-renormalizable terms in the potential, as for instance in Starobinsky inflation [2]. Fundamentally, such terms have to be understood as remnants in an effective description of some underlying theory of gravity and matter, such as quantum gravity or string theory, but specific top-down justifications of suitable forms of the potential are usually hard to come by.

Alternatively, if chaotic-type potentials, which have been ruled out by data as single-field models, can somehow be resurrected, then the burden of explaining these potentials

does not have to fall on quantum gravity. Motivated by this observation, we begin with a Higgs-inspired classical potential,

$$V_{\text{cl}}(\psi) = M^4 \left(1 - 2 \frac{\psi^2}{v^2} + \frac{\psi^4}{v^4} \right) \quad (1.1)$$

with two parameters, M and v , assumed to be positive. While the only known scalar to have been discovered to date is the standard-model Higgs particle, it is well-known that this type of an inflaton potential, by itself, is found to be inconsistent with cosmological observations. To make matters worse, even renormalization-group improvements do not suffice to make Higgs-like potentials compatible with data [3–5]. The only observationally consistent formulations proposed up until now have been based on a scalar field non-minimally coupled to the Ricci scalar [6, 7], modifying the kinetic term of the Higgs field. Non-minimal coupling terms, however, mean that one is forced to modify the nature of the standard model at high energies [8], amongst other issues [9].

In the present work, we will preserve the simple nature of a minimally coupled field with a quartic classical potential (1.1). Applying a canonical formalism of effective theory which, crucially, remains valid in non-adiabatic regimes. Heuristically, this formalism includes effects of higher time derivatives in the quantum effective action without requiring a derivative expansion. We emphasize that this notion of non-adiabatic behavior refers to the background state of the scalar field, describing its homogeneous contribution, rather than its inhomogeneous modes which may well remain largely adiabatic if the slow-roll regime is sufficiently long. By these methods, the classical potential will be quantum extended to a two-field model with a specific potential derived from (1.1). The second field, φ , will be shown to be an authentic field degree of freedom representing quantum fluctuations of the background inflaton, ψ . Our application of quantum fluctuations in the context of inflation is different from the way they appear in stochastic or eternal inflation [10–15]. We are using back-reaction effects implied by quantum fluctuations in the deterministic evolution of wave functions, rather than stochastic properties implied by fluctuations for the measurement process. Our model could certainly be extended by including suitable stochastic terms in the equations of motion [11, 16–19], but we will not attempt to do so in the present paper. As such, it is subject to uncertainty relations that will be used to obtain important lower bounds on its initial value. Initial evolution is then non-adiabatic, but it automatically sets the stage for a long slow-roll phase (in a so-called waterfall regime of the two-field model) that is consistent with observational constraints. A final non-adiabatic phase automatically ends inflation with just the right number of e -folds in a large region of the parameter space.

Coefficients of the two-field potential are determined by the same two parameters, M and v , that appear in the single-field model (1.1). In addition, there are new coefficients derived from moments of the inflaton state, such as parameters for non-Gaussianity of the background state. In inflation models, this is a new kind of non-Gaussianity different from what one usually refers to in primordial fluctuations during inflation. In our case, non-Gaussianity is present already in the wave function of the homogeneous quantum inflaton field (referred to here as the background state), and not only in the perturbation spectrum. It is therefore possible to put constraints on the two-field potential based on known properties of states, or conversely, to determine conditions on suitable inflaton states based on observational constraints. An important finding is that constraints on the spectral index, its running, and the tensor-to-scalar ratio prefer small background non-Gaussianity.

In section 2, we present a review of relevant methods of non-adiabatic quantum dynamics, which have appeared in various forms in fields as diverse as quantum field theory, quantum chaos, quantum chemistry, and quantum cosmology. The same section presents a comparison with Gaussian methods and shows how non-adiabatic dynamics can include non-Gaussian states. These methods are applied to cosmology in section 3, focusing on Higgs-like inflation. The results are, however, more general and can easily be adapted to any potential. This section will demonstrate the importance of going beyond Gaussian dynamics, including higher-order moments, and maintaining non-adiabatic regimes. A detailed cosmological analysis, including numerical simulations and analytical approximations, is performed in section 4, where observational implications are discussed. The derivations in the present paper justify the more concise physical discussion presented in [20].

2 Canonical effective potentials

Our construction is based on canonical effective methods for non-adiabatic quantum dynamics, which in a leading-order treatment has appeared several times independently in various fields [21–26], including quantum chaos, quantum chemistry, and quantum cosmology, but has only recently been worked out to higher orders using systematic methods of Poisson manifolds [27, 28]. While higher orders go beyond Gaussian dynamics, the leading-order effects are closely related to Gaussian approximations and can therefore be used for an illustration of the method.

Throughout this paper, the term “adiabatic” will by default refer to the concepts discussed in the present section, unless otherwise stated. That is, adiabatic behavior is by definition realized when it is possible to capture crucial physical phenomena in a derivative expansion, for instance in a quantum effective action or, as used below, in the equations of motion for expectation values and moments of a state. The behavior is non-adiabatic when a derivative expansion does not faithfully capture the dynamics. In this case, new non-classical degrees of freedom play an important role, which may be given by auxiliary fields in a non-local effective action, or independent moments of a quantum state. In general, this notion has no relationship with the concept of adiabatic modes which is often used in cosmology. Later on, we will however briefly use an adiabatic combination of fields when deriving observables in the context of multi-field inflation.

2.1 Relation to the time-dependent variational principle

In order to illustrate our claim that quantum fluctuations can provide an independent degree of freedom that can influence the inflationary dynamics, we first consider a canonical formulation of the time-dependent variational principle for Gaussian states.

The most general parametrization of Gaussian fluctuations around the homogeneous field ψ can be represented by the wave function [21]

$$\begin{aligned} \Psi(\psi'|\psi, \pi_\psi, \varphi, \pi_\varphi) = & \frac{1}{(2\pi\varphi^2)^{1/4}} \exp\left(-\frac{1}{4}\varphi^{-2}(1-2i\varphi\pi_\varphi)(\psi'-\psi)^2\right) \\ & \times \exp(i\pi_\psi(\psi'-\psi)) \exp\left(-\frac{1}{2}i\varphi\pi_\varphi\right). \end{aligned} \quad (2.1)$$

The notation is such that Ψ is a wave function depending on a free variable ψ' for any choice of the parameters ψ , π_ψ , φ and π_φ , which determine a specific ψ' -dependent wave function.

Despite its lengthy form, this variational wave function has some useful properties: it is normalized, $\langle \Psi | \Psi \rangle = 1$, and has basic expectation values

$$\langle \Psi | \hat{\psi} | \Psi \rangle = \psi, \quad \langle \Psi | \hat{\pi}_\psi | \Psi \rangle = \pi_\psi \quad (2.2)$$

and variances

$$\langle \Psi | (\hat{\psi} - \psi)^2 | \Psi \rangle = \varphi^2, \quad \langle \Psi | (\hat{\pi}_\psi - \pi_\psi)^2 | \Psi \rangle = \pi_\varphi^2 + \frac{1}{4\varphi^2} \quad (2.3)$$

where operators are defined with respect to the dependence of Ψ on ψ' . Moreover, Ψ obeys the conditions

$$i\langle \Psi | \partial / \partial \psi | \Psi \rangle = \pi_\psi, \quad i\langle \Psi | \partial / \partial \varphi | \Psi \rangle = \pi_\varphi \quad (2.4)$$

$$\langle \Psi | \partial / \partial \pi_\psi | \Psi \rangle = 0, \quad \langle \Psi | \partial / \partial \pi_\varphi | \Psi \rangle = 0. \quad (2.5)$$

The equations of motion for the variational parameters, ψ , φ , π_ψ and π_φ , are given by the variation of the action

$$\begin{aligned} S &= \int dt \langle \Psi | (i\partial_t - \hat{H}) | \Psi \rangle \\ &= \int dt \left(i\dot{\psi} \langle \Psi | \partial / \partial \psi | \Psi \rangle + i\dot{\varphi} \langle \Psi | \partial / \partial \varphi | \Psi \rangle - \langle \Psi | \hat{H} | \Psi \rangle \right) \end{aligned} \quad (2.6)$$

using the chain rule. The identities obeyed by Ψ therefore allow us to write the action in canonical form,

$$S = \int dt \left(\dot{\psi} \pi_\psi + \dot{\varphi} \pi_\varphi - H_G \right) \quad (2.7)$$

where we defined the Gaussian Hamiltonian $H_G = \langle \Psi | \hat{H} | \Psi \rangle$. The variation of this action gives Hamilton's equations

$$\dot{\psi} = \frac{\partial H_G}{\partial \pi_\psi}, \quad \dot{\pi}_\psi = -\frac{\partial H_G}{\partial \psi}, \quad \dot{\varphi} = \frac{\partial H_G}{\partial \pi_\varphi}, \quad \dot{\pi}_\varphi = -\frac{\partial H_G}{\partial \varphi}. \quad (2.8)$$

For example, if we consider the Hamilton operator

$$\hat{H} = \frac{1}{2} \hat{\pi}_\psi^2 + M^4 \left(1 - 2 \frac{\hat{\psi}^2}{v^2} + \frac{\hat{\psi}^4}{v^4} \right) \quad (2.9)$$

with the Higgs-like potential, the Gaussian Hamiltonian is

$$H_G = \frac{1}{2} \pi_\psi^2 + \frac{1}{2} \pi_\varphi^2 + \frac{1}{8\varphi^2} + M^4 \left(1 - 2 \frac{\psi^2}{v^2} + \frac{\psi^4}{v^4} + 6 \frac{\psi^2 \varphi^2}{v^4} - 2 \frac{\varphi^2}{v^2} + 3 \frac{\varphi^4}{v^4} \right). \quad (2.10)$$

2.2 Canonical effective methods

While the Gaussian approximation is useful in a wide range of applications a more general class of states is relevant for our application to inflation where non-Gaussianities should be included in the analysis. Canonical effective methods [29, 30] provide a good alternative because they allow for generally non-Gaussian states while still retaining the canonical structure that makes Gaussian states attractive. Importantly, it is not required to find a specific representation of non-Gaussian states as wave functions, which would be much more involved than (2.1). Instead, one can formulate states of a quantum system in terms of expectation

values and moments assigned by a generic state to the basic operators $\hat{\psi}$ and $\hat{\pi}_\psi$. The evolution of a state is then formulated as a dynamical system for the basic expectation values $\psi = \langle \hat{\psi} \rangle$ and $\pi_\psi = \langle \hat{\pi}_\psi \rangle$ as well as the moments

$$\Delta(\psi^a \pi_\psi^b) = \left\langle (\hat{\psi} - \langle \hat{\psi} \rangle)^a (\hat{\pi}_\psi - \langle \hat{\pi}_\psi \rangle)^b \right\rangle_{\text{Weyl}}, \quad (2.11)$$

using Weyl (or completely symmetric) ordering in order to avoid overcounting degrees of freedom.

The basic expectation values and moments inherit a Poisson structure from the commutator,

$$\{\langle \hat{A} \rangle, \langle \hat{B} \rangle\} = \frac{1}{i\hbar} \langle [\hat{A}, \hat{B}] \rangle, \quad (2.12)$$

augmented by the Leibniz rule in an application to moments. The equations of motion for some phase space function, $F(\psi, \pi_\psi, \Delta(\cdot))$, are then given in the form of the usual Hamilton's equations,

$$\dot{F}(\psi, \pi_\psi, \Delta(\cdot)) = \{F, H_Q\} \quad (2.13)$$

with a quantum Hamiltonian $H_Q = \langle \hat{H} \rangle$ defined as the expectation value of the Hamilton operator $\hat{H}$ in a generic (not necessarily Gaussian) state. For a Hamiltonian of the form $\hat{H} = \frac{1}{2}\hat{\pi}_\psi^2 + \hat{V}(\psi)$, this definition implies the quantum Hamiltonian

$$H_Q = \langle \hat{H} \rangle = \frac{1}{2}\pi_\psi^2 + \frac{1}{2}\Delta(\pi_\psi^2) + V(\psi) + \sum_{n=2}^{\infty} \frac{1}{n!} \frac{\partial^n V}{\partial \psi^n} \Delta(\psi^n). \quad (2.14)$$

The formulation of the system in terms of expectation values and moments allows for a systematic canonical analysis at the semiclassical level. Written directly for moments as coordinates on the quantum phase space, the Poisson structure, based on (2.12) together with the Leibniz rule, is rather complicated. For instance, one can see that the Poisson bracket of two moments is not constant and not linear in general [29, 31]. Using moments as coordinates on a phase space therefore leads to a more complicated inflationary analysis lacking a clear separation between configuration and momentum variables. It is then unclear how to determine kinetic and potential energies or a unique relationship between specific phenomena and individual degrees of freedom.

In order to make the semiclassical analysis more clear, it is preferable to choose a coordinate system on phase space that puts the Poisson bracket in canonical form as in the variables used in (2.10), but possibly extended to higher orders in moments. The Darboux theorem [32] or its extension to Poisson manifolds [33] guarantees the existence of such coordinates, but explicit constructions are in general difficult. For second-order moments, the moment phase space is 3-dimensional and can be handled more easily than in the general context. In this case, a canonical mapping has been found several times independently [21–24]. It is accomplished by the coordinate transformation

$$\Delta(\pi_\psi^2) = \pi_\phi^2 + \frac{U}{\varphi^2}, \quad \Delta(\psi\pi_\psi) = \varphi\pi_\varphi, \quad \Delta(\psi^2) = \varphi^2 \quad (2.15)$$

where $\{\varphi, \pi_\varphi\} = 1$. The parameter $U = \Delta(\psi^2)\Delta(\pi_\psi^2) - \Delta(\psi\pi_\psi)^2$ is a conserved quantity (or a Casimir variable of the algebra of second-order moments), restricted by Heisenberg's uncertainty relation to obey the inequality $U \geq \hbar^2/4$. Direct calculations show that the

transformation (2.15) is a canonical realization of the algebra of second-order moments. At this stage we already have a departure from the Gaussian states, because the uncertainty for a pure Gaussian equals $\hbar^2/4$, while we retain the uncertainty as a free (but bounded) parameter.

Additional non-Gaussianity parameters, relevant for inflation, are revealed by an extension of the canonical mapping to higher-order moments. Considering higher order semiclassical corrections implies more canonical degrees of freedom. (For a single classical degree of freedom, the moments up to order N form a phase space of dimension $D = \sum_{j=2}^N (j+1) = \frac{1}{2}(N^2+3N-4)$.) A canonical mapping for these higher-order semiclassical degrees of freedom has only recently been derived in [27, 28] up to the fourth order. For the relevant moments, the results are

$$\Delta(\pi_\psi^2) = \sum_{i=1}^5 \pi_{\varphi_i}^2 + \sum_{i>j} \frac{U}{(\varphi_i - \varphi_j)^2} \quad (2.16)$$

$$\Delta(\psi^2) = \sum_{i=1}^5 \varphi_i^2 \quad (2.17)$$

$$\Delta(\psi^3) = C \sum_{i=1}^5 \varphi_i^3 \quad (2.18)$$

$$\Delta(\psi^4) = C^2 \sum_{i=1}^5 \varphi_i^4 + \sum \varphi_i^2 \varphi_j^2 \quad (2.19)$$

while all other moments up to fourth order can be derived from the relevant ones using suitable Poisson brackets. There are now five canonical pairs, $(\varphi_i, \pi_{\varphi_i})$ and two Casimir variables, U and C , forming a 12-dimensional phase space of moments.

In order to parametrize the entire fourth-order semiclassical phase space we had to introduce a total of five pairs of canonical degrees of freedom and two Casimir variables, U and C . In principle, we could consider all ten non-constant semiclassical degrees of freedom, but in order to keep the analysis simple, we take inspiration from some more terrestrial applications [28, 34, 35] and choose a moment closure, thereby approximating higher-order moments in terms of lower-order ones. In particular, we choose $\Delta(\pi_\psi^2) = \pi_\varphi^2 + U/\varphi^2$, $\Delta(\psi^2) = \varphi^2$, $\Delta(\psi^3) = a_3$ (or, alternatively, $a_3\varphi^3$) and $\Delta(\psi^4) = a_4\varphi^4$. This closure corresponds to (2.16) written in higher dimensional spherical coordinates with the assumption that the angular momenta are small enough to be ignored. The parameter values $U = \hbar^2/4$, $a_3 = 0$ and $a_4 = 3$ correspond to the Gaussian case. We can therefore think of this closure as describing the non-Gaussianities by three parameters, U , a_3 and $\delta = a_4 - 3$, while maintaining the same number of degrees of freedom as in the Gaussian case.

Considering a Higgs-inspired matter field coupled to a classical and isotropic space-time background with spatial metric $h_{ij} = a(t)^2 \delta_{ij}$ in terms of proper time t , the standard Lagrangian

$$L = \int d^3x \sqrt{\det h} \left(\frac{1}{2} \dot{\psi}^2 - \frac{1}{2} h^{ij} \partial_i \psi \partial_j \psi - V(\psi) \right) \quad (2.20)$$

is first reduced to homogeneous form by assuming spatially constant ψ and integrating:

$$L_{\text{hom}} = \frac{1}{2} a(t)^3 V_0 \dot{\psi}^2 - a(t)^3 V_0 V(\psi). \quad (2.21)$$

The new parameter V_0 , defined as the coordinate volume of the spatial region in which inflation takes place, does not have physical implications but merely ensures that the combination $a(t)^3 V_0$ represents the spatial volume in a coordinate-independent way. (The value of $a(t)^3 V_0$ would be determined by the maximum length scale on which approximate homogeneity may be assumed in the early universe just before inflation [36, 37].) This Lagrangian implies the scalar momentum

$$\pi_\psi = \frac{\partial L_{\text{hom}}}{\partial \dot{\psi}} = a(t)^3 V_0 \dot{\psi} \quad (2.22)$$

such that the Hamiltonian is given by

$$H = \frac{1}{2a(t)^3 V_0} \pi_\psi^2 + a(t)^3 V_0 V(\psi). \quad (2.23)$$

Quantizing the scalar field, using our explicit potential (1.1), the Hamilton operator is

$$\hat{H} = \frac{1}{2a(t)^3 V_0} \hat{\pi}_\psi^2 + a(t)^3 V_0 M^4 \left(1 - \frac{\hat{\psi}^2}{v^2} \right)^2, \quad (2.24)$$

keeping the background scale factor $a(t)$ classical. The closure we choose here implies the reduced version

$$H_Q^{\text{closure}} = \frac{1}{2a(t)^3 V_0} \pi_\psi^2 + \frac{1}{2a(t)^3 V_0} \pi_\varphi^2 + \frac{U}{2a(t)^3 V_0 \varphi^2} + a(t)^3 V_0 M^4 \left(1 + \left(\frac{6\varphi^2}{v^4} - \frac{2}{v^2} \right) \psi^2 + \frac{\psi^4}{v^4} - 2\frac{\varphi^2}{v^2} + \frac{a_4 \varphi^4}{v^4} + 4\frac{a_3 \varphi^3 \psi}{v^4} \right) \quad (2.25)$$

of the quantum Hamiltonian. Hamilton's equations generated by H_Q^{closure} are, as usual, deterministic, even though here they contain variables representing quantum fluctuations and higher moments. This dynamics presents an approximation of the deterministic evolution of a wave function that is implicitly determined by the moments. We therefore do not include stochastic effects of fluctuations that would be present if the inflaton were somehow measured while inflation is still going on.

While parameterizing some higher moments through a moment closure is required for a tractable model, keeping at least one quantum degree of freedom, φ , independent is crucial for a description of non-adiabatic phases. In this way, our quantum Hamiltonian goes beyond effective potentials of low-energy type, in particular the Coleman-Weinberg potential [38]. As shown in [39], it is possible to derive the Coleman-Weinberg potential from a field-theory version of (2.25) if one minimizes the Hamiltonian with respect to φ . This step eliminates all independent quantum degrees of freedom and, in the traditional treatment, is equivalent to ignoring non-adiabatic effects by using a low-order truncation of the derivative expansion, in addition to the semiclassical expansion also applied here. In this sense, by including the new variable φ as an authentic degree of freedom we retain non-adiabatic information of our dynamics.

In our cosmological scenario, this degree of freedom will be relevant at the beginning and end of inflation. Since the long, intermediate phase of slow-roll inflation remains by necessity adiabatic, a traditional low-energy effective action or a derivative expansion of a quantum field theory for the inflaton may be applied. As shown in [29, 30, 40], the background contribution of such an effective theory [41] is equivalent to an adiabatic approximation

applied to moment corrections in a quantum Hamiltonian. All relevant phases are therefore included in our formalism.

The effective Hamiltonian (2.25) is very similar to the Gaussian Hamiltonian (2.10), which also retains an independent quantum variable, but it is more general because of the presence of the new parameters U , a_3 and a_4 . As shall be shown later, the characteristics of our inflationary phase depend crucially on these parameters. In particular for a Gaussian state, inflation never ends, but if we consider small non-Gaussianities parametrized by U , a_3 and a_4 , we can obtain a phenomenologically viable inflationary phase. Moreover, these parameters are determined by the quantum state of the early universe, and so constraining them with data would shed light on the character of the quantum state of the early universe.

3 Two-field model

After our transformation to canonical moment variables, we can uniquely extract an effective potential from (2.25),

$$\begin{aligned} \frac{1}{M^4} V_{\text{eff}}(\psi, \varphi) &= 1 + \frac{U}{2M^4 a^6 V_0^2 \varphi^2} + \left(6 \frac{\varphi^2}{v^4} - \frac{2}{v^2} \right) \psi^2 + \frac{\psi^4}{v^4} - 2 \frac{\varphi^2}{v^2} + \frac{4a_3 \psi}{v^4} + a_4 \frac{\varphi^4}{v^4} \\ &\approx 1 + 2 \left(\frac{\varphi^2 - \varphi_c^2}{\varphi_c^2} \right) \frac{\psi^2}{v^2} + \frac{4a_3 \psi}{v^4} + \frac{\psi^4}{v^4} - \frac{2}{3} \frac{\varphi^2}{\varphi_c^2} + a_4 \frac{\varphi^4}{v^4}, \end{aligned} \quad (3.1)$$

where $\varphi_c^2 := v^2/3$. By construction, the second field, φ , represents the quantum fluctuation associated with the classical field ψ . As explained earlier, the additional parameters, U , a_3 and a_4 describe a possibly non-Gaussian quantum state of the background inflaton.

3.1 Initial conditions and the trans-Planckian problem

In the second line of the equation, we ignored the U -term $U/(2M^4 a^6 V_0^2 \varphi^2)$ in an approximation valid for sufficiently large scale factors (or, rather, averaging volumes $a^3 V_0$). The origin of this term is purely quantum and represents a potential barrier that enforces Heisenberg's uncertainty relation for the fluctuation variable φ . This term can be easily ignored after a few e -folds of inflation, but at early times its presence necessitates φ to start out at large values. The subsequent non-adiabatic phase will be crucial for our model, and therefore this term alleviates our need to fine-tune the initial condition for φ .

The main effect of this repulsive term in the potential is to push out φ to large values to begin with, after which we are always able to neglect it throughout inflation. The initial φ obtained in this way is indeed consistent with requirements on inflation models. In particular, we can easily obtain the initial condition $\varphi > \varphi_c$ of hybrid inflation [42]: we expect the initial φ to be large and can therefore restrict the effective potential (3.1) to the term quartic in φ , together with the U -term relevant at early times. This restricted potential has a local minimum at

$$\varphi = \sqrt[6]{\frac{U v^4}{4 a^6 V_0^2 M^4 a_4}}. \quad (3.2)$$

We do not know much about the volume $a^3 V_0$ of the initial spatial region that was meant to expand in an inflationary way. But in order to avoid the trans-Planckian problem [43–45], we should require that $a^3 V_0 > \ell_{\text{P}}^3$. This lower bound implies the upper bound

$$\varphi_{\text{ini}} < \frac{1}{\ell_{\text{P}}} \sqrt[6]{\frac{U v^4}{4 a_4 M^4}} \quad (3.3)$$

for (3.2). For parameters of the order $v \sim \mathcal{O}(M_P)$ and $M^4 \ll M_P^4$, as common in hybrid models and used in our analysis to follow, the upper bound on φ_{ini} is much greater than φ_c .

3.2 Waterfall: phase transitions

Our effective potential (3.1), depending on the classical field ψ and its fluctuation, φ is of the hybrid-inflation type. These models typically produce a blue-shifted tilt when one starts with a large φ and small ψ [42]. Inflation in this scenario essentially relies on the near-constant vacuum energy of ψ . However, there is an alternative scenario in the same model, the so-called waterfall regime [46, 47], realized at a later stage in our model in which φ has moved to and stays close to a minimum while ψ gradually inches away from its vacuum value that has by then become an unstable equilibrium position.

As we will show, initial conditions for the waterfall regime to take place are generated in our extension of the model by a non-adiabatic phase in which φ is still large. The subsequent waterfall regime then generates a significant number of e -folds and leads to a red-shifted tilt for a wide range of parameters. For this scenario to take place, it is important that our effective potential differs from the traditional hybrid one in that we have an $a_4\varphi^4$ term as well as a Z_2 -breaking term $a_3\psi$, which is assumed to be small but not exactly zero. The latter term relieves us of the burden of supplying a non-zero initial value for ψ , which is required to start the dynamics of the waterfall regime, as we shall demonstrate later. Because both new terms depend on state parameters in our semiclassical approximation, the resulting description of inflation is characterized by an intimate link between observational features and properties of quantum states.

Another difference with the traditional hybrid model is that the hierarchy between our set of parameters is more rigid, leaving less room for tuning and ambiguity and making our results more robust. The traditional potential has three parameters which can be adjusted independently, while in our case only two (non-state) parameters are independent. This is so because we do not have a generic two-field model but rather a single-field model accentuated by its quantum fluctuation. As opposed to the traditional hybrid model [46], we have two phase transitions characterized by non-adiabatic behavior, and the majority of e -folds are created in between.

As in the original hybrid model, we start with some $\varphi > \varphi_c$ with φ quickly rolling down to its minima under an effective φ^4 term. This phase is driven by a simplified potential of the form

$$\frac{V_{\text{eff}}^\varphi}{M^4} = 1 - \frac{2}{3} \frac{\varphi^2}{\varphi_c^2} + a_4 \frac{\varphi^4}{v^4} \quad (3.4)$$

since ψ sits in its local minimum at the origin during this time and therefore all ψ -terms can be ignored. Once φ crosses φ_c , the new true minima of ψ are displaced from the origin due to a tachyonic term in its effective potential, of the form

$$\frac{V_{\text{eff}}^\psi}{M^4} = 1 + 2 \left(\frac{\varphi^2 - \varphi_c^2}{\varphi_c^2} \right) \frac{\psi^2}{v^2} + \frac{4a_3\psi}{v^4} + \frac{\psi^4}{v^4} - \frac{2}{3} \frac{\varphi^2}{\varphi_c^2} + a_4 \frac{\varphi^4}{v^4}. \quad (3.5)$$

Due to the a_3 term, the Z_2 symmetry of ψ is broken and the field starts slowly rolling away from the origin. This gradual change enables φ to closely follow its vacuum expectation value, φ_* . (Its gradual nature also means that the back-reaction of homogeneous fluctuations φ on the homogeneous expectation value ψ is small, justifying our semiclassical approximation. The combined system of ψ and φ has a pronounced effect on the back-ground space-time, driving its expansion. However, since energy densities always remain

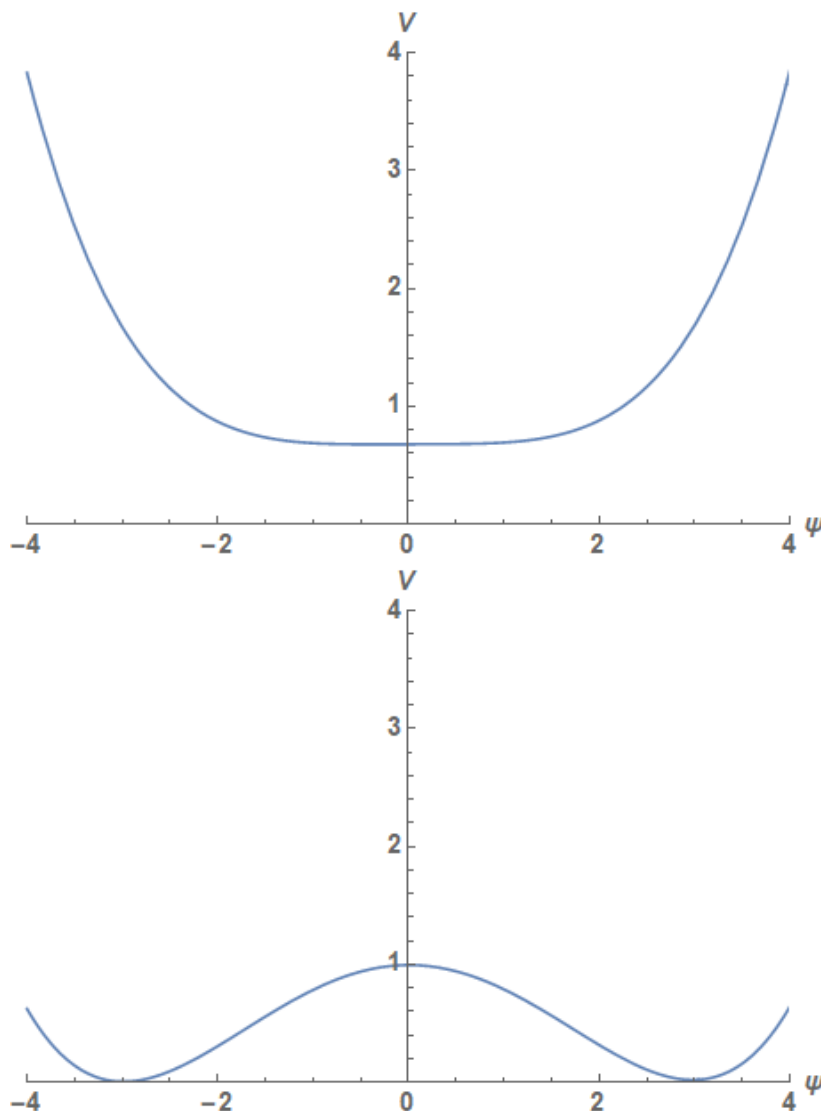


Figure 1. Shape of the potential $V(\psi)$ for constant φ at early (top) and late times (bottom), defined relative to the time when φ crosses φ_c .

sub-Planckian, our model is semiclassical also in the sense of quantum gravity and we are justified in keeping the scale factor a unquantized.) Eventually, φ_* approaches zero but never reaches it due the uncertainty principle, thereby almost restoring the symmetry for φ ; this is the second phase transition mentioned above. As shown in figures 1 and 2, φ causes the traditional phase transition when it crosses φ_c , and then the slow roll of ψ down its tachyonic hilltop will end in a second phase transition. The whole process is clarified further by examining how the effective potential changes in time, shown in figures 3 and 4.

The hilltop phase generates the dominant number of e -folds, and it ends automatically when ψ reaches its new minimum. This is a new feature compared to the traditional hybrid inflation and relies on the existence of a φ^4 term in our effective potential. Our model is not a variant of the original hybrid model [48], such as the inverted-hybrid model [49] or a modified hilltop model [50], or having corrections to the potential coming from supergravity-

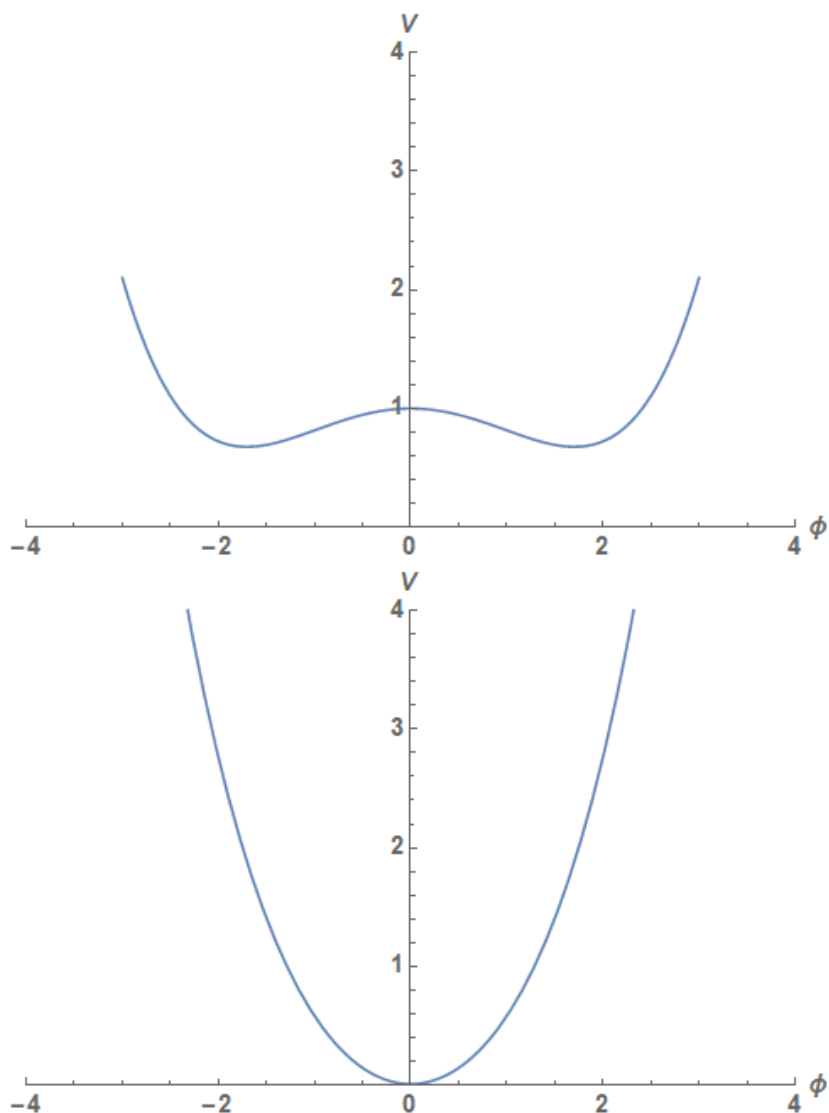


Figure 2. Shape of the potential $V(\varphi)$ for constant ψ at early (top) and late times (bottom), defined relative to the time when φ crosses φ_c .

embedding of the model [51]; rather, we start with a Higgs-like model and include effects from an initial quantum state that turn it into a hybrid model with some additional terms.

3.3 UV-completion and the swampland

One of the conceptual requirements for inflation models is that they should have a well-defined quantum completion. One way to implement this is to derive specific forms of inflationary potentials from string theory constructions as was done, for instance, in the case of natural inflation. Another recent idea has been that of the swampland, a complement of the string landscape, which stems from the fact that not all low-energy effective field theories can be consistently completed in the ultraviolet into a quantum theory of gravity [52, 53]. In order for an effective field theory to be consistent, it would have to satisfy the eponymous swampland constraints. This is a much more general way in which quantum gravity may restrict the

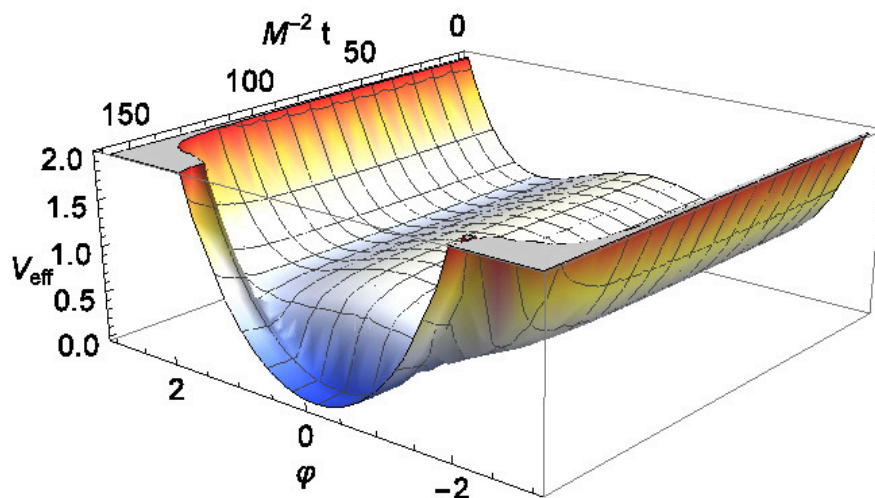


Figure 3. After a brief non-adiabatic phase when it rolls down a steep potential wall, φ traces its minimum for the majority of inflation. The growth of ψ^2 moves the φ -minima closer to zero, causing another non-adiabatic phase that ends with an approximate symmetry restoration for φ . The parameters used are $v = 3$, $a_3 = 0.05$ and $\delta = 0.1$.

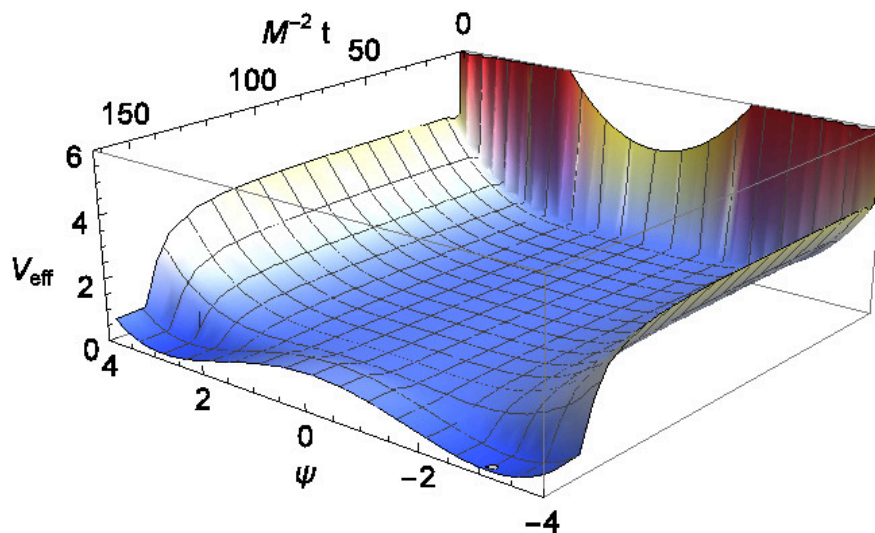


Figure 4. During the initial non-adiabatic phase, a phase transition akin to traditional hybrid models occurs. Reflection symmetry in the potential is slightly broken by the a_3 -term (which is not apparent in the figure due to its smallness). This non-Gaussianity term drives ψ to its new stable point where ψ^2 approaches v^2 . The parameters are the same as figure 3.

form of the potential, amongst other things, in the low-energy effective field theory used as the starting point for inflation. More specifically, it has been argued that many models of (at least) single-field inflation are not consistent with the swampland conjectures since the latter require either a large value for the slope of the potential, $|V'|/V > \mathcal{O}(1)$, or large tachyonic directions, $V''/V < -\mathcal{O}(1)$ [54, 55].

Taken together, these conjectures severely restrict the lifetime of metastable (quasi-) de Sitter spacetimes that can be built from string theory. In order to obtain an estimate for the numbers of order one that appear in one of them, the so-called de-Sitter conjecture, one has to resort to fundamental properties of quantum gravity such as the absence of eternal inflation [56, 57] or the trans-Planckian censorship conjecture [58, 59]. The latter has put a more concrete bound on the duration of inflation which, when combined with the observed power spectrum, imposes severe constraints on the allowed models for inflation. It has been shown that only hilltop type of models, which generically allow for a small slow-roll parameter ϵ but a big η , are the ones that survive amongst all single-field models unless one invokes additional degrees of freedom as in non-Bunch Davies initial states or warm inflation. Even for hilltop potentials, which seem to be the most compatible with the swampland, one has to resort to an arbitrary steepening of the potential to end inflation so as not to have too many e -folds since that would once again make the model incompatible with the constraints. To date, there are no string theory realizations of any such single-field potential that can abruptly stop inflation after a finite amount of time.

The remarkable feature of our new model is that it is able to give a viable inflationary cosmology as well as a graceful exit with a tachyonic (p)reheating, all starting from a Higgs-like single-field potential as the main input. We are using only standard quantum mechanics in a non-adiabatic semiclassical approximation and do not have to rely on unknown features of quantum gravity. In addition, by virtue of the fact that the classical field ψ plays the role of the inflaton relevant for observable scales, this model is essentially of the hilltop type which has recently been shown to be preferred by the swampland and to be able to ameliorate the η -problem [60]. Quantum effects imply that the single-field classical potential is, upon quantization, no longer a single-field model that would have to be tuned in order to avoid having too many e -folds of inflation or require any additional mechanism to achieve stability against radiative corrections [61]. Moreover, our detailed derivations below reveal that the model maintains a large value of the slow-roll parameter η throughout inflation (in addition to a small ϵ , as is usually the case for a prototype hilltop model). Indeed, it is when the value of η becomes too large that inflation ends in this model, once again thanks to effects of quantum fluctuations of the classical field (as opposed to a generic second field). All of this is possible even though we start with a single-field model with a monomial potential, but then take into account the effects of quantum fluctuations in a systematical manner.

4 Analysis

The effective Hamiltonian (2.25) describes a two-field model with standard kinetic terms in an expanding universe and an interaction potential similar to hybrid models. A numerical analysis can be applied directly to Hamilton's equations for ψ and ϕ generated by H_Q^{closure} , (2.25), using suitable initial values. We will present such solutions in comparison with a slow-roll approximation to be developed first.

4.1 Slow-roll approximation

For inflationary applications of (2.25), we are interested in a long phase of slow roll that can be generated by ψ staying near its initially stable and then metastable equilibrium position at $\psi = 0$. As long as $\psi^2 \ll v^2$ and $\varphi^2 \approx \varphi_*^2$ is near a local minimum, the slow-roll approximation can be used and evaluated analytically. This phase is adiabatic and therefore does not require all terms in (2.25) that are implied by semiclassical methods for non-adiabatic quantum dynamics. However, as we have already seen, the remaining terms are essential in achieving

suitable initial values for the slow-roll phase and to end it early enough. Throughout this analysis, we will also assume small background non-Gaussianity. As our results will show, this assumption is justified by observational constraints on the spectral index.

Given these conditions, the slow-roll parameters can be approximated as

$$\epsilon_\varphi \equiv \frac{1}{2}M_{\text{P}}^2 \left(\frac{V_\varphi}{V} \right)^2 \approx \frac{1}{2}M_{\text{P}}^2 \left(\frac{M^4}{P} \right)^2 \left(\frac{4\varphi}{3\varphi_c^2} \left(1 - \frac{3\psi^2}{v^2} \right) - \frac{4a_4\varphi^3}{v^4} \right)^2 \quad (4.1)$$

$$\epsilon_\psi \equiv \frac{1}{2}M_{\text{P}}^2 \left(\frac{V_\psi}{V} \right)^2 \approx \frac{1}{2}M_{\text{P}}^2 \left(\frac{M^4}{P} \right)^2 \left(\frac{4\psi}{v^2} \left(\frac{\varphi^2}{\varphi_c^2} + \frac{\psi^2}{v^2} - 1 \right) + \frac{4a_3}{v^4} \right)^2 \quad (4.2)$$

$$\eta_{\varphi\varphi} \equiv M_{\text{P}}^2 \frac{V_{\varphi\varphi}}{V} = -\frac{M^4}{P} \left(\frac{4}{3\varphi_c^2} \left(1 - \frac{3\psi^2}{v^2} \right) - \frac{12a_4\varphi^2}{v^2} \right) \quad (4.3)$$

$$\eta_{\psi\psi} \equiv M_{\text{P}}^2 \frac{V_{\psi\psi}}{V} = \frac{M^4}{P} \frac{4}{v^2} \left(\frac{\varphi^2 - \varphi_c^2}{\varphi_c^2} + \frac{3\psi^2}{v^2} \right) \quad (4.4)$$

$$\eta_{\psi\varphi} \equiv M_{\text{P}}^2 \frac{V_{\psi\varphi}}{V} = \frac{M^4}{P} \frac{8\psi\varphi}{v^2\varphi_c^2}, \quad (4.5)$$

where $V_\varphi = \partial V/\partial\varphi$ and $V_\psi = \partial V/\partial\psi$, iterated for higher derivatives. The constant P is the initial potential energy, evaluated when $\varphi \approx \varphi_c$ and $\psi \approx 0$. In the following we set $M_{\text{P}} = 1$. We will see later that small background non-Gaussianity ensures that $\varphi^2/\varphi_c^2 - 1 \ll 1$. Along with the adiabatic approximation for φ , this inequality can ensure that ϵ_ψ and $\eta_{\psi\psi}$ are very small. However, $\eta_{\varphi\varphi}$ is not necessarily small, even though $\ddot{\varphi} \ll 3H\dot{\varphi}$ and $\dot{\varphi}^2 \ll V$.

Our equations of motion, under slow roll, then read

$$\frac{3H\dot{\varphi}}{M^4} = \frac{4\varphi}{3\varphi_c^2} \left(1 - \frac{3\psi^2}{v^2} \right) - \frac{4a_4\varphi^3}{v^4} \quad (4.6)$$

$$\frac{3H\dot{\psi}}{M^4} = -\frac{4\psi}{v^2} \left(\frac{\varphi^2 - \varphi_c^2}{\varphi_c^2} + \frac{\psi^2}{v^2} \right) - \frac{4a_3}{v^4}. \quad (4.7)$$

where we can make M implicit by rescaling $t \rightarrow t/M^2$. The regime covered by our approximations can be split into two phases followed by an end phase.

4.1.1 Phase 1

In early stages, we have $\psi^2 \ll v^2$ and can thus ignore the term $3\psi^2/v^2$ in (4.6). Therefore, the constant $\varphi^2 \approx \varphi_*^2 \approx 3\varphi_c^2/a_4$ is a solution. Adiabaticity ensures that we can expand the equation of motion around the critical point φ_* where $V_\varphi(\varphi_*) = 0$:

$$\dot{\varphi} \approx -\frac{1}{3H} V_{\varphi\varphi}(\varphi_*)(\varphi - \varphi_*). \quad (4.8)$$

Defining $\varphi' := d\varphi/dN$ where N is the number of e -folds, we obtain

$$\varphi' \approx -\eta_{\varphi\varphi}(\varphi = \varphi_*, \psi \approx 0)(\varphi - \varphi_*). \quad (4.9)$$

For small background non-Gaussianity, we have $a_4 = 3 + \delta$ with $\delta \ll 1$. Choosing the initial value $\varphi(0) = \varphi_c$ for Phase 1 therefore implies

$$\varphi_1(N) \approx \frac{\varphi_c\delta}{2a_4} \exp(-\eta_{\varphi\varphi}(\varphi_*, 0)N) + \varphi_*. \quad (4.10)$$

Note that small non-Gaussianity also implies $\varphi_*^2 = \varphi_c^2 + O(\delta) + O(\psi^2)$.

We can expect $\varphi^2/\varphi_c^2 - 1 \approx -\delta/a_4$ to be much bigger than ψ^2/v^2 at early times. This reduces the second equation of motion, (4.7), to

$$\psi' \approx \frac{1}{P} \frac{4}{v^2} \left(\frac{\delta}{a_4} \psi - \frac{a_3}{v^2} \right) \quad (4.11)$$

which is solved by

$$\psi_1(N) \approx -\frac{a_3 a_4}{\delta v^2} \left(\exp \left(\frac{4\delta}{v^2 a_4 P} N \right) - 1 \right) \quad (4.12)$$

for an initial ψ_1 at the origin. To summarize, Phase 1 is characterized mathematically by the possibility to ignore the ψ^2/v^2 terms in (4.6) and (4.7).

4.1.2 Phase 2

As ψ moves away from its metastable position at $\psi = 0$, the terms ψ^2/v^2 in the equations of motion will eventually have noticeable effects even while they may still be small. In particular, the local minimum of φ at

$$\varphi_*(\psi(t))^2 = \frac{v^4}{3\varphi_c^2 a_4} \left(1 - \frac{3\psi(t)^2}{v^2} \right) \quad (4.13)$$

is then time-dependent. The solution for φ in Phase 2 can therefore be obtained directly from (4.10) by inserting the time-dependent ψ and φ_* ,

$$\varphi_2(N) = \varphi_1(N)|_{\psi \rightarrow \psi(N)}, \quad (4.14)$$

using the solution for $\psi(N) \equiv \psi_2(N)$ to be derived now. As implied by adiabaticity, we still have $\varphi^2 \approx \varphi_*^2$, tracking the local minimum.

Our phase now is described by the first two terms of (4.7) dominating over the a_3 -term. Therefore,

$$\begin{aligned} \psi' &\approx -\frac{1}{P} \frac{4\psi}{v^2} \left(\frac{\varphi_*(\psi(t))^2 - \varphi_c^2}{\varphi_c^2} + \frac{\psi^2}{v^2} \right) \\ &= \frac{1}{P} \frac{4\psi}{v^2} \left(\frac{\delta}{a_4} + \frac{2\psi^2}{v^2} + O(\delta\psi^2/v^2) \right). \end{aligned} \quad (4.15)$$

which is solved by

$$\psi_2(N) \approx -\text{sgn}(a_3) \sqrt{\frac{\delta}{(2a_4/v^2 + \delta/\psi_g^2) \exp(-8\delta(N - N_g)/(v^2 P a_4)) - 2a_4/v^2}}. \quad (4.16)$$

(Although a_3 does not appear in our approximate equation (4.15), its sign determines the direction in which ψ starts moving as a consequence of reflection symmetry breaking.) Here, the subscript “g” denotes the value of solutions at the “gluing” point of the two phases, defined as the point where the cubic term in (4.7) is on the order of the a_3 -term; see figure 6 below for an illustration.

4.1.3 End phase

Even though Phase 1 and Phase 2 are sufficient to describe the majority of inflation, finding the point at which inflation ends requires a qualitatively different approximation compared with the above two phases. The physics is also quite different. To see this, note that if we extend the approximations of Phase 2 too far, we arrive at two wrong conclusions. First, ψ will eventually cross the point $\psi^2 = v^2/3$, such that the two minima of $V_{\text{eff}}(\varphi)$ meet at $\varphi_* = 0$. Second, this behavior causes φ to approach zero, such that the field ψ ends up at its new $V_{\text{eff}}(\psi)$ -minimum, $\psi_{\text{min}} = -v$ (assuming a_3 is positive). The former ($\varphi \rightarrow 0$) is forbidden by the uncertainty principle, embodied in our U -term in V_{eff} neglected so far in the slow-roll analysis, and the latter is erroneous since it implies that once everything has settled, H^2 , which is proportional to V_{eff} during slow roll, would seem to approach a negative value $4a_3\psi/v^4 < 0$.

However, this last conclusion certainly cannot be correct because our classical potential (1.1), a complete square $V_{\text{cl}}(\psi) = M^4(1 - \psi^2/v^2)^2$, is positive semidefinite. Therefore, it is quantized to a positive, self-adjoint operator $\hat{V}$ which cannot possibly have a negative expectation value $V_{\text{eff}} = \langle \hat{V} \rangle$ in any admissible state. In terms of moments used in our canonical effective description, after ψ crosses the value $v^2/3$, the fluctuation variable φ shrinks. Therefore, according to our moment closure introduced after equation (2.16), the variance $\Delta(\psi^2) = \varphi^2$ as well as the fourth-order moment $\Delta(\psi^4) = a_4\varphi^4$ approach zero, while $\Delta(\psi^3) = a^3$ has so far been assumed constant. This latter assumption violates higher-order uncertainty relations for small φ .

We will not require a precise form of such higher-order uncertainty relations, or a specific decreasing behavior of $\Delta(\psi^3)$ because, referring to positivity, we know that the magnitude of the a_3 -term in the potential is not allowed to be larger than the sum of the rest of the terms in V_{eff} . (But see the next subsection for numerical examples with decreasing $\Delta(\psi^3)$.) This observation places an implicit bound on non-Gaussianity parameters when our potential energy decreases at the end of inflation. Taking this outcome into account, our effective potential eventually becomes

$$\frac{V_{\text{eff}}}{M^4} \approx \left(1 - \frac{\psi^2}{v^2}\right)^2 + \frac{2}{3} \frac{\varphi^2}{\varphi_c^2} \left(\frac{3\psi^2}{v^2} - 1\right) + \frac{U}{2M^4 a^6 V_0^2 \varphi^2}, \quad (4.17)$$

where we have neglected the φ^4 and a_3 terms for small fluctuations. The corrected values φ_* of the two φ -minima are now

$$\varphi_* \approx \pm \left(\frac{u}{K(\psi^2)} \varphi_c^2\right)^{1/4} \quad (4.18)$$

where

$$u = \frac{U}{M^4 a^6 V_0^2} \quad \text{and} \quad K(\psi^2) = \frac{4}{3} \left(\frac{3\psi^2}{v^2} - 1\right). \quad (4.19)$$

Since u is extremely small after 60 e -folds, we have $|\varphi_*| \ll 1$. The symmetry restoration for φ is therefore only an approximate one. In addition, we neglected the $O(\delta\psi^2/v^2)$ -term in (4.15), but kept δ/a_3 . These two terms become comparable around $\psi^2 = v^2/3$ for our chosen parameters. However, as we will see later in a comparison with numerical solutions, setting $\varphi = 0$ and using the $\psi(N)$ expression of Phase 2 during the end phase gives a sufficiently accurate number of e -folds. This also means that only a negligible amount of e -folds forms in the non-adiabatic phases of our evolution, and while non-adiabatic effects are crucial for the beginning and end of inflation, they do not affect observations directly.

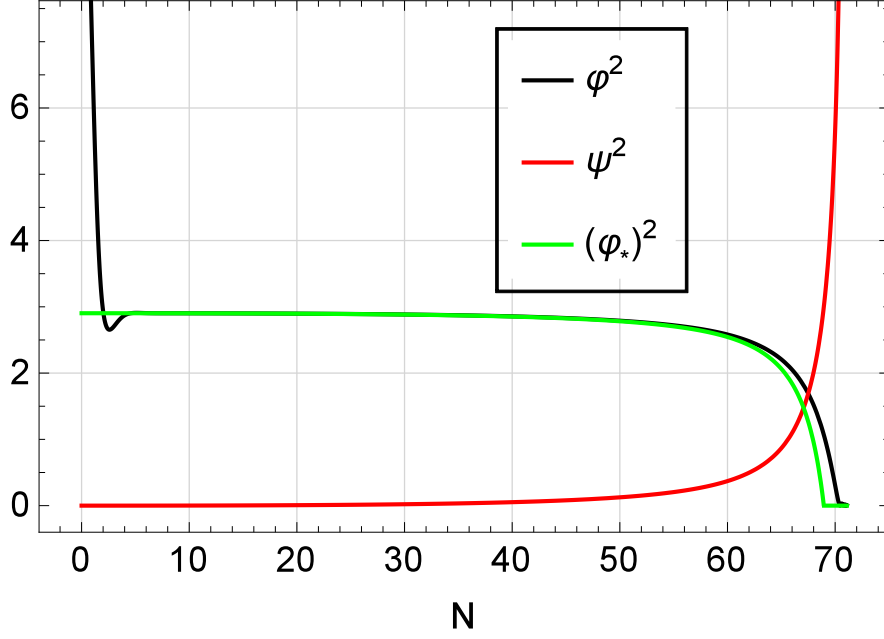


Figure 5. Overview of full numerical evolution. The field ψ remains small during inflation while φ follows its vacuum expectation value φ_* very closely throughout the whole evolution. After inflation ends, ψ^2 approaches v^2 , a value cut off in this presentation. While the fields may take Planckian values, of the order one in natural units, except for very early times they hover near their potential minima where they imply sub-Planckian energy densities. Quantum-gravity effects are therefore negligible during inflation. The field ψ^2 increases at the end of inflation, but it merely approaches its new minimum seen in figure 4 and is not a run-away solution.

4.2 Comparison of analytical and numerical solutions

Our analytical solutions were obtained with certain approximations, but they generally agree well with numerical solutions of the full equations,

$$\ddot{\varphi} + 3H\dot{\varphi} = \frac{4\varphi}{3\varphi_c^2} \left(1 - \frac{3\psi^2}{v^2} \right) - \frac{4a_4\varphi^3}{v^4} \quad (4.20)$$

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{4\psi}{v^2} \left(\frac{\varphi^2 - \varphi_c^2}{\varphi_c^2} + \frac{\psi^2}{v^2} \right) - \frac{4a_3}{v^4}, \quad (4.21)$$

in situations relevant for inflation. To be specific, we choose parameters $v = 3$, $\delta = 0.1$ and $a_3 = 0.05$ in our numerical solutions. Figure 5 shows a representative example of full numerical evolution. To test our analytical assumptions, figure 6 shows the magnitudes of individual terms that contribute to the equation of motion (4.21) for ψ , while figures 7 and 8 compare analytical and numerical solutions of both equations.

Cosmological parameters relevant for inflation are shown in the next figures, figure 9 for the slow-roll parameter $\eta_{\psi\psi}$ which eventually ends inflation, figure 10 for the spectral index according to both analytical and numerical solutions, as well as its running in figure 11. As shown by these figures, the parameters easily imply solutions compatible with observational constraints. It is also shown how $\eta_{\psi\psi}$ increases at an opportune time to end inflation with just the right number of e -folds in order to avoid the trans-Planckian problem.

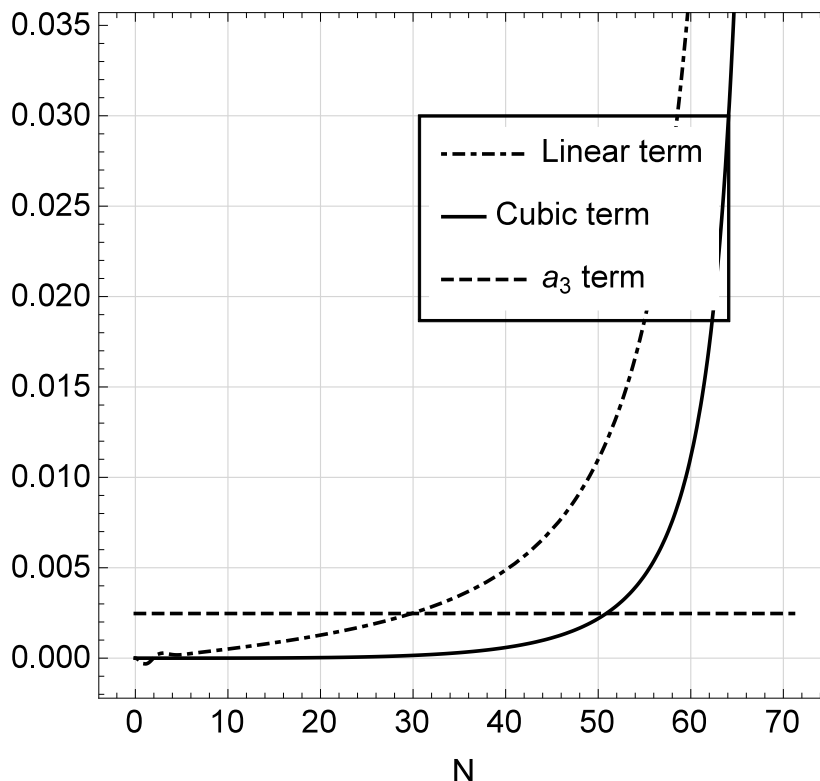


Figure 6. The magnitudes of individual terms in (4.7) as functions of N . The term ψ^3/v^4 in (4.7) approaches the order of a_3/v^4 around $N = 50$, marking the transition point to Phase 2.

The role of non-Gaussianity parameters can also be studied. For instance, parameterizing $a_3 = 0.01\varphi^3$ instead of a constant $a_3 = 0.05$ leads to comparable results, as shown for the number of e -folds in figure 12. The effects of different choices of $\delta = a_4 - 3$ on the spectral index and the tensor-to-scalar ratio (computed as $r \approx 16\epsilon_\sigma$, σ being the effective adiabatic field [46]) are shown in figures 13 and 14. An important new result is that the non-Gaussianity parameters effectively control the onset and duration of inflation, such that observationally preferred numbers of e -folds can be obtained for reasonable choices of background non-Gaussianity. In particular, only small deviations from a nearly Gaussian ground state are required.

4.3 Analytical results for cosmological observables

As we have argued previously, our system is essentially a two-field model, such that we may directly apply tools from multi-field inflation to predict the number of e -folds N (starting from the crossing of $\phi = \phi_c$) and the spectral index. Using our analytical solutions for the background variables, we may obtain approximate analytical expressions for the observables, which are based on perturbative inhomogeneity. The standard treatment of inflation quantizes the inflaton fields, subject to a given potential, on an expanding space-time and computes power spectra from correlation functions of inhomogeneous modes. Here, we have already used quantum properties to generate our extended 2-field potential. As a consequence, the variable φ , derived from quantum fluctuations of ψ , cannot give rise to a quantum field that could imply correlation functions to be used in a multi-field calculation of power spectra. In

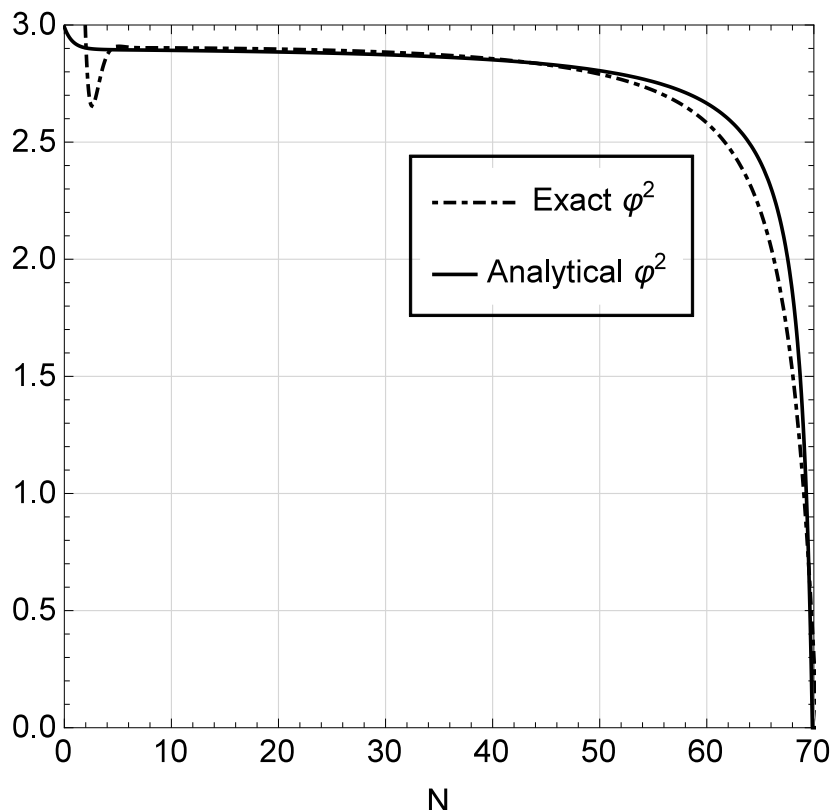


Figure 7. Comparison of analytical and numerical solutions for $\varphi(N)$. Our analytical solution for $\varphi(N)$ agrees well with the full numerical one, justifying the adiabatic approximation during inflation.

a field quantization of our model, there would be only one field operator, $\hat{\psi}$, rather than two quantum fields.

Nevertheless, we are able to formulate our model in a multi-field manner even for perturbative inhomogeneity. In our formalism, we would describe the full system of background variables and perturbative modes within the same setting of canonical effective theory. As before, such a framework would be based on moments which, now, also include the sought-after correlation functions of modes. While a complete treatment is well beyond the scope of the present paper, it is not difficult to see that the correct field degrees of freedom would be present. In particular, instead of deriving correlation functions for a quantized fluctuation field φ , which does not exist in our model, we can describe relevant correlation functions through higher moments: standard correlation functions are quadratic expressions in modes of φ , which as a fluctuation is itself quadratic in the original field ψ . Suitable fourth-order moments of modes of the field ψ , which is associated with a quantum field, can therefore be used as correlation functions for the derived field φ . Since higher-order moments are subdominant for near-Gaussian states, as encountered here, the mode dynamics does not include terms beyond those relevant for the required correlation functions. We are therefore able to apply standard methods from multi-field inflation.

4.3.1 Perturbation modes

In our model, both the classical background variable ψ and its quantum fluctuation φ undergo slow-roll evolution in different phases of the dynamics. Therefore, they should both

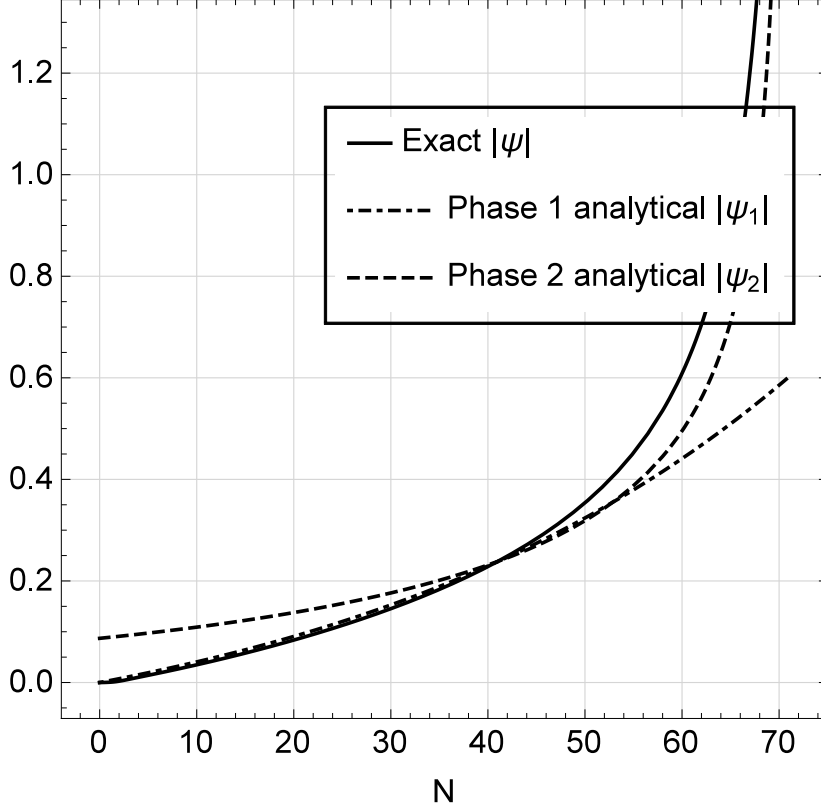


Figure 8. Comparison of analytical and numerical solutions for $\psi(N)$. The analytical solution agrees extremely well with the exact one in Phase 1 (before $N = 50$), while small deviations occur in ψ_2 occur Phase 2 (after about $N = 50$).

contribute to the curvature perturbation once inhomogeneous modes are included and one can write down the effective adiabatic field σ as a combination of both these fields, ψ and ϕ . (Here, we use the term “adiabatic” in its standard meaning applied to modes of perturbative inhomogeneity.)

In terms of the adiabatic field σ , consider the spectral index at around horizon exit,

$$n_s = 1 - 6\epsilon_\sigma + 2\eta_{\sigma\sigma}. \quad (4.22)$$

At early times, using the slow-roll approximation for ϕ and small ψ , we have

$$\epsilon_\sigma = \epsilon_\psi + \epsilon_\phi \approx 0 + O(\psi^2, \delta^2, a_3^2). \quad (4.23)$$

For $\eta_{\sigma\sigma}$ we have [46]

$$\eta_{\sigma\sigma} = \eta_{\phi\phi} \cos^2 \theta + \eta_{\psi\psi} \sin^2 \theta + 2\eta_{\phi\psi} \sin \theta \cos \theta \quad (4.24)$$

where θ is defined such that

$$\cos \theta = \frac{\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\psi}^2}}, \quad \sin \theta = \frac{\dot{\psi}}{\sqrt{\dot{\phi}^2 + \dot{\psi}^2}}. \quad (4.25)$$

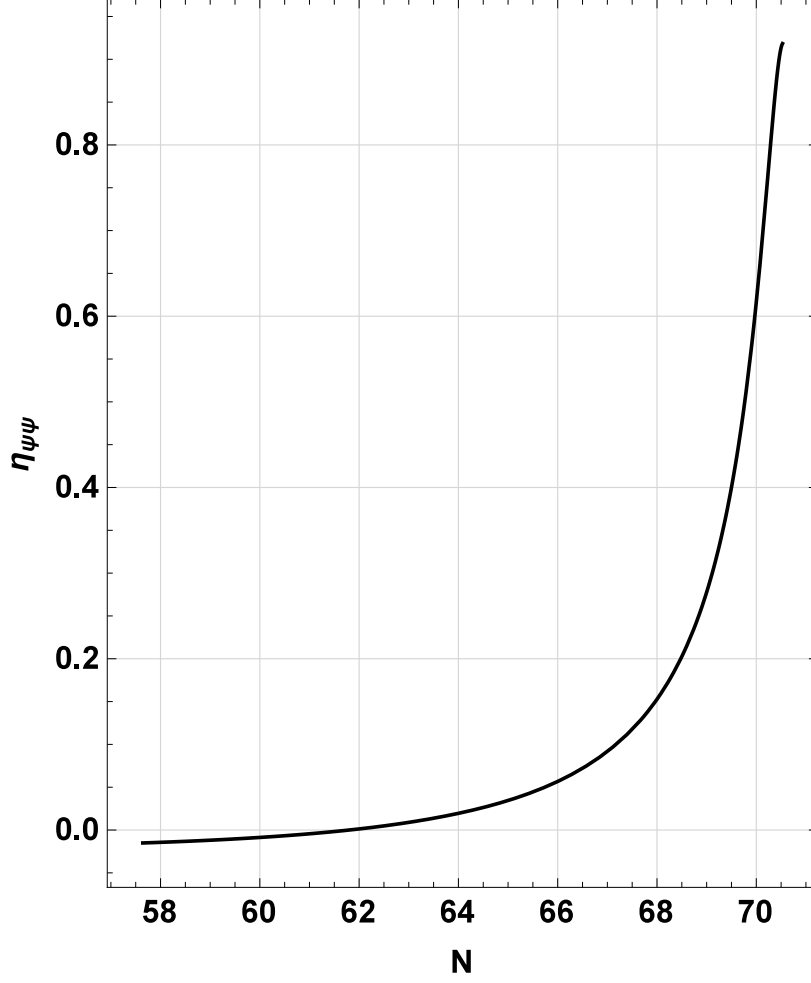


Figure 9. Late time behavior (Phase 2) of $\eta_{\psi\psi}(N)$ obtained from analytical solutions for $\psi(N)$ and $\varphi(N)$. The slow-roll assumption starts being violated around $N \sim 70$, effectively ending inflation.

Based on the slow roll equations of motion for ψ and $\dot{\varphi} \approx \dot{\varphi}_* = -3\psi(a_4\varphi_*)^{-1}\dot{\psi}$ we obtain

$$\cos \theta \approx -\frac{3\psi}{a_4\varphi_*} \sin \theta, \quad \sin \theta \approx 1, \quad (4.26)$$

where we used $V_\psi \gg V_\varphi \approx 0$. To leading order of ψ , we therefore have

$$\eta_{\varphi\varphi} \cos^2 \theta \approx 0 + O(\delta^2, \alpha_3^2, \psi^2) \quad (4.27)$$

$$\eta_{\varphi\psi} \sin \theta \cos \theta \approx 0 + O(\psi^2) \quad (4.28)$$

$$\eta_{\psi\psi} \sin^2 \theta \approx -\frac{4\delta}{a_4 P v^2} + O(\psi^2), \quad (4.29)$$

such that

$$n_s \approx 1 - \frac{8\delta}{a_4 P v^2}. \quad (4.30)$$

Evaluating

$$P \equiv V(\varphi_*(\psi=0), \psi=0) = 1 - \frac{1}{a_4} \approx \frac{2}{3} + \frac{\delta}{9} + O(\delta^2) \quad (4.31)$$

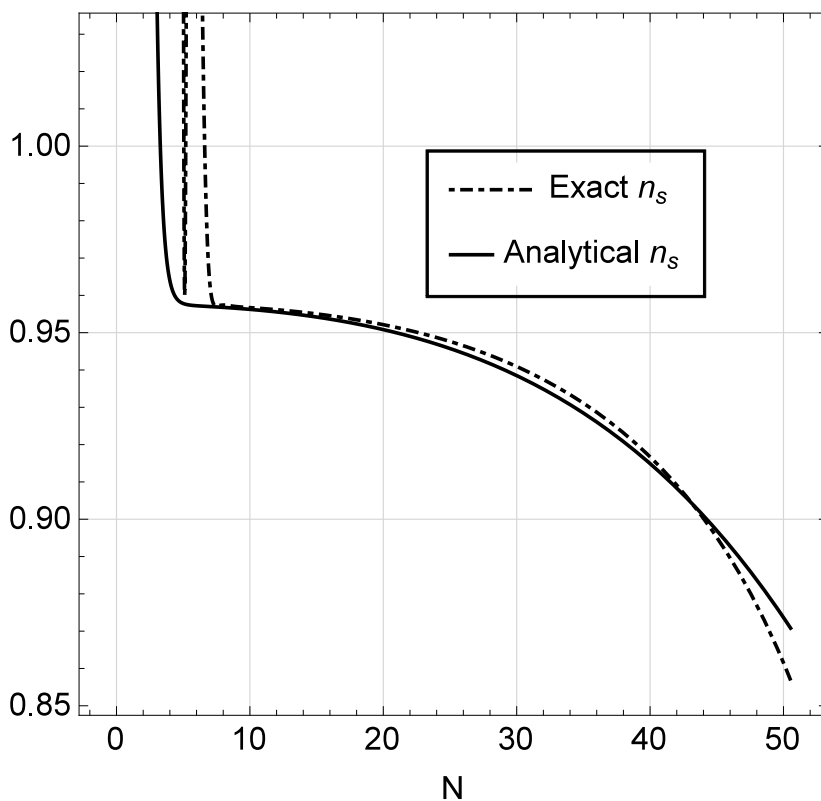


Figure 10. Analytical and numerical solutions for the spectral index $n_s(N)$ in Phase 1. Since Hubble exit takes place at least a $\Delta N \sim 60$ prior to the end of inflation, it can only occur in Phase 1. Importantly, $n_s \approx 0.96$ at $\Delta N \sim 60$.

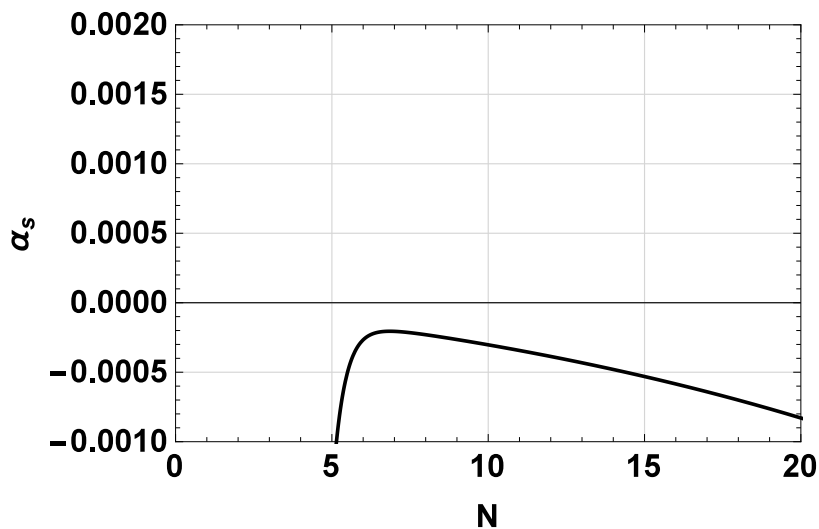


Figure 11. Analytical solution for the running $\alpha_s \approx dn_s/dN$ [62] at early times, using a non-Gaussianity parameter $a_3 = 0.05$. Estimating Hubble exit at $N \sim 10$, α_s is well within Planck's upper bound on the magnitude ($\sim 10^{-3}$).

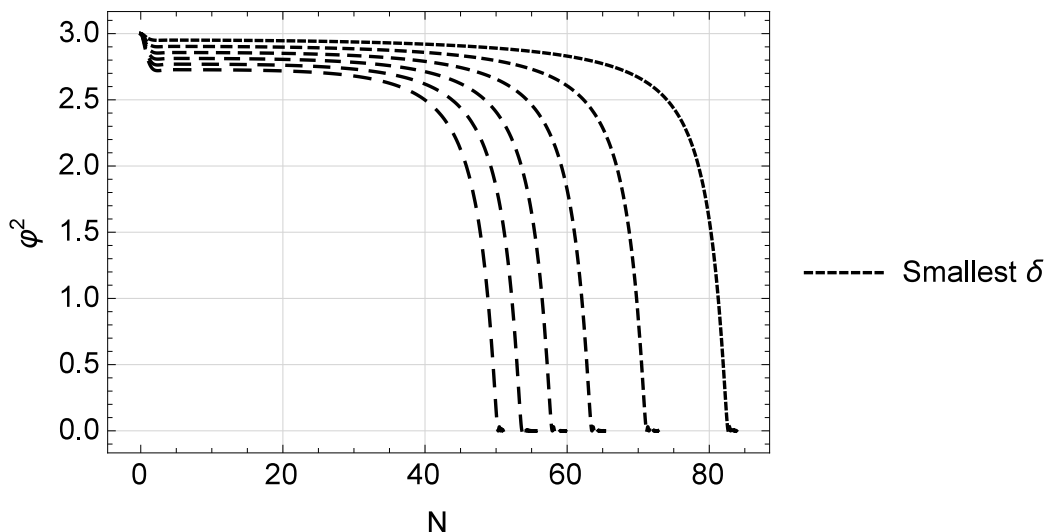


Figure 12. Evolution of $\varphi(N)^2$, from numerical solutions using $a_3 = 0.01\varphi^3$. Inflation ends at N_e where $\varphi(N_e) \approx 0$. Different curves correspond to different values of a_4 , or $\delta = a_4 - 3$, where $\delta = 0.05, 0.1, 0.15, 0.2, 0.25, 3$. Smaller δ increase the duration of inflation.

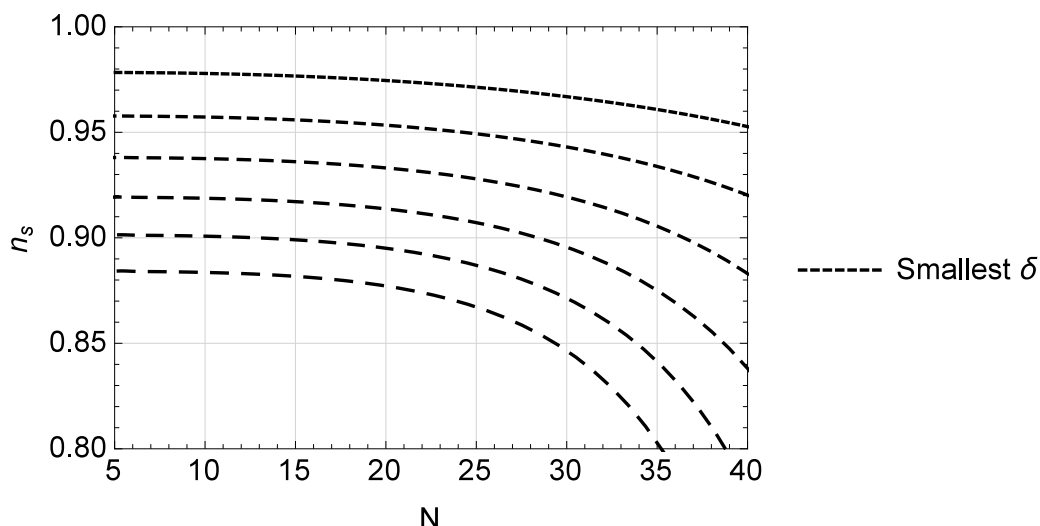


Figure 13. Spectral index $n_s(N)$ as a function of e -folds N at Hubble exit from numerical solutions, using $a_3 = 0.01\varphi^3$. Different curves correspond to different values of a_4 , or $\delta = a_4 - 3$, where $\delta = 0.05, 0.1, 0.15, 0.2, 0.25, 3$. Smaller δ brings the spectral index closer to one.

leads to the final expression

$$n_s \approx 1 - 12 \frac{\delta}{a_4 v^2} + O(\delta^2). \quad (4.32)$$

Imposing a slow-roll condition such as $\eta_{\psi\psi} \sim 10^{-2}$ requires $v^2/\delta \sim O(10^2)$, which implies typical values of n_s in the range $0.9 < n_s < 1$.

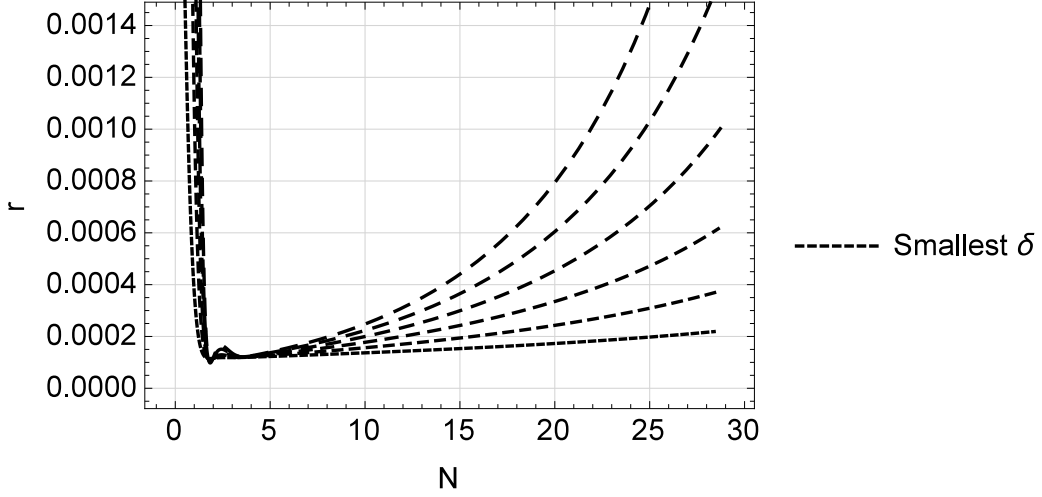


Figure 14. Tensor-to-scalar ratio $r(N)$ as a function of e -folds at Hubble exit from numerical solutions, using $a_3 = 0.01\varphi^3$. Different curves correspond to different values of a_4 , or $\delta = a_4 - 3$, where $\delta = 0.05, 0.1, 0.15, 0.2, 0.25, 3$. Smaller δ decrease r .

4.3.2 Number of e -folds

Now, for total number of e -folds N_e we first need to find the value ψ_e of ψ at which inflation ends. Approximately, this stage occurs when

$$\eta_{\psi\psi}(\varphi_*, \psi_e) = \frac{V_{\psi\psi}}{V}|_{\varphi=\varphi_*, \psi=\psi_e} \approx 1 \quad (4.33)$$

during the end phase. Under the approximation $\varphi \approx 0$, we have

$$\frac{V_{\psi\psi}}{V} \approx \frac{4}{v^2} \frac{3\psi^2/v^2 - 1}{(1 - \psi^2/v^2)^2}. \quad (4.34)$$

Then $V_{\psi\psi} \approx V(\varphi_*, \psi)$ gives

$$\frac{\psi_e^2}{v^2} \approx 1 + \frac{6}{v^2} \pm 2\sqrt{\frac{9}{v^4} + \frac{2}{v^2}} \quad (4.35)$$

$$= 1 + \frac{6}{v^2} \left(1 - \sqrt{1 + \frac{2v^2}{9}} \right), \quad (4.36)$$

where we chose the minus sign in the second line. From the above expression we see that typically $\psi_e^2/v^2 - 1/3 \sim O(10^{-1})$. Then using

$$\Delta\psi \sim -\frac{V_\psi}{V}\Delta N \sim O(1)\Delta N, \quad (4.37)$$

we see that beyond $\psi^2/v^2 = 1/3$, we do not get many e -folds before reaching the point $\eta_{\psi\psi} \approx 1$, effectively ending inflation. In terms of the total number of e -folds, it is therefore justified to approximate

$$\psi_2(N)^2 \approx v^2/3 \quad \text{such that} \quad \varphi_*^2 = 0 \quad (4.38)$$

as the end point of inflation.

Since our analytical solution consists of ψ_1 and ψ_2 , to find the total number of e -folds N_e at $\psi_2^2 = v^2/3$, we must first find the number of e -folds N_g at the gluing point. By definition of the latter,

$$\psi_g^3 \equiv \psi_1(N_g)^3 = -a_3. \quad (4.39)$$

Denoting

$$\eta \equiv |\eta_{\psi\psi}(\varphi_1, \psi_1)| \approx \frac{4\delta}{a_4 P v^2} \approx \frac{6\delta}{a_4 v^2} + O(\delta^2), \quad (4.40)$$

we have

$$2\eta = 1 - n_s. \quad (4.41)$$

Using (4.12),

$$\exp(\eta N_g) = \frac{v^2}{\psi_g^2} \frac{\delta}{a_4} + 1 \quad (4.42)$$

which, inserted in (4.16), using (4.41) and setting $\psi_2(N_e)^2 = v^2/3$, implies

$$N_e = \frac{1}{1 - n_s} \left(\log \left(\frac{2}{v^2} + \frac{1 - n_s}{12} \chi \right) + 2 \log \left(\frac{1 - n_s}{12} \chi v^2 + 1 \right) - \log \left(\frac{2}{v^2} + \frac{1 - n_s}{4} \right) \right), \quad (4.43)$$

where $\chi \equiv v^2/\psi_g^2$. The relationship (4.43) is illustrated in figure 15.

Aside from the parameter v that appears in common Higgs-like or hybrid models, our observables depend on two new parameters a_3 and δ which describe the non-Gaussianity of the background state. Background non-Gaussianity effectively controls the amount of non-adiabatic evolution due to its modulation on the shifting of local φ -minima at φ_* . The dependence of the number of e -folds on the non-Gaussianity parameter a_3 is shown in figure 16, using the analytical solutions.

The dependence (4.43) of N_e on n_s is more complicated than in non-minimal Higgs models, but it is nevertheless related. To facilitate a comparison, we rewrite the expression as

$$N_e \approx \frac{f(1 - n_s, v, a_3)}{1 - n_s} \quad (4.44)$$

where the function f describes a weak, logarithmic dependence on $1 - n_s$. In non-minimal Higgs inflation, the analog of the function $f(1 - n_s, v, a_3)$ is constant ($f = 2$) [6]. Here, the function increases logarithmically with growing $1 - n_s$, taking values in the range $1 \lesssim f(1 - n_s, v, a_3) \lesssim 5$ for typical parameter values considered in our analysis. (An abbreviated derivation of (4.44) can be found in [20].)

5 Conclusions

Typically, potentials for the inflaton field are postulated so as to match existing observations. On the other hand, one of the most remarkable successes of inflation is that it explains the large-scale structure of the universe as originating from quantum vacuum fluctuations. It is inconceivable to quantize the fluctuations of the inflaton field *alone* without taking into account the quantum corrections to the background field potential. In other words, one cannot simply express the inflationary potential in terms of expectation values of the homogeneous background field, but should also take fluctuations and higher moments of the quantum state into account. It is customary to express the resulting effective potential in a derivative expansion (of the Coleman-Weinberg type); however, this method is not

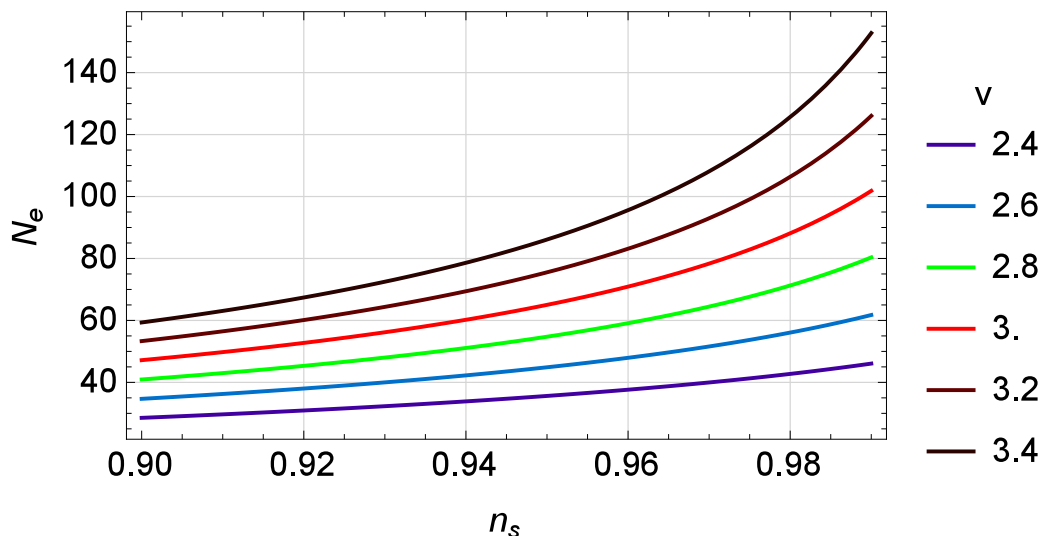


Figure 15. The number of e -folds, N_e , increases as a function of the spectral index n_s , using the approximate relation (4.43). The function is shown for varying parameters v in the potential, while $a_3 = 0.05$. As a function of the non-Gaussianity parameters, the number of e -folds decreases; see figure 16. (Note that in the analytical relation (4.32), the variation of n_s mirrors the non-Gaussianity ratio $\delta/(a_4 v^2)$.)

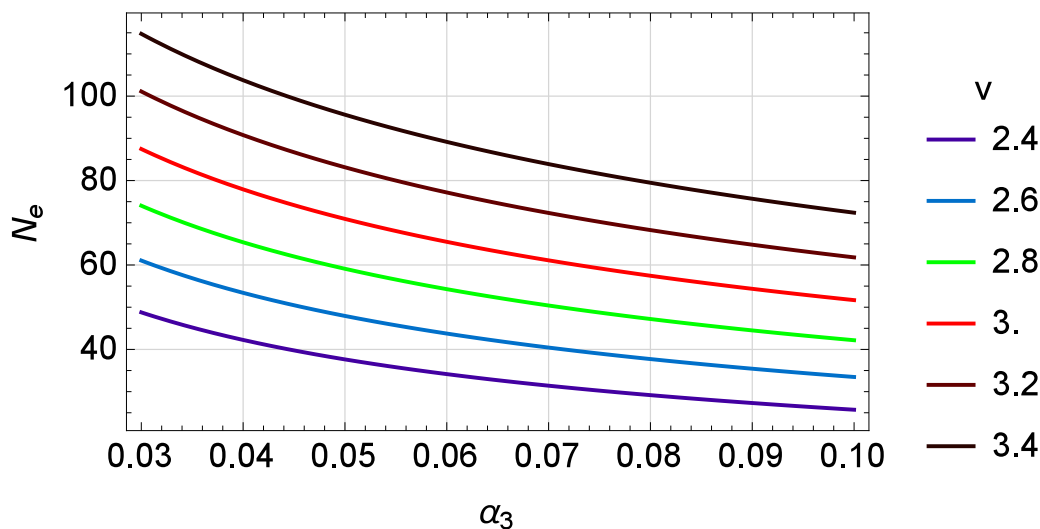


Figure 16. The number of e -folds, N_e , decreases with the amount of non-Gaussianity, parameterized by a_3 , shown here for fixed $n_s \approx 0.96$, $\delta = 0.1$ and using (4.43). Background non-Gaussianities increases the departure from adiabatic evolution, effectively ending inflation earlier than desired.

sufficient if one has to consider non-adiabatic evolution of quantum fluctuations. An adiabatic approximation is certainly valid during a slow-roll regime, but, as shown here, it can miss important features at the beginning and the end of slow-roll. Non-adiabaticity can play a crucial role setting up the initial conditions for a slow roll phase as well as help ending it at the right time. We have presented a more general procedure for calculating the effects of such non-adiabatic evolution in the context of early-universe cosmology.

Using non-adiabatic effective methods, we have constructed an observationally consistent extension of Higgs-like inflation by introducing non-adiabatic quantum effects in a semiclassical approximation, although our formalism is applicable more generally for any inflationary potential. As shown, these effects imply that the classical potential is not only corrected in its coefficients but is also amended by new terms for independent quantum degrees of freedom, in particular the quantum fluctuation of the Higgs field. The original single-field model is therefore turned into a multi-field model. The multi-field terms incorporate quantum corrections of the background field, corresponding to backreaction of radiative corrections. Since the single-field potential is renormalizable, our quantum scenario is robust from the perspective of quantum field theory.

New interaction terms in the multi-field potential have coupling constants that depend on the background state, parameterizing its non-Gaussianity. They imply two new non-adiabatic phases that cannot be seen in low-energy potentials or in cosmological studies based completely on slow-roll approximations. In particular, an initial non-adiabatic phase, combined with the uncertainty relation for the fluctuation degree of freedom, sets successful initial conditions for inflation to take place, and a second non-adiabatic phase ends inflation after the right number of e -folds. In an indirect way, observational constraints show that background non-Gaussianity should be small, but it must be non-zero for the non-adiabatic phases to be realized. (The observational input we use here is not a limit on statistical non-Gaussianity in the inhomogeneity spectrum. Rather, the new link between the number of e -folds and background non-Gaussianity, shown in figure 16, makes it possible to use readily available limits on the number of e -folds and, in conjunction with figure 15, the spectral index in order to limit quantum non-Gaussianity of the background state of the inflaton.) Our model is highly constrained because this non-Gaussianity is bounded from below, but we are nevertheless able to derive successful inflation in the range of parameters available to us.

Our model presents a new picture on the role of the quantum state in inflationary cosmology. Quantum fluctuations do not only provide the seeds of structure as initial conditions for perturbative inhomogeneity, they also play a crucial role in guiding the inflationary dynamics of the background state. With further analysis and observations, it may be possible to further constrain the quantum state of the inflaton based on cosmological investigations.

Acknowledgments

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