Analytical Power Loss Sensitivity Analysis in Distribution Systems

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***Abstract*—Distribution systems are experiencing various structural changes with increased integration of distributed energy resources (DERs), electric vehicles (EVs), and energy storage systems (ESSs). These changes are forcing utilities to carefully track operational parameters such as voltage states, line flows and system power losses. Studying the sensitivity of power loss in the system aids in several downstream applications such as optimal DER and EV management. Conventional methods for studying the impact DER and EV integration on losses are based on classical methods that utilize either perturb and observe methods or the system Jacobian for deriving the sensitivity of losses with respect to complex power changes in the system. These classical methods are slow and lack flexibility. Therefore, the development of a quick, yet accurate method for analyzing system losses is essential for enhancing system efficiency. The present work tackles this problem and proposes a new low-complexity analytical approach to compute the sensitivity of individual line and system losses to changes in complex power at any system nodes. The proposed approach is tested on the IEEE 69 distribution system and verified against classical load flow based method. Results indicate that the proposed method is accurate and has an edge compared to classical load flow based method in terms of computational complexity and execution time.**

***Index Terms*—Distribution Networks, Power Loss, Sensitivity Analysis, Computational Efficiency, Distributed Generation**

I. Introduction

The rapid growth and integration of distributed energy resources (DERs), electric vehicles (EVs), and energy storage systems (ESSs) brings along new dimensions of uncertainty and variability in distribution systems. The variability in power at different system nodes can cause additional power loss in the system, which in turn may result in significant economic losses [1]. Power loss analysis is an essential aspect of distribution grid planning and control applications. This includes but is not limited to system loss minimization applications that exploit vehicle to grid (V2G) based reactive power compensation [2], electric vehicle charging scheduling [3], DER placement and sizing [4], and DER active and reactive power curtailment [5]. Thus, understanding the sensitivity of losses to power changes is crucial for improving the efficiency of the system.

In general, sensitivity analysis is a powerful tool for distribution system planning. For instance, voltage sensitivity analysis helps utilities understand the impact of system dynamics on voltage states, which can be used for voltage violation detection [6], [7] or photovoltaic (PV) hosting capacity analysis [8].

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Similarly, loss sensitivity has gained increased attention as a planning tool for optimal placement and sizing of DERs [1], [9], [4], [10], [11], [12]. In [9], authors present an analytical method to find the optimal locations for integrating DER in a distribution system using bus admittance matrix, generation information, and load distribution of the system. Similarly, [4] considers voltage profile improvement and loss indices for optimal sizing and placement of multiple DER units in the system. It has been shown that the integration of DERs helps improve system voltage stability margin [13] and reduce system losses [14]. Among the existing methods for power loss sensitivity, classical Newton-Raphson method is the most widely adopted. This approach is based on computing the inverse of the Jacobian matrix of the system, which then can be used to guide maximum nodal sensitivity criteria for ranking candidate nodes based on loss sensitivity [15]. Other approaches for power loss sensitivity include perturb and observe methods such as incremental [16], proportional-sharing, and Z-bus loss allocation [17]. The complexity of all these methods increases with the increase in network size. For instance, incremental sharing method requires detailed derivation for networks with size larger than 4 nodes [16]. Similarly, computing the domains and commons in Z-bus allocation method is complex for large networks and requires memory [17]. Therefore, prior work on loss sensitivity analysis is generally based on static methods that require recomputing system states for a given change in power injection or consumption, making these methods computationally expensive and impractical for real-time applications.

Understanding the impact of DER, EV, and ESS penetration especially in the context of future transactive energy markets require quick methods for computing system loss sensitivity in order to guide optimal planning and operational control strategies. Therefore, this work presents a computationally efficient analytical method for quantifying losses in the system. The proposed loss sensitivity analysis (LSA) utilizes existing system topology knowledge and analyzes the impact of changes in power injection/consumption at any node in the network. The major scientific contributions of this work include:

- A new computationally efficient analytical method is proposed to compute loss sensitivity at any line in the network due to changes in active and reactive power injections at multiple consumer locations.
- The proposed approach does not rely on load flow com-

2021 IEEE Power & Energy Society General Meeting (PESGM) | 978-1-6654-0507-2/21/$31.00 ©2021 IEEE | DOI: 10.1109/PESGM46819.2021.9637980

putations, which makes it applicable to real-time planning and operational control applications. The accuracy of the proposed method is validated against Newton-Raphson based power flow method and offers over 94% accuracy in computing loss sensitivity.
- The complexity of the proposed analytical approach is significantly lower than classical load flow-based methods with an order faster execution time.

II. Background: Loss Sensitivity Analysis

Consider a distribution system with $\mathcal{L}$ lines $\mathcal{N}$ nodes consisting of 1 slack bus and n_{PV} PV (generator) buses and n_{PQ} PQ (load) buses. Change in complex power at any node (ΔS) will result in a change of line losses at all system lines. In this paper, the terms node/bus and edge/line are used interchangeably. Computing the change in line losses is related to the voltage change at system nodes for a given ΔS. Load flow algorithm can be used to compute the change in voltage magnitude ($|\Delta V_k|$) and angle ($\Delta \delta_k$) of node $k \in \mathcal{N}$ given a change in power injection/consumption ΔS. In this case, the known system parameters are the real and reactive power of PV and PQ nodes and the unknown parameters are the voltage magnitude and angle as represented by the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, i.e.,

$$\boldsymbol{x} = [\delta_2 ... \delta_{\mathcal{N}} | V_2 ... V_{\mathcal{N}}]^{'}, \quad \boldsymbol{y} = [P_2 ... P_{\mathcal{N}} | Q_2 ... Q_{\mathcal{N}}]^{'} \tag{1}$$

Here, $\mathcal{N} = n_{PV} + n_{PQ} + 1$, δ is the phase angle of PV and PQ nodes, and V represents the voltage magnitude. Node 1 is not included in the analysis as it represents the slack bus with a stable voltage of $1\angle 0$ p.u. Since power ($\boldsymbol{y}$) is the non-linearly related to voltage ($\boldsymbol{x}$) via the power flow equations, the change in power ($\Delta \boldsymbol{y}_j$) can be approximated as

$$\Delta \boldsymbol{y}_j = \boldsymbol{J}_j \Delta \boldsymbol{x}_j, \tag{2}$$

where, $\boldsymbol{J}_j$ is the Jacobian on each iteration j and the power mismatch vector is $\Delta \boldsymbol{y}_j = \boldsymbol{y} - \boldsymbol{f}(\boldsymbol{x_j})$ with $\boldsymbol{f}(\boldsymbol{x_j})$ representing the power flow equations. The elements of the Jacobian matrix are the partial derivatives of power flow equations with respect to δ and V. Using the Jacobian matrix, one can calculate the unknowns voltage magnitude and angle of nodes of the system

$$\Delta \boldsymbol{x} = \boldsymbol{J}^{-1} \Delta \boldsymbol{y} \tag{3}$$

For determining the voltage change associated with a given change in power at system nodes, the change in edge current flows in the system can be calculated based on the admittance matrix $\boldsymbol{Y}$ as,

$$\boldsymbol{\Delta I} = \boldsymbol{Y} \cdot \boldsymbol{\Delta V} \tag{4}$$

Now, consider the line $l \in \mathcal{L}$ connecting nodes $m, n \in \mathcal{N}$ with impedance of $Z_{mn} = R_{mn} + jX_{mn}$. Let node m be the origin node and n be the destination as illustrated in Fig. 1, where power flows from the origin to the destination node. Calculating the change in current flow from origin to destination allows calculating the change in active and reactive power loss through that line (ΔL_{mn}) as [18],

$$\Delta L_{mn} = |\Delta I_{mn}|^2 Z_{mn} \tag{5}$$

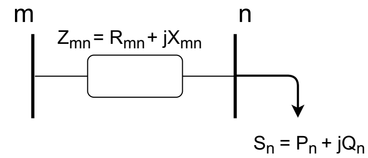

Fig. 1. Illustration of power flow from origin to destination node.

5 can also be used to determine the change in total system active and reactive power loss (ΔL_s) as

$$\Delta L_s = \sum_{l \in \mathcal{L}} |\Delta I_l|^2 Z_l \tag{6}$$

The applicability of this method depends on the availability of the Jacobian matrix of the system. In many cases, it may not be easy to obtain the Jacobian matrix accurately. Even if the Jacobian matrix is accessible, recomputing system states for a given change in power profile in the system adds to the computational complexity. Therefore, section III presents a new analytical method that is accurate and computationally efficient for loss sensitivity analysis in distribution systems.

III. Analytical Loss Sensitivity Analysis

The change in complex power drawn/injected at any node $a \in \mathcal{N}$ results in changes in node voltages and line losses. The node where complex power changes is called an actor node (a) and the node where voltage is monitored is referred to as observation node (m). The line where complex power loss is monitored is referred to as an observation line (l). When complex power at a changes from S_a to $S_a + \Delta S_a$, then voltage at node m changes from V_m to $V_m + \Delta V_{ma}$ and complex power loss at line l changes from L to $L^{'}$. If we consider a single actor node, then, the change in active and reactive power loss (ΔL_{mn}) at line l is given by Theorem 1.

Theorem 1. *For a radial distribution network, the change in complex power loss at a line l_{mn} due to complex power change at an actor node a can be approximated as:*

$$\Delta L_{mn} \approx Z_{mn} \frac{S^*_m}{V^*_m}^{2} \left[\frac{1+k_1}{1+k_2}^{2} - 1 \right] \tag{7}$$

where $k_1 = \frac{\Delta S^*_m}{S^*_m}$ is the ratio of power change to base load at the origin node m of line mn; $k_2 = -\frac{\Delta S^*_a Z_{ma}}{V^*_a V^*_m}$ is the ratio of voltage change to the base voltage at origin node m of the line mn; ΔS^*_a is the conjugate of change in power at actor node a that causes voltage change and consequently the loss change; Z_{ma} is the impedance of the path shared by origin node m and actor node a from the source node as illustrated in Fig. 2 by the bold arrow; S^*_m and V^*_m are the conjugate base load and base voltage at origin node m, respectively. Fig. 2 pictorially illustrates the terms described above.

Proof. Complex power loss at line l with impedance Z_{mn} can be computed using Joule's first law. Let I_k be the complex current drawn by load connected to node k, and i_{mn} be the complex line current flowing through line l. The complex power loss at line l connecting nodes m and n can be written as,

$$L_{mn} = |i_{mn}|^2 Z_{mn}. \tag{8}$$

Total current flowing thought line l from $\mathcal{N}$ nodes can be written as,

$$i_{mn} = \sum_{k \in \mathcal{N}} I_k. \tag{9}$$

Substituting (9) in (8) we get

$$L_{mn} = \left| \sum_{k \in \mathcal{N}} I_k \right|^2 Z_{mn}. \tag{10}$$

Now if current drawn by all nodes changes from I_k to $I_k^{'}$, change in losses (L_{mn}) due to change in current of multiple nodes can be written as,

$$L_{mn}^{'} = \left| \sum_{k \in \mathcal{N}} I_k^{'} \right|^2 Z_{mn}. \tag{11}$$

where $I_k^{'} = I_k + \Delta I_k$. The effective change in line losses ($\Delta L_{mn} = L_{mn}^{'} - L_{mn}$) can be expressed as,

$$\Delta L_{mn} = \left[\left| \sum_{k \in \mathcal{N}} I_k + \Delta I_k \right|^2 - \left| \sum_{k \in \mathcal{N}} I_k \right|^2 \right] Z_{mn} \tag{12}$$

The current drawn by node k can be written as $\frac{S_k^*}{V_k^*}$, where S_k^* is the complex conjugate of power drawn/injected at node k, and V_k^* is complex conjugate of voltage at node k. Therefore,

$$\Delta L_{mn} = \left[\left| \sum_{k \in \mathcal{N}} \frac{S_k^* + \Delta S_k^*}{V_k^* + \Delta V_k^*} \right|^2 - \left| \sum_{k \in \mathcal{N}} \frac{S_k^*}{V_k^*} \right|^2 \right] Z_{mn} \tag{13}$$

Here, ΔV_k^* is the complex conjugate of voltage change at node k due to complex power change at actor node. This voltage change can be approximated using our analytical voltage sensitivity analysis (VSA) as [19]

$$\Delta V_{ma} \approx - \frac{\Delta S_a^* Z_{ma}}{V_a^*} \tag{14}$$

where V_a^* is the complex conjugate of voltage at actor node a and Z_{ma} is the mutual path impedance between observation node m and actor node a. For a given complex load in a radial distribution network at node n, the origin node is m, through an impedance of Z_{mn}. Furthermore, the change in voltage

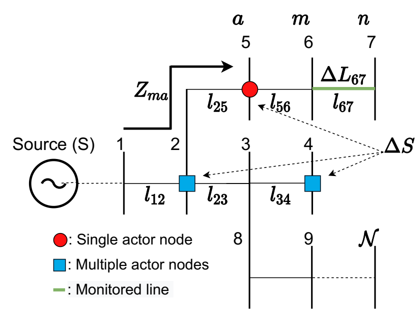

Fig. 2. Illustration of LSA method.

can be caused by nodes other than m. As such, ΔV_k becomes ΔV_{ma} Therefore, (13) reduces to,

$$\begin{aligned}
\Delta L_{mn} &= \left[\left| \frac{S_m^* + \Delta S_m^*}{V_m^* + \Delta V_{ma}^*} \right|^2 - \left| \frac{S_m^*}{V_m^*} \right|^2 \right] Z_{mn} \\
&= \left[\left| \frac{S_m^*(1 + \frac{\Delta S_m^*}{S_m^*})}{V_m^*(1 + \frac{\Delta V_{ma}^*}{V_m^*})} \right|^2 - \left| \frac{S_m^*}{V_m^*} \right|^2 \right] Z_{mn} \\
&= Z_{mn} \left| \frac{S_m^*}{V_m^*} \right|^2 \left[\left| \frac{1 + \frac{\Delta S_m^*}{S_m^*}}{1 + \frac{\Delta V_{ma}^*}{V_m^*}} \right|^2 - 1 \right] \\
&= Z_{mn} \left| \frac{S_m^*}{V_m^*} \right|^2 \left[\left| \frac{1 + \frac{\Delta S_m^*}{S_m^*}}{1 + \frac{\Delta S_a^* Z_{ma}}{V_a^* V_m^*}} \right|^2 - 1 \right] \\
&= Z_{mn} \left| \frac{S_m^*}{V_m^*} \right|^2 \left[\left| \frac{1 + k_1}{1 + k_2} \right|^2 - 1 \right]
\end{aligned}$$

with $k_1 = \frac{\Delta S_m^*}{S_m^*}$ representing the ratio of power change to base load at the origin node m of line mn and $k_2 = -\frac{\Delta S_a^* Z_{ma}}{V_a^* V_m^*}$ is the ratio of voltage change to the base voltage at origin node m. □

To compute the loss change for the overall system, we can use Theorem 1 to analytically compute loss change for each line of the network and then aggregate it to obtain the cumulative loss change. In modern distribution systems, power varies at multiple locations across the network and thus Corollary 1 provides an analytical expression to obtain total loss change under this scenario.

Corollary 1. *The change in system complex power loss (ΔL_s) due to multiple actor nodes ($a \in \mathcal{A}$) is the cumulative change in losses due to each actor node and is approximated as,*

$$\Delta L_{mn} \approx \sum_{l \in \mathcal{L}} Z_{mn} \left| \frac{S_m^*}{V_m^*} \right|^2 \left[\left| \frac{1 + k_1}{1 + k_2} \right|^2 - 1 \right] \tag{15}$$

where $\mathcal{L}$ is the set containing all the lines of the network; $k_2 = -\sum_{a \in \mathcal{A}} \frac{\Delta S_a^* Z_{ma}}{V_a^* V_m^*}$ is the ratio of voltage change to the base voltage at origin node m of the branch mn.

To illustrate the LSA method using single and multiple actor nodes, consider again the system illustrated in Fig. 2. Case 1

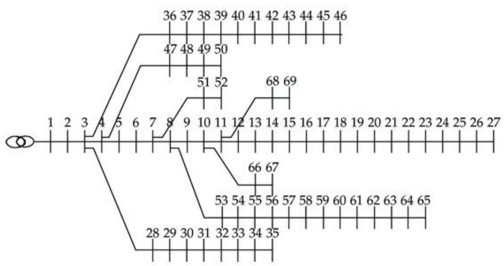

Fig. 3. IEEE 69 bus test system.

represents a single actor node changing its complex power as highlighted by a red circle in the figure. The complex power loss ΔL_{67} is monitored at line l_{67} connecting nodes 6 and 7, highlighted by green color. In this case, the actor node is 5 and ΔL_{67} is due to only node 5. However, the proposed method is generic and all nodes in the network can be actor nodes. ΔL_{67} can be computed using Theorem 1 as follows,

$$\Delta L_{67} = Z_{67} \, {\frac{S_6^*}{V_6^*}}^2 \left[{\frac{1+\frac{\Delta S_6^*}{S_6^*}}{1+\frac{\Delta V_{65}^*}{V_6^*}}}^2 - 1 \right] \tag{16}$$

where, Z_{67} is the impedance of line l_{67} and ΔV_{65}^* is the change in voltage at node 6 due to complex power change at node 5, which can be computed using (14). In the second case, two actor nodes (2 and 4) are considered and the same line loss (ΔL_{67}) is monitored. In this case, the cumulative effect of complex power change at multiple actor nodes on ΔL_{67} can be computed using Corollary 1 considering only line l_{67}. Then the term ΔV_{65}^* in (16) becomes $\frac{\Delta V_{62}^*+\Delta V_{64}^*}{V_6^*}$, which captures the voltage change due to complex power change at nodes 2 and 4. This can be computed for all line losses in the system, which provides the change in overall system losses ΔL_s. This method is generic to all choices of actor nodes, observation nodes, and observation lines. In the following section, the method is validated on an the IEEE 69 bus test system.

IV. Simulation and Results

The proposed loss sensitivity method is verified via simulation on the IEEE 69 bus test system using an intel i7 processor based PC. The IEEE 69 bus test system comprises of 69 nodes and 68 edges with a nominal voltage of 12.66 kV as shown in Fig. 3. The results of the proposed loss sensitivity method are compared to a classical Newton-Raphson based method presented in section II. In this section, three scenarios are explored to demonstrate the effectiveness of the proposed method. It is assumed that all nodes in the system can be active consumers, i.e., they can change their complex power. The proposed method can compute change in real and reactive power loss but we are reporting real power losses due to space limitations. For the first case study, the impact of active power change of a single actor node on a single line is examined. The actor nodes are chosen randomly, i.e., 4, 15, 19, and 23. Table I shows a comparison between change in active power loss at the monitored line (in kW) and the respective execution time

TABLE I
Active power loss change due to single actor node.

Actor node	Monitored line	ΔL_{mn} (kW)		ET (S)	
		Sim.	LSA	Sim.	LSA
4	4-47	0.0039	0.0039	0.1975	0.0297
35	34-35	0.0045	0.0045	0.3772	0.0831
15	15-16	0.0352	0.0350	0.2081	0.0446
19	19-20	0.0381	0.0379	0.2054	0.0449
46	45-46	0.0038	0.0040	0.3731	0.0891
23	23- 24	0.0631	0.0628	0.2015	0.0431

(ET) using classical Newton-Raphson based method (Sim.) and the proposed (LSA) method based on Theorem 1. The results show that the proposed method is not only accurate in estimating line loss but also computationally more efficient with an order faster execution time. It is important to note that the computational advantage of the proposed method amplifies as the network size grows. In addition, one can compute the total losses in the network. If, for instance, a 50 kW is injected at node 19, then the corresponding total active power loss would be 0.6411 and 0.6402 kW using classical Newton-Raphson and the proposed method, respectively. The second case deals with computing the sensitivity of line losses in the system due to multiple actor nodes changing their active and reactive power simultaneously. The reactive power change is chosen in such a way that it is reasonably less than the active power change in order to reflect real-world scenarios. Table II shows the actor nodes and the respective amount of complex power change. Negative active or reactive power change represents a decrease in power consumption or increase in injection. Positive active or reactive power change represents increase in consumption or decrease in injection. This setup could represent a PV DER connected to node 15 and an EV charging station connected to node 41. Fig. 4 shows the change in active power losses at all system lines due to the actor nodes in table II. The figure illustrates that negative power withdrawal (or increased injection) can result in negative power loss at the respective lines in the system. For example, lines 12 and 61 show a slight reduction in active power loss. This result is expected since the line flows in the system are reduced due to increased injections at nodes 15 and 23, which means more of the demand at those nodes is being supplied locally. Again, execution time (in sec.) for this case is 0.23 using simulation and 0.045 using the proposed method. The last case presents the value of the proposed method in real-time system loss monitoring. For this case, a hypothetical PV generation profile is used to represent the trend of power change at node 33. The PV profile is not shown due to space limitations. The change in active system loss is illustrated in

TABLE II
Complex power change at multiple actor nodes.

Node	ΔP (kW)	ΔQ (kVAr)
15	-40	0
23	-14	2
41	18	6

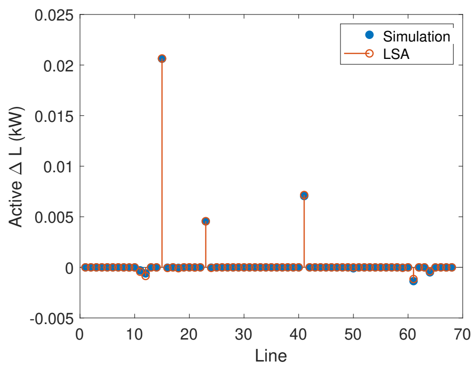

Fig. 4. Change in line active power loss.

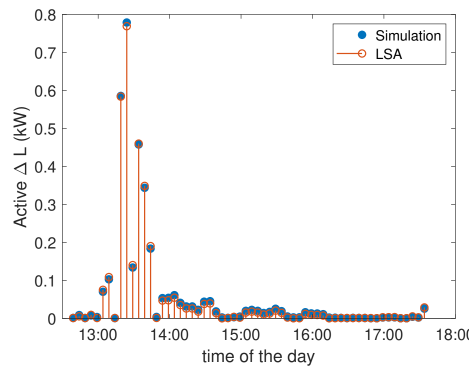

Fig. 5. Change in system overall active power loss.

Fig. 5. In this case, the time step at which the PV generation gets recorded is 15 minutes. However, the proposed method is general to accommodate smaller time steps. For this case, the execution time using simulation is 5.91 sec whereas it is 0.11 sec using LSA.

V. Conclusion

Modern distribution systems are characterized by increasing penetration of DERs, EVs, and ESSs. This necessitates the need for developing efficient and quick methods for analyzing the performance and monitoring losses of the system, which can help utilities in their management and planning operations. Classical methods of loss sensitivity can become computationally expensive, especially with increase in network size and are not generally suitable for real time applications. For this reason, this paper proposes a novel computationally efficient analytical method for quickly computing the sensitivity of system losses to changes in nodal complex power. The method is validated against existing Newton-Raphson based load flow method and results demonstrate that the method is accurate with significant saving in execution time. This method can empower several execution-time sensitive applications in modern grids including EV/DER management.

VI. Acknowledgment

This material is based upon work partly supported by National science foundation under award # 1855216.

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