Transaction Pricing Mechanism Design and Assessment for Blockchain

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Abstract

The importance of transaction fees in maintaining blockchain security and sustainability has been confirmed by extensive research, although they are not mandatory in most current blockchain systems. To enhance blockchain in the long term, it is crucial to design effective transaction pricing mechanisms. Different from the existing schemes based on auctions with more consideration about the profit of miners, we resort to game theory and propose a correlated equilibrium based transaction pricing mechanism through solving a pricing game among users with transactions, which can achieve both the individual and global optimum. To avoid the computational complexity exponentially increasing with the number of transactions, we further improve the game-theoretic solution with an approximate algorithm, which can derive almost the same results as the original one but costs significantly reduced time. We also propose a truthful assessment model for pricing mechanism to collect the feedback of users regarding the price suggestion. Extensive experimental results demonstrate the effectiveness and efficiency of our proposed mechanism.

Keywords: blockchain, transaction pricing, game theory, correlated equilibrium, peer prediction

1. Introduction

The world has witnessed a great deal of attention injected into the field of blockchain technology from both academia and industry since Bitcoin was introduced as a representative concept of blockchain in 2008. The most important contribution of blockchain is that it can achieve distributed trust without any centralized coordination, which further highlights an attractive feature of blockchain that all recorded information, such as transactions and smart contracts, inside blocks on the main chain cannot be arbitrarily modified or repudiated. This delicately designed technology achieving distributed security enables wide applications of blockchain in various directions, such as blockchain-based database [1, 2], blockchain-witnessed trustworthy cloud service [3, 4], blockchain-assisted admission control in cognitive radio network [5], and blockchain-driven internet of things [6, 7, 8].

To maintain the aforementioned attractive feature, a large number of nodes are involved to reach consensus on who should append the newly generated block to the main chain so as to guarantee the stability and security of the whole blockchain network, which might incur massive costs for participated nodes [9], e.g., computation cost and communication cost, according to specific consensus algorithms. As an incentive for their work, the node who finally wins the accounting right will receive a reward, which usually comes from two sources, including the blockchain system and the information-record owners. The first part is generally predefined when the blockchain system is initially designed, which is relatively stable. While the second part is determined by the record owners as a sort of handling fee. As *transaction* is a representative type of information record in blockchain, we study its pricing problem as an example in this paper[1] and refer to its owner as a *user*. In most prevailing blockchain systems, such as Bitcoin [10] and Ethereum [11], the transaction fee is optional, thus making it unpredictable and seemingly trivial. However, as pointed out in [12, 13, 14], transaction fees from users have a significant influence on the system security of blockchain, which becomes even more prominent in blockchain systems with decreasing block rewards.

Being aware of this, many researchers analyze specific relationships between transaction fees and various security metrics of blockchain systems

[1]Pricing of other sorts of information records can be tackled in a similar way.

with the help of game theory [15, 16, 17, 18]. Other existing work is devoted to transaction pricing mechanism design based on auctions [14, 19, 20] from the perspective of miners' profit. In contrast, our paper designs a transaction pricing mechanism from the perspective of users, providing price suggestions to realize both global and individual rationality.

However, it is challenging to design such a transaction pricing mechanism due to the following two reasons. First, from the perspective of users, the ultimate goal of everyone is to get his transaction included in a valid block on the main chain by paying the transaction fee as low as possible. This is hard to achieve since they have to compete with each other on price with incomplete information about the offers from other competitors. Second, from the perspective of the blockchain system, if malicious competitions among users enforce an excessive pricing bar or extremely long waiting time, users who cannot afford it will stop using it, which impedes the sustainable development of blockchain.

To address the above challenges, we resort to game theory to model the coexisted competition and collaboration among users and take advantage of the concept of correlated equilibrium [21] to achieve both individual and global optimum. To be specific, we consider a platform for price recommendation with users inputting the sizes and time sensitivities of their transactions, which can efficiently calculate the optimal pricing strategies for all users with the best utilities, thus solving the first challenge. In addition, as the recommended prices are derived according to the real-time parameters of all transactions, the expenses of users will not increase uncontrollably for the cumulative impact of malicious bidding, which therefore overcomes the second challenge.

After the pricing mechanism design, a reliable assessment model reflecting the users' feedback can quantify the performance of our proposed scheme and inspire the future design. One method is to collect the opinions of users regarding whether he/she will take the suggested price or not. However, collecting truthful feedback from users is not trivial, given the existence of malicious users submitting unreliable information, which may seriously impact the assessment accuracy and further hinder the implementation of the proposed pricing mechanism. To address this issue, we design a truthful assessment model for the pricing mechanism based on the private-prior peer prediction theory. Instead of collecting the users' opinions directly, our truthful assessment model requires them to submit a random peer's belief about the price before and after the pricing game. Users' trustworthiness can be

then calculated through the strictly proper scoring rule. To avoid the negative impact of unreliable reports, we design a scheme considering only honest users' feedback so as to guarantee the truthfulness of the assessment model.

In summary, our contributions in this paper are as follows (a preliminary version of this paper is published in [22]):

- We propose a *pricing game* to sketch the transaction pricing competition among users in blockchain, where the possibility of each transaction being included is defined to help depict the individual utility of each user.
- To achieve individual rationality with the maximized utility, we leverage *correlated equilibrium* to integrate it to the global optimal objective for securing the interests of all users, which comes into an optimization problem with exponential complexity in the number of transactions.
- To overcome the weakness on computational cost, we propose an approximate algorithm with divided optimum achieved parallelly for speeding up the calculation process, which is numerically evaluated to demonstrate its effectiveness and efficiency.
- We design a truthful assessment model for the pricing mechanism to collect the feedback of users with respect to the price suggestion, which is incentive compatible for encouraging users to behave honestly.

The rest of this paper is organized as follows. We investigate the most related work on blockchain transaction pricing in Section 2. In Section 3, we formulate the problem of transaction fee determination as a pricing game, where all users hope to maximize their individual utility. To achieve the goal, we take advantage of the correlated equilibrium to analyze the pricing game from a global perspective in Section 4, which is summarized as an optimization problem and approximately solved with higher efficiency in Section 5. In Section 6, we design an assessment model to collect the feedback of the pricing mechanism. We evaluate our proposed solution in Section 7 and conclude the whole paper in Section 8.

2. Related Work

Although paying transaction fees is not mandatory in most existing blockchian systems, a large number of studies have indicated that it plays a major role

in the security of blockchain. In [12], a financial reasoning was conducted to demonstrate the unsustainability of blockchain with zero or infinitesimal transaction fees. Further, as a counterintuitive conclusion, Carlsten *et al.* [13] proved that whether rewards of miners are coming from blockchain system or transaction fees significantly affects the system security since there exists nearly no equilibrium with favorable security properties.

Besides, game theory is widely adopted to analyze the impact of transaction fees on blockchain. Correlating the issue with simple static partial equilibrium, Houy [15] analyzed that it is equivalent to keep a fixed transaction fee or let the decentralized market to determine the unit price with a fixed block size. He *et al.*[23] explored the interaction between security and the decision of fees in the manner of equilibrium, proving that the primary state of the system determines the result regarding the interplay of users and miners. Focusing on the owner-less characteristic of blockchain, Huberman *et al.* [16] provided closed form formulas on the relationship between the transaction fees and waiting times through formulating user behavior as a queuing game. Similarly, a queuing game with non-preemptive priority was employed in [17] to depict the dynamics in memory pool of blockchain with transactions flowing in and out, where five types of Nash equilibrium were found. In [18], Easley *et al.* constructed a game-theoretic model to analyze the evolution of transaction fees in Bitcoin from a market perspective. Based on Lyapunov optimization and large deviation theory, the transaction selection mechanism is proposed in [24], which maximizes the utility of both system and miners.

With knowing the importance of transaction fees, more research work has been conducted to design various pricing schemes in recent. Most of the existing pricing mechanisms take advantage of auction to find the optimal price setting strategy, with a focus on maximizing the profit of miners. In [14], Lavi *et al.* figured out two challenges in Bitcoin related to the decreasing block reward and limited block size, based on which they analyzed the applicability of monopolistic auction in this scenario due to its immunity to untrusted auctioneers. In particular, all transactions included in a block pay the same lowest bid instead of the current pay-your-bid approach in Bitcoin, which can decouple the above two challenges. As a theoretical supplement to the monopolistic auction mechanism in [14], Yao [19] proved its approximate incentive-compatibility and further demonstrated its dominance compared to a traditional auction mechanism named Random Sampling Optimal Price auction (RSOP) [25]. Besides, Basu *et al.* [20] proposed a novel transac-

tion pricing mechanism based on the generalized second price auction, which was demonstrated to be resistant to arbitrary manipulation as the derived bidding satisfied truthfulness.

In summary, the existing work related to transaction fees in blockchain have revealed its importance, most of which utilized game theory to conduct analysis; other closely related work on pricing mechanism design mainly relies on auctions, aiming at improving the efficiency and profit of miners. In contrast, our work leverages game theory to model the competitive and collaborative relationships among users considering the size and time sensitivity of each transaction, which can result in both global and individual optimum, thus maintaining a more sustainable and lively blockchain ecosystem.

3. Problem Formulation

In this paper, we consider the *mempool* of a blockchain system with all transactions from users, where the miners will select a set of transactions to be included in their individual block. Since there will be only one valid block at the end of each round of mining, we focus on the selection of transactions in this single block from a global perspective[2]. As the blockchain network prevails, increasingly large amount of transactions are generated, streaming into the mempool to be included in the valid block. However, the size of a block is limited, yielding the competition among transactions with respected to the transaction fees.

To describe this competitive system, we denote all transactions in the current mempool as $\{tx_1, tx_2, \cdots, tx_n\}$, where n is the total number of transactions from users. Each transaction has a specific size as s_i, which is fixed once the transaction is appearing in the mempool. The total size of all transactions included in one block cannot exceed the maximum limitation. Considering that transactions can have different time sensitivity, we assume that each transaction tx_i come with a time tag T_i indicating its remaining time to be included in a valid block; otherwise, the user launching this transaction will suffer from a big loss. In our paper, we aim to study how to design a pricing mechanism to provide unit price suggestions for all transactions in

[2]Transactions included in different blocks owned by different miners might be heterogeneous due to the network transmission delay, which will be considered in our future work.

the current mempool considering both competitive prices provided by other transactions and their various emergency levels (i.e., time sensitivity).

Given a unit price v_i for tx_i with size s_i, the miner will get the payment of $v_i s_i$ for including tx_i in the valid block. In particular, we characterize the miner's behavior of including transactions in the valid block with a probabilistic model which is inspired by the *Discrete Choice Model* presented in [26]. Formally, we define the transaction inclusion probability as follows.

Defination 1 (Transaction Inclusion Probability)**.** *With the unit price vector from all transactions in the current mempool, denoted as* $\boldsymbol{v}$*, the probability of* tx_i *with unit price* v_i *being included in the valid block is*

$$p_i(\boldsymbol{v}) = \frac{\exp(a_i v_i - b)}{\sum_{j=1}^{n} \exp(a_j v_j - b)}, \tag{1}$$

where $a_i > 0$ *and* $b > 0$ *are parameters related to each* tx_i *and the block, respectively.*

The above definition based on discrete choice model can macroscopically describe the transaction inclusion event from both the perspectives of the miner (block) and the user (transaction). Generally, the probability of a transaction tx_i being included is positively proportional to its provided unit price v_i. In practical, the miner would be more willing to include those transactions with higher unit price as they can bring more profit for a length-limited block; and the transactions eager to be included are inclined to come with higher unit price to attract the miner's attention.

In addition, a_i and b in (1) can reflect the randomness during the process of including transactions in a block, which may come from both the transaction side and the block side. For example, since the size of the block is limited, the miner cannot always select the remaining highest unit price provider in the current mempool especially when the size of the block left cannot cover the length of this transaction with the highest unit price, which can be captured by the parameter a_i dependent on tx_i at this point, denoted by $a_i = \psi(s_i)$; similarly, other factors of the block, such as the generation location, can also impact the transaction inclusion results.

It is worth mentioning that the values of a_i and b could be obtained by querying the transaction and statistically calculating the historical block information.

With the definition of inclusion probability, we can calculate the payoff of the user, which is defined as individual utility in the following.

Defination 2 (Individual Utility)**.** *The expected individual utility of a user publishing tx_i with size s_i and unit price p_i can be calculated by*

$$U_i(\boldsymbol{v}) = p_i(\boldsymbol{v})\big(\phi(T_i) - v_i s_i\big), \tag{2}$$

where T_i is the time tag of tx_i and $\phi(\cdot)$ is a non-decreasing function with T_i. In particular, $\phi(\cdot)$ can be defined as

$$\phi(T_i) = \frac{\alpha_i}{1 + \exp(-\beta_i T_i)}, \tag{3}$$

where $\alpha_i, \beta_i > 0$ are scalars for tx_i.

In (2), the individual utility of a user is mainly dependent on two parts, i.e., the profit of the transaction being successfully included before its deadline and the total cost that the user needs to pay for the transaction. The cost part is obvious to be the unit price multiplying the size of the transaction. For the profit part defined in (3), we consider that, generally, the sooner the transaction gets included, the higher profit the user can gain, so the increasing remaining time to be included for a transaction can bring more profit for this user; while this sort of advantage of time length left cannot last forever, which is reflected in the upper limitation of $\phi(T_i)$ as α_i. In addition, the increasing speed of $\phi(T_i)$ with respect to T_i is unique for each transaction tx_i, decided by β_i.

Note that since the range of $\phi(T_i)$ is $[\frac{\alpha_i}{2}, \alpha_i]$, we assume that $\alpha_i \geq 2v_i s_i$ to guarantee the individual utility defined in (2) is non-negative.

According to the above definition, it can be seen that the individual utility of each transaction is not only related to its own posted unit price but also the unit prices provided by other transactions in the current mempool. In order to depict this interdependent relationship among all transactions, we take advantage of the non-cooperation game to further model this problem as a *Pricing Game*.

Defination 3 (Pricing Game)**.** *All users with transactions*[3] *in the current mempool form a pricing game where any user with tx_i is a game player,*

[3]Here we consider each user only has one transaction. For a user with multiple transactions, we treat each transaction individually, regarding there is a corresponding user behind each one.

exerting the strategy to provide a unit price v_i and getting the payoff of the individual utility $U_i(\boldsymbol{v})$.

As the individual utility of any user is collectively decided by all the unit prices, we can specifically express it as $U_i(v_i, \boldsymbol{v}_{-i})$ where $\boldsymbol{v}_{-i} = (v_1, \cdots, v_{i-1}, v_{i+1}, \cdots, v_n)$ denotes the unit prices provided by other users for their transactions $\{tx_1, \cdots, tx_{i-1}, tx_{i+1}, \cdots, tx_n\}$. As a rational and utility-driven player, any user wants to maximize the individual utility $U_i(v_i, \boldsymbol{v}_{-i})$. However, it is not feasible for any user to achieve this goal without knowing the offers from others. Therefore, in this paper, we start from a global perspective to help all users make the decision on how to provide reasonable unit prices to their transactions to get them included in the valid block before the deadline and achieve maximum payoffs as well.

4. Game Theoretic Solution

In the previous section, we propose the pricing game to characterize the unit price decision problem of transactions among all users, which leaves the individual utility maximization as a challenge. In this section, we first derive it as a correlated equilibrium, and then analyze this problem from a macro perspective to achieve the global optimum for all users, followed by the final solution.

Without loss of generality, we assume that the strategy space of users is discrete, denoted by $\mathcal{V}$, and with the size of V. According to the individual utility maximization requirement of each user, we can get the correlated equilibrium of the pricing game as follows.

Defination 4 (Correlated Equilibrium)**.** *For our proposed pricing game, there exists a correlated equilibrium $F(\boldsymbol{v})$, which is a unique probability distribution over the space $\mathcal{V}^n$ denoting all possible combinations of unit prices provided by all users, if and only if for any user with the strategy $v_i \in \mathcal{V}$, it satisfies*

$$\sum_{\boldsymbol{v}_{-i} \in \mathcal{V}^{n-1}} F(v_i, \boldsymbol{v}_{-i}) \Big(U_i(v_i, \boldsymbol{v}_{-i}) - U_i(v_i', \boldsymbol{v}_{-i}) \Big) \geq 0, \tag{4}$$

where $v_i' \in \mathcal{V}$ is any strategy other than v_i.

According to the above definition, one can see that under the correlated equilibrium $F(\boldsymbol{v})$, any user has no motivation to deviate from the current

strategy v_i when others are fixed to $\boldsymbol{v}_{-i}$. In other words, any user can thus maximize the individual utility as long as each user sets v_i according to $\boldsymbol{v}$ sampled from $F(\boldsymbol{v})$. Since $F(\boldsymbol{v})$ is a probability distribution, we have the constraints $F(\boldsymbol{v}) \geq 0$ and $\sum_{\boldsymbol{v}\in\mathcal{V}^n} F(\boldsymbol{v}) = 1$. Combined with the above inequality (4), it is easy to calculate a correlated equilibrium through solving a linear programming problem, which could generate a set of results as multiple correlated equilibria. In order to find the best one, we introduce the following global objective of social welfare for the pricing game.

Defination 5 (Social Welfare)**.** *For a specific correlated equilibrium of the pricing game $F(\boldsymbol{v})$, the social welfare is defined as the expected total utilities of all users, i.e., $\sum_{\boldsymbol{v}\in\mathcal{V}^n} F(\boldsymbol{v}) \sum_{i=1}^{n} U_i(\boldsymbol{v})$.*

Therefore, to derive the best pricing strategy for each user, we can solve the best correlated equilibrium for the pricing game from a global perspective, which can be summarized into the following optimization problem.

$$\max : \sum_{\boldsymbol{v}\in\mathcal{V}^n} F(\boldsymbol{v}) \sum_{i=1}^{n} U_i(\boldsymbol{v}) \tag{5}$$

$$\text{s.t.: } F(\boldsymbol{v}) \geq 0, \tag{6}$$

$$\sum_{\boldsymbol{v}\in\mathcal{V}^n} F(\boldsymbol{v}) = 1, \tag{7}$$

$$\sum_{\boldsymbol{v}_{-i}\in\mathcal{V}^{n-1}} F(v_i, \boldsymbol{v}_{-i}) \Big(U_i(v_i, \boldsymbol{v}_{-i}) - U_i(v_i', \boldsymbol{v}_{-i}) \Big) \geq 0,$$

$$\forall v_i, v_i' \in \mathcal{V}. \tag{8}$$

For simplicity, we refer this optimization problem as *social welfare maximization problem* as the objective function in (5) is to maximize the social welfare of the pricing game. It is worth mentioning that the first two constraints (6) and (7) are coming from the definition of probability distribution, and the last constraint (8) is to guarantee the individual utility maximization presented in Definition 4. As mentioned above, this optimization problem is exactly a linear programming problem, where the variable is the probability distribution over all possible combinations of unit prices, i.e., $F(\boldsymbol{v})$. Thus we can employ existing algorithms to solve it in an efficient manner, such as dual-simplex and interior-point, which will cost polynomial time in the numbers of variables and constraints. However, it is not realistic to directly adopt

the existing algorpaithms to solve our aforementioned optimization problem because the computational cost in our case is non-polynomial, which can be demonstrated by the following Theorem.

Theorem 6. *Directly using the existing algorithms to solve the social welfare maximization problem has computational cost exponentially increasing with the number of transactions n.*

Proof. Given the number of transactions n and the size of strategy space V, the number of variables in the social welfare maximization problem, i.e., $F(\boldsymbol{v}), \boldsymbol{v} \in \mathcal{V}^n$, is V^n since the probability distribution is over all possible combinations of unit prices. For the number of constraints, it is clear that (7) is a single constraint, while constraint (6) is held for every variable $F(\boldsymbol{v})$, so its total number is also V^n; and the number of constraints according to (8) is nV. Therefore, both the numbers of variables and constraints are $O(V^n)$.

In this case, even though the computational cost of the existing algorithms for solving the linear programming problem is polynomial time in the numbers of variables and constraints, directly applying them on our social welfare maximization problem will lead to exponential computational complexity in the number of transactions because the numbers of variables and constraints are exponentially increasing with n as mentioned above. □

As shown in the above Theorem, as the increase of the number of transactions n, the number of variables will increase significantly, offsetting the efficiency of employing existing algorithms with polynomial computational complexity. To overcome this challenge, we propose an approximate algorithm to maximize the social welfare of the pricing game based on the existing linear programming algorithms, which is introduced in the following section.

5. An Approximate Algorithm

To decrease the computational cost of directly employing classical linear programming algorithms for solving our problem, we need to eliminate the impact of exponential relationship between the numbers of variables and constraints and the number of transactions. In this section, we achieve this goal through proposing an approximate algorithm which controls the exponentially increasing numbers of variables and constraints to an acceptable level.

In general, our main idea is to divide all the current transactions into small sets where the social welfare is maximized locally in a smaller pricing game to approximate the global optimization objective. Considering that transactions coming with the same time tag will compete with each other more severely as they have the same remaining time to be included in the valid block, we divide all transactions in the current mempool into τ sets, where τ is the number of different time tags of transactions. By this means, the original social welfare maximization problem in (5)-(8) can be divided into τ sub-problems achieving local social welfare maximization for transactions with the same time tag, where the number of transactions in each sub-problem is defined as n_t depending on the time tag. Formally, we can express the t-th sub-problem as follows,

$$\max : \sum_{\boldsymbol{v}_t \in \mathcal{V}^{n_t}} F_t(\boldsymbol{v}_t) \sum_{i=1}^{n_t} U_{t,i}(\boldsymbol{v}_t) \tag{9}$$

$$\text{s.t.} : F_t(\boldsymbol{v}_t) \geq 0, \tag{10}$$

$$\sum_{\boldsymbol{v}_t \in \mathcal{V}^{n_t}} F_t(\boldsymbol{v}_t) = 1, \tag{11}$$

$$\sum_{\boldsymbol{v}_{t,-i} \in \mathcal{V}^{n_t-1}} F_t(v_{t,i}, \boldsymbol{v}_{t,-i}) \Big(U_{t,i}(v_{t,i}, \boldsymbol{v}_{t,-i}) - U_{t,i}(v'_{t,i}, \boldsymbol{v}_{t,-i}) \Big) \geq 0, \forall\ v_{t,i}, v'_{t,i} \in \mathcal{V}, \tag{12}$$

where $\boldsymbol{v}_t$ is the unit price vector of n_t transactions; $F_t(\boldsymbol{v}_t)$ is a correlated equilibrium of the small pricing game among them; $U_{t,i}$ and $v_{t,i}$ are respectively the utility and unit price of the i-th transaction while $\boldsymbol{v}_{t,-i}$ is the unit price vector except $v_{t,i}$.

Obviously, the above sub-problem has the same components as the original one, which will output the best correlated equilibrium $F_t(\boldsymbol{v}_t)$ for maximizing the local social welfare in each small set of transactions. Thus, after solving all sub-problems with the existing linear programming algorithms, we can derive an approximate solution through combining all the solutions of sub-problems, which means $F(\boldsymbol{v}) = (F_1(\boldsymbol{v}_1), F_2(\boldsymbol{v}_2), \cdots, F_\tau(\boldsymbol{v}_\tau))$. By this means, the computational cost of solving the social welfare maximization problem come into an acceptable level, which is demonstrated in the following theorem.

Theorem 7. *Assuming that the number of transactions with the same time tag has a maximum limitation as $\bar{n}$, which is much less than n since the time tags of transactions could be much diverse; and that the number of different time tags τ is polynomially increasing with n. Then our proposed approximate solution can solve the social welfare maximization problem in polynomial time.*

Proof. Using the existing algorithms, we can solve the above sub-problem with the computational cost of $O(V^{n_t})$ according to Theorem 6. As we assume that $n_t \leq \bar{n} \ll n$, we have $O(V^{n_t}) \leq O(V^{\bar{n}})$ where the latter item can be regarded as constant with respect to n. In addition, since τ increases with n in a polynomial manner, the overall computational cost of our proposed approximate solution is $\tau O(V^{\bar{n}})$ which is polynomial in n. □

6. Truthful Assessment Model of the Pricing Mechanism

To have a reliable assessment of the derived pricing mechanism, we design a truthful scheme in this section to collect feedback from users with respect to the received price suggestion.

6.1. Overview of the Truthful Assessment Model

With the pricing mechanism we proposed above, each user can get a price suggestion in each pricing game. For such a price, it is not guaranteed that it will be accepted by users. Therefore, it is necessary to design a model to obtain the users‘ opinions of accepting the price suggestion or not. However, it is nontrivial to collect users' honest feedback since there may exist malicious users submitting unreliable information which will heavily affect the accuracy of assessment and further impede the implementation of the proposed pricing mechanism.

To address the above issues, we utilize the private-prior peer prediction theory [27, 28, 29] to build the truthful assessment model for the pricing mechanism. On the one hand, we need to calculate the trustworthiness of user i, which is denoted as H_i ($i \in \{1, 2, \cdots, N\}$, with N denoting the number of users in total); on the other hand, we need to design a reasonable scheme to determine the quality of the proposed pricing mechanism that is defined as the acceptance rate of honest users, denoted as W. The main idea of the peer prediction model is to require user i to report both the prior and posterior opinions (i.e., probability) of a random peering user j accepting

the offered price. Based on the collected opinions, we can take advantage of the strictly proper scoring rule to calculate the trustworthiness of users for calculating the effective acceptance rate.

First, we define some parameters to describe the process of reporting opinions. We term C as the acceptance status of the pricing mechanism, where $C = a$ means that this price will be accepted while $C = r$ refers to a rejection. We define S_i as the judgement of user i towards the price, and $x_i \in \{0, 1\}$ stands for the opinion of user i.

To describe whether users' judgments are correct and reports are truthful, we introduce the following four definitions. Let P^1_{cj} and P^0_{cj} represent the probability of correct judgement of users. More specifically, P^1_{cj} is the conditional probability that user i makes the positive judgement when the price is accepted, i.e., $P(S_i = 1|C = a)$; and P^0_{cj} means that user i makes the negative judgement when the price is rejected, i.e., $P(S_i = 0|C = r)$. When user i honestly reports the opinion, i.e., $S_i = 1$ given $C = a$ or $S_i = 0$ for $C = r$, we define the probability of honestly reporting as $P^1_{hr} = P(x_i = 1|C = a)$ and $P^0_{hr} = P(x_i = 0|C = r)$.

Besides, we define $P(C = a)$ as the probability of the price being accepted from the perspective of historical records, which can be obtained by averaging the sum of the probabilities of the price being accepted in all previous rounds. The calculation of $P(C = a)$ is

$$P(C = a) = \frac{1}{K-1} \sum_{k=1}^{K-1} P_k(C = a), \tag{13}$$

where K is the total number of pricing rounds by now.

$P(S_i = 1)$ represents the judgment that the price being taken by user i, however, it is private information so we can't get it directly from user i. In our model, we use the historical choice records of user i toward the prices, denoted as z_i, to estimate the value of $P(S_i = 1)$. When user i takes the price, $z_i = 1$; otherwise, $z_i = 0$. Then, we can define $P(S_i = 1)$ as follows:

$$P(S_i = 1) = P(z_i = 1) = \frac{1}{K-1} \sum_{k=1}^{K-1} z_i = 1 \tag{14}$$

6.2. Private-prior Peer Prediction Model

We deploy the private-prior peer prediction model to get two different belief reports before and after experiencing the pricing game. Then we calculate any user's trustworthiness through a strictly proper scoring rule.

6.2.1. Prior Belief

Before the pricing game, user i is required to report the opinion that how likely user j will take the price based on his/her observation, which is denoted as δ_{ij} and is calculated as follows:

$$\begin{aligned}\delta_{ij} &= P(x_j = 1) \\ &= P(x_j = 1|C = a)P(C = a) + P(x_j = 1|C = r)P(C = r) \\ &= P^1_{cj}(i)P(C = a) + (1 - P^0_{cj}(i))P(C = r).\end{aligned} \tag{15}$$

6.2.2. Posterior Belief

After receiving the pricing suggestion derived from the correlated equilibrium based pricing mechanism, user i reports another opinion of user j based on his/her own experience, thus there are two possible situations, i.e., $S_i = 1$ and $S_i = 0$. First we denote the general form of the posterior report $\hat{\delta}_{ij}$:

$$\begin{aligned}\hat{\delta}_{ij} &= P(x_j = 1|S_i) \\ &= P(x_j = 1|C = a)P(C = a|S_i) + P(x_j = 1|C = r)P(C = r|S_i).\end{aligned} \tag{16}$$

Then we calculate $\hat{\delta}_{ij}$ in two different situations. When $S_i = 1$, the posterior belief is calculated by

$$\begin{aligned}\hat{\delta}_{ij}(1) &= P(x_j = 1|S_i = 1) \\ &= P(x_j = 1|C = a)P(C = a|S_i = 1)+ \\ &\quad P(x_j = 1|C = r)P(C = r|S_i = 1) \\ &= P^1_{hr}(i)\frac{P^1_{cj}(i)P(C = a)}{P(S_i = 1)}+ \\ &\quad (1 - P^0_{hr}(i))\frac{P^0_{cj}(i)P(C = r)}{P(S_i = 1)}.\end{aligned} \tag{17}$$

When $S_i = 0$, we have

$$\begin{aligned}\hat{\delta}_{ij}(0) &= P(x_j = 1|S_i = 0) \\ &= P(x_j = 1|C = a)P(C = a|S_i = 0)+ \\ &\quad P(x_j = 1|C = r)P(C = r|S_i = 0) \\ &= P^1_{hr}(i)\frac{(1 - P^1_{cj}(i))P(C = a)}{P(S_i = 0)}+ \\ &\quad (1 - P^0_{hr}(i))\frac{(1 - P^0_{cj}(i))P(C = r)}{P(S_i = 0)}.\end{aligned} \tag{18}$$

6.2.3. Opinion Inference

We can deduce the real opinion of user i based on the two submitted reports , i.e., δ_{ij} and $\hat{\delta}_{ij}$. If $\delta_{ij} < \hat{\delta}_{ij}$, we believe that user i will accept the price since his/her experience of the pricing game is better than the prior idea, i.e., $x_i = 1$. Otherwise, user i will reject the price due to his/her worse experience, i.e., $x_i = 0$.

6.3. Strictly Proper Scoring Rule

We adopt the the quadratic scoring rule to calculate the trustworthiness of each user, which is one of the strictly proper scoring rules utilized in [28, 29]. The general form of the quadratic scoring rule is:

$$\begin{aligned} L(y, x = 1) &= 2y - y^2, \\ L(y, x = 0) &= 1 - y^2, \end{aligned} \tag{19}$$

where y is the belief of a user, and x is the true opinion towards the price suggestion of the user.

Based on our analysis about the procedure of reporting, we can calculate H_i via δ_{ij} and $\hat{\delta}_{ij}$ as

$$H_i = \theta L(\delta_{ij}, x_j) + (1 - \theta) L(\hat{\delta}_{ij}, x_j) + \eta, \tag{20}$$

where θ is a parameter determining the weights of prior and posterior reports to the trustworthiness, with $\theta \in [0, 1]$; and η is a parameter to make trustworthiness stable and can be calculated as

$$\eta = -\frac{1}{N} \sum_{n=1}^{N} \left(\theta L(\delta_{ij}, x_j) + (1 - \theta) L\left(\hat{\delta}_{ij}, x_j\right) \right). \tag{21}$$

6.4. Assessment Result

After the calculation of H_i, we can know whether users are honest and their data can be further utilized to assess the proposed pricing mechanism. Specifically, we can distinguish honest users and filter out dishonest users based on their H_i: when $H_i \geq h$, user i can be considered as honest; otherwise, user i is malicious. Here h is the threshold value and can be calculated by averaging all H_i of users in each round.

We let $\mathcal{H}$ represent the set of honest users, and for each honest user, where each has an opinion towards the price as $x_i = 1$ or $x_i = 0$; and we let $\mathcal{X}$ as the set of honest users with opinions $x_i = 1$. Then we define ψ and ξ

as the sizes of $\mathcal{H}$ and $\mathcal{X}$ respectively. The calculation of the acceptance rate of the pricing mechanism is

$$W = \frac{\xi}{\psi}. \tag{22}$$

Theorem 8. *The above truthful assessment model for the pricing mechanism is incentive compatible.*

Proof. According to (20), we can calculate the expectation of H_i as

$$\begin{aligned} E(H_i) = & E\left(\theta L\left(\delta_{ij}, x_j\right)\right) + E\left((1-\theta) L\left(\hat{\delta}_{ij}, x_j\right)\right) + E(\eta)) \\ = & \theta\left(1-\frac{1}{N}\right) E(L(\delta_{ij}, x_j)) + \\ & (1-\theta)\left(1-\frac{1}{N}\right) E\left(L\left(\hat{\delta_{ij}}, x_j\right)\right) - \\ & \frac{1}{N} \sum_{n=1, n \neq i}^{N}\left(\theta L\left(\delta_{ij}, x_j\right) + (1-\theta) L\left(\hat{\delta}_{ij}, x_j\right)\right). \end{aligned} \tag{23}$$

We first set $p_1 = P(x_j = 1)$ and $p_2 = P(x_j = 1|S_i)$, and by applying the binary quadratic scoring rule to (23), we can have

$$\begin{aligned} E\left(H_i\right) = & \theta\left(1-\frac{1}{N}\right)\left(p_1\left(2\delta_{ij} - \delta_{ij}^2\right) + \left(1-p_1\right)\left(1-\delta_{ij}^2\right)\right) + \\ & (1-\theta)\left(1-\frac{1}{N}\right)\left(p_2\left(\hat{\delta}_{ij} - \hat{\delta_{ij}}^2\right) + \left(1-p_2\right)\left(1-\hat{\delta_{ij}}^2\right)\right) - \\ & \frac{1}{N} \sum_{n=1, n \neq i}^{N}\left(\theta L\left(\delta_{ij}, x_j\right) + (1-\theta) L\left(\hat{\delta}_{ij}, x_j\right)\right). \end{aligned}$$

Then we can calculate the partial derivatives with respect to δ_{ij} and $\hat{\delta}_{ij}$

$$\frac{\partial E(H_i)}{\partial \delta_{ij}} = \theta\left(1-\frac{1}{N}\right)(2p_1 - 2\delta_{ij}), \tag{24}$$

$$\frac{\partial E(H_i)}{\partial \hat{\delta}_{ij}} = (1-\theta)\left(1-\frac{1}{N}\right)(2p_2 - 2\hat{\delta}_{ij}). \tag{25}$$

To get the optimal value, we let $\frac{\partial E(H_i)}{\partial \delta_{ij}} = 0$ and $\frac{\partial E(H_i)}{\partial \hat{\delta}_{ij}} = 0$, and we can have $\delta_{ij} = p_1$ and $\hat{\delta}_{ij} = p_2$.

Then we take the second partial derivatives of $E(H_i)$ regarding δ_{ij} and $\hat{\delta}_{ij}$:

$$\frac{\partial^2 E\left(H_i\right)}{\partial \delta_{ij}^2} = -2\theta\left(1 - \frac{1}{N}\right) < 0, \tag{26}$$

$$\frac{\partial^2 E\left(H_i\right)}{\partial \hat{\delta}_{ij}^2} = -2\left(1-\theta\right)\left(1 - \frac{1}{N}\right) < 0. \tag{27}$$

□

Based on the above analysis, when $\delta'_{ij} = p_1$ and $\hat{\delta}'_{ij} = p_2$ are satisfied, $E(H_i)$ can have the maximum value. It means that only if user i submits his/her beliefs of the pricing mechanism honestly can he/she get the maximum value of trustworthiness. Here we consider that every user expects the submitted opinion to be valued, leading them the motivation of getting get higher trustworthiness. This proves that our proposed truthful assessment model for the pricing mechanism is incentive-compatible to encourage users to behave truthfully.

7. Experimental Evaluation

In this section, we evaluate our proposed pricing mechanism and its truthful assessment model through simulation experiments.

7.1. Performance Evaluation of the Pricing Mechanism

We implement our experiments using MATLAB R2019a in Windows 10 running on Intel i7 processor with 16 GB RAM and 512 GB SSD. For parameters related to transactions, we randomly choose $s_i \in [100, 300]$ KB and $T_i \in [10, 30]$ min. Other parameters are set as $n = 500$, $\alpha_i = 3000$, $\beta_i = 0.01$, $a_i = \frac{s_i}{100}$, $\tau = 200$, and $n_t \in \{1, 2, 3\}$ unless otherwise specified. Note that all our experiments are repeated 20 times to have the average results.

In order to demonstrate that our proposed approximate solution in Section 5 can bring similar results for the optimization problem in a more efficient manner, we compare the experimental results returned by traditional solution with the interior-point method and our proposed one. In detail, we set the number of transactions $n \in \{5, 6, \cdots, 16\}$ and run both the traditional

and approximate algorithms to obtain the computational efficiency and the results of optimization problem, i.e., maximized social welfare and the unit price vector with the highest probability as the correlated equilibrium.

As shown in Fig. 1, the computational cost of the traditional algorithm solving the linear programming problem increases exponentially with the number of transactions n, while that of our proposed approximate solution is linearly changing with n, which is consistent with the analysis results presented in Theorems 6 and 7. Besides, we present the optimization results in Tables 1 and 2, where only the results of $n = 5$ to 8 are reported to avoid redundancy and the results in other cases have the similar trend. As can be seen from Table 1, even though our proposed approximate solution cannot obtain the exact same maximized social welfare compared to the traditional one with the accurate constraints, we can have approximate values fluctuating around the accurate ones with lower computational cost in the long run. Since the optimal correlated equilibrium for social welfare maximization is a probability distribution over all possible combinations of unit price vector, we regard that the combination with the highest probability is the most appropriate one, which is presented in Table 2. One can see that all cases we examined come with the same results.

From the above analysis on our numerical comparison results, we can have the conclusion that our proposed approximate solution can solve the social welfare maximization problem efficiently and obtain similar results to the traditional one.

Then, we evaluate the performance of our proposed approximate solution with respect to the maximized social welfare when $n = 500$ under different parameter settings of a_i, α_i, β_i, and τ.

To begin with, we examine the impact of transaction inclusion probability defined in Definition 1 on the maximized social welfare, especially the impact of a_i. As mentioned in Section 3, here we assume that a_i is a function of s_i, i.e., the size of tx_i, which is set as $a_i = \psi(s_i) = \frac{si}{X}$ with $X = s_i/a_i$ being a parameter changing from 100 to 1000 with an interval of 100. As shown in Fig. 2, we can obtain that the maximized social welfare changes with s_i/a_i

Table 1: Maximized social welfare.

Solution	n=5	n=6	n=7	n=8
Traditional	709.25	1293.53	876.43	712.17
Approximate	713.57	1284.19	916.59	692.05

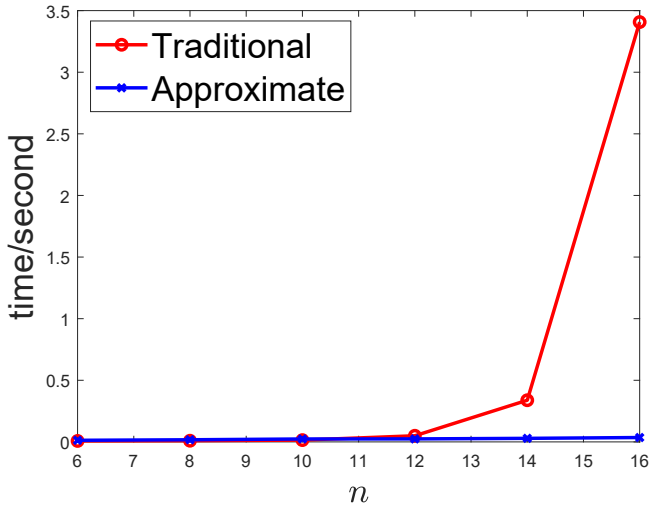

Figure 1: Computational cost comparison changing with n.

in a relatively complicated manner which seems like a combination of the exponential and stable trend. This is reasonable since (1) indicates that the relationship between probability and a_i is exponent divided by the sum of exponents, which could present this sort of complicated curve and further affect the social welfare through the individual utility as defined in (2). It is worth noting that the impact of b has also been checked but brings subtle influence on the maximized social welfare, which is because this parameter is block-related with an equivalent impact on all users, thus finally resulting in an offsetting of its impact.

Table 2: Unit price vector with the highest probability.

Solution	**Traditional**	**Approximate**
n=5	(10, 5, 10, 10, 10)	(10, 5, 10, 10, 10)
n=6	(10, 5, 5, 10, 10, 5)	(10, 5, 5, 10, 10, 5)
n=7	(10, 10, 5, 10, 5, 5, 10)	(10, 10, 5, 10, 5, 5, 10)
n=8	(10, 5, 5, 5, 10, 10, 5, 10)	(10, 5, 5, 5, 10, 10, 5, 10)

Next, we explore the impact of individual utility on the maximized social welfare. In particular, we inspect the influences of α_i and $beta_i$ and report the corresponding experimental results in Figs. 3 and 4, respectively.

For α_i, we change it from 3000 to 8000 with an interval of 500. As can

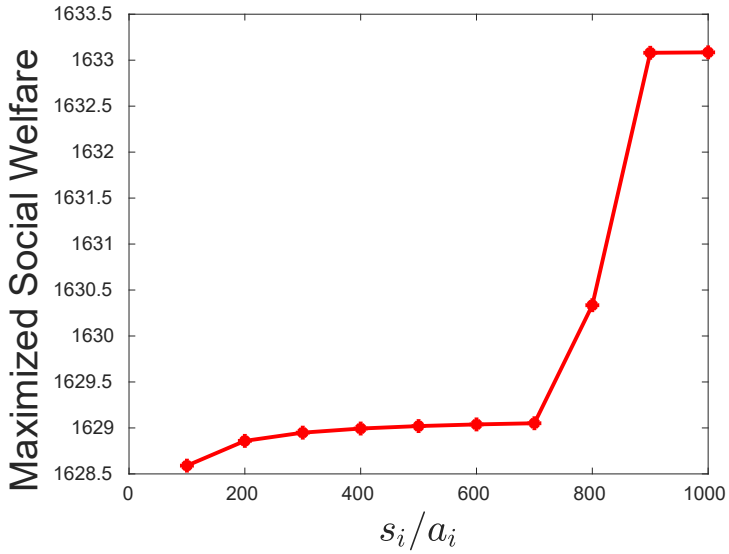

Figure 2: Maximized social welfare changing with s_i/a_i.

be seen from Fig. 3, the maximized social welfare increases linearly with α_i. This is because individual utility in Definition 2 is linearly related to α_i when other parameters are fixed. Thus, the maximized social welfare denoting the total utilities of all users has a linear relationship with α_i.

And for β_i, we set it as 0.01 to 0.1 with an interval of 0.01. According to Fig. 4, one can see that the maximized social welfare increases with β_i in a non-linear manner. In fact, it is a part of the "S"-shape curve. As shown in definition 2, we define that the individual utility is linear to the profit function $\phi(\cdot)$ which is a sigmoid function with respect to β_i as shown in (3). So the maximized social welfare also presents this trend with β_i.

From the above two figures, we can get the conclusion that the maximized social welfare will be affected by α_i and β_i in a similar way that the individual utility U_i gets influenced.

Finally, we study whether changing the number of transactions in subsets will affect the optimization result or not. To be specific, we achieve this by adjusting the number of different T_i of transactions, i.e., τ. As presented in Fig. 5, the maximized social welfare has no obvious changing trend with respect to τ, which helps demonstrate the stability and robustness of our proposed approximate solution.

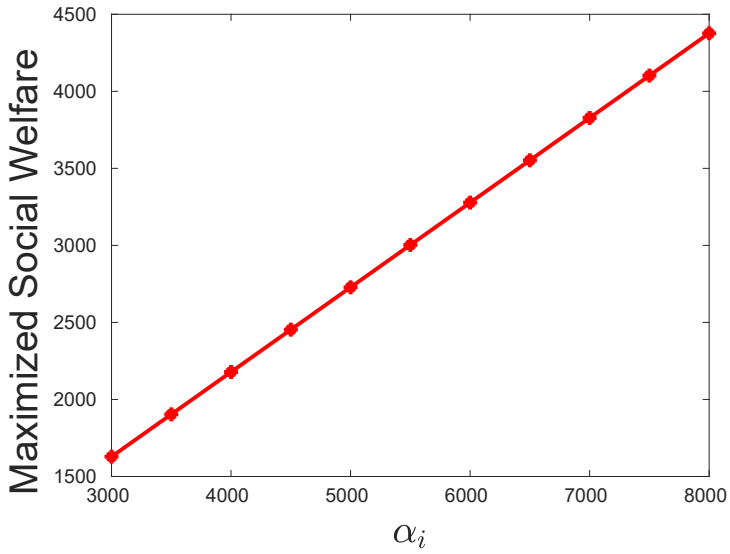

Figure 3: Maximized social welfare changing with α_i.

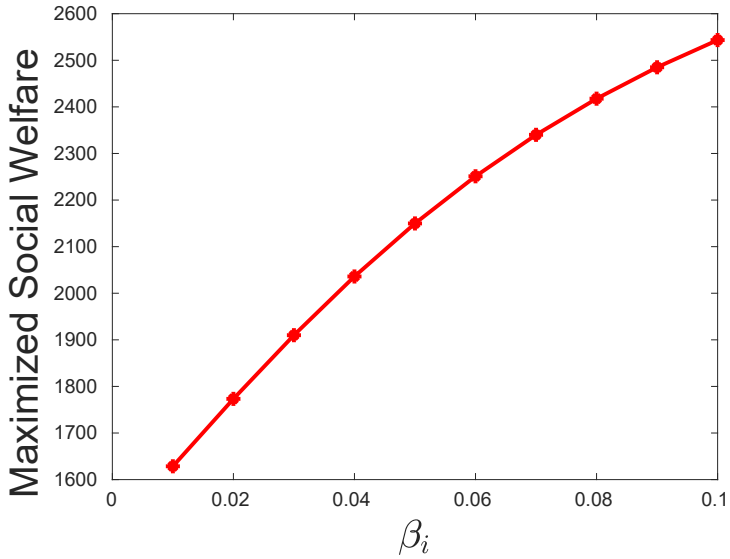

Figure 4: Maximized social welfare changing with β_i.

7.2. Evaluation of the Truthful Assessment Model

In this subsection, we design simulation experiments to evaluate our proposed truthful assessment model and verify its incentive capability. We first

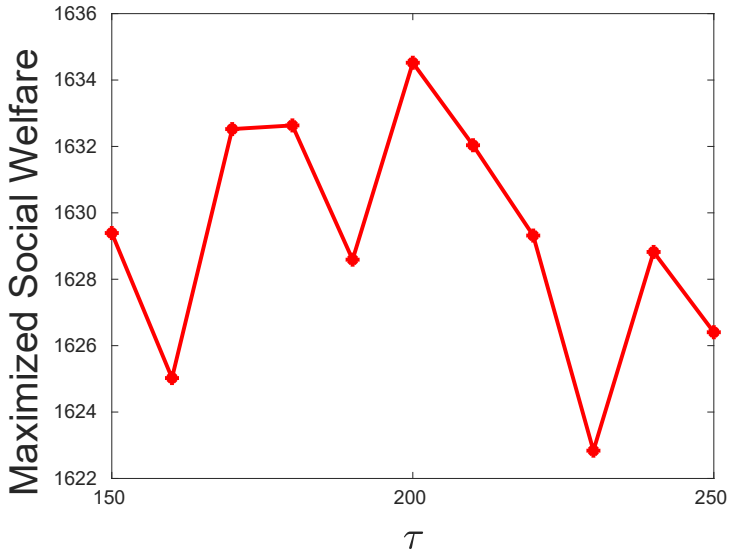

Figure 5: Maximized social welfare changing with τ.

examine the impacts of $P(S_i = 1)$ and θ on H_i in the similar experiment setting, and then we calculate the accumulative trustworthiness of different types of users.

As for the calculation of trustworthiness, the parameter θ indicates the importance of prior belief and that of posterior belief, so our first experiment is to find how will H_i change with θ. We consider two different users, i.e., honest user i and malicious user j . For user i, we let $P^1_{cj}(i) = P^0_{cj}(i) = 0.8$, $P^1_{hr}(i) = P^0_{hr}(i) = 1$, $P(C = a) = 0.8$ and $P(S_i = 1) = 0.6$. As for user j, we set $P^1_{hr}(j) = P^0_{hr}(j) = 0.1$, and the values of other parameters are set the same as user i. The above setting means that both user i and user j can make the right judgement after the price suggestion is received, while user i will report truthful opinions but user j will not. According to the results shown in Fig. 6, we can see that the trustworthiness of user i changes slightly with θ, while the trustworthiness of user j increases when θ is larger. The reason is that the posterior belief is submitted after the pricing game, so it contains more information than the information prior belief. For honest user i, he/she will report all the beliefs honestly, so both prior belief and posterior belief are important and reliable to the calculation of trustworthiness. But for user j, since the posterior belief is generated based on the experience of the pricing mechanism, user j may submit the wrong posterior belief intentionally to

obtain a higher trustworthiness.

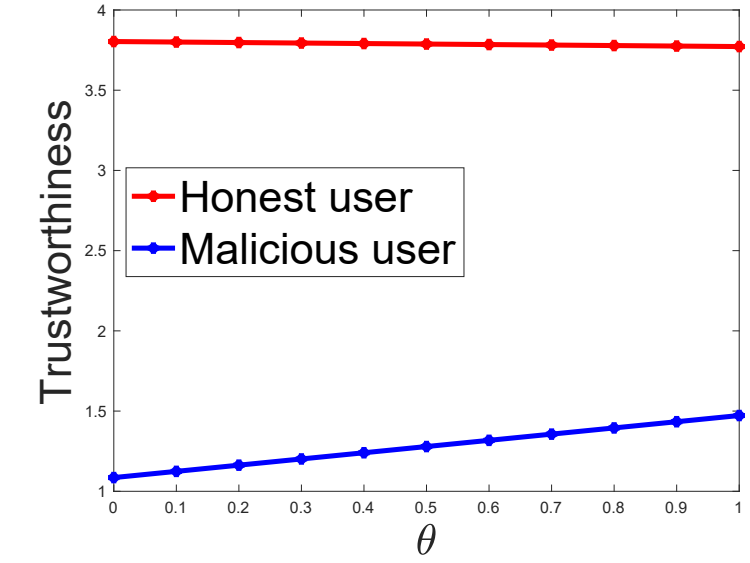

Figure 6: Trustworthiness changing with θ.

With a similar experimental setting to the above, we conduct experiments to analyze the impact of a user's judgment of the price on the trustworthiness. According to the results in Fig. 7, we find that the trustworthiness of honest user i does not fluctuate much as $P(S_i = 1)$ changes, while the trustworthiness of malicious user j is negative and increases when $P(S_i = 1) < 0.3$ and then keeps stable. The results indicate that our model can adapt to the situation of numerous users with different judgments. Besides, our model is based on the assumption that $P(S_i = 1) \in (0, 1]$ is randomly distributed, and our experimental results effectively support the validity of this assumption.

Finally, we calculate the accumulative trustworthiness of multiple users. We assume that there exist 80% honest users and 20% malicious users. As for the honest users, they will submit their reports truthfully, hence we set $P^1_{hr}(i), P^0_{hr}(i) \sim U[0.6, 0.8]$ for them, with $U[u_1, u_2]$ representing the uniform distribution between u_1 and u_2. While for malicious users, they usually won't report their real beliefs, so we set $P^1_{hr}(i), P^0_{hr}(i) \sim U[0.1, 0.2]$ for them. For both honest users and malicious users, we assume that they can make the right judgements with relatively high probability, so we set $P^1_{cj}(i), P^0_{cj}(i) \sim U[0.6, 0.8]$. As for $P(S_i = 1)$, we assume it is randomly selected from $[0, 1]$ for all users. We also set $\theta = 0.5$ and $P(C = a) = 0.4$. Then, we calculate the

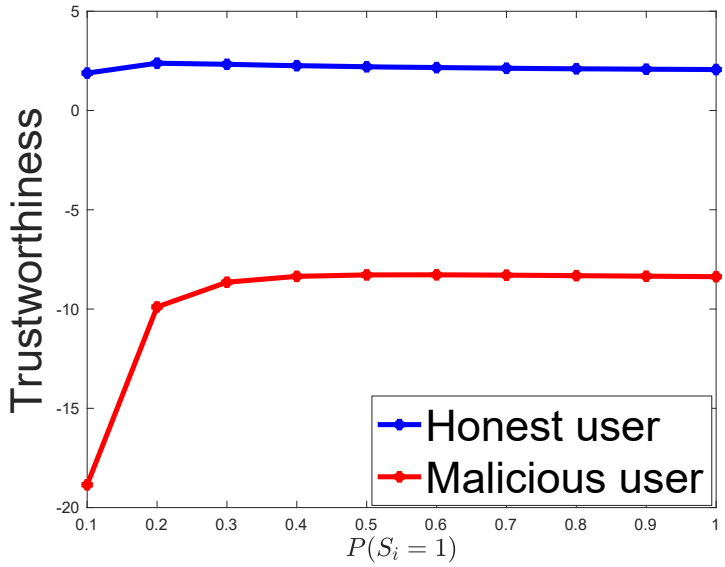

Figure 7: Trustworthiness changing with $P(S_i = 1)$.

accumulative trustworthiness of 10 users in 100 rounds, and get the average of accumulative trustworthiness in each round for both malicious and honest users. The results reported in Fig. 8, which shows that the honest users' accumulative trustworthiness is positive and goes up, while the accumulative trustworthiness of malicious users is negative and goes down, proving that our proposed assessment model is incentive capable and can encourage the users to behave honestly according to the rules.

8. Conclusion

In this paper, we study the transaction pricing issue in blockchain with the help of game theory from the perspective of users and propose a correlated equilibrium based pricing mechanism for transactions. To be specific, we first model the transaction inclusion competition among users as a pricing game, and then utilize the concept of correlated equilibrium to maximize the individual utility of each user through unifying it with achieving the global optimum. To overcome the weakness of exponential complexity in the original solution, we propose an approximate algorithm to yield polynomial time cost. In addition, we design a truthful assessment model for our proposed price mechanism to collect the feedback of users regarding the price sug-

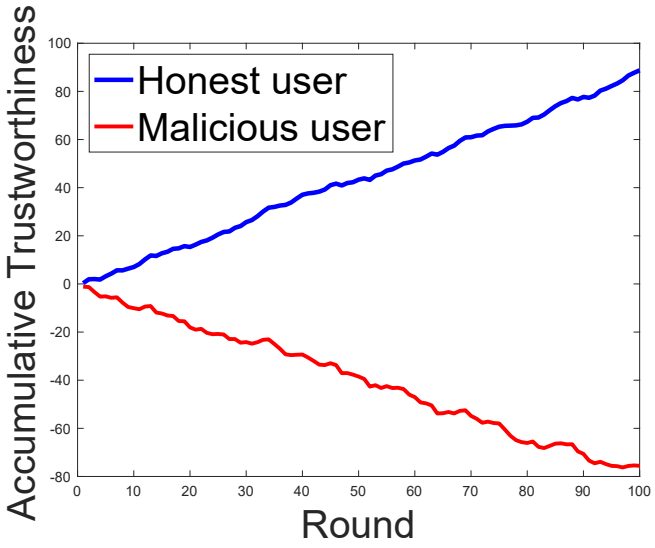

Figure 8: Accumulative trustworthiness changing with rounds.

gestion. Finally, we conduct extensive simulations to evaluate our proposed pricing mechanism and the corresponding truthful assessment model.

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