Editors' Suggestion

Stability of Superdiffusion in Nearly Integrable Spin Chains

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Superdiffusive finite-temperature transport has been recently observed in a variety of integrable systems with non-Abelian global symmetries. Superdiffusion is caused by giant Goldstone-like quasiparticles stabilized by integrability. Here, we argue that these giant quasiparticles remain long-lived and give divergent contributions to the low-frequency conductivity $\sigma(\omega)$, even in systems that are not perfectly integrable. We find, perturbatively, that $\sigma(\omega) \sim \omega^{-1/3}$ for translation-invariant static perturbations that conserve energy and $\sigma(\omega) \sim |\log \omega|$ for noisy perturbations. The (presumable) crossover to regular diffusion appears to lie beyond low-order perturbation theory. By contrast, integrability-breaking perturbations that break the non-Abelian symmetry yield conventional diffusion. Numerical evidence supports the distinction between these two classes of perturbations.

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Introduction.—Many paradigmatic quantum models in one dimension are integrable, including the Heisenberg and Hubbard models; thus, *approximate* integrability is ubiquitous in one-dimensional quantum materials and strongly influences their response properties [1]. Although integrable models are in some sense "exactly solvable," their finite-temperature and nonequilibrium dynamical properties have proved difficult to compute exactly. However, recent years have seen enormous progress in our understanding of the dynamics of integrable systems [2,3], due to the development of improved numerical techniques [4–15], as well as recent experiments [16–23] and new theoretical methods such as generalized hydrodynamics [24–50]. Perhaps the most surprising outcome of these developments has been the discovery that superdiffusive spin transport [4,11], with the dynamical scaling exponent $z = \frac{3}{2}$ and scaling functions [12] that appear to lie in the Kardar-Parisi-Zhang (KPZ) universality class [51], is generic in integrable spin chains that are invariant under a continuous non-Abelian symmetry [13,52,53]. Superdiffusive spin transport was also observed in recent inelastic neutron scattering experiments [23]. Although superdiffusion is not yet fully understood, it has become clear (from multiple lines of argument) that the degrees of freedom responsible for it are "giant quasiparticles" of the integrable model [41,54–58], corresponding to large solitonic wave packets made up of Goldstone modes [53,59]. These giant quasiparticles consist of local rotations of the non-Abelian vacuum and therefore cost little energy. Because these systems are integrable, giant quasiparticles are stable even at finite temperature and therefore nontrivially influence spin and charge transport.

The fate of superdiffusion when integrability is weakly broken is a question of great experimental relevance [23]. Away from integrability, neither exact solutions nor infinitely stable quasiparticles exist; the theoretical tools we have to deal with this regime are still primitive [58,60–66]. A natural framework to address weakly decaying quasiparticles is the Boltzmann equation [67,68], which relies on the matrix elements of generic local operators between eigenstates of the integrable system. These matrix elements are in general unknown, despite some recent progress [40,69]; their asymptotic form is known in a few special limits, such as slowly fluctuating noise [70] or atom losses [71,72]. In the present Letter, we show that symmetry considerations constrain the effects of integrability breaking and determine the fate of superdiffusion in the limit of weak integrability breaking. In particular, local perturbations that are invariant under the global non-Abelian symmetry *cannot* scatter giant quasiparticles, because these quasiparticles look locally like the vacuum. For a giant quasiparticle of size ℓ, the fastest possible decay allowed by symmetry is ℓ^{-2}; phase-space constraints can suppress this further. As a consequence, *within* perturbation theory, it appears that some form of anomalous diffusion survives: for symmetry-preserving noise, one has a logarithmically divergent diffusion constant, whereas for symmetry-preserving Hamiltonian perturbations, the KPZ scaling is unaffected by the perturbation to leading order. By contrast, perturbations that break the non-Abelian symmetry can immediately dismember giant quasiparticles and restore diffusion. The distinction between these two types of behavior is clearly visible in our numerical studies of spin transport.

Heisenberg spin chain.—Although our analysis will extend to other isotropic integrable models, we will focus for concreteness on the perturbed Heisenberg spin chain

$$H = H_0 + gV, \qquad H_0 = J\sum_n \vec{S}_n \cdot \vec{S}_{n+1}. \qquad (1)$$

Here, $\vec{S}_j$ are spin-$\frac{1}{2}$ operators on site j, V is a possibly time-dependent integrability-breaking perturbation with g small, and we will set $J = 1$ in the following. Spin transport in the integrable limit $g = 0$ is known to be *superdiffusive*, while energy transport is purely ballistic. For simplicity, we will focus on the high-temperature regime, although our conclusions carry over to arbitrary finite temperatures. We will characterize spin transport by the ac spin conductivity, which at high temperature is given by the Kubo formula

$$\sigma(\omega) = \beta \int_0^\infty dt e^{i\omega t}\langle J(t) j_0(0)\rangle_\beta, \qquad (2)$$

with the spin current $j_n = -i(S_n^+ S_{n+1}^- - \text{H.c.})$, $J = \sum_n j_n$, and with $\langle \ldots \rangle_\beta$ referring to equilibrium ensemble average at temperature β^{-1}.

We first summarize the key ingredients responsible for spin superdiffusion in the model (1) in the integrable $g = 0$ limit. Here, the ac conductivity scales as $\sigma(\omega) \sim \omega^{-1/3}$, corresponding to a dynamical exponent $z = 3/2$. This exponent can be understood by analyzing the quasiparticles of the Hamiltonian H_0. Those excitations are built starting from a reference ferromagnetic state ("vacuum") and adding (nontrivially dressed) magnons and bound states thereof called "strings." Strings are labeled by a quantum number $s = 1, 2, \ldots$ (with $s = 1$ corresponding to single magnons) and a continuous momentum k_s that will play little role in our analysis. The thermodynamic properties of these quasiparticles are known exactly from the thermodynamic Bethe ansatz solution of H_0: for example, their density in an equilibrium finite-temperature Gibbs state scales as $\rho_s \sim 1/s^3$. At large s, they have been identified with macroscopically large solitons made out of interacting Goldstone modes (slow modulations of the vacuum orientation) above the ferromagnetic vacuum [57,59]. While it might seem counterintuitive that long-wavelength modes matter to high-temperature physics, their stability is protected by integrability. Their properties follow from ferromagnetic Goldstone mode physics: a string s has width (size) s and is made out of Goldstone modes with momentum $\sim s^{-1}$; thus they have a small energy density $\sim s^{-2}$ using the dynamical exponent $z = 2$ of ferromagnetic Goldstone modes. The corresponding velocity is $v_s \sim 1/s$, so large-s strings move slowly. The energy carried by each string is suppressed as $\varepsilon_s \sim s^{-1}$ (consistent with the intuition that they are "soft" excitations), so we immediately see that energy transport in this model is ballistic (since the quasiparticles have a finite velocity), and is dominated by small strings.

Superdiffusion in the integrable limit.—We now turn to spin transport, which is dominated by large strings instead. We give an argument for spin superdiffusion that is reformulated from Refs. [41,56] in a way that is physically more transparent. When propagating in vacuum, s strings carry a spin $m_s = s$ (their number of magnons). In thermal states, strings are "screened" via nonperturbative dressing due to interaction effects and effectively become neutral [73,74]; their dressed, or effective, magnetization vanishes $m_s^{\text{dr}} = 0$ in any thermal state with no net magnetization. Specifically, an s string is screened when it encounters an $s' > s$ string (just as a magnon is screened when it passes through a domain wall [41]); such collisions set the "lifetime" for an s string to transport magnetization. The density of strings bigger than s is $\rho_{s'>s} = \sum_{s'>s} \rho_s \sim 1/s^2$; thus, an s string moving at speed $v_s \sim 1/s$ first encounters a larger string on a timescale $\tau_s^0 \sim (1/\rho_{s'>s})/v_s \sim s^3$. This characteristic timescale was identified using different approaches in previous works [41,56,75] and underlies the physics of superdiffusion. At a given time t, such that $t \gg \tau_s^0$, small strings are screened, namely, $m_s^{\text{dr}} = 0$, and do not contribute to transport. As a result, as time increases, spin transport is dominated by strings with larger and larger s, namely, such that $\tau_s^0 \sim s^3 \sim t$.

We now apply this logic to the Kubo formula (2). At a time t inside the integral in (2), strings with $s \le t^{1/3}$ have been completely screened and carry no net current, whereas those with $s \ge t^{1/3}$ still carry their original current $j_s \sim v_s m_s = \mathcal{O}(1)$, where we used $v_s m_s \sim s^{-1} \times s$. As the screening gives rise to exponential decay of their contributions to spin current [76], one can write $\langle J(t) j_0(0)\rangle \sim \sum_s \rho_s (m_s v_s)^2 e^{-t/\tau_s^0}$. Plugging into the Kubo formula,

$$\sigma(\omega) \sim \int_0^\infty dt e^{i\omega t} \sum_{s\ge 1} s^{-3} e^{-t/s^3} \sim \omega^{-1/3}. \qquad (3)$$

As expected, the conductivity diverges at low frequency with an exponent corresponding to $z = 3/2$. The precise form of the exponential cutoff (3) is not important and can be replaced with any integrable function of t/s^3. However, Eq. (3) has an appealing physical interpretation where each string gives a Lorentzian (diffusive) contribution to the conductivity of width $1/\tau_s^0 \sim s^3$.

Quasiparticle lifetimes.—We now consider the effects of (small) integrability-breaking perturbations $g \neq 0$, which preserve the total z component of the magnetization $S^z = \sum_n S_n^z$ and may preserve the energy, but break all other conservation laws. We incorporate integrability-breaking terms by writing a Boltzmann equation for the quasiparticles, with a collision integral—incorporating quasiparticle scattering and decay—computed perturbatively using Fermi's golden rule

[67,68]. This approach is valid in the "Boltzmann regime" at long times and small g, with tg^2 fixed; we will discuss the roles of higher-order terms in perturbation theory below. At the level of our analysis, the main effect of the perturbation V is to give a new, finite lifetime τ_s, with decay rates $\Gamma_s = \tau_s^{-1}$ of order $\mathcal{O}(g^2)$. Starting from a representative thermal state $|\rho\rangle$, containing all strings with thermal densities ρ_s, those decay rates are obtained by summing over all possible accessible states $|n\rangle$ compatible with the residual conservation laws (energy and/or quasimomentum conservation), with a rate given by the square of the matrix element $|\langle n|V|\rho\rangle|^2$ with the appropriate density of state factors. The ac spin conductivity is then given by

$$\sigma(\omega) \sim \int_0^\infty dt e^{i\omega t} \sum_{s\geq 1} s^{-3} e^{-t/\tau_s^0} e^{-t\Gamma_s}. \tag{4}$$

As we will see below, different classes of perturbations lead to very different scalings of the rates Γ_s with s, with some perturbations allowing for long-lived giant strings.

Symmetry-breaking noisy perturbations.—We first consider a generic perturbation that breaks the spin-rotation symmetry. In this case, we expect the matrix element of the perturbation to be either $\mathcal{O}(1)$ or possibly to increase with s (since, for example, the charge of an s string is $m_s = s$). If we further assume that the integrability breaking is due to temporally fluctuating noise, then the density of available states is clearly nonvanishing, since an s string can scatter by changing its momentum. (This might not be the dominant process, of course.) In this case, $\Gamma_s \sim s^\alpha$ with $\alpha > 0$ in Eq. (4) leading to a finite dc value $\sigma^{\rm dc} = \lim_{\omega\to 0} \sigma(\omega)$. This is the expected behavior of integrability breaking, which generically should lead to diffusive transport for residual conserved charges. Note that Eq. (4) predicts that the finite-time diffusion constant $D(t) = (\beta/\chi) \int_0^t \sum_n \langle j_n(t) j_0(0)\rangle_\beta$, with χ the spin susceptibility, approaches its asymptotic value very quickly, with exponential convergence in time. While anomalous transport is washed out by the integrability-breaking perturbation in this case, remnants of the anomalous exponent $z = 3/2$ can be observed in the dependence of the diffusion constant on the integrability-breaking parameter g. If we convolve the anomalous integrable scaling $\omega^{-1/3}$ by a Lorentzian of width $\Gamma \sim g^2$, we immediately find

$$\sigma^{\rm dc} = \chi D \sim g^{-2/3}. \tag{5}$$

To leading order, the small g dependence of the susceptibility can be ignored, and we see that the diffusion constant scales in a nonanalytic way with g. Similar nonanalytic dependences of diffusion constants were reported in Refs. [67,77].

To check these predictions, we consider a noisy perturbation $V(t) = \sum_n \eta_n(t) S_n^z$, with η_n some classical correlated noise $\langle \eta_n(t)\eta_{n'}(t')\rangle = \gamma\delta(t-t')f(n-n')$, and $\gamma \sim g^2$ the strength of the noise. Such correlated noise can be expected to model generic external perturbations at large scales. We implemented time evolution using a Lindblad approach and matrix product operators (MPO) [78]. We considered both uncorrelated $[f(n) = \delta_{n,0}]$ and correlated $[f(n) \sim e^{-|n|/\xi}]$ noisy perturbations. Our results are consistent with diffusive transport and a diffusion constant scaling as $D \sim (\gamma/\xi)^{-1/3}$, as expected from (5) (Fig. 1) and with a relaxation time $\sim 1/\gamma$ [78].

Symmetric noisy perturbations.—We now turn to a much more interesting class of perturbations that preserve the spin-rotation SU(2) symmetry of H_0. Intuitively, the action of such perturbations on large strings should be suppressed since those are smooth vacuum rotations. In fact, Goldstone physics implies that the matrix elements of any SU(2)

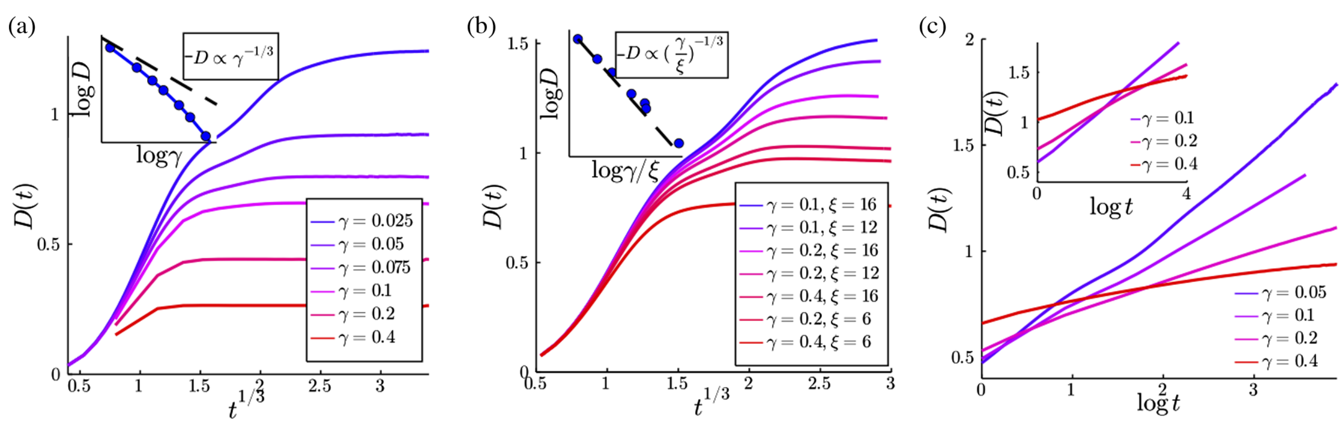

FIG. 1. Noisy perturbations. (a) Apparent diffusion constant $D(t)$ vs time t, for noise coupling to S_i^z with spatially uncorrelated fluctuations. The diffusion constant rapidly saturates to a value plotted in the inset vs noise strength $\gamma \sim g^2$. (b) Same as (a), but for noise with a spatial correlation length ξ. Inset: scaling of the diffusion constant compared to the prediction $D \sim (\gamma/\xi)^{-1/3}$. (c) Apparent logarithmic divergence of $D(t)$ vs t for noise that preserves SU(2) symmetry: both for noise coupling to the energy density (main panel) and for noise coupling to the energy current density (inset).

invariant must be suppressed with s at least as energy is, as $\varepsilon_s \sim 1/s$. The matrix element for large strings to decay is thus suppressed, so we can expect large strings to be long-lived, leading potentially to anomalous transport.

In order to compute the scaling of the decay rates Γ_s, we need to consider the processes involved and the accessible density of states more carefully. For concreteness, we first consider the case of noise coupling to energy: $V(t) = \sum_n \eta_n(t)\vec{S}_n \cdot \vec{S}_{n+1}$ with η_n some uncorrelated white noise as before. As this perturbation breaks both energy and momentum conservation, the leading processes will be single particle-hole excitations where a given string s with momentum k_s will scatter into another mode with new momentum k_s' [67,70] (causing rapidity or momentum diffusion [70]). Strings s can also decay into multiple smaller strings while preserving S^z. Now the size of the Brillouin zone for an s string goes as $\sim 2\pi/s$, as $k_s \sim s^{-1}$, but the noise perturbation allows for processes scattering across different Brillouin zones, therefore the accessible density of states for a single particle-hole process does not scale with s [78]. As a result, the decay rates for an s string is set solely by the matrix element of the perturbation and scales as its square, namely, $\Gamma_s \sim |\langle k_s|V|k_s'\rangle|^2 \sim 1/s^2$. The late scaling can be deduced by the known low-energy limit of the matrix element $\lim_{k_s' \to k_s} \langle k_s|V|k_s'\rangle = \varepsilon_s^{\mathrm{dr}} \sim s^{-1}$ [40,79], where $\varepsilon_s^{\mathrm{dr}}$ is the dressed energy of the string. Giant strings are long-lived, but decay before they get screened. As this order in perturbation theory, this leads to a log divergence of $\sigma(\omega)$

$$\sigma(\omega) \sim \sum_s \frac{1}{s^3} \frac{s^{-2}}{\omega^2 + s^{-4}} \sim |\log \omega|, \tag{6}$$

which can be seen by noticing that the sum is approximately given by $\sum_{s \geq 1}^{\Lambda}(1/s)$ for $\Lambda \sim 1/\omega^{1/2}$. Remarkably, giant strings are long-lived enough to lead to superdiffusive transport, albeit in the weaker form of logarithmic corrections to diffusion. Similar log-diffusion scalings have recently been observed up to long times in various isotropic spin chains and was interpreted in the framework of the KPZ equation in Ref. [58], though the relation to integrability has remained controversial [66].

We have checked this prediction using MPO simulations by considering uncorrelated noise coupling to either energy density $\vec{S}_n \cdot \vec{S}_{n+1}$ or energy current density $\vec{S}_n \cdot (\vec{S}_{n+1} \times \vec{S}_{n+2})$ (Fig. 1). In both cases, our results are consistent with logarithmic diffusion $D(t) \sim \log t$ for weak enough perturbations. For stronger perturbations, we do observe a trend toward saturation in $D(t)$, indicating that higher-order processes in perturbation theory or nonperturbative processes can eventually restore ordinary diffusion.

Static perturbations.—Finally, we consider static, translation-invariant SU(2)-preserving perturbations. The argument about suppressed matrix elements for large strings still applies, but the main difference with the noisy perturbation considered above is that energy and (quasi) momentum conservation greatly constrains the allowed processes contributing to Γ_s. In particular, single particle-hole processes are not allowed. Because of the dispersion mismatch between different strings, two-particle umklapp scattering processes do not seem capable of relaxing momentum either. Thus, relaxation occurs by scattering processes with three or more of them. While we have little analytic control over such processes, we find numerically that the superdiffusive behavior with time-dependent diffusion constant $D(t) \sim t^{1/3}$ ($z = 3/2$) persists up to all the timescales we access numerically, even for sizable perturbations (Fig. 2). This suggests that the decay rate falls off as $\Gamma_s \sim 1/s^3$ (or faster), so that decay is no longer parametrically faster than screening. The bound $\Gamma_s \leq 1/s^2$ from Goldstone physics continues to hold, but appears not to be saturated in this case. As a result, we find that the anomalous scale $z = 3/2$ is remarkably robust to isotropic integrability-breaking perturbations. We do expect that at long enough times, higher-order processes in perturbation theory will eventually take over and lead to regular

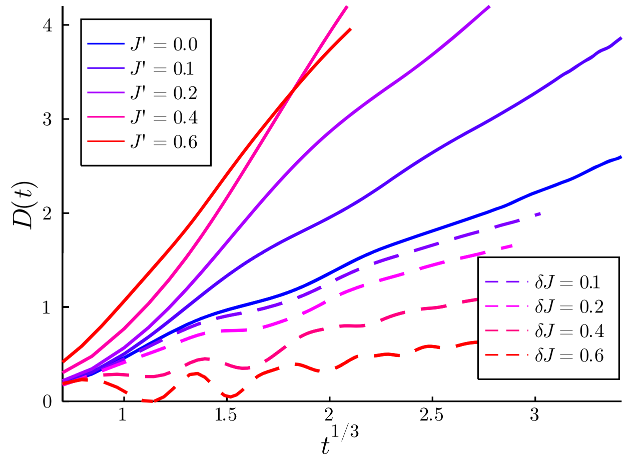

FIG. 2. Static perturbations. Time-dependent diffusion constant $D(t)$ vs $t^{1/3}$ for next-nearest neighbor couplings ($gV = J' \sum_n \vec{S}_n \cdot \vec{S}_{n+2}$, solid lines) and staggered fields [$gV = \delta J \sum_n (-1)^n \vec{S}_n \cdot \vec{S}_{n+1}$, dashed lines] of various strengths. The KPZ behavior $z = 3/2$ seems to persist in all cases.

TABLE I. Perturbative predictions for the ac spin conductivity of perturbed Heisenberg spin chains at intermediate frequencies, depending on whether the SU(2) symmetry is preserved (✓) or broken (✗).

Symmetry	SU(2) ✓	SU(2) ✗
Energy ✓ Crystal momentum ✓	$\sigma(\omega) \sim \omega^{-1/3}$	$\sigma(\omega) \sim \sigma^{\mathrm{dc}}$
Energy ✗ Crystal momentum ✓	$\sigma(\omega) \sim \lvert\log \omega\rvert$	$\sigma(\omega) \sim \sigma^{\mathrm{dc}}$
Crystal momentum ✗ (Disorder)	$\sigma(\omega) \sim \sigma^{\mathrm{dc}}$	$\sigma(\omega) \sim \sigma^{\mathrm{dc}}$

diffusion, though we were not able to access this crossover regime numerically. We refer the reader to the Supplemental Material for additional numerical results for other perturbations breaking either translation invariance (disorder) or SU(2) [78]. Our final results are summarized in Table I.

Discussion.—We have discussed the fate of anomalous spin diffusion in spin chains with non-Abelian symmetries in the presence of weak integrability-breaking perturbations. Since anomalous transport is due to thermally dressed "Goldstone solitons," its fate depends on the symmetries of the integrability-breaking perturbation. When the perturbation breaks the non-Abelian symmetry, the Goldstone solitons are immediately unstable and diffusion immediately sets in. When the perturbation preserves the symmetry, however, it cannot effectively scatter Goldstone solitons, so some form of anomalous diffusion persists. We have argued that this should give rise to a diffusion constant that diverges at least logarithmically at low frequencies and perhaps faster, depending on phase-space constraints. These symmetry considerations directly extend to other Lie-group symmetry. While we expect—and our numerics suggest—that regular diffusion is eventually restored on some long timescale, the dependence of this timescale on the integrability-breaking parameter g lies outside the scope of low-order perturbation theory. Identifying the relevant decay channels and understanding the crossover to regular diffusion is an interesting task for future work. Another interesting question is whether the mechanisms explored here can manifest themselves, e.g., as anomalously large long-time tails in classical hydrodynamics, as suggested in Ref. [66].

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