Computation and Stability Analysis of Periodically Stationary Pulses in a Short Pulse Laser

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Abstract

A significant challenge in the modeling of short pulse fiber lasers is that with each successive generation there has been a dramatic increase in the amount by which the pulse varies over each round trip. Therefore, lumped rather than averaged models are required to accurately compute the periodically stationary (breather) solutions generated by these lasers.

We use a spectral method to assess the linear stability of periodically stationary pulses in lumped models. This approach extends previous work by Menyuk and Wang [3] on stationary pulses in averaged models. We first present a gradient based optimization method inspired by the work of Ambrose and Wilkening [1] to compute periodically stationary pulses. Then, we use Floquet theory to characterize the linear stability of the pulses obtained using optimization in terms of the spectrum of the monodromy operator, $\mathcal{M}$, obtained by linearization of the round trip operator about a periodically stationary pulse. Building on work of Zweck et al. [5], we prove the existence and uniqueness of $\mathcal{M}$ using the theory of evolution semigroups [4]. Then, we prove that the essential spectrum of $\mathcal{M}$ is equal to the essential spectrum of the asymptotic monodromy operator, $\mathcal{M}_\infty$. Finally, since $\mathcal{M}_\infty$ acts as a multiplication operator in the Fourier domain, we are able to obtain an explicit formula for the essential spectrum of $\mathcal{M}$.

For the numerical implementation, we consider an experimental stretched pulse laser designed by Kim et al. [2]. We present results showing agreement between the essential spectrum obtained using a matrix discretization of $\mathcal{M}$ and the formula. We also perform parameter continuation studies in which we observe bifurcations from stable to unstable periodically stationary pulses. Our work represents the first stability analysis of periodically stationary pulses in a realistic lumped model of an experimental short pulse laser.

References

[1] D. M. Ambrose and J. Wilkening. Computing time-periodic solutions of nonlinear systems of partial differential equations. In *Hyperbolic Problems: Theory, Numerics and Applications (In 2 Volumes)*, pages 273–280. World Scientific, 2012.

[2] H. Kim, P. Qin, Y. Song, H. Yang, J. Shin, C. Kim, K. Jung, C. Wang, and J. Kim. Sub-20-attosecond timing jitter mode-locked fiber lasers. *IEEE J. Sel. Top. Quantum Electron*, 20(5):260–267, 2014.

[3] C. R. Menyuk and S. Wang. Spectral methods for determining the stability and noise performance of passively modelocked lasers. *Nanophotonics*, 5(2):332–350, 2016.

[4] A. Pazy. *Semigroups of linear operators and applications to partial differential equations*, volume 44. Springer Science & Business Media, 2012.

[5] J. Zweck, Y. Latushkin, J. L. Marzuola, and C. K. R. T. Jones. The essential spectrum of periodically-stationary solutions of the complex Ginzburg-Landau equation. *J. Evol. Equ.*, pages 1–17, 2020.