

A Bi-Level Framework for Expansion Planning in Active Power Distribution Networks

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Abstract—This paper presents a new framework for multistage expansion planning in active power distribution networks, in which the distribution system operator (DSO) considers active network management by clearing the local energy market at the distribution level. The proposed model is formulated as a bi-level optimization problem, where the upper level minimizes the net present value of the total costs imposed to DSO associated with the investment and maintenance of the network assets as well as the network operation, while the lower level on clearing the local energy market captures the participation of distributed energy resource (DER) owners and demand aggregators to maximize the social welfare. The expansion plans consider the investments in DER owners' assets as well as variety of network assets in which the profitability of DER owners' investment is guaranteed. The Karush-Kuhn-Tucker optimality conditions and the strong duality theory are employed through which the model is converted to a mixed integer linear programming optimization problem. The implementation of the suggested model on the 24-bus and 86-bus distribution test systems validates its performance and efficacy in making cost-effective planning decisions.

Index Terms—Active power distribution network, active network management, expansion planning, local energy market, distribution system operator (DSO).

NOMENCLATURE

Indices (Sets)

$b, d (B)$	Distribution network buses.
$i (I)$	Alternatives for investment, including I^l , I^k and I^u describing alternatives of feeders, transformers, and DGs, respectively.

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$k (K)$

$l (L)$

$o, s (O)$

$t, \tau (T)$

$u (U)$

$\omega (\Omega)$

BL, BS, BST

BT

BU

ϑ^l, γ_b^l

Parameters

$C_{k,i}^{IK}, C_{l,i}^{IL}, C_b^{IS}$

$C_{u,i}^{IST}, C_{u,i}^{IU}$

$C_{k,i}^{MK}, C_{l,i}^{ML}$

$C_{u,i}^{MST}, C_{u,i}^{MU}$

$\bar{D}_{b,t,o,\omega}^{up}, \bar{D}_{b,t,o,\omega}^{dn}$

$G_{b,d,i}^l, B_{b,d,i}^l$

IRR, RoI

$\overline{IB}_t^{DSO}, \overline{IB}_t^{DER}$

Types of transformer which include K^{NTR} and K^{ETR} denoting the new and existing transformers, respectively.

Feeder types including L^R, L^F, L^{NA} , and L^{NR} respectively associated with replaceable feeders, fixed feeders, newly added feeders, and newly replaced feeders.

Operating scenarios.

Planning problem time stages.

Types of DGs including U^Ψ and U^C related to PV and conventional generators, respectively.

Uncertainty scenarios.

Sets of load points, substations, and candidate buses for ESSs installation.

Set of transfer buses.

Set of candidate buses for installing DGs including BU^C and BU^Ψ for conventional and PV generators, respectively.

Sets of lines with type l and buses connected to bus b by feeder l , respectively.

Investment costs of transformers, feeders, and substations, respectively (\$/km, \$).

Investment costs of ESSs and DGs, respectively (\$/MW).

Maintenance costs of transformers and feeders, respectively (\$/yr).

ESSs and DGs maintenance costs (\$/yr).

Maximum bidding powers of increase and decrease in the nodal base load (MW).

Real and imaginary segments of the network admittance matrix (p.u.).

Internal rate of return of DER owners and annual rate of interest (%).

Investment budgets of DSO and DER owners (\$).

LF_o	Load factor of operating scenario o .	$p_{b,i,t,o,\omega}^k$	Transformers injected power to network (MW).
$P_{b,t,\omega}^D$	The nodal peak load (MW).	$p_{b,t,o,\omega}^d$	Net demand of load points in local energy market (MW).
$\bar{P}_{b,t,o}^{ST,ch}, \underline{P}_{b,t,o}^{ST,ch}$	Maximum and minimum bidding powers of ESSs to local energy market associated with their charging and discharging (MW).	$p_{b,d,i,t,o,\omega}^l$	Power flow through feeder of type l (MW).
$\bar{P}_{b,t,o}^{ST,dch}, \underline{P}_{b,t,o}^{ST,dch}$	Bidding power of DGs to local energy market (MW).	$p_{b,t,o,\omega}^{ST,ch}, p_{b,t,o,\omega}^{ST,dch}$	Accepted charging and discharging power of ESSs in local energy market (MW).
$P_{b,i,t,o}^u$	Maximum power flow of transformers and feeders (MW).	$p_{b,i,t,o,\omega}^u$	DGs' accepted power in local energy market (MW).
$\bar{P}_i^k, \bar{P}_i^l$	System power factor and penetration limit of DERs.	$v_{b,t,o,\omega}, \delta_{b,t,o,\omega}$	Voltage magnitude and voltage angle of system nodes (p.u., rad).
PF, PL	Base power and base voltage (MVA, kV).	$x_{b,i,t}^k, x_{b,d,i,t}^l$	Binary variables; 1 if transformers and feeders are installed, respectively.
S^B, V^B	DGs' maximum capacity and PV generators' power availability (MVA, MW).	$x_{b,t}^S, x_{b,t}^{ST}, x_{b,i,t}^u$	Binary variables; 1 if substations, ESSs, and DGs are installed, respectively.
$\bar{S}_i^u, P_{b,t,o,\omega}^\Psi$	ESSs' minimum and maximum capacity (MVA).	$y_{b,i,t,o}^k, y_{b,d,i,t,o}^l$	Binary variables; 1 if transformers and feeders are utilized, respectively.
$\underline{S}^{ST}, \bar{S}^{ST}$	Buses' minimum and maximum voltage magnitude (p.u.).	$y_{b,t,o}^{ST}, y_{b,i,t,o}^u$	Binary variables; 1 if ESSs and DGs are utilized, respectively.
$\underline{V}_b, \bar{V}_b$	Duration of operating scenario o (Hour).	$y_{t,o,\omega}^D$	Binary variable; 1 if load control capability is utilized.
Δ_o	Price elasticity (own elasticity if $s = o$, cross elasticity otherwise).	$a_{b,t,o}^{ST,ch}, a_{b,t,o}^{ST,dch}$	Binary variables indicating the charging/ discharging conditions of ESSs.
$\varepsilon_{o,s}$	Efficiency of ESSs' charging/discharging.	$\alpha_{b,d,t,o}^l, Tn_b$	Binary variables associated with the connection of feeders and transfer nodes in the spanning tree.
$\eta^{ST,ch}, \eta^{ST,dch}$	Wholesale market electricity price (\$/MWh).	$\beta_{b,d,t,o}^l$	Binary variable; 1 if b is parent node of d .
$\lambda_{t,o,\omega}^T$	Average market electricity price over the operating scenarios (\$/MWh).	$\lambda_{b,t,o,\omega}^D$	DLMP of distribution network buses (\$/MWh).
$\bar{\lambda}_{t,\omega}^T$	Deviation of market electricity price from its average.	$\partial^{U/ST}, \kappa, \lambda^D$	Lagrange coefficients of the lower level problem.
$\tilde{\lambda}_{t,\omega}^T$	Length of feeders (km).	$\rho^l, v, \gamma, \phi, \xi$	
$\Lambda_{b,d}$	Bidding price of DGs, ESSs and loads to local energy market (\$/MWh).	$\mu^{u/l/ch/dch}$	
$\pi_{b,i,t,o}^u, \pi_{b,t,o}^{ST}, \pi_{b,t,o}^D$	Weight of uncertainty scenario ω .	$\mu^{d-up/d-dn}$	
Π_ω			
Functions and Variables			
$c_{t,o}^{E_DSO}$	Energy costs (\$).		
$c_t^{I_DSO}, c_t^{M_DSO}$	Investment and maintenance costs related to the DSO's assets (\$).		
$c_t^{I_DER}, c_t^{M_DER}$	Investment and maintenance costs related to the assets of DER owners (\$).		
$c_{t,o}^{Op_DER}, Inc_{t,o}^{DER}$	Operation costs and income of DER owners (\$).		
$d_{b,t,o,\omega}^{up}, d_{b,t,o,\omega}^{dn}$	Accepted increase and decrease in load points' base demand through load control in local energy market (MW).		
NPV^{DSO}	NPV of total costs associated with DSO (\$).		
NPV^{DER}	NPV of total costs associated with DER owners (\$).		

I. INTRODUCTION

EXPANSION planning of the power distribution network assets jointly with distributed energy resources (DERs) is critical and has been addressed extensively in [1]–[4]. The proliferation of DERs has introduced new challenges to distribution network expansion planning (DNEP) practices due to the introduction of intensified distributed uncertainties across the network. CIGRE introduced the active distribution network (ADN) concept [5] which is able to relieve the DERs' uncertainty and improve the network performance in long-term planning and short-term operation horizons.

ADN offers a new paradigm in controlling DERs—i.e., distributed generators (DGs), responsive loads and energy storage systems (ESSs)—and managing the electricity flows in the network using a flexible topology, called active network

management [5]. Additionally, ESSs offer new opportunities in ADN and can be utilized in active network management in order to improve the decision making on the network expansion planning [6], [7]. In order to address the challenges arisen by DERs in the DNEP problem, authors in [8] and [9] considered resource active management in ADN planning. In [8], a dynamic planning approach has been proposed for investment decision making on DGs. In [9], a multi-configuration optimal power flow (OPF) technique has been proposed in which the role of active network management on the DGs investment planning is evaluated. In these references, the investment plan of the network assets is disregarded. References [10] and [11] have jointly determined the expansion plans for the DGs and network assets; however, the network operation is considered independently of the planning problem. In other words, a two-step method is developed in which DNEP is addressed in the former step and the network management is addressed in the latter. In order to effectively achieve the expansion plan, the network management should be simultaneously integrated in the planning problem [9]. Authors in [12] have addressed optimal schedules of ESSs in ADN planning through a tri-level optimization problem in which distribution system operator (DSO) directly manages ESSs' operation. An ADN planning framework based on multi-load scenarios was proposed in [13]–[15], where DSO jointly plans for DERs and the network assets, while active network management is judiciously integrated in the DNEP. In reference [16], the effect of stakeholders in DNEP problem is addressed using a tri-level model which is solved based on benders decomposition. In this reference, the distribution company and the DG owners expand their assets. Furthermore, the resources operation management is handled by independent DSO through OPF aiming at minimizing the system operation costs.

Based on the reviewed literature, some references addressed the active network management practice independently of the DNEP problem. On the other hand, DSO directly manages the network and DERs operation through the OPF-based mechanisms. However, it is required that the active network management is handled through a market-based mechanism that accurately captures the role of different actors in the operation of the distribution network. In [17], the impact of non-network solutions (i.e., existing DERs in the network) on the DSO's network expansion plans is considered by modeling the bidirectional interactions between the involved stakeholders through a deregulated retail market. This reference, however, concentrated majorly on the control of DERs through retail market to prevent unnecessary investments on the network assets (and not the joint multistage expansion of the network assets and DERs), where the model is solved using a tri-layer iterative method. Table I summarizes different aspects of the reviewed literature on DNEP compared to the proposed model in this paper.

In order to achieve an efficient joint expansion plan of the network assets and DERs, it is required that besides the technical aspects of the DERs, the DER owners' and demand aggregators' objectives, constraints, and monetary transactions are captured in the active network management practices. To fill in the knowledge gap, an efficient framework is proposed in this paper in which active network management is addressed in a multistage

TABLE I
SUMMARY OF THE REVIEWED LITERATURE: STATE-OF-THE-ART VS. THE PROPOSED MODEL

Model	Network assets planning	DERs planning		Active network management	Local energy market	Solution technique
		DGs	ESSs			
[3],[4]	✓	✓	✗	✗	✗	MILP-MP
[6],[13]	✓	✓	✗	✓	✗	MIP-MP*
[8],[9]	✗	✓	✗	✓	✗	Heuristic
[10],[11]	✓	✓	✗	Independent of planning	✗	Heuristic
[12]	✓	✓	✓	✓	✗	Heuristic
[14]	✓	✓	✓	✓	✗	MILP-MP
[15]	✓	✗	✓	✓	✗	MIP-MP
[16]	✓	✓	✗	✓	✗	Benders decomposition
[17]	✓	✗	✗	✓	Deregulated retail market	Iterative tri-layer decomposition
Proposed model	✓	✓	✓	✓	✓	Converting Bi-level to MILP-MP

*MIP-MP: Mixed integer programming-mathematical programming.

DNEP formulation, where the local energy market in the ADN is captured. The local energy market is cleared through operating and uncertainty scenarios which are characterized according to the generation portfolios and the demand profiles.

In this framework, investments in DER owners assets including photovoltaic (PV) generators, conventional gas turbines, and ESSs are considered jointly with distribution network assets (i.e., substations, transformers, and feeders) in the planning level where the minimum acceptable rate of return (MARoR) of DER owners is considered and the attained expansion plan guarantees the profitability of DER owners' investment. Moreover, for each scenario of operation, the optimal utilization of DGs, ESSs, and responsive loads across the ADN as well as the optimal network configuration are decided through active network management.

In the proposed model, the distribution network agents including DSO, DER owners, and demand aggregators participate in the local energy market. The local energy market is managed by DSO in the distribution level in order to efficiently manage the operation of DERs and achieve the optimal bidding to participate in the day-ahead (DA) wholesale electricity market.

In order to address the active management of resources in DNEP through clearing the local energy market (i.e., two problems in different time scales), this paper proposes a bi-level optimization model. The upper level problem makes the investment decisions for each time stage of the planning problem and the optimal network configuration is decided for each operating scenario. In the lower level problem, DSO clears the local energy market and the optimal DERs' operation is achieved in each operating scenario. The proposed model effectively addresses the correlated uncertainty in the intermittent resources using a set of scenarios. With the Karush-Kuhn-Tucker (KKT) optimality conditions and the strong duality theory applied, the suggested bi-level optimization model is converted to a mixed integer linear programming (MILP) formulation that can be effectively solved by the off-the-shelf commercial solvers. The main contributions of this paper are summarized as follows,

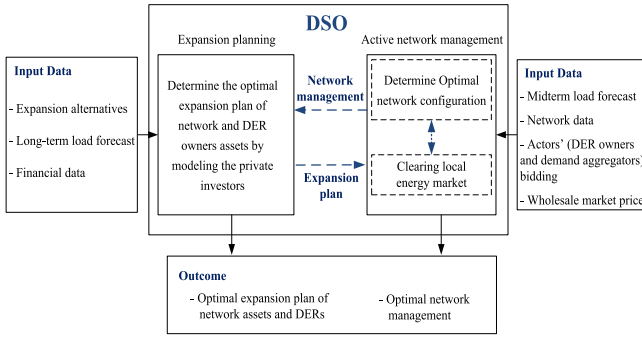


Fig. 1. General architecture of the proposed multi-stage DNEP through the network active management.

- 1) An active network management scheme for the joint multistage expansion planning of the network and DER owners' assets is suggested that captures the optimal network configuration and clears the local energy market at the distribution level through operating scenarios.
- 2) The participation of different actors including DSO, DER owners, and demand aggregators in the local energy market is modeled in which actors' objective and constraints and monetary transactions are taken into account.
- 3) A mathematical programming-based model is approached to efficiently solve the proposed problem using off-the-shelf commercial solvers. In this regard, the model is formulated as a bi-level optimization problem which is converted to an MILP model through a linearization technique, the KKT optimality conditions, and the strong duality theory.

II. METHODOLOGY

A. General Structure

The DSO takes investment decisions for the network assets while the expansion plan of DER owners' assets (including DGs and ESSs) is attained through a multistage expansion planning problem by respecting private asset owners and taking the role of active network management into account. The general architecture of the introduced model is illustrated in Fig. 1. The network assets include feeders, sub-transmission substations, and transformers. On the other hand, ESSs, PV generation plants, and conventional gas turbines are the network flexible resources invested by DER owners that the DSO should effectively consider. In order to capture the private asset owners in the joint expansion planning problem, the model considers the MARoR of DER owners and it is guaranteed that the investment plan of DERs is profitable from the DER owners' perspective [4]. In other words, the expansion plans attained for DERs are feasible because the technical constraints of the network are considered by the DSO and the project is profitable from the DER owners' standpoint. On the other hand, in order to attain an efficient expansion plan by utilizing DERs as flexible resources, DSO achieves the optimal operation of the network and DERs through an active management strategy.

Several scenarios associated with the network operation are characterized according to the generation portfolios and demand profiles at each planning time stage. Furthermore, by modeling the correlated uncertainties across the system (including wholesale market price, PV generation, and load demand), several uncertainty scenarios are defined. For each scenario of operation, the optimal network configuration and optimal utilization of DERs across the network are decided through active network management. In this paper, by modeling the actors' behavior in a restructured market at the distribution level, the optimal operation of DERs is obtained by clearing the local energy market. Actors of the local energy market are DSO, DER owners, and the demand aggregators who participate in the local energy market to determine the schedule of their assets and resources. DER owners and demand aggregators submit their power bids and the associated price to the local energy market, with which the DSO clears the market aiming at maximizing the social welfare by considering distribution locational marginal price (DLMP) signal. In the proposed model, the expansion of DERs is modeled by considering the profitability of private investors (i.e., the DER owners). In this regard, the DERs' optimal technologies and locations are attained in addition to the expansion plan of the network assets. It should be noted that the obtained expansion plan of DERs is feasible from both technical and economical perspectives. The reason lies in the fact that the MARoR of the private investors is achieved and the technical constraints associated with the network operation are modeled by DSO in the problem. On the other hand, in the active network management scheme, DSO does not directly control DERs and DER owners and demand aggregators realize their optimal dispatch by participating in the local energy market cleared by the DSO.

To model this problem, a bi-level optimization approach is used in this paper. The model can be considered as a Stackelberg game in which the upper level deals with the planning problem as a leader, and at the lower level, the local energy market is cleared as the follower. The conceptual architecture of the proposed bi-level problem is depicted in Fig. 2. In the upper level, the investment decisions on the network and DER owners' assets are determined at each planning time stage. Furthermore, the optimal radial configuration of the network is attained in this level for each operating scenario. The objective in this level is to minimize the net present value (NPV) of the total costs imposed to DSO associated with the expansion planning, maintenance and operation of the network. In the upper level problem, the profitability of the private agents' investments on expanding the DERs in the network is preserved, where their investment, maintenance and operation costs as well as their income from selling power to DSO are modeled. In addition, this level is subject to several logic constraints of the planning problem and some associated with the operation of the network assets in the active management scheme. Note that the network radiality is preserved using a graph-based technique where the DSO and DER owners' investment budgets are accounted for.

In the lower level, DSO handles active management of the resources by clearing the local energy market in order to determine the optimal operation of DGs and ESSs as well as the optimal utilization of responsive loads and maximize the social

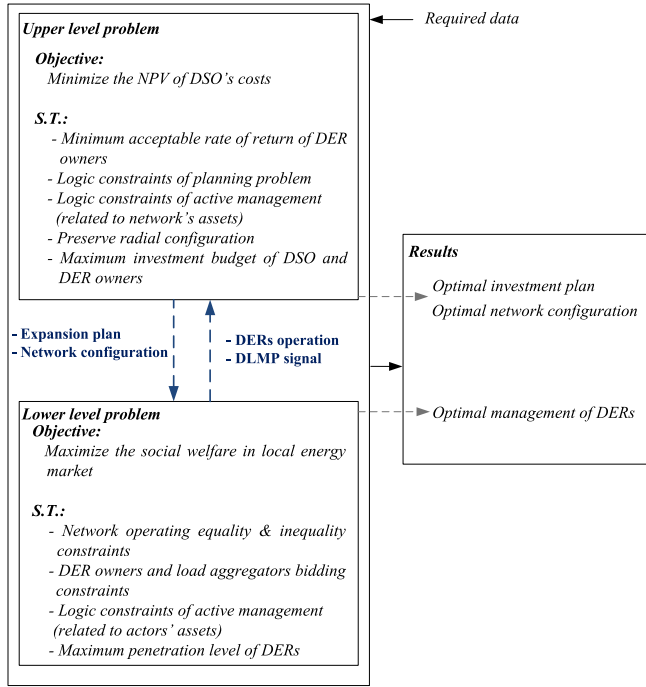


Fig. 2. The proposed bi-level approach for active network management through clearing the local energy market in the DNEP problem.

welfare. DSO and other market participants find equilibrium in the local energy market with a DLMP signal. In other words, DER owners and demand aggregators submit their power bids and the associated price to the local energy market, and then the DSO clears the market and determines DLMP and the actors' optimal operation (i.e., the accepted power of DGs, ESSs' charging and discharging and the amount of increase and decrease in the elastic loads) for maximized social welfare. It is worth noting that the decision variables in the upper level problem are to be parameters for the lower level problem.

The lower level problem is solved for each operating and uncertainty scenario, and therefore, the local energy market is cleared correspondingly for each operating and uncertainty scenario. The operational constraints of the underlying distribution network and the logic and other constraints related to bidding of DER owners and demand aggregators are addressed in the lower level problem. In summary, the expansion plan of the network assets and DERs at each planning time stage and the optimal network configuration at each operating scenario are determined in the upper level, while the DGs, ESSs, and responsive loads' optimal schedules are achieved via active management of resources in the lower level problem.

B. Mathematical Formulation

Based on the proposed model, the DSO solves a bi-level problem for integrating active network management in its investment decisions by clearing the local energy market.

1) *Upper Level Problem*: The upper level problem's objective is to minimize the NPV of the total costs imposed to DSO, associated with investment and maintenance of the network assets

as well as the network operation, through the planning horizon. The objective function is presented in (1) and is comprised of three main parts, namely investment, maintenance, and network operation costs, which are described in (2)–(4), respectively.

$$NPV^{DSO} = \sum_{t \in T} \frac{1}{(1 + RoI)^{t-1}} \times \left(c_t^{I-DSO} + c_t^{M-DSO} + \sum_{o \in O} c_{t,o}^{E-DSO} \right) \quad (1)$$

Equation (2) represents the annual investment cost of the network assets and includes the costs for adding or replacing feeders, constructing or reinforcing substations, and investing on new transformers installation. Annual maintenance cost of the network assets is presented in (3). The cost of the network operation is calculated in (4) which is equal to the cost of energy purchases from the upstream grid as well as the purchase cost of supplied energy by DGs and ESSs subtracted by the payments by the demand aggregator and ESSs for their consumed energies which is calculated based on DLMP. It is worth noting that the product of resources' power and DLMP makes the objective function non-linear. To linearize the objective function, the strong duality theory and some of the KKT conditions, presented in Appendix A, are used. The linearized form is presented later in equation (58).

$$c_t^{I-DSO} = \sum_{l \in \{L^{NR}, L^{NA}\}} \sum_{i \in I^l} \sum_{(b,d) \in \vartheta^l} C_{l,i}^{IL} \cdot \Lambda_{b,d} \cdot x_{b,d,i,t}^l + \sum_{b \in BS} C_b^{IS} \cdot x_{b,t}^S + \sum_{i \in I^k} \sum_{b \in BS} C_{k,i}^{IK} \cdot x_{b,i,t}^k, \forall t \in T \quad (2)$$

$$c_t^{M-DSO} = \sum_{o \in O} \left(\sum_{l \in L} \sum_{i \in I^l} \sum_{(b,d) \in \vartheta^l} C_{l,i}^{ML} \cdot y_{b,d,i,t,o}^l + \sum_{k \in K} \sum_{i \in I^k} \sum_{b \in BS} C_{k,i}^{MK} \cdot y_{b,i,t,o}^k \right), \forall t \in T \quad (3)$$

$$c_{t,o}^{E-DSO} = \sum_{\omega \in \Omega} \Pi_{\omega} \cdot \Delta_o \times \left(\sum_{k \in K} \sum_{i \in I^k} \sum_{b \in BS} \lambda_{t,o,\omega}^T \cdot p_{b,i,t,o,\omega}^k + \sum_{u \in U} \sum_{i \in I^u} \sum_{b \in BU} \lambda_{b,t,o,\omega}^D \cdot p_{b,i,t,o,\omega}^u + \sum_{b \in BST} \lambda_{b,t,o,\omega}^D \left(p_{b,t,o,\omega}^{ST,dch} - p_{b,t,o,\omega}^{ST,ch} \right) - \sum_{b \in BL} \lambda_{b,t,o,\omega}^D \cdot p_{b,t,o,\omega}^D \right), \quad \forall t \in T, o \in O \quad (4)$$

a) *DER owners' investment constraints*: Investing on and utilizing DERs can postpone the expansion of network assets including feeders and substations. Therefore, it is necessary for the DSO to capture investment and optimal utilization of DERs in the DNEP problem. To address the expansion of DERs in the proposed model, DSO considers the economic perspective of

DER owners as private investors. Different economic perspectives of private investors are addressed in [4]. In this paper, the internal rate of return is considered as the economic indicator of DER owners' investment project.

The investment, maintenance and operation costs of DERs imposed to the owners as well as their income from selling energy in the local energy market is considered through (5)–(8), respectively. The investment cost in (5) is associated with the installation of DGs and ESSs through planning horizon. The NPV of DER owners' costs is formulated in (9). In order to guarantee the profitability of DER owners' investment, the MARoR which is specified by the investors should be achieved. In this regard, the NPV of DER owners' costs should be lower than zero if the internal rate of return is set to MARoR which is adhered in (10).

$$c_t^{I_DER} = \sum_{u \in U} \sum_{i \in I^u} \sum_{b \in BU} C_{u,i}^{IU} \cdot PF \cdot \bar{S}_i^u \cdot x_{b,i,t}^u + \sum_{b \in BST} C^{IST} \cdot PF \cdot \bar{S}^{ST} \cdot x_{b,t}^{ST}, \forall t \in T \quad (5)$$

$$c_t^{M_DER} = \sum_{o \in O} \left(\sum_{u \in U} \sum_{i \in I^u} \sum_{b \in BU} C_{u,i}^{MU} \cdot y_{b,i,t,o}^u + \sum_{b \in BST} C^{MST} \cdot y_{b,t,o}^{ST} \right), \quad \forall t \in T \quad (6)$$

$$c_{t,o}^{Op_DER} = \sum_{\omega \in \Omega} \Pi_{\omega} \cdot \Delta_o \times \left(\sum_{u \in U} \sum_{i \in I^u} \sum_{b \in BU} C_{u,i}^{Op} \cdot p_{b,i,t,o,\omega}^u + \sum_{b \in BST} C^{Op_ST} \left(p_{b,t,o,\omega}^{ST,dch} + p_{b,t,o,\omega}^{ST,ch} \right) \right), \quad \forall t \in T, o \in O \quad (7)$$

$$Inc_{t,o}^{DER} = \sum_{\omega \in \Omega} \Pi_{\omega} \cdot \Delta_o \times \left(\sum_{u \in U} \sum_{i \in I^u} \sum_{b \in BU} \lambda_{b,t,o,\omega}^D \cdot p_{b,i,t,o,\omega}^u + \sum_{b \in BST} \lambda_{b,t,o,\omega}^D \left(p_{b,t,o,\omega}^{ST,dch} - p_{b,t,o,\omega}^{ST,ch} \right) \right), \quad \forall t \in T, o \in O \quad (8)$$

$$NPV^{DER} = \sum_{t \in T} \frac{1}{(1 + IRR)^{t-1}} \times \left(c_t^{I_DER} + c_t^{M_DER} + \sum_{o \in O} \left(c_{t,o}^{Op_DER} - Inc_{t,o}^{DER} \right) \right) \quad (9)$$

$$NPV^{DER}|_{IRR=MAROR} \leq 0 \quad (10)$$

Constraint (10) guarantees the feasibility of the DER owners' investment with respect to their income and operation, maintenance, and investment costs. In this regard, DERs are installed at locations where the plan is feasible for both DSO and DER owners, respectively from technical and economical standpoints.

b) Logic constraints of the joint multistage DNEP: Throughout the planning horizon, only one alternative can be invested. Equations (11) and (12) respectively describe this condition for feeders and transformers. Furthermore, this condition should be considered for DER owners' assets including DGs, and ESSs which is adhered through (13) and (14), respectively.

$$\sum_{t \in T} \sum_{i \in I^l} x_{b,d,i,t}^l \leq 1, \quad \forall l \in \{L^{NR}, L^{NA}\}, (b, d) \in \vartheta^l \quad (11)$$

$$\sum_{t \in T} \sum_{i \in I^k} x_{b,i,t}^k \leq 1, \quad \forall b \in BS \quad (12)$$

$$\sum_{t \in T} \sum_{i \in I^u} x_{b,i,t}^u \leq 1, \quad \forall u \in U, b \in BU \quad (13)$$

$$\sum_{t \in T} x_{b,t}^{ST} \leq 1, \quad \forall b \in BST \quad (14)$$

DSO can invest in transformers if its substation already has been reinforced or constructed which is adhered by (15).

$$x_{b,i,t}^k \leq \sum_{\tau=1}^t x_{b,\tau}^S, \forall b \in BS, i \in I^k, t \in T \quad (15)$$

The annual investment costs are limited by the DSO's and DER owners' investment budgets through (16) and (17), respectively.

$$c_t^{I_DSO} \leq \overline{IB}_t^{DSO}, \forall t \in T \quad (16)$$

$$c_t^{I_DER} \leq \overline{IB}_t^{DER}, \forall t \in T \quad (17)$$

c) Logic constraints for network assets utilization: The utilization constraints of the fixed feeders, new replaced or added feeders, and replaceable feeders are modeled through (18)–(20), respectively.

$$y_{b,d,i,t,o}^l \leq 1, \quad \forall l \in L^F, (b, d) \in \vartheta^l, i \in I^l, t \in T, o \in O \quad (18)$$

$$y_{b,d,i,t,o}^l \leq \sum_{\tau=1}^t x_{b,d,i,\tau}^l, \quad \forall l \in \{L^{NR}, L^{NA}\}, (b, d) \in \vartheta^l, i \in I^l, t \in T, o \in O \quad (19)$$

$$y_{b,d,i,t,o}^l \leq 1 - \sum_{\tau=1}^t \sum_{i \in I^l} x_{b,d,i,\tau}^{L^{NR}}, \quad \forall l \in L^R, (b, d) \in \vartheta^l, i \in I^l, t \in T, o \in O \quad (20)$$

Transformers can be utilized only when they have been installed in the current or previous time stages.

$$y_{b,i,t,o}^k \leq \sum_{\tau=1}^t x_{b,i,\tau}^k, \forall b \in BS, i \in I^k, t \in T, o \in O \quad (21)$$

It is worth noting that the utilization of transformers and feeders are attained for each operating scenario. In other words, the network optimal configuration is determined for each operating scenario. It should be noted that in the Stackelberg game, the upper level problem's variables are known for the lower level problem and the local energy market clearance in the lower level problem is done on the determined network configuration achieved in the upper level problem.

In order to obtain the optimal network configuration, the radial structure of the network should be preserved. The island operation of the distribution network in the normal operating status is here assumed not allowed. Two sets of constraints associated with spanning tree (22)–(25) and transfer nodes model (26)–(29) are applied to ensure the radial network operation. Constraints (22)–(25) ensure that the distribution network is structured as a spanning tree connected to the main substation, irrespective to the direction of the power flows [18]. Constraints (26)–(29) enforce the conditions for the radial network operation with transfer nodes. The transfer nodes have neither load nor generation and are used to interconnect the other nodes across the network [19].

$$\alpha_{b,d,t,o}^l = \sum_{i \in I^l} y_{b,d,i,t,o}^l, \quad \forall l \in L, (b,d) \in \vartheta^l, \quad t \in T, o \in O \quad (22)$$

$$\alpha_{b,d,t,o}^l = \beta_{b,d,t,o}^l + \beta_{d,b,t,o}^l, \quad \forall l \in L, (b,d) \in \vartheta^l, \quad t \in T, o \in O \quad (23)$$

$$\beta_{b,d,t,o}^l = 0, \quad \forall l \in L, (b,d) \in \vartheta^l, b \in BS, \quad t \in T, o \in O \quad (24)$$

$$\sum_{d \in \gamma_b^l} \beta_{b,d,t,o}^l = 1, \quad \forall l \in L, (b,d) \in \vartheta^l, b \in BL, \quad t \in T, o \in O \quad (25)$$

$$Tn_b \geq \alpha_{b,d,t,o}^l, \quad \forall l \in L, (b,d) \in \vartheta^l, b \in BT, \quad t \in T, o \in O \quad (26)$$

$$Tn_b \geq \alpha_{d,b,t,o}^l, \quad \forall l \in L, (b,d) \in \vartheta^l, b \in BT, \quad t \in T, o \in O \quad (27)$$

$$\sum_{l \in L} \sum_{(b,d) \in \vartheta^l} \alpha_{b,d,t,o}^l \geq 2Tn_b, \quad \forall b \in BT, \quad t \in T, o \in O \quad (28)$$

$$\sum_{l \in L} \sum_{(b,d) \in \vartheta^l} \alpha_{b,d,t,o}^l = |B| - |BS| - \sum_{b \in BT} (1 - Tn_b), \quad \forall t \in T, o \in O \quad (29)$$

As described in [6], (30) presents a generic ESS model. Furthermore, the net increase and decrease in responsive loads power should be equal to zero over the operating scenarios as modeled in (31).

$$\sum_{o \in O} \Delta_o \left(\eta^{ST,ch} p_{b,t,o,\omega}^{ST,ch} - (1/\eta^{ST,dch}) p_{b,t,o,\omega}^{ST,dch} \right) = 0, \quad \forall b \in BST, t \in T, \omega \in \Omega \quad (30)$$

$$\sum_{o \in O} \left(d_{b,t,o,\omega}^{up} - d_{b,t,o,\omega}^{dn} \right) = 0, \quad \forall b \in BL, t \in T, \omega \in \Omega \quad (31)$$

2) Lower Level Problem Formulation: The lower level problem is solved for each operating and each uncertainty scenario, in which the local energy market is cleared and the optimal behavior of the market actors is attained to maximize the social welfare. In this paper, the optimal schedules of DGs, ESSs, and responsive loads are attained through clearing the local energy market. The objective of maximizing the social welfare is equivalent to minimizing the total payments made to the sellers minus the total payments made by the buyers as formulated in (32). $\pi_{b,i,t,o}^u$, $\pi_{b,t,o}^{ST}$, and $\pi_{b,t,o}^D$ are respectively the bidding price of the DGs, ESSs, and load aggregators for their generated, charging/discharging, and consumed power, which are attained based on their cost and utility functions.

$$\begin{aligned} Obj_{LL} = & \sum_{k,i,b} \lambda_{b,t,o,\omega}^T \cdot p_{b,i,t,o,\omega}^k + \sum_{u,i,b} \pi_{b,i,t,o}^u \cdot p_{b,i,t,o,\omega}^u \\ & + \sum_{b \in BST} \pi_{b,t,o}^{ST} \times \left(p_{b,t,o,\omega}^{ST,dch} - p_{b,t,o,\omega}^{ST,ch} \right) \\ & - \sum_{b \in BL} \pi_{b,t,o}^D \cdot p_{b,t,o,\omega}^D, \end{aligned} \quad \forall t \in T, o \in O, \omega \in \Omega \quad (32)$$

a) Logic constraints for DGs and ESSs utilization: Logic constraints associated with the utilization of DERs is expressed in (33)–(35) which should be considered by DSO in local energy market. The simultaneous charging and discharging of ESSs at each operating and each uncertainty scenario is forbidden which is adhered by (35). The indices in front of equations indicate the corresponding Lagrange coefficients.

$$y_{b,i,t,o}^u \leq \sum_{\tau=1}^t x_{b,i,\tau}^u \cdot \partial_{u,b,i,t,o}^U, \quad \forall u \in U, b \in BU, i \in I^u, t \in T, o \in O \quad (33)$$

$$y_{b,t,o}^{ST} \leq \sum_{\tau=1}^t x_{b,\tau}^{ST} \cdot \partial_{b,t,o}^{ST}, \quad \forall b \in BST, t \in T, o \in O \quad (34)$$

$$a_{b,t,o,\omega}^{ST,ch} + a_{b,t,o,\omega}^{ST,dch} = y_{b,t,o}^{ST} \cdot \kappa_{b,t,o,\omega}, \quad \forall b \in BST, t \in T, o \in O, \omega \in \Omega \quad (35)$$

b) DER owners' constraints: This section addresses the DER owners' constraints which DSO should consider in performing the local energy market. The accepted power of the

DG units and the accepted charging and discharging power of the ESSs are respectively limited through (36)–(39) based on the bidding power of their owners. It should be noted that the owners submit their power bids based on DERs' capacity (i.e., $\bar{S}_i^u$ for DGs and $\underline{S}^{ST}$, $\bar{S}^{ST}$ for ESSs).

$$0 \leq p_{b,i,t,o,\omega}^u \leq y_{b,i,t,o,\omega}^u \cdot P_{b,i,t,o}^u : \underline{\mu}_{b,i,t,o,\omega}^u, \bar{\mu}_{b,i,t,o,\omega}^u$$

$$\forall u \in U, b \in BU, i \in I^u, t \in T, o \in O, \omega \in \Omega \quad (36)$$

$$0 \leq p_{b,i,t,o,\omega}^u \leq y_{b,i,t,o}^u \cdot \text{Max} \{ P_{b,i,t,o,\omega}^\Psi, P_{b,i,t,o}^u \} :$$

$$\underline{\mu}_{b,i,t,o,\omega}^\Psi, \bar{\mu}_{b,i,t,o,\omega}^\Psi$$

$$\forall u \in U^\Psi, b \in BU^\Psi, i \in I^u, t \in T, o \in O, \omega \in \Omega \quad (37)$$

$$a_{b,t,o,\omega}^{ST,ch} \cdot \underline{P}_{b,t,o}^{ST,ch} \leq p_{b,t,o,\omega}^{ST,ch} \leq a_{b,t,o,\omega}^{ST,ch} \cdot \bar{P}_{b,t,o}^{ST,ch} : \underline{\mu}_{b,t,o,\omega}^{ch}, \bar{\mu}_{b,t,o,\omega}^{ch}$$

$$\forall b \in BST, t \in T, o \in O, \omega \in \Omega \quad (38)$$

$$a_{b,t,o,\omega}^{ST,dch} \cdot \underline{P}_{b,t,o}^{ST,dch} \leq p_{b,t,o,\omega}^{ST,dch} \leq a_{b,t,o,\omega}^{ST,dch} \cdot \bar{P}_{b,t,o}^{ST,dch} :$$

$$\underline{\mu}_{b,t,o,\omega}^{dch}, \bar{\mu}_{b,t,o,\omega}^{dch}$$

$$\forall b \in BST, t \in T, o \in O, \omega \in \Omega \quad (39)$$

c) *Load control model of load aggregators:* In order to address the load control in the proposed active resource management scheme, responsive loads should be modeled and incorporated in clearing the local energy market (i.e., in the lower level problem). The change in the elastic loads is modeled as increase and decrease in the base load represented by $d_{b,t,o,\omega}^{up}$ and $d_{b,t,o,\omega}^{dn}$, respectively. The net demand of each load point can be obtained by (40). The accepted amounts of increase or decrease in the base load levels are limited to the associated maximum bidding amounts which are respectively adhered in (41) and (42).

$$p_{b,t,o,\omega}^D = LF_o \cdot P_{b,t,\omega}^D + d_{b,t,o,\omega}^{up} - d_{b,t,o,\omega}^{dn} : \xi_{b,t,o,\omega}$$

$$\forall b \in BL, t \in T, o \in O, \omega \in \Omega \quad (40)$$

$$0 \leq d_{b,t,o,\omega}^{up} \leq \bar{D}_{b,t,o,\omega}^{up} : \underline{\mu}_{b,t,o,\omega}^{d-up}, \bar{\mu}_{b,t,o,\omega}^{d-up}$$

$$\forall b \in BL, t \in T, o \in O, \omega \in \Omega \quad (41)$$

$$0 \leq d_{b,t,o,\omega}^{dn} \leq \bar{D}_{b,t,o,\omega}^{dn} : \underline{\mu}_{b,t,o,\omega}^{d-dn}, \bar{\mu}_{b,t,o,\omega}^{d-dn}$$

$$\forall b \in BL, t \in T, o \in O, \omega \in \Omega \quad (42)$$

The maximum shiftable load (maximum bidding powers of increase and decrease in the nodal base load) at each operating scenario is estimated based on the demand elasticity which express customers' sensitivities to electricity price changes. In this regard, the maximum capability of the load modification is evaluated based on own and cross price elasticity factors and the change in the electricity price [20]. The reference price signal is assumed to be the average substation price over the operating scenarios. The maximum increase and decrease amounts in the nodal base load are respectively defined in (43)

and (44),

$$\bar{D}_{b,t,o,\omega}^{up} = \text{Max} \left\{ 0, LF_o \cdot P_{b,t,\omega}^D \sum_s \varepsilon_{o,s} \times \tilde{\lambda}_{t,s,\omega}^T \right\},$$

$$\forall b \in BL, t \in T, o \in O, \omega \in \Omega \quad (43)$$

$$\bar{D}_{b,t,o,\omega}^{Dn} = \text{Max} \left\{ 0, -LF_o \cdot P_{b,t,\omega}^D \sum_s \varepsilon_{o,s} \times \tilde{\lambda}_{t,s,\omega}^T \right\},$$

$$\forall b \in BL, t \in T, o \in O, \omega \in \Omega \quad (44)$$

where $\tilde{\lambda}_{t,o,\omega}^T$ is deviation of the market price from average price at each operating scenario, represented in (45).

$$\tilde{\lambda}_{t,o,\omega}^T = (\lambda_{t,o,\omega}^T - \bar{\lambda}_{t,\omega}^T) / \bar{\lambda}_{t,\omega}^T, \forall t \in T, o \in O, \omega \in \Omega \quad (45)$$

The linear form of relations (43) and (44) are presented in (46) and (47) using the big-M method [1] where $y_{t,o,\omega}^D$ is a binary variable which demonstrates the utilization of load responses at each planning time stage and operating and uncertainty scenario.

$$-M(1 - y_{t,o,\omega}^D) \leq \bar{D}_{b,t,o,\omega}^{up} - LF_o \cdot P_{b,t,\omega}^D \sum_s \varepsilon_{o,s} \times \tilde{\lambda}_{t,s,\omega}^T$$

$$\leq M(1 - y_{t,o,\omega}^D), \forall b \in BL, t \in T, o \in O, \omega \in \Omega \quad (46)$$

$$-M(1 - y_{t,o,\omega}^D) \leq \bar{D}_{b,t,o,\omega}^{Dn}$$

$$+ LF_o \cdot P_{b,t,\omega}^D \sum_s \varepsilon_{o,s} \times \tilde{\lambda}_{t,s,\omega}^T \leq M(1 - y_{t,o,\omega}^D),$$

$$\forall b \in BL, t \in T, o \in O, \omega \in \Omega \quad (47)$$

d) *Network operational constraints:* In this paper, a linear model of the power flow based on square of the voltage magnitude is used (its performance and associated errors are discussed in [21]). Equation (48) guarantees the power balance at each node. The flow of power across the distribution feeders is represented in (49) considering the feeders installed between nodes b and d . This equation is non-linear due to the product of binary variable y , which indicates feeder utilization status. The linear form of (49) is formulated in (50) in which M is a large-enough positive number.

$$\sum_{l \in L} \sum_{i \in I^l} \sum_{d \in \gamma_b^l} p_{b,d,i,t,o,\omega}^l + p_{b,t,o,\omega}^{ST,ch} + p_{b,t,o,\omega}^D = \sum_{k \in K} \sum_{i \in I^k} p_{b,i,t,o,\omega}^k$$

$$+ \sum_{u \in U} \sum_{i \in I^u} p_{b,i,t,o,\omega}^u + p_{b,t,o,\omega}^{ST,dch} : \lambda_{b,t,o,\omega}^D,$$

$$\forall b \in B, t \in T, o \in O, \omega \in \Omega \quad (48)$$

$$y_{b,d,i,t,o}^l$$

$$\times \left[p_{b,d,i,t,o,\omega}^l + S^B \left(\frac{1}{2} G_{b,d,i}^l (v_{b,t,o,\omega}^2 - v_{d,t,o,\omega}^2) \right) \right] = 0$$

$$\forall l \in L, b \in B, d \in \gamma_b^l, i \in I^l, t \in T, o \in O, \omega \in \Omega \quad (49)$$

$$-M(1 - y_{b,d,i,t,o}^l) \leq p_{b,d,i,t,o,\omega}^l$$

$$\begin{aligned}
& + S^B \left(\frac{1}{2} G_{b,d,i}^l (v_{b,t,o,\omega}^2 - v_{d,t,o,\omega}^2) \right. \\
& \left. - B_{b,d,i}^l (\delta_{b,t,o,\omega} - \delta_{d,t,o,\omega}) \right) \leq M (1 - y_{b,d,i,t,o}^l) : \\
& \underline{\rho}_{b,d,i,t,o,\omega}^l, \bar{\rho}_{b,d,i,t,o,\omega}^l \\
& \forall l \in L, b \in B, d \in \gamma_b^l, i \in I^l, t \in T, o \in O, \omega \in \Omega
\end{aligned} \quad (50)$$

The operational constraints of the network are described by (51)–(54). In order to preserve the model linearity, constraints (53) and (55) are written based on the voltage magnitude square. Furthermore, the voltage at the root nodes is set to $1.05 \angle 0$ in equations (55) and (56).

$$\begin{aligned}
& -y_{b,d,i,t,o}^l \cdot \bar{P}_i^l \leq p_{b,d,i,t,o,\omega}^l \leq y_{b,d,i,t,o}^l \cdot \bar{P}_i^l : \\
& \underline{\mu}_{b,d,i,t,o,\omega}^l, \bar{\mu}_{b,d,i,t,o,\omega}^l \\
& \forall l \in L, b \in B, d \in \gamma_b^l, i \in I^l, t \in T, o \in O, \omega \in \Omega
\end{aligned} \quad (51)$$

$$\begin{aligned}
& -y_{b,i,t,o}^k \cdot \bar{P}_i^k \leq p_{b,i,t,o,\omega}^k \leq y_{b,i,t,o}^k \cdot \bar{P}_i^k : \underline{\mu}_{b,i,t,o,\omega}^k, \bar{\mu}_{b,i,t,o,\omega}^k \\
& \forall k \in K, b \in BS, i \in I^k, t \in T, o \in O, \omega \in \Omega
\end{aligned} \quad (52)$$

$$\begin{aligned}
& (\underline{V}_b)^2 \leq v_{b,t,o,\omega}^2 \leq (\bar{V}_b)^2 : \underline{v}_{b,t,o,\omega}, \bar{v}_{b,t,o,\omega} \\
& \forall b \in B, t \in T, o \in O, \omega \in \Omega
\end{aligned} \quad (53)$$

$$\begin{aligned}
& -\pi \leq \delta_{b,t,o,\omega} \leq \pi : \underline{\gamma}_{b,t,o,\omega}, \bar{\gamma}_{b,t,o,\omega} \\
& \forall b \in B, t \in T, o \in O, \omega \in \Omega
\end{aligned} \quad (54)$$

$$\begin{aligned}
& v_{b,t,o,\omega}^2 = (1.05)^2 : v_{b,t,o,\omega} \\
& \forall b \in BS, t \in T, o \in O, \omega \in \Omega
\end{aligned} \quad (55)$$

$$\begin{aligned}
& \delta_{b,t,o,\omega} = 0 : \gamma_{b,t,o,\omega} \\
& \forall b \in BS, t \in T, o \in O, \omega \in \Omega
\end{aligned} \quad (56)$$

Finally, the DERs' penetration level is limited by (57).

$$\begin{aligned}
& \sum_{u \in U} \sum_{i \in I^u} \sum_{b \in BU} p_{b,i,t,o,\omega}^u + \sum_{b \in BST} p_{b,t,o,\omega}^{ST,d-ch} \leq PL \\
& \times \sum_{b \in BL} LF_o \cdot P_{b,t,\omega}^D : \bar{\phi}_{t,o,\omega} \\
& \forall t \in T, o \in O, \omega \in \Omega
\end{aligned} \quad (57)$$

C. Solution Method

1) *Conversion of the Proposed Bi-Level Model to MILP*: In the followings three main steps for converting the proposed problem into an MILP are described.

i) *Substitution of the lower level problem with associated KKT conditions*

In the introduced model, the DER owners and demand aggregators' biddings are known parameters for the DSO and the

lower level problem is thus linear and convex. As a result, the lower level problem can be substituted with its KKT optimality conditions. In this case, the lower level problem is substituted with a set of constraints imposed to the upper level problem. Using the associated Lagrange equation of the lower level problem, the KKT optimality conditions are obtained as presented in Appendix A. This equivalent model is a mathematical model with equilibrium constraints (MPEC) for which the objective function in (1) and associated KKT complementary conditions in (a15)–(a38) are non-linear. Therefore, they need to be linearized to convert the MPEC problem to an MILP model [22]. Linearizing KKT complementary conditions and the upper level objective function are respectively discussed next in parts *ii* and *iii*.

i) *Linearizing the KKT complementary conditions*

The KKT complementary conditions presented in (a15)–(a38) are nonlinear. These relations are linearized using sufficiently large numbers and auxiliary binary variables [22] which are described in Appendix A.

ii) *Linearization of the upper level objective function*

The objective function of the upper level problem is nonlinear because of the terms $\lambda_{b,t,o,\omega}^D \cdot p_{b,i,t,o,\omega}^u$, $\lambda_{b,t,o,\omega}^D \cdot (p_{b,t,o,\omega}^{ST,dch} - p_{b,t,o,\omega}^{ST,ch})$ and $\sum_{b \in BL} \lambda_{b,t,o,\omega}^D \cdot p_{b,t,o,\omega}^D$ in equation (4). As mentioned earlier, the lower level problem is linear, and therefore convex from the DSO's viewpoint. Based on the strong duality theory, the objective of the lower level problem is equal to its dual in the optimum point [22]. Using the strong duality theory and the KKT optimality conditions presented in Appendix A, the linearized form of the non-linear terms are derived as follows:

$$\begin{aligned}
& \sum_{u,i,b} \lambda_{b,t,o,\omega}^D \cdot p_{b,i,t,o,\omega}^u - \sum_{b \in BST} \lambda_{b,t,o,\omega}^D \cdot p_{b,t,o,\omega}^{ST,ch} \\
& + \sum_{b \in BST} \lambda_{b,t,o,\omega}^D \cdot p_{b,t,o,\omega}^{ST,dch} - \sum_{b \in BL} \lambda_{b,t,o,\omega}^D \cdot p_{b,t,o,\omega}^d = \\
& - \sum_{k,i,b} \lambda_{b,t,o,\omega}^T \cdot p_{b,i,t,o,\omega}^k + \sum_b \lambda_{b,t,o,\omega}^D \cdot LF_o \cdot P_{b,t,\omega}^D \\
& + \sum_{b \in BL} \bar{\mu}_{b,t,o,\omega}^{d-up} \bar{D}_{b,t,o,\omega}^{up} \\
& + \sum_{b \in BL} \bar{\mu}_{b,t,o,\omega}^{d-dn} \bar{D}_{b,t,o,\omega}^{dn} + \sum_{b \in BS} v_{b,t,o,\omega} \cdot (1.05)^2 \\
& + \sum_b \underline{v}_{b,t,o,\omega} \cdot (\underline{V}_b)^2 \\
& - \sum_b \bar{v}_{b,t,o,\omega} (\bar{V}_b)^2 - \sum_b \delta_{b,t,o,\omega} \cdot \pi - \sum_b \bar{\delta}_{b,t,o,\omega} \cdot \pi
\end{aligned} \quad (58)$$

where $\bar{\mu}_{b,t,o,\omega}^{d-up}$, $\bar{\mu}_{b,t,o,\omega}^{d-dn}$, $\lambda_{b,t,o,\omega}^D$, $(\underline{v}_{b,t,o,\omega}, \bar{v}_{b,t,o,\omega})$, $(\delta_{b,t,o,\omega}, \bar{\delta}_{b,t,o,\omega})$ and $v_{b,t,o,\omega}$ are the corresponding Lagrange coefficients of equations (41), (42), (48), (53)–(55), respectively.

TABLE II
DATA OF LOAD, DURATION, AND ENERGY COST OF THE STUDIED
OPERATING SCENARIOS

Operating Scenario	I	II	III	IV
Hours per year	2190	2920	1095	2555
Load factor	0.4	0.7	1	0.3
Purchased energy cost (\$/MWh)	50	65	80	40

The final MILP model of the proposed framework is summarized as follows:

Objective:	Minimize (1): Three parts of (1) are defined through (2)-(4), the nonlinear parts of the objective function are substituted using (58).
Constraints:	- Upper level problem constraints: (5)-(31). - KKT optimality conditions of the lower level problem: (a1)-(a14), (35), (40), (48), (55), (56). - Linearized form of the KKT complementary conditions of (a15)-(a38).

2) *Uncertainty Modeling of the System Load, DERs and Wholesale Electricity Prices:* In order to model the related uncertainties, i.e., in characterizing the variations in demand for electricity, PV power generation, and wholesale electricity price, a scenario-driven stochastic optimization model is applied [6]. The uncertainty modeling for each scenario of operation is conducted according to the PV output power, electricity price, and load curves which are achieved through hourly historical data.

III. CASE STUDY

A. Test Systems, Data, and Assumptions

Two test systems are selected to evaluate the effectiveness of the proposed model. Test system 1 is a 20 kV 24-bus distribution network introduced in [23]. In order to validate the scalability of the proposed model, the 11.4 kV 86-bus distribution network (introduced in [24] and modified in [25]) is chosen as test system 2. In the following, the main characteristics of the two test networks and the required information are presented.

Test System 1: Test system 1 is the 20 kV 24-bus distribution network. The network has 20 load points which can be fed through 4 substations and 33 feeders. Feeders' data and load points' peak demand are presented in [3]. The capacity of the existing transformers and feeders are 7.5 and 3.94 MVA, respectively. Furthermore, two zones with different solar irradiation are defined in the network: Zone A contains nodes (1, 4–6, 9, 13–15, 17–20, 22, 24) while the other nodes are located in Zone B.

Test System 2: The 11.4 kV 86-bus distribution network consists of 83 load points, 94 lines, and 3 substations (buses 101, 102, and 103). The one-line diagram and the network data are taken from [25]. In this network, the capacity of existing transformers and feeders are 12 and 6.28 MVA, respectively. Additionally, the load points' peak demand for three planning time stages is demonstrated in Table XII in Appendix B. In order to evaluate the solar irradiation for PV installation in the network, two zones, namely Zone A and B, are considered: Zone

TABLE III
ESSs AND PVs CHARACTERISTICS

ESSs			PV units	
$\bar{S}^{ST}$	C^{IST}	$\eta^{ST, ch}, \eta^{ST, d-ch}$	$\bar{S}_i^{\psi}$	$C_{\psi, i}^{IU}$
(MVA)	(M\$/MW)		(MVA)	(M\$/MW)
0.25	1.800	0.95	1.0/1.65	0.850

TABLE IV
MAINTENANCE COST, LIFE TIME, AND CANDIDATE LOCATIONS FOR
INSTALLATION OF THE NETWORK'S AND DER OWNERS' ASSETS IN BOTH TEST
SYSTEMS

Assets	Feeders	Transformers	DGs	ESSs
Maintenance costs	\$450	From [26] & [25] for TS1 & TS2	5% of IC	2.5% of IC
Life time (year)	25	15	25	10
Candidate places for installation	TS1	Based on [3]	BU^{ψ} : 1,4,5,9	7,8,16,18
			12,14,15,17,19	
	TS2	Based on [25]	BU^{ψ} : 2,9,12,20	4,6,20,32
			34,42,54,69,71	
			BU^C : 3,7,10,15,16,17,20	
			BU^C : 3,10,21,30	54,65,68
			36,60,75,80,82	

*IC: Investment cost, TS1: Test system 1, TS2: Test system 2.

TABLE V
NETWORK VARIABLES AND ASSOCIATED ERRORS OF THE LINEAR POWER
FLOW MODEL FOR TIME STAGE III AND OPERATING SCENARIO 3

Case	Minimum Voltage	Maximum loading	Maximum error		MAE*	
			Voltage (%)	Active flow (%)	Voltage (p.u)	Active flow (MW)
A	0.99	74%	0.07	2.46%	0.00008	0.2227
B	0.98	76%	0.09	2.37%	0.00008	0.2210
C	0.97	82%	0.07	2.52%	0.00007	0.2240
D	0.99	74%	0.08	2.55%	0.00008	0.2228

*MAE: Mean absolute error of the linear power flow.

A includes nodes (37–64, 72–77, 81–83, 103), while others are located at Zone B.

In the model, the planning time horizon is 15 years for which three planning time stages with duration of 5 years are defined. For each time stage, 4 scenarios of operation are defined where active network management is considered. The operating scenarios' durations and load factors in addition to the purchased power cost at each operating scenario can be accessed from Table II. The detailed properties of the investment alternatives related to the network assets for test system 1 and test system 2 are respectively presented in [26] and [25], while the data of the conventional DGs are taken from [26]. The ESSs data including their capacity, investment cost, and charging/discharging efficiencies as well as PV units' alternatives capacities and installation costs are presented in Table III. In addition, the maintenance cost, life time, and candidate locations for installation of network assets and DER owners' assets in the test systems are represented in Table IV. It is worth noting that the locations which are not suitable candidates for DSO or DER owners should be discarded from the list of candidate nodes for DERs installation. The data associated with available solar power at Zone A in both test

TABLE VI
INVESTMENT DECISIONS AT EACH TIME STAGE FOR CASES A–C

Case	A			B			C		
Time stage	I	II	III	I	II	III	I	II	III
DERs' installation including ESS, CG, and PV	ESS (7 & 16) CG1 (3)* PV2 (4)	ESS (18) CG1 (10) PV1 (12)	CG2 (17) PV1 (14)	CG1 (10) CG2 (7) PV2 (9)	ESS (16) PV1 (12)	ESS (8 & 18) CG1 (3) CG2 (15) PV2 (14)	—	—	—
Network assets installation including NS, NTr, NAF, and NRF	NS (23) NTr1 (23) NAF1(7, 23) NAF1(5, 24) NAF1(4, 9) NAF1(4, 7) NAF1(3, 10) NAF1(2, 3) NAF2(1, 9) NRF2(1, 21)	NS (24) NTr1 (24) NAF1(18, 24) NAF1(17, 22) NAF1(15, 17) NAF2(14, 18) NAF1(11, 23) NAF1(6, 13) NAF1(4, 16) NAF1(2, 3) NAF1(1, 14) NRF1(8, 22)	NAF1(20, 24) NAF1(15, 19) NAF1(5, 6) NAF2(3, 23)	NS (23) NTr1 (23) NAF1(7, 23) NAF2(5, 24) NAF1(4, 15) NAF1(4, 9) NAF1(3, 10) NAF1(2, 3) NRF1(8, 22) NRF2(1, 21)	NAF1(17, 22) NAF1(15, 17) NAF1(11, 23) NAF1(6, 13) NAF1(4, 16) NAF2(3, 23) NAF1(2, 12) NAF1(1, 14) NAF2(1, 9)	NS (24) NTr1 (24) NAF1(20, 24) NAF1(18, 24) NAF1(15, 19) NAF2(14, 18) NAF1(10, 16) NAF1(5, 6)	NTr2 (22) NAF2(17, 22) NAF2(15, 17) NAF1(10, 16) NAF2(7, 8) NAF1(14, 18) NAF1(5, 9) NAF2(5, 6) NAF2(4, 16) NAF2(4, 15) NAF2(2, 3) NRF1(8, 22) NRF2(1, 21)	NS (23 & 24) NTr2 (23) NTr1 (24) NAF2(18, 24) NAF1(14, 18) NAF2(11, 23) NAF2(10, 23) NAF1(7, 11) NAF1(6, 13) NAF2(3, 10) NAF1(2, 12) NAF2(1, 9)	NTr2 (21) NAF1(20, 24) NAF1(15, 19) NAF1(5, 24)

*The numerals after the symbol of assets indicate the number of associated alternative and the numerals in parenthesis indicate the installation location.
CG: conventional generator, NRF: newly replaced feeder, NAF: newly added feeder, NTr: new transformer, NS: new substation.

TABLE VII
COMPARISON OF THE SYSTEM COSTS (IN MILLION \$) FOR CASES A–C

Case	<i>t</i>	Investment	Maintenance	Operation	Total Costs	Total Costs' NPV
		DSO/DO*	DSO/DO	DSO/DO	DSO/DO	DSO
A	I	1.02/4.90	0.048/0.98	35.24/12.24	36.31/17.31	1168.6
	II	2.04/4.12	0.098/3.26	51.20/17.29	53.34/24.67	
	III	1.02/2.92	0.12/3.72	88.71/32.30	89.85/38.94	
B	I	1.21/4.98	0.051/1.22	39.41/15.76	40.67/21.96	1269.9
	II	1.92/2.48	0.089/2.35	59.87/19.92	61.88/24.75	
	III	1.15/4.95	0.13/3.79	98.94/37.65	100.22/46.39	
C	I	1.42/-	0.052/-	45.45/-	46.922/-	1462.9
	II	2.65/-	0.11/-	69.62/-	72.38/-	
	III	1.05/-	0.13/-	115.05/-	116.23/-	

*DO: DER Owners.

TABLE VIII
COMPARISON OF DER OWNERS' PROFIT AND SYSTEM OPERATION COST IN CASE A AND CASE D

Time stages	DER owners profit (k\$)		System operation costs (M\$)	
	Case A	Case D	Case A	Case D
I	552	328	34.90	34.68
II	780	524	50.69	50.48
III	1088	782	85.67	85.41

systems are taken from [27] while it is assumed that available power at Zone B of the test systems is 90% of that in Zone A. To model the role of responsive loads, both own and cross price elasticity factors are assumed to be 4%.

The lower and upper limits on the bus voltage magnitude are 0.95 and 1.05 p.u., respectively. The base power is assumed 1 MVA while the base voltage for the test systems 1 and 2 are 20 kV and 11.4 kV, respectively. The penetration level of DGs and ESSs, rate of interest, network power factor (for both test systems), and MARoR of DER owners are assumed to be 0.4, 10%, 0.9, and 12.5%, respectively. Furthermore, investment budget for DSO and DER owners in both tests systems are \$4M and \$6M at each time stage, respectively. With the historical PV output power [27] and load data [26] as well as the wholesale electricity prices [28], 27 uncertainty scenarios are considered in each operating condition.

TABLE IX
INVESTMENT DECISIONS OF THE PROPOSED MODEL (CASE A) ON TEST SYSTEM 2 AT EACH TIME STAGE

Time stage	I	II	III
DERs' installation including ESS, CG, and PV	ESS (4 & 32) CG2 (21) PV2 (34)	ESS (7 & 18) CG1 (60) PV2 (9)	CG2 (10) PV1 (54)
Network assets installation including NS, NTr, NAF, and NRF	NTr1 (101) NAF1(21, 65) NAF2(26, 78) NAF1(68, 78) NAF1(68, 69) NAF1(36, 66) NAF1(66, 67) NRF2(102, 61)	NTr1 (102) NAF1(21, 70) NAF2(70, 71) NAF2(42, 72) NAF1(72, 73) NAF1(73, 74) NAF1(48, 75) NAF1(75, 76) NAF1(76, 77) NRF1(8, 22)	NS(103) NAF2(78, 79) NAF1(79, 80) NAF1(81, 82) NAF1(82, 83) NAF1(80, 103) NAF1(83, 103)

TABLE X
COMPARISON OF THE SYSTEM COSTS IN TEST SYSTEM 2 (IN MILLION \$) FOR CASES A–C

Case	<i>t</i>	Investment	Maintenance	Operation	Total Costs	Total Costs' NPV
		DSO/DO	DSO/DO	DSO/DO	DSO/DO	DSO
A	I	1.90/3.79	0.68/2.36	76.10/28.06	78.68/34.21	2112.4
	II	2.70/3.34	0.90/4.59	102.67/34.91	106.27/42.84	
	III	1.90/2.38	1.07/4.97	137.98/52.16	140.95/59.51	
B	I	2.06/3.88	0.71/2.83	81.34/28.92	84.11/35.63	2281.9
	II	2.85/3.42	0.95/5.79	108.34/38.58	112.14/47.79	
	III	2.19/4.76	1.09/6.07	151.23/55.96	154.51/66.79	
C	I	2.23/-	0.75/-	90.80/-	93.78/-	2503.7
	II	3.42/-	0.98/-	115.44/-	119.84/-	
	III	2.01/-	1.12/-	164.86/-	167.99/-	

B. Numerical Results and Analysis

With the proposed bi-level model applied on the test systems (i.e., integrating active network management in DNEP problem through clearing local energy market at the distribution level), the results are extracted and investigated (Case A). In order to perceive the impact of active network management when incorporated in the DNEP problem, the results of Case A are compared with the case that DSO optimizes its expansion plan regardless of the network management and then manages the

TABLE XI

THE NUMBER OF BINARY AND CONTINUOUS VARIABLES IN THE PROPOSED MODEL FOR (TEST SYSTEM 1/TEST SYSTEM 2)

# of upper level variables	B^*	$\left T \right \left(\sum_{l \in \{L^{NR}, L^{AF}\}} I^l g^l + BS + I^K BS + \sum_{u \in U} I^u BU^u + BST \right) + T O \left(\sum_{l \in L} I^l g^l + \sum_{k \in K} I^k BS + 3 \sum_{l \in L} g^l + BT \right)$ (2670/6156)
	C	$4 T + 3 T O $ (48/48)
# of lower level variables	B	$ T O \left(\sum_{u \in U} I^u BU^u + BST \right) + 2 T O O BST + T O O $ (3060/4872)
	C	$ T O O \left(\sum_{k \in K} I^k BS + \sum_{u \in U} I^u BU^u + 2 BST + \sum_{l \in L} I^l g^l + 2 B + 3 BL \right)$ (66672/187236)
# of dual variables	B	$ T O \left(2 \sum_{u \in U} I^u BU^u + 2 BST \right) + T O O \left(4 \sum_{u \in U} I^u BU^u + 8 BST + 8 \sum_{l \in L} I^l g^l + 4 \sum_{k \in K} I^k BS + 8 B + 4 BL + 2 \right)$ (292320/696696)
	C	$ T O \left(\sum_{u \in U} I^u BU^u + BST \right) + T O O \left(5 B + 5 BST + 2 \sum_{u \in U} I^u BU^u + 4 \sum_{l \in L} I^l g^l + 5 BL + 2 \sum_{k \in K} I^k BS + 2 BS + 1 \right)$ (175986/457536)

*B: Binary variables, C: Continuous variables.

resources through local energy market after the investment decisions are determined (Case B). To illustrate the positive impact of considering DER owners' assets and responsive loads in the joint expansion planning problem and in active network management scheme, Case C is introduced which represents the conditions in which DER owners' assets (i.e., DGs and ESSs) do not exist in the planning problem alternatives and load control is not included in active management scheme; therefore, the network assets are taken into account in the planning level and the optimal configuration of the network is determined in the operation level. Finally, in order to perceive the positive impact of modeling the local energy market in DNEP problem using bi-level framework, Case D is introduced in which DSO directly schedules DERs through economic dispatch using a single level framework. In the following, we evaluate the performance of the proposed model when applied to test system 1, and then to test system 2 for scalability validation in large-scale test systems.

1) *Case Study 1: 24-Bus Distribution Test System:* The main outcome of the introduced model is the optimal expansion plan of the network's and DER owners assets for each planning time stage and the optimal network configuration and operation schedule of DERs for each operating scenario. The network variables at the third planning time stage and operating scenario 3 (i.e., peak load of the network) for case scenarios A to D are

TABLE XII
LOAD POINTS' PEAK DEMAND (MVA)

Bus #	Time stage			Bus #	Time stage		
	I	II	III		I	II	II
1	0.570	0.673	0.751	43	0.282	0.333	0.371
2	0.113	0.134	0.149	44	0.149	0.176	0.196
3	0.101	0.119	0.133	45	0.412	0.487	0.543
4	0.206	0.244	0.272	46	1.096	1.295	1.444
5	0.737	0.871	0.972	47	0.628	0.743	0.828
6	0.352	0.417	0.465	48	0.563	0.666	0.743
7	0.565	0.668	0.745	49	0.449	0.530	0.592
8	0.148	0.174	0.194	50	0.261	0.308	0.344
9	0.295	0.349	0.389	51	0.636	0.751	0.838
10	0.332	0.392	0.437	52	0.174	0.206	0.229
11	0.277	0.327	0.365	53	0.618	0.730	0.814
12	0.590	0.697	0.778	54	0.165	0.195	0.218
13	0.690	0.815	0.909	55	0.052	0.062	0.069
14	0.719	0.850	0.948	56	0.972	1.149	1.282
15	0.510	0.603	0.672	57	0.049	0.058	0.065
16	0.470	0.556	0.620	58	0.360	0.425	0.474
17	0.944	1.115	1.244	59	0.527	0.623	0.695
18	0.348	0.411	0.458	60	0.221	0.261	0.291
19	0.165	0.195	0.218	61	0.935	1.105	1.233
20	0.074	0.087	0.097	62	0.542	0.640	0.714
21	0.566	0.668	0.746	63	0.803	0.949	1.059
22	0.148	0.174	0.195	64	0.696	0.823	0.918
23	0.566	0.668	0.746	65	0.286	0.338	0.377
24	0.295	0.349	0.389	66	3.436	4.091	4.582
25	0.592	0.699	0.780	67	0.714	0.850	0.952
26	0.140	0.165	0.184	68	0.146	0.175	0.197
27	0.893	1.055	1.177	69	0.440	0.528	0.594
28	0.475	0.562	0.626	70	0.000	0.350	0.395
29	0.305	0.361	0.402	71	0.000	0.392	0.444
30	0.140	0.165	0.184	72	0.000	0.598	0.679
31	0.829	0.980	1.093	73	0.000	0.781	0.888
32	0.209	0.247	0.276	74	0.000	0.164	0.186
33	0.376	0.445	0.496	75	0.000	0.139	0.159
34	0.972	1.148	1.281	76	0.000	0.274	0.315
35	0.821	0.970	1.082	77	0.000	0.174	0.201
36	0.444	0.524	0.585	78	0.000	0.000	1.676
37	0.593	0.701	0.782	79	0.000	0.000	0.052
38	0.184	0.218	0.243	80	0.000	0.000	0.183
39	0.315	0.372	0.415	81	0.000	0.000	1.423
40	0.003	0.004	0.004	82	0.000	0.000	0.705
41	0.015	0.018	0.020	83	0.000	0.000	0.743
42	0.376	0.445	0.496				

presented in Table V. In this table, the maximum error and the mean absolute error of the adopted linear power flow method, introduced in Section II-B, are presented when compared to those of the precise AC power flow based on the Newton-Raphson technique.

The results of the expansion problem in Cases A, B, and C at each time stage are presented in Tables VI and VII. It should be noted that investment decisions in Case D are found somehow near to those in Case A (hence, its results associated with expansion planning decisions are not presented in two aforementioned tables). As demonstrated, in Case C without DERs, the operation cost is found higher than that in the other two cases due to the purchased power from the upstream network. Furthermore, the investment costs on the network assets' (i.e., new transformers, substations, and feeders) in Case C are higher compared to Case A & Case B, in which besides the network assets, DERs are invested by private investors. The reason lies in the fact that DSO requires to install higher-capacity transformers and feeders to meet the growing demand. In other words, considering DERs provides an opportunity for DSO to supply the growing load

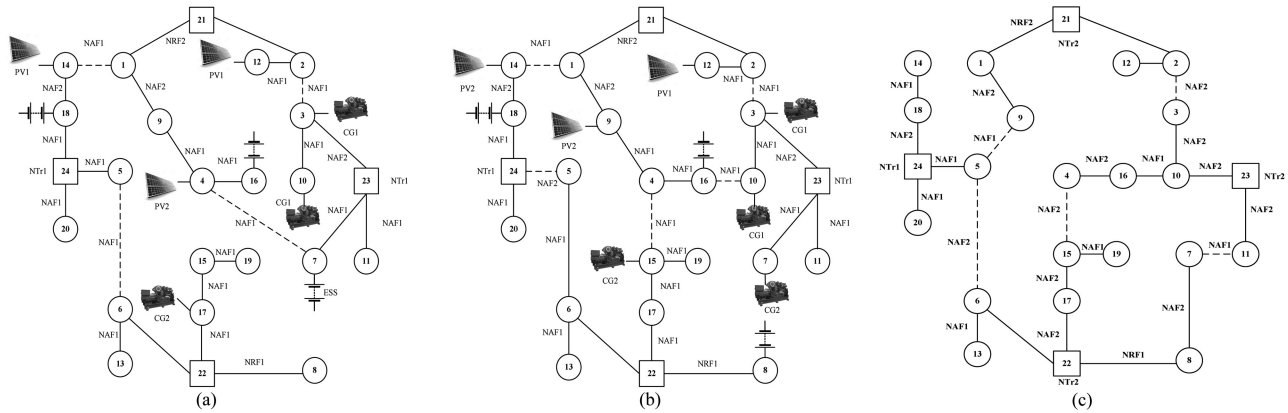


Fig. 3. Optimal network and DERs arrangement for (a) Case A, (b) Case B, and (c) Case C.

through investing in DERs which leads to lower investment in network assets in the planning horizon. In addition, incorporating active management of resources in the DNEP problem (i.e., Case A) leads to lower investment costs of the network assets expansion compared to Case B in which resources are managed independently of the DNEP problem. In summary, considering DER owners' assets in the planning problem as well as managing DERs through active management scheme in the DNEP problem result in lower investment costs for the network assets' expansion. In other words, it leads to investment postponement of the network assets during the planning horizon. It should be noted that, in Case A & Case B, PV generators are mostly installed at the Zone A with better solar power capability. The study results show that the presence of DERs in these two cases leads to the lower investment costs on network assets and total operation costs reduction over the planning horizon in comparison with Case C, as a consequence of an investment on DERs and their joint utilization in the DNEP problem.

In order to investigate the positive impacts of incorporating active network management in DNEP through the local energy market, i.e., the proposed bi-level framework, the results in Case A and Case B are compared. As it can be seen, the NPV of the total costs of DSO in Case A is about 8% lower in comparison with that in Case B. Therefore, the proposed framework in this paper leads to more economic investment and utilization plan compared to Case B, in which the active network management is addressed independently from the DNEP problem. The final configurations of the ADN (operating scenario 4 at the third time stage) in the three cases are illustrated in Fig. 3. The feeders that are not utilized in this operating scenario are denoted by dashed lines.

To investigate the active network management strategies in DNEP problem, the optimal schedule of DERs, including conventional and PV DGs as well as ESSs and responsive loads, in Case A is summarized in Fig. 4. Besides the optimal DERs' schedule, in active management scheme the optimal radial configuration of the network is achieved for each operating scenario through deciding on the utilization of feeders (including existing and newly added or replaced feeders). An optimal network configuration for Cases A to C is depicted in Fig. 3 corresponding to the operating scenario 4 at

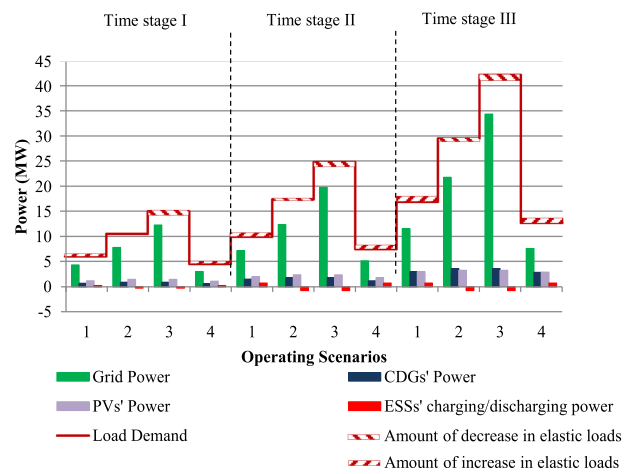


Fig. 4. Optimal operation of DERs through planning time stages and operating scenarios in Case A.

planning time stage III. The numerical results show that in case of disregarding the load control, the operation costs of DSO has increased which leads to 0.4% increase in DSO's costs NPV. It shows the positive impact of utilizing responsive loads.

Analyzing the above-mentioned cases shows the positive impacts of integrating active network management through clearing the local energy market in the DNEP as well as considering the expansion planning of DER owners' asset besides the network assets. In order to further analyze the impact of considering local energy market, the profit of the DER owners and system operation costs in Case A (i.e., the proposed approach) is compared with Case D in which the local energy market is disregarded and DSO directly manages the DERs through economic dispatch. Based on the results presented in Table VIII, it is concluded that DER owners' profit is decreased in case of dispatching them directly by DSO. On the other hand, higher operation cost is incurred to the system when local energy market is modeled. It should be noted that in the proposed approach (i.e., Case A), the increase in the DER owners' profit is more than the increase in the system operation costs, highlighting that implementing the proposed approach leads to an enhanced social welfare.

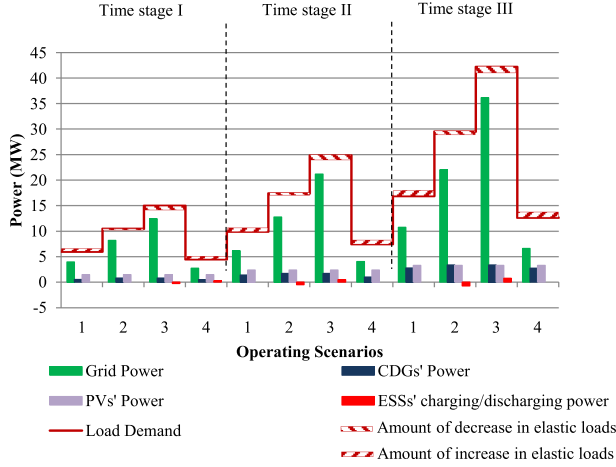


Fig. 5. Optimal operation of DERs through planning time stages and operating scenarios in Case D.

In addition, the DERs schedules in Case D are depicted in Fig. 5. As it can be seen, the ESSs are not utilized effectively in Case D and DSO mainly utilizes PV units because of their lower operation costs. The cost data of the conventional DGs, PVs and ESSs are presented in [26] and [29] while the utility function of the load is taken from [30].

2) *Case Study 2: 11.4kV 86-Bus Distribution Test System:* In order to validate the scalability of the proposed model, it is implemented on the 11.4kV 86-bus test system. Table IX shows the investment decisions associated with the network's and DER owners' assets at each planning time stage in Case A. Considering the previously-defined cases, the associated system costs are summarized in Table X for Cases A to C. The results of this test system demonstrate the effectiveness of the proposed model in achieving cost-effective planning decisions. In addition, as previously investigated, considering DER owners' assets in the DNEP problem and active network management leads to lower investment in network assets throughout the planning horizon. As it can be seen, integrating active network management through clearing local energy market (Case A) leads to about 8.5% reduction in the total DSO costs' NPV in comparison with that in Case B.

The simulations are carried out on a PC with Intel Corei7-4770, 3.40 GHz CPU and 32 GB of RAM. The CPLEX 12 solver in GAMS 24.1.3 programming environment is used to solve the optimization problem. The optimality gap representative of the solution accuracy is set equal to 0.5%. The computational time is highly dependent on the number of variables and the optimality gap. The number of continuous and binary variables in the upper level and lower level problems as well as dual variables of the lower level problem are summarized in Table XI. In this table, the first and second numbers in parenthesis at each row show the number of variables in test system 1 and test system 2, respectively.

The computation time of the proposed simulation approach for test system 1 is 8.9 h while it is increased to 15.8 h in case of implementation on test system 2. It shows that the proposed approach is practically appropriate for long-term DNEP problem.

IV. CONCLUSION

A bi-level optimization framework was presented in this paper to address an active network management in multistage expansion planning of distribution network's and DER owners' assets. In the upper level problem, the optimal expansion plan of the network's and DER owners' assets at each time stage as well as the optimal network configuration at each operating scenario are extracted in which the profitability of DER owners is guaranteed. Furthermore, DSO clears the local energy market in the lower level in order to achieve the optimal schedule of DERs in each operating scenario. To model the prevailing uncertainties in demands, PVs' power generation and electricity prices, a scenario-driven stochastic optimization model was used. The application of the proposed MILP model on the 24-bus and 86-bus test systems was evaluated. Comparing the results of the proposed model with the case where DNEP and active network management are handled independently, it is revealed that the DSO achieves more economic expansion and operation plans in the former. Furthermore, the proposed active network management based on clearing local energy market leads to enhanced social welfare compared to the case in which DSO schedules DERs through economic dispatch. Finally, the results demonstrated the positive impacts of the DERs in power distribution networks in both planning and operation decisions.

APPENDIX

A. Lower Level KKT's Optimality Conditions

In this part, the KKT optimality conditions of the lower level problem introduced in Part *i* of Section in Section II-C and the linearized form of the KKT complementary conditions mentioned in Part *ii* of Section in Section II-C are presented. The KKT optimality conditions are obtained using the associated Lagrange equation of the lower level problem which are listed in the following.

$$\pi_{b,i,t,o}^u - \lambda_{b,t,o,\omega}^D - \mu_{b,i,t,o,\omega}^u + \bar{\mu}_{b,i,t,o,\omega}^u + \bar{\phi}_{t,o,\omega} = 0 \quad (\text{a1})$$

$$-\pi_{b,t,o}^{ST} + \lambda_{b,t,o,\omega}^D - \mu_{b,t,o,\omega}^{ch} + \bar{\mu}_{b,t,o,\omega}^{ch} = 0 \quad (\text{a2})$$

$$\pi_{b,t,o}^{ST} - \lambda_{b,t,o,\omega}^D - \mu_{b,t,o,\omega}^{d-ch} + \bar{\mu}_{b,t,o,\omega}^{d-ch} + \bar{\phi}_{t,o,\omega} = 0 \quad (\text{a3})$$

$$-\pi_{b,t,o}^D + \lambda_{b,t,o,\omega}^D - \xi_{b,t,o,\omega} = 0 \quad (\text{a4})$$

$$\xi_{b,t,o,\omega} - \mu_{b,t,o,\omega}^{d-up} + \bar{\mu}_{b,t,o,\omega}^{d-up} = 0 \quad (\text{a5})$$

$$-\xi_{b,t,o,\omega} - \mu_{b,t,o,\omega}^{d-dn} + \bar{\mu}_{b,t,o,\omega}^{d-dn} = 0 \quad (\text{a6})$$

$$\lambda_{b,t,o,\omega}^T - \lambda_{b,t,o,\omega}^D - \mu_{b,i,t,o,\omega}^k + \bar{\mu}_{b,i,t,o,\omega}^k = 0 \quad (\text{a7})$$

$$\sum_{d \in \gamma_b^l} \frac{1}{2} G_{b,d,i}^l \cdot S^B \cdot \left(\rho_{d,b,i,t,o,\omega}^l - \rho_{b,d,i,t,o,\omega}^l + \bar{\rho}_{d,b,i,t,o,\omega}^l - \bar{\rho}_{b,d,i,t,o,\omega}^l \right) - \underline{v}_{b,t,o,\omega} + \bar{v}_{b,t,o,\omega} - v_{b,t,o,\omega} \big|_{b \in BS} = 0 \quad (\text{a8})$$

$$\sum_{d \in \gamma_b^l} B_{b,d,i}^l \cdot S^B \cdot \left(\rho_{d,b,i,t,o,\omega}^l - \rho_{b,d,i,t,o,\omega}^l + \bar{\rho}_{d,b,i,t,o,\omega}^l - \bar{\rho}_{b,d,i,t,o,\omega}^l \right)$$

$$-\bar{\rho}_{b,d,i,t,o,\omega}^l) - \gamma_{b,t,o,\omega} + \bar{\gamma}_{b,t,o,\omega} - \gamma_{b,t,o,\omega} |_{b \in BS} = 0 \quad (\text{a9})$$

$$\lambda_{b,t,o,\omega}^D - \bar{\mu}_{b,d,i,t,o,\omega}^l + \bar{\mu}_{b,d,i,t,o,\omega}^l - \underline{\rho}_{b,d,i,t,o,\omega}^l + \bar{\rho}_{b,d,i,t,o,\omega}^l = 0 \quad (\text{a10})$$

$$\partial_{u,b,i,t,o}^U - \bar{\mu}_{b,i,t,o,\omega}^u \cdot P_{b,i,t,o}^u = 0 \quad (\text{a11})$$

$$\partial_{b,t,o}^{ST} + \kappa_{b,t,o,\omega} = 0 \quad (\text{a12})$$

$$\underline{\mu}_{b,t,o,\omega}^{ch} \cdot P_{b,t,o}^{ST,ch} - \bar{\mu}_{b,t,o,\omega}^{ch} \cdot \bar{P}_{b,t,o}^{ST,ch} - \kappa_{b,t,o,\omega} = 0 \quad (\text{a13})$$

$$\underline{\mu}_{b,t,o,\omega}^{d-ch} \cdot P_{b,t,o}^{ST,d-ch} - \bar{\mu}_{b,t,o,\omega}^{d-ch} \cdot \bar{P}_{b,t,o}^{ST,d-ch} - \kappa_{b,t,o,\omega} = 0 \quad (\text{a14})$$

Relations (35), (40), (48), (55), (56)

$$\partial_{u,b,i,t,o}^U \geq 0 \perp \sum_{\tau=1}^t x_{b,i,\tau}^u - y_{b,i,t,o}^u \geq 0 \quad (\text{a15})$$

$$\partial_{b,t,o}^{ST} \geq 0 \perp \sum_{\tau=1}^t x_{b,\tau}^{ST} - y_{b,t,o}^{ST} \geq 0 \quad (\text{a16})$$

$$\mu_{b,d,i,t,o,\omega}^l \geq 0 \perp y_{b,d,i,t}^l \bar{P}_i^l + p_{b,d,i,t,o,\omega}^l \geq 0 \quad (\text{a17})$$

$$\bar{\mu}_{b,d,i,t,o,\omega}^l \geq 0 \perp y_{b,d,i,t}^l \bar{P}_i^l - p_{b,d,i,t,o,\omega}^l \geq 0 \quad (\text{a18})$$

$$\rho_{b,d,i,t,o,\omega}^l \geq 0 \perp \left(\frac{M(1 - y_{b,d,i,t}^l) + p_{b,d,i,t,o,\omega}^l}{S^B \cdot \left(G_{b,d,i}^l (v_{b,t,o,\omega}^2 - v_{d,t,o,\omega}^2) \right)} \right) \geq 0 \quad (\text{a19})$$

$$\bar{\rho}_{b,d,i,t,o,\omega}^l \geq 0 \perp \left(\frac{-S^B \cdot \left(G_{b,d,i}^l (v_{b,t,o,\omega}^2 - v_{d,t,o,\omega}^2) \right)}{-B_{b,d,i}^l (\delta_{b,t,o,\omega} - \delta_{d,t,o,\omega})} \right) \geq 0 \quad (\text{a20})$$

$$\mu_{b,i,t,o,\omega}^k \geq 0 \perp y_{b,i,t,o}^k \bar{P}_i^k + p_{b,i,t,o,\omega}^k \geq 0 \quad (\text{a21})$$

$$\bar{\mu}_{b,i,t,o,\omega}^k \geq 0 \perp y_{b,i,t,o}^k \bar{P}_i^k - p_{b,i,t,o,\omega}^k \geq 0 \quad (\text{a22})$$

$$v_{b,t,o,\omega} \geq 0 \perp -V_b^2 + v_{b,t,o,\omega}^2 \geq 0 \quad (\text{a23})$$

$$\bar{v}_{b,t,o,\omega} \geq 0 \perp (\bar{V}_b)^2 - v_{b,t,o,\omega}^2 \geq 0 \quad (\text{a24})$$

$$\gamma_{b,t,o,\omega} \geq 0 \perp \pi + \delta_{b,t,o,\omega} \geq 0 \quad (\text{a25})$$

$$\bar{\gamma}_{b,t,o,\omega} \geq 0 \perp \pi - \delta_{b,t,o,\omega} \geq 0 \quad (\text{a26})$$

$$\mu_{b,i,t,o,\omega}^u \geq 0 \perp p_{b,i,t,o,\omega}^u \geq 0 \quad (\text{a27})$$

$$\bar{\mu}_{b,i,t,o,\omega}^u \geq 0 \perp y_{b,i,t,o}^u \cdot P_{b,i,t,o}^u - p_{b,i,t,o,\omega}^u \geq 0 \quad (\text{a28})$$

$$\bar{\mu}_{b,i,t,o,\omega}^\Psi \geq 0 \perp y_{b,i,t,o}^u \cdot \text{Max} \{ P_{b,i,t,o,\omega}^\Psi, P_{b,i,t,o,\omega}^u \} - p_{b,i,t,o,\omega}^u \geq 0 \quad (\text{a29})$$

$$\mu_{b,t,o,\omega}^{ch} \geq 0 \perp -a_{b,t,o,\omega}^{ST,ch} \cdot P_{b,t,o}^{ST,ch} + p_{b,t,o,\omega}^{ST,ch} \geq 0 \quad (\text{a30})$$

$$\bar{\mu}_{b,t,o,\omega}^{ch} \geq 0 \perp a_{b,t,o,\omega}^{ST,ch} \cdot \bar{P}_{b,t,o}^{ST,ch} - p_{b,t,o,\omega}^{ST,ch} \geq 0 \quad (\text{a31})$$

$$\mu_{b,t,o,\omega}^{dch} \geq 0 \perp -a_{b,t,o,\omega}^{ST,d-ch} \cdot P_{b,t,o}^{ST,dch} + p_{b,t,o,\omega}^{ST,dch} \geq 0 \quad (\text{a32})$$

$$\bar{\mu}_{b,t,o,\omega}^{dch} \geq 0 \perp a_{b,t,o,\omega}^{ST,dch} \cdot \bar{P}_{b,t,o}^{ST,dch} - p_{b,t,o,\omega}^{ST,dch} \geq 0 \quad (\text{a33})$$

$$\mu_{b,t,o,\omega}^{d-up} \geq 0 \perp d_{b,t,o,\omega}^{up} \geq 0 \quad (\text{a34})$$

$$\bar{\mu}_{b,t,o,\omega}^{d-up} \geq 0 \perp \bar{D}_{b,t,o,\omega}^{up} - d_{b,t,o,\omega}^{up} \geq 0 \quad (\text{a35})$$

$$\mu_{b,t,o,\omega}^{d-dn} \geq 0 \perp d_{b,t,o,\omega}^{dn} \geq 0 \quad (\text{a36})$$

$$\bar{\mu}_{b,t,o,\omega}^{d-dn} \geq 0 \perp \bar{D}_{b,t,o,\omega}^{dn} - d_{b,t,o,\omega}^{dn} \geq 0 \quad (\text{a37})$$

$$\bar{\phi}_{t,o,\omega} \geq 0 \perp \left(\frac{PL \times \sum_{b \in BL} L F_o P_{b,t,\omega}^D}{-\sum_{u \in U} \sum_{i \in I^u} \sum_{b \in BU} p_{b,i,t,o,\omega}^u - \sum_{b \in BST} p_{b,t,o,\omega}^{ST,d-ch}} \right) \geq 0 \quad (\text{a38})$$

The KKT complementary conditions in (a15)–(a38) are nonlinear. The equivalent linear form of each complementary condition is achieved via a set of binary variables z and sufficiently-large M parameters. As an example, in the following the linearized form of (a15) using the mentioned procedure is addressed. The linearized form of other KKT complementary conditions can be similarly derived.

$$\sum_{\tau=1}^t x_{b,i,\tau}^u - y_{b,i,t,o}^u \geq 0 \quad (\text{a39})$$

$$\partial_{u,b,i,t,o}^U \geq 0 \quad (\text{a40})$$

$$\sum_{\tau=1}^t x_{b,i,\tau}^u - y_{b,i,t,o}^u \leq (1 - \bar{z}_{u,b,i,t,o}^U) \times M \quad (\text{a41})$$

$$\partial_{u,b,i,t,o}^U \leq \bar{z}_{u,b,i,t,o}^U \times M' \quad (\text{a42})$$

B. Load Points' Peak Demand in Test System 2

The nodal peak demand in test system 2 for three time stages is presented in Table XII.

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