

# Neutron Star Cooling on Strong Magnetic Field : Neutrino -Antineutrino Pair Emission and Direct Urca Processes

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We study neutrino and anti-neutrino emissions through the  $\nu\bar{\nu}$ -pair emission and the direct Urca processes in the strong magnetic field. The strong magnetic fields supply additional momentum and the neutrino emissivity and largely enhance the emissivity. We solve exact wave functions for protons and electrons in the states described with Landau levels. Both processes in strong magnetic fields are expected to contribute significantly to the cooling of the magnetars.

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## 1. Introduction

Magnetic fields in neutron stars play important roles in the interpretation of many observed phenomena. Magnetars, which are associated with super strong magnetic fields, [1, 2] have properties different from normal neutron stars. Thus, phenomena related to magnetars can provide a lot of information about the physics of the magnetic field.

Many authors have paid attention into cooling processes of neutron stars (NS) because it gives important information on neutron star structure [3]. Neutron stars are cooled by neutrino emission, and a magnetic field is expected to affect the emission mechanism largely because a strong magnetic field can supply energy and momentum into cooling processes.

In this work we study two kind of the processes, the  $\nu\bar{\nu}$ -pair synchrotron radiation  $p(e^-) \rightarrow p(e^-) + \nu + \bar{\nu}$ , and the direct Urca (DU) process  $(n \rightarrow p + e^- + \bar{\nu}_e, p + e^- \rightarrow n + \nu_e)$  when the temperature is lower than 1 keV. We apply the quantum theoretical approach to the  $\nu\bar{\nu}$ -pair productions in the strong magnetic field and calculate it through the transition between the different Landau levels for electrons and protons.

## 2. Formalism

Here, we briefly explain our formalism.

In this work we assume the neutron-star matter composing proton, neutron and electron. and a uniform magnetic field along the  $z$ -direction,  $\mathbf{B} = (0, 0, B)$ , and take the electro-magnetic vector potential  $A^\mu$  to be  $A = (0, 0, xB, 0)$  at the position  $\mathbf{r} \equiv (x, y, z)$ . The relativistic proton (electron) wave function  $\psi$  is obtained from the following Dirac equation:

$$\left[ \alpha_z p_z - i\alpha_x \partial_x + \alpha_y (p_y - eB_x) + (M - U_s) \beta - \frac{\kappa}{M} B \Sigma_z + U_0 \right] \psi(x, p_z, s) = E \psi(x, p_z, s), \quad (2.1)$$

where  $M$  is the proton (electron) mass,  $\kappa$  is the AMM,  $e$  is the particle charge, and  $E$  is the single particle energy written as

$$E(n, p_z, s) = \sqrt{p_z^2 + (\sqrt{2|e|Bn + M^2} - s\kappa B/M)^2} + U_0. \quad (2.2)$$

with  $n$  being the Landau level number. In the above equations,  $U_s$  and  $U_0$  are the scalar and vector mean-fields which are given by the relativistic mean-field approach.

In order to control the proton fraction of the NS matter we introduce the two kinds of the iso-vector interaction channels [4], the Lorentz scalar and Lorentz vector channels, which are given by the following Lagrangian density as

$$\mathcal{L}_{IV} = -\frac{C_s^{IV}}{2M_N^2} (\bar{\psi} \tau \psi)^2 - \frac{C_v^{IV}}{2M_N^2} (\bar{\psi} \gamma \tau \psi)^2, \quad (2.3)$$

where  $M_N$  is the nucleon mass, and  $C_s^{IV}$  and  $C_v^{IV}$  are the coupling constants.

We use the parameter-set PM1 [5] for the symmetric matter and give the three parameter-sets, SF1, SF2 and SF3, which fix the symmetry energy of 32 MeV at the saturation density. The SF1 includes only the Lorentz vector channel ( $C_s^{IV} = 0$ ), the SF2 includes only the iso-vector Lorentz

scalar channel ( $C_\nu^{IV} = 0$ ), and the SF3 includes the negative value of the Lorentz vector channel ( $C_\nu^{IV} < 0 < C_s^{IV}$ ).

The weak interaction part of the Lagrangian density is written as

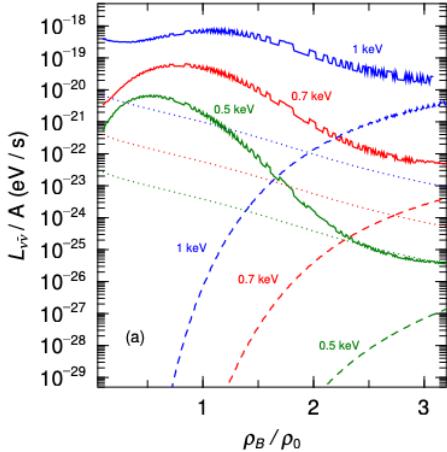
$$\mathcal{L}_W = G_F \bar{\psi}_\nu \gamma_\mu (1 - \gamma_5) \psi_\nu \sum_\alpha \bar{\psi}_\alpha \gamma_\mu (c_V - c_A \gamma_5) \psi_\alpha, \quad (2.4)$$

where  $\psi_\nu$  is the neutrino field,  $\psi_\alpha$  is the field of the particle  $\alpha$  which indicates the proton and electron, and the  $c_V$  and  $c_A$  are the weak vector and axial coupling constants dependent on each channel.

The detailed expressions to calculate the neutrino emissivity are written in Ref. [6] for the  $\nu\bar{\nu}$ -pair emission process and in Ref. [7] for the DU process. For the  $\nu\bar{\nu}$ -pair emission process, energy intervals between two states with different Landau numbers are much larger than the temperature, so that we cannot use the Sommerfeld expansion at the low temperature limit. On the other hand, for the DU process, the initial particle is a neutron which does not stay in the Landau level, and the energy interval is continuous, so that we use this expansion in the calculations.

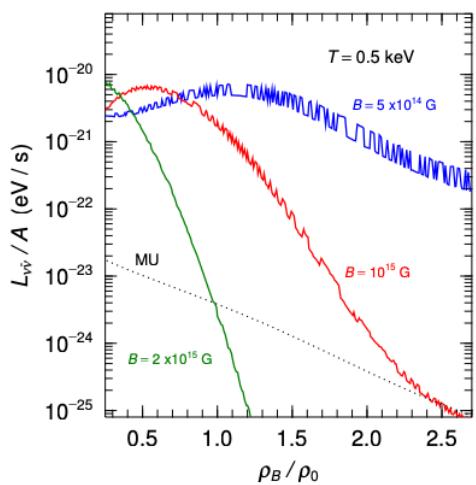
### 3. Results in $\nu\bar{\nu}$ -pair Emission Process

In Fig. 1, we show the density dependence of the luminosities in the  $\nu\bar{\nu}$ -pair emission by using the parameter-set, PM1-SF1 for the mean-fields. For comparison, we give the  $\bar{\nu}$ -luminosities from the modified Urca (MU) process [8]



**Figure 1:** Density dependence of the  $\nu\bar{\nu}$ -pair emission luminosity per nucleon for  $B = 10^{15}$  G at  $T = 0.5, 0.7$  and  $1$  keV (from bottom to top). The solid and dashed lines represent the contributions from protons and electrons. The dotted lines indicate the neutrino luminosities from the MU process.

We see that in the strong magnetic field, thus, the neutrino luminosities become larger than those from the MU process particularly in low density regions. The calculation results include fluctuations. The density dependence of the factor  $f(E_i)[1 - f(E_f)]$  does not smoothly vary for



**Figure 2:** Solid lines show the density dependence of the  $\nu\bar{\nu}$ -pair emission luminosity per nucleon for  $B = 5 \times 10^{14}$  G,  $10^{15}$  G and  $2 \times 10^{15}$  G at  $T = 0.5$  keV. The dotted lines indicate the neutrino luminosity from the MU process.

strong magnetic fields because the energy intervals between the initial and final states are larger than the temperature.

In Fig. 2 we show the density dependence of the  $\nu\bar{\nu}$ -luminosities at  $B = 5 \times 10^{14}$  G,  $B = 10^{15}$  G and  $B = 2 \times 10^{15}$  G. We see that as the magnetic field strength decreases, the luminosity increases in the density region,  $\rho_B/\rho_0 \gtrsim 1$ . The  $\nu\bar{\nu}$ -pairs are produced via the transition of the proton and electron between different Landau levels. As the magnetic field strength increases, the momentum transfer from the magnetic field becomes larger, and the energy interval between the initial and final states is also larger. The former effect enhances the emission rate, but the latter effect suppresses it. In the present calculations the latter effect dominates in the higher density region.

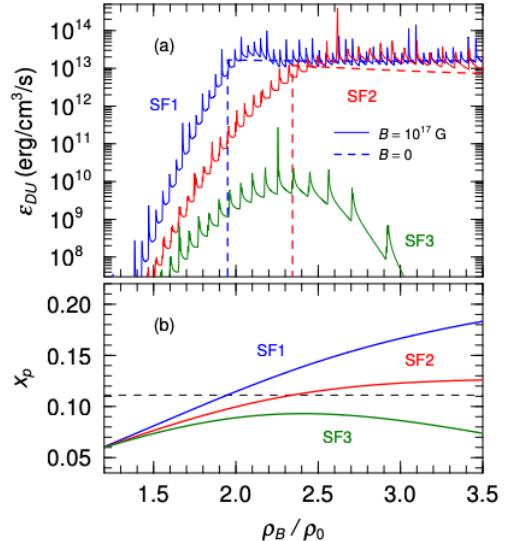
#### 4. Results in Direct Urca Process

Next, we show the results on the neutrino and anti-neutrino emissivities per volume in the DU process. Because of energy-momentum conservation and Fermi statistics for NS matter composed of protons, neutrons and electrons, the DU process occurs in a density region, where the proton fraction,  $x_p$ , is larger than  $1/9$ ,  $x_p \geq 1/9$ : this condition cannot be achieved in low density regions.

In Fig. 3 we show the density dependence of the neutrino emissivity in the DU process (a) and that of the proton fraction in NS matter,  $x_p$  (b). In Fig. 3(a), the dashed line indicates the results for  $B = 0$ , which appear in the density region of  $x_p \geq 1/9$ ;  $x_p = 1/9$  is denoted by the horizontal dashed line in the lower panel (b).

Here, we define  $\rho_{DU}$  as the critical density at which the proton fraction  $x_p = 1/9$ . When  $B = 0$ , the results of the neutrino emissivity for SF1 and SF2 suddenly appear at the density  $\rho_{DU}$ . They increase very rapidly and become almost flat as the density increases. When  $B = 10^{17}$  G, the emissivity appears at a density of  $\rho < \rho_{DU}$  and becomes larger gradually with increasing density. In the density region  $\rho \geq \rho_{DU}$ , they undergo large oscillations though their local minima almost agree with the  $B = 0$  results.

For SF3, on the other hand, the proton fraction does not exceed  $1/9$ , and the DU does not occur when  $B = 0$ . When  $B = 10^{17}$  G, however, neutrinos are emitted, and thus, we can confirm that the magnetic-field increases the neutrino emission. In addition, the peak position of the neutrino emissivity agrees



**Figure 3:** (a) The density dependence of the neutrino emissivity per volume in the DU process at  $T = 0.50$  keV and (b) that of the proton fraction of NS matter at  $B = 0$ . The vertical lines represent the results with the SF1, SF2 and SF3, respectively. In the upper panel (a) the solid and dashed lines indicate the results when  $B = 10^{17}$  G and  $B = 0$ , respectively.

with that of the proton fraction, namely the effect of the magnetic field is related to the proton fraction and can appear when the proton fraction is close to 1/9.

## 5. Summary

We have used a relativistic quantum approach to study neutrino and anti-neutrino emission in the  $\nu\bar{\nu}$ -pair production and DU processes from NS matter with strong magnetic fields. We see that the magnetic field has a role to largely enhance the neutrino emission in both processes.

The  $\nu\bar{\nu}$ -pair emission process has a much larger effect than that of the MU process in strong magnetic fields. We can conclude that the  $\nu\bar{\nu}$ -pair emission process is dominant in the low density region,  $\rho_B \lesssim \rho_0$ , for a cooling process of magnetars whose magnetic field strength is  $10^{14} - 10^{15}$  G. Therefore, our results suggest that one needs to introduce the  $\nu\bar{\nu}$ -pair emission process when calculating the cooling rate of magnetars.

In the high density region,  $\rho_B \gtrsim 3\rho_0$ , the direct Urca process appears, and its contribution is much larger than that of the  $\nu\bar{\nu}$ -pair emission. If the proton fraction satisfies the DU condition  $x_p \geq 1/9$ , the neutrino emissivities are not much different from the case of  $B = 0$ , and the magnetic field does not significantly amplify the emission, though it causes very large fluctuations in the density dependence of the neutrino emissivity. In the usual forbidden region  $x_p < 1/9$ , however, the magnetic field contributes to the emission and changes the kinematical condition.

The present results demonstrate that the magnetic field very largely contribute to the neutrino emission processes, and that quantum calculations are necessary to describe the momentum transfer from the magnetic field exactly.

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