A Spatio-temporal Analysis of Cellular-based IoT Networks under Heterogeneous Traffic

Praful D. Mankar, Zheng Chen, Mohamed A. Abd-Elmagid, Nikolaos Pappas, and Harpreet S. Dhillon

***Abstract*—In this paper, we consider a cellular-based Internet of things (IoT) network consisting of IoT devices that can communicate directly with each other in a device-to-device (D2D) fashion as well as send real-time status updates about some underlying physical processes observed by them. We assume that such real-time applications are supported by cellular networks where cellular base stations (BSs) collect status updates over time from a subset of the IoT devices in their vicinity. We characterize two performance metrics: i) the network throughput which quantifies the performance of D2D communications, and ii) the Age of Information which quantifies the performance of the real-time IoT-enabled applications. Concrete analytical results are derived using stochastic geometry by modeling the locations of IoT devices as a bipolar Poisson Point Process (PPP) and that of the BSs as another Independent PPP. Our results provide useful design guidelines on the efficient deployment of future IoT networks that will jointly support D2D communications and several cellular network-enabled real-time applications.**

***Index Terms*—Age of Information, cellular networks, device-to-device communication, IoT networks, and stochastic geometry.**

I. Introduction

The emerging paradigm of Internet of Things (IoT) is particularly appealing for enabling a variety of real-time applications involving features such as local decision making and remote monitoring. For example, IoT networks can play a vital role in the efficient detection and management of natural disasters by deploying multiple sensors over disaster-prone areas. In such applications, the sensors may be dual-purpose edge devices communicating with each other over direct links for local processing of data while also communicating with a remote base station (BS) to share updates about the random process that they are monitoring. Motivated by such applications, this paper develops a novel analytical model for IoT networks, which includes both proximity-based device-to-device (D2D) communications and remote monitoring via cellular BSs. From the perspective of D2D communication, it is beneficial to maximize the network throughput. However, ensuring the freshness of information is important for designing the real-time remote monitoring applications, which the conventional metrics, such as data rate and delay, fail to capture. Recently, a new metric, termed *Age of Information* (AoI), was introduced in [2] for accurately quantifying the freshness of information updates received through some random medium such as wireless channels. This work analyzes the AoI performance for the cellular-IoT enabled remote monitoring applications and also investigates its interplay with the achievable data rates for the proximity-based D2D links.

Related work. We utilize the concept of AoI to quantify the freshness of information at the BSs regarding random processes monitored by IoT devices [3]. The authors of [2] characterized temporal mean performance of AoI for various queue disciplines for a single source-destination pair. A series of works then focused on characterizing the temporal mean of AoI or other age-related metrics for different variations of queue disciplines [4]. Another important line of research is to develop AoI-optimal policies in order to effectively utilize the resources in a variety of communication networks including, D2D communications [5], [6], broadcast networks [7], [8], and IoT networks [9]–[15]. The interplay between throughput and AoI was also investigated in [6] and [16] for wireless networks with heterogeneous traffic. While the aforementioned works provide a thorough understanding of the temporal statistics of AoI, they are fundamentally limited in their ability to provide insights about the spatial disparity in the AoI performance that is inherently present in wireless networks. This is primarily because each receiver perceives a different signal and interference environments, which cannot be studied using approaches considered in the above works. Therefore, it is important to accurately model the spatial distribution of wireless devices to analyze the impact of spatial variations on the achievable AoI.

Stochastic geometry tools have received significant attention for modeling the spatial distribution of wireless nodes [17]. Recently, the authors of [18]–[21] applied stochastic geometry tools to perform spatio-temporal analysis of AoI for D2D communication by modeling the locations of D2D pairs using a bipolar Poisson point process (PPP) [18]–[20], and for cellular-based IoT networks by modeling the locations of the IoT devices and BSs using independent PPPs [21]. Specifically, they derived bounds on the spatio-temporal mean AoI [18], the spatio-temporal mean peak AoI [19], [21], and the spatial distribution of the temporal mean peak AoI [20]. In contrast to these analyses that considered AoI as the only performance quantifying metric, this paper presents a joint spatio-temporal analysis of AoI and throughput for cellular-based IoT networks.

Contributions. We present a stochastic geometry-based analysis of cellular-based IoT networks which includes: i) the D2D communications between IoT devices, and ii) the transmission of status updates from the IoT devices to the BSs regarding the random processes they are sensing. We assume that the locations of the IoT devices follow a bipolar PPP and the locations of the BSs follow an independent PPP. For this setup, we first derive the moments of the transmission success probabilities for both the D2D communication and the status update links.

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Next, we characterize the D2D network throughput, and the spatial moments of the temporal mean AoI (which capture the spatial disparity in the AoI performance of the status update links) as a function of maximum allowed BS-device distance. Our numerical results validate the analytical findings and also highlight the impact of maximum allowed BS-device distance on the mean AoI for different system design parameters.

II. System Model

We consider a cellular-based IoT network wherein the IoT devices can exchange messages in a D2D fashion and also send status updates regarding some random processes to their associated BSs. The D2D links of IoT devices are assumed to be randomly distributed according to a bipolar PPP wherein the transmitting IoT devices form a PPP $\Phi_{\rm d}$ with intensity $\lambda_{\rm d}$. Their designated receiving IoT devices are independently located at distance $R_{\rm d}$ in uniformly random directions. The locations of the BSs are also assumed to follow an independent PPP $\Phi_{\rm b}$ with intensity $\lambda_{\rm b}$. The status updates from the IoT devices contain timestamped measurements of independent random processes observed in their vicinity (i.e., each IoT device will have a separate AoI process associated with its measurements). We consider the maximum mean signal strength based cell association policy. Thus, the IoT device is served by its nearest BS. The IoT devices associated with a given BS at $\mathbf{x}\in\Phi_{\rm b}$ lie within its Poisson Voronoi (PV) cell which is

$$V_{\mathbf{x}}=\{\mathbf{y}\in\mathbb{R}^2:\|\mathbf{x}-\mathbf{y}\|\leq\|\mathbf{z}-\mathbf{y}\|,\mathbf{z}\in\Phi_{\rm b}\}.$$

To alleviate interference, we consider that the IoT devices (those are not scheduled for reporting status updates) choose to transmit regular packets in D2D fashion with probability $q_{\rm d}$ in a given time slot. Further, we assume that each BS schedules one device for status updating from its associated IoT devices in a uniformly random fashion across the transmission slots to avoid the collision at the BSs. We consider that the BSs and receiving devices do not have channel state information and the IoT devices transmit the updates and regular messages at fixed rates. Fig. 1 shows a typical realization of the considered system model. Let $\Psi_{\rm b}$ and $\Psi_{\rm d}$ denote the sets of the locations

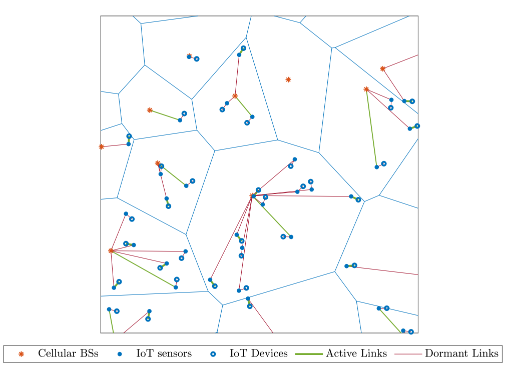

Figure 1. A typical realization of the cellular-based IoT network.

of active IoT devices transmitting status updates and regular messages, respectively, in a given slot. Thus,

$$\Psi_{\rm b}=\{U(V_{\mathbf{x}}\cap\Phi_{\mathbf{d}}):\mathbf{x}\in\Phi_{\rm b}\},\tag{1}$$

where $U(A)$ represents the point selected uniformly at random from the set A. We assume $\lambda_{\rm d}\gg\lambda_{\rm b}$ to avoid $V_{\mathbf{x}}\cap\Phi_{\mathbf{d}}=\emptyset$ for $\forall\mathbf{x}\in\Phi_{\rm b}$. Because of the stationarity of PPP, the points within the set $V_{\mathbf{x}}\cap\Phi_{\rm d}$ are uniformly distributed in $V_{\mathbf{x}}$. Therefore, $\Psi_{\rm b}$ represents the Type I user point process defined in [22]. From Slivnyak's theorem, we know that conditioning on a point of PPP at $\mathbf{x}$ is equivalent to adding $\mathbf{x}$ to the PPP. Therefore, we focus on the analysis of the status update received at the typical BS placed at the origin o of the BS PPP $\Phi_{\rm b}\cup\{o\}$, thus the PV cell V_o represents the typical cell. Let $\mathbf{y}\sim U(V_o)$ denote the location of the typical IoT device scheduled for the status update and let $R_{\rm b}=\|\mathbf{y}\|$ denote its distance from the typical BS placed at o.

Similarly, we perform the D2D communication analysis from the perspective of the typical designated receiving IoT device placed at o by adding a transmitting IoT device at $\mathbf{z}\equiv(R_{\rm d},0)$ (paired with the typical designated receiver) to the PPP $\Phi_{\rm d}$. For the ease of exposition, we focus on the interference-limited scenario. The signal-to-interference ratios (SIRs) at the typical D2D receiver and the typical BS are

$$\mathrm{SIR_d}=\frac{h_oR_{\rm d}^{-\alpha}P_{\rm d}}{I_{\rm d}}\text{ and }\mathrm{SIR_b}=\frac{h_oR_{\rm b}^{-\alpha}P_{\rm b}}{I_{\rm b}},$$

where

$$I_{\rm s}=\sum\nolimits_{\mathbf{x}\in\Phi_{\rm d}}h_{\mathbf{x}}\|\mathbf{x}\|^{-\alpha}[P_{\rm d}\mathbb{1}(\mathbf{x}\in\Psi_{\rm d})+P_{\rm b}\mathbb{1}(\mathbf{x}\in\Psi)]$$

such that $\Psi=\Psi_{\rm b}$ for $\mathrm{s}=\mathrm{d}$ and $\Psi=\tilde{\Psi}_{\rm b}=\Psi_{\rm b}\setminus\{\mathbf{y}\}$ for $\mathrm{s}=\mathrm{b}$ where $\mathbf{y}\in V_o$ is the location of the typical scheduled IoT device. $P_{\rm b}$ and $P_{\rm d}$ represent the fixed power levels for the transmission of a status update and a regular packet, respectively. Note that α denotes the path-loss exponent and $h_{\mathbf{x}}$ denotes the fading coefficient associated with the link from IoT device at $\mathbf{x}$. By assuming independent Rayleigh fading, we model $\{h_{\mathbf{x}}\}$ as independent unit mean exponential random variables. The IoT devices are assumed to transmit the information, containing either regular messages or status updates, in time slotted manner over the same frequency band. This paper characterizes the achievable D2D network throughput and the average AoI of the status updates observed at the typical BS. The AoI of updates received at the BS is defined by the elapsed time from the generation of the latest received status update [2]. Thus, the AoI measured by the BS related to the status updates from device $\mathbf{y}$ at time slot k is

$$A_{\mathbf{y},k}=k-S_{\mathbf{y},k},\tag{2}$$

where $S_{\mathbf{y},k}$ represents the time stamp of the generation of the latest received update until time slot k. We assume that the IoT device always generate a fresh status update just before its transmission. In case of a transmission failure, the update is dropped. As a result, the AoI drops to one whenever a successful transmission occurs.

The scheduling probability of an IoT device in V_o is equal to $\frac{1}{K_{V_o}}$ where K_{V_o} is the number of IoT devices associated with the typical BS placed at o. Since the IoT devices and BSs follow independent PPPs, the probability mass function (`pmf`) of K_{V_o} can be tightly approximated by [23]

$$\mathbb{P}[K_{V_o}=n]=\frac{\Gamma(n+c)}{\Gamma(n+1)\Gamma(c)}\frac{c^c\left(\lambda_{\rm d}/\lambda_{\rm b}\right)^n}{(c+\lambda_{\rm d}/\lambda_{\rm b})^{n+c}},\tag{3}$$

where $c=3.575$ and $\Gamma(\cdot)$ is the gamma function.

III. Success Probability Analysis

In the following, we derive the successful transmission probabilities of the regular D2D messages and status updates.

A. Success probability for the typical D2D link

The success probability of the regular message transmission on D2D link is defined as the probability that SIR is above a threshold $\beta_{\rm d}$. Thus, the success probability for the typical designated D2D receiver can be determined as

$$p_{\rm d} = \mathbb{P}[{\rm SIR}_{\rm d} > \beta_{\rm d}] = \mathbb{E}_{I_{\rm d}}\left[\exp\left(-\beta_{\rm d} R_{\rm d}^{\alpha} I_{\rm d}/P_{\rm d}\right)\right].$$

Recall that the point process $\Psi_{\rm b}$ of IoT devices with active status updates is selected conditioning on the BS PPP $\Phi_{\rm b}$. Thus, we can interpret $\Psi_{\rm b}$ as the dependent thinning of the PPP $\Phi_{\rm d}$ for given $\Phi_{\rm b}$. Despite this dependent thinning, the process of remaining points in $\Phi'_{\rm d} = \Phi_{\rm d} \setminus \Psi_{\rm b}$ can be accurately approximated using homogeneous PPP with density $\lambda'_{\rm d} = \lambda_{\rm d} - \lambda_{\rm b}$ because of the assumption $\lambda_{\rm d} \gg \lambda_{\rm b}$. The exact distribution of $\Psi_{\rm b}$ is difficult to derive because of the dependent thinning. However, one can see that $\Psi_{\rm b}$ is equal in distribution with the point process obtained by the uniformly random displacement of points $\mathbf{x} \in \Phi_{\rm b}$ within their PV cells $V_{\mathbf{x}}$. Therefore, using this argument and the stationarity of $\Phi_{\rm b}$, one can approximate $\Psi_{\rm b}$ with a homogeneous PPP of density $\lambda_{\rm b}$. As we will discuss in Section III-B, handling $\Psi_{\rm b}$ in the analysis of the success probability for the typical BS is a more complicated. This is because of the need to separately characterize the serving link distance and the interference power. Unlike the well-known downlink analysis of a cellular network, where the setup can be simplified using Slivnyak's theorem, we need to develop an appropriate approximation in this case. The point process $\Psi_{\rm d}$ can be directly interpreted as the unconditional thinning of point process $\Phi'_{\rm d}$ with probability $q_{\rm d}$. Thus, $\Psi_{\rm d}$ will also closely resemble PPP with density $q_{\rm d}\lambda'_{\rm d}$. Using the above arguments, the point processes $\Psi_{\rm b}$ and $\Psi_{\rm d}$ can be considered as independent of each other. We can segregate the interference power as $I_{\rm d} = I_{\Psi_{\rm d}} + I_{\Psi_{\rm b}}$ where

$$I_{\Psi_{\rm d}} = \sum\nolimits_{\mathbf{x}\in\Psi_{\rm d}} h_{\mathbf{x}} \|\mathbf{x}\|^{-\alpha} P_{\rm d} \text{ and } I_{\Psi_{\rm b}} = \sum\nolimits_{\mathbf{x}\in\Psi_{\rm b}} h_{\mathbf{x}} \|\mathbf{x}\|^{-\alpha} P_{\rm b}.$$

Since $\Psi_{\rm d}$ and $\Psi_{\rm b}$ are considered to be independent, we can evaluate the success probability as

$$p_{\rm d} = \mathcal{L}_{I_{\Psi_{\rm d}}}(\beta_{\rm d} R_{\rm d}^{\alpha}/P_{\rm d})\mathcal{L}_{I_{\Psi_{\rm b}}}(\beta_{\rm d} R_{\rm d}^{\alpha}/P_{\rm d}), \tag{4}$$

where $\mathcal{L}_X(\cdot)$ is the Laplace transform (LT) of random variable X. The LT of $I_{\Psi_{\rm d}}$ can be determined as

$$\mathcal{L}_{I_{\Psi_{\rm d}}}(s) = \mathbb{E}_{\Psi_{\rm d}} \prod\nolimits_{\mathbf{x}\in\Psi_{\rm d}} \frac{1}{1 + sP_{\rm d}\|\mathbf{x}\|^{-\alpha}},$$

where the above equality follows due to the independent Rayleigh fading. Further, using the probability generating functional (PGFL) of the PPP $\Psi_{\rm d}$, we can obtain [17]

$$\mathcal{L}_{I_{\Psi_{\rm d}}}(s) = \exp\left(- \pi q_{\rm d} \lambda'_{\rm d} \frac{(sP_{\rm d})^{\delta}}{{\rm sinc}(\delta)}\right),$$

where $\delta = \frac{2}{\alpha}$. Similarly, we can obtain $\mathcal{L}_{I_{\Psi_{\rm b}}}(s) = \exp\left(-\pi\lambda_{\rm b}(sP_{\rm d})^{\delta}/{\rm sinc}(\delta)\right)$. The following lemma provides the success probability of regular transmission which we obtained by substituting the LT of both $I_{\Psi_{\rm d}}$ and $I_{\Psi_{\rm b}}$ at $s = \beta_{\rm d} R_{\rm d}^{\alpha}/P_{\rm d}$ in (4).

Lemma 1. *The success probability of the typical D2D link is*

$$p_{\rm d} = \exp\left(- \pi q_{\rm d}\lambda'_{\rm d} \frac{\beta_{\rm d}^{\delta} R_{\rm d}^2}{{\rm sinc}(\delta)} - \pi\lambda_{\rm b} \frac{(\beta_{\rm d} P_{\rm b}/P_{\rm d})^{\delta} R_{\rm d}^2}{{\rm sinc}(\delta)}\right). \tag{5}$$

B. Success probability for the typical BS

The success probability of the status updates is defined as the probability that ${\rm SIR}_{\rm b}$ is above a threshold $\beta_{\rm b}$. Similar to Section III-A, this success probability can be derived by averaging over the space. However, this spatially averaged success probability is not very useful to characterize the performance of non-linear metrics, such as AoI, as it will be evident shortly. For this reason, the distribution of the conditional success probability, termed *meta distribution* [24], is required. Therefore, we derive the moments of meta distribution in the following. Given $\Phi = \Phi_{\rm d} \cup \Phi_{\rm b}$, the conditional success probability of the update from the IoT device at $\mathbf{y} \in V_o$ is

$$p_{\rm b}(\mathbf{y}, \Phi) = \mathbb{P}[{\rm SIR}_{\rm b} > \beta_{\rm b} | \mathbf{y}, \Phi] = \exp\left(-\beta_{\rm b} R_{\rm b}^{\alpha} I_{\rm b}/P_{\rm b}\right).$$

While $\mathbf{y}$ is already included in Φ, we explicitly condition $p_{\rm b}$ on $\mathbf{y}$ to indicate the IoT device at $\mathbf{y}$ is scheduled for the status update. Given Φ, the conditional success probability requires the joint characterization of evolution of $\Psi_{\rm d}$ and $\tilde{\Psi}_{\rm b}$. However, given the complexity of characterizing $\tilde{\Psi}_{\rm b}$ even for a fixed time instance, as presented in [22], we presume that it is difficult to analytically model the joint evolution of $\Psi_{\rm d}$ and $\tilde{\Psi}_{\rm b}$. Therefore, we perform the conditional success probability analysis by considering the interference powers received from the IoT devices belonging to $\Psi_{\rm d}$ and $\tilde{\Psi}_{\rm b}$ separately (independently) for a given serving link distance $R_{\rm b}$. Because of this, the random activities of an IoT device at $\mathbf{x} \in \Phi_{\rm d}$ for transmitting the status updates and the regular messages can be modeled independently of each other.

Let $\tilde{\Phi}_{\rm d} = \Phi_{\rm d} \setminus (\Phi_{\rm d} \cap V_o)$. Recall that each BS is assumed to schedule its associated users uniformly at random in a given slot. This implies that the probability of an IoT device at $\mathbf{x} \in V_{\mathbf{z}}$ transmitting a status update is $q_{\rm b}(V_{\mathbf{z}}) = K_{V_{\mathbf{z}}}^{-1}$ for $\mathbf{z} \in \tilde{\Phi}_{\rm d}$. Since $V_o \sim V_{\mathbf{z}}$, the expected status update activity of a typical IoT device in $\tilde{\Phi}_{\rm d}$ can be obtained as $q_{\rm b} = \mathbb{E}[K_{V_o}^{-1}]$, which can be evaluated using the pmf of K_{V_o} given in (3). The IoT devices, that are not scheduled for status update, are assumed to transmit regular messages with probability $q_{\rm d}$. Hence, to accurately capture this, we consider that the IoT devices in $\Phi_{\rm d}$ transmits regular messages with probability $q'_{\rm d} = q_{\rm d}(1 - q_{\rm b})$. Thus, we can obtain the conditional success probability as

$$p_{\rm b}(\mathbf{y}, \Phi) = \prod\nolimits_{\mathbf{x}\in\tilde{\Phi}_{\rm d}} \left(\frac{q_{\rm b}}{1 + \beta_{\rm b} R_{\rm b}^{\alpha}\|\mathbf{x}\|^{-\alpha}} + 1 - q_{\rm b}\right) \times \prod\nolimits_{\mathbf{x}\in\Phi_{\rm d}} \left(\frac{q'_{\rm d}}{1 + \beta_{\rm b} R_{\rm b}^{\alpha}\|\mathbf{x}\|^{-\alpha}\frac{P_{\rm d}}{P_{\rm b}}} + 1 - q'_{\rm d}\right).$$

The distribution of $R_{\rm b}$ can be accurately approximated as

$$f_{R_{\rm b}}(r) = 2\pi c_2 \lambda_{\rm b} \exp(-\pi c_2 \lambda_{\rm b} r^2), \tag{6}$$

where $c_2 = \frac{9}{7}$ [25]. Since it is difficult to directly derive the meta distribution, we determine its b-th moment as $M_b =$

$$\mathbb{E}_{\mathbf{y},\Phi}[p_{\rm b}(\mathbf{y}, \Phi)^b] = \mathbb{E}_{R_{\rm b}}\Bigg[\underbrace{\mathbb{E}\prod\nolimits_{\mathbf{x}\in\tilde{\Phi}_{\rm d}} \left(1 - \frac{q_{\rm b}}{1 + \beta_{\rm b}^{-1} R_{\rm b}^{-\alpha}\|\mathbf{x}\|^{\alpha}}\right)^b}_{\mathcal{A}(\tilde{\Phi}_{\rm d})}$$

$$\times \underbrace{\mathbb{E}\prod\nolimits_{\mathbf{x}\in\Phi_{\rm d}}\left(1-\frac{q_{\rm d}'}{1+\beta_{\rm b}^{-1}R_{\rm b}^{-\alpha}\|\mathbf{x}\|^{\alpha}\frac{P_{\rm b}}{P_{\rm d}}}\right)^{b}}_{\mathcal{B}(\Phi_{\rm d})}\Bigg]. \quad (7)$$

Using [24, Theorem 1], we obtain $\mathcal{B}(\Phi_{\rm d}) =$

$$\exp\left(-\pi\lambda_{\rm d}\frac{(\beta_{\rm b}P_{\rm d}/P_{\rm b})^{\delta}R_{\rm b}^{2}}{\text{sinc}(\delta)}\sum\nolimits_{n=1}^{\infty}\binom{b}{n}\binom{\delta-1}{n-1}{q_{\rm d}'}^{n}\right).$$

To evaluate $\mathcal{A}(\tilde{\Phi}_{\rm d})$, we require the distribution of $\tilde{\Phi}_{\rm d}$ as seen from the typical BS at o. In [22], the pair correlation function (pcf) for the interferers of point process defined in (1) (i.e. $\tilde{\Psi}_{\rm b}$) with respect to the typical BS placed at o is derived as $g(r) = 1-\exp(-\pi c_1\lambda_{\rm b}r^2)$, where $c_1 = \frac{12}{5}$. Similar to [26], we use this pcf to approximate $\tilde{\Phi}_{\rm d}$ using non-homogeneous PPP with density $\lambda_{\rm d}g(r)$. Thus, using the PGFL of this non-homogeneous PPP, we get $\mathcal{A}(\tilde{\Phi}_{\rm d}) =$

$$\exp\left(-2\pi\lambda_{\rm d}\int_0^{\infty}g(r)\left(1-\left[1-\frac{q_{\rm b}}{1+\beta_{\rm b}^{-1}R_{\rm b}^{-\alpha}r^{\alpha}}\right]^{b}\right)r{\rm d}r\right).$$

Finally, by substituting $\mathcal{A}(\tilde{\Phi}_{\rm d})$ and $\mathcal{B}(\Phi_{\rm d})$ in (7) and then using (6), we obtain the moments of $p_{\rm b}(\mathbf{y},\Phi)$ as given in Lemma 2.

Lemma 2. *The b-th moment of the conditional success probability observed by the typical BS is*

$$M_b = \int_0^{\infty}2\pi c_2\lambda_{\rm b}\exp\left(-\pi c_2\lambda_{\rm b}r^2-\pi\lambda_{\rm d}\mathcal{G}(r,b)\right)r{\rm d}r, \quad (8)$$

$$\text{where } \mathcal{G}(r,b) = r^2\frac{(\beta_{\rm b}P_{\rm d}/P_{\rm b})^{\delta}}{\text{sinc}(\delta)}\sum_{n=1}^{\infty}\binom{b}{n}\binom{\delta-1}{n-1}{q_{\rm d}'}^{n}$$
$$+2\int_0^{\infty}g(v)\left(1-\left[1-\frac{q_{\rm b}}{1+\beta_{\rm b}^{-1}(v/r)^{\alpha}}\right]^{b}\right)v{\rm d}v.$$

IV. Throughput and Average AoI

In this section, we characterize the D2D network throughput and the moments of AoI using results derived in Section III. The network throughput is measured as the average number of successfully delivered information bits per unit area per second per Hertz. The effective probability of an IoT device transmitting the regular messages is $q_{\rm d}' = q_{\rm d}(1-q_{\rm b})$. Thus, using (5), the throughputs of the typical D2D link and the D2D network can be determined as

$$\mathcal{T}_{\rm L} = q_{\rm d}'\log_2(1+\beta_{\rm d})p_{\rm d} \text{ and } \mathcal{T}_{\rm N} = \lambda_{\rm d}\mathcal{T}_{\rm L}$$

Now we derive the moments of (location-dependent) mean AoI for the typical IoT device-BS link. Fig. 2 depicts a representative sample path of the AoI. Let $Y_{\mathbf{y},k}$ and $X_{\mathbf{y},k}$ denote the sum of AoI (i.e., area of shaded region) and the time difference between the successful reception of the k-th and the $(k+1)$-th updates from the device at $\mathbf{y}$. Thus,

$$Y_{\mathbf{y},k} = \sum\nolimits_{k=t_k}^{t_{k+1}}A_{\mathbf{y},k} \text{ and } X_{\mathbf{y},k} = \sum\nolimits_{i=1}^{M_{\mathbf{y}}}T_{\mathbf{y},i}, \quad (9)$$

where $T_{\mathbf{y},i}$ denotes the time between two consecutive scheduling instances and $M_{\mathbf{y}}$ denotes the random number of attempted transmissions between two successfully received status updates. The mean AoI is charaterized here similarly to [27] wherein the authors determine the mean AoI for the case of a single point-to-point link. For N transmissions slots with

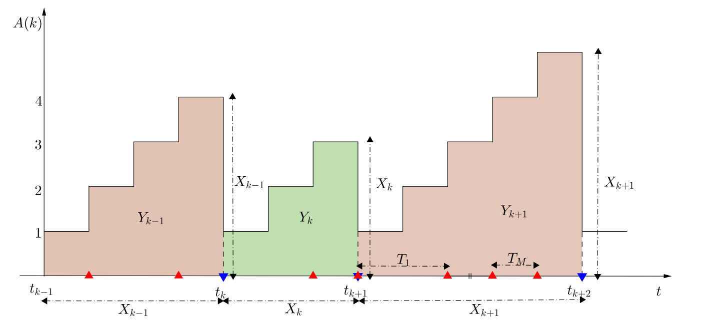

Figure 2. Sample path of $A_{\mathbf{y},k}$. The red upward and blue downward arrows show the transmission attempts and successful transmissions, respectively.

K successful updates, the average AoI for the device at $\mathbf{y}$ is $\Delta_N(\mathbf{y}) =$

$$\frac{1}{N}\sum\nolimits_{k=1}^{N}A_{\mathbf{y},k} = \frac{1}{N}\sum\nolimits_{k=1}^{K}Y_{\mathbf{y},k} = \frac{K}{N}\frac{1}{K}\sum\nolimits_{k=1}^{K}Y_{\mathbf{y},k}.$$

Using $\lim\limits_{N\to\infty}\frac{K}{N} = \frac{1}{\mathbb{E}[X_{\mathbf{y},k}]}$ and $\lim\limits_{K\to\infty}\frac{1}{K}\sum\limits_{k=1}^{K}Y_{\mathbf{y},k} = \mathbb{E}[Y_{\mathbf{y},k}]$, we get $\Delta(\mathbf{y}) = \lim\limits_{N\to\infty}\Delta_N(\mathbf{y}) = \mathbb{E}[Y_{\mathbf{y},k}]/\mathbb{E}[X_{\mathbf{y},k}]$. We obtain the relation between $Y_{\mathbf{y},k}$ and $X_{\mathbf{y},k}$ as

$$Y_{\mathbf{y},k} = \sum\nolimits_{m=1}^{X_{\mathbf{y},k}}m = \frac{1}{2}X_{\mathbf{y},k}(X_{\mathbf{y},k}+1).$$

Finally, we obtain

$$\Delta(\mathbf{y}) = \frac{1}{2}\frac{\mathbb{E}\left[X_{\mathbf{y},k}(X_{\mathbf{y},k}+1)\right]}{\mathbb{E}[X_{\mathbf{y},k}]} = \frac{\mathbb{E}[X_{\mathbf{y},k}^2]}{2\mathbb{E}[X_{\mathbf{y},k}]}+\frac{1}{2}. \quad (10)$$

Thus, the first two moments of $X_{\mathbf{y},k}$ are sufficient to evaluate the mean AoI. However, the distribution of $X_{\mathbf{y},k}$ is not identical for the IoT devices spread across the network for the following reasons. The distribution of $X_{\mathbf{y},k}$ of an IoT device-BS link depends on its scheduling and the success probabilities. In particular, for a given $\Phi = \Phi_{\rm b}\cup\Phi_{\rm d}$ and the IoT device at $\mathbf{y}\in V_o$, the scheduling probability $1/K_{V_o}$ and conditional success probability $p_{\rm b}(\mathbf{y},\Phi)$ characterize the distributions of $T_{\mathbf{y},i}$ and $M_{\mathbf{y}}$, respectively, which determines its mean AoI through $X_{\mathbf{y},k}$. This implies that the mean AoI observed by the IoT at $\mathbf{y}$ is conditioned on the locations of the IoT devices and the BSs. Hence, we refer the the mean AoI of IoT device at $\mathbf{y}$ as the *conditional mean AoI* denoted by $\Delta(\mathbf{y},\Phi)$. Our focus is to derive the distribution of the conditional mean AoI.

Conditional Mean AoI: For a device at $\mathbf{y}$ given Φ, the probability that the attempted transmission is successful is $p_{\rm b}(\mathbf{y},\Phi)$ and the probability that it is scheduled for the status update is $\zeta_{V_o} = 1/K_{V_o}$. Thus, the pmfs of $T_{\mathbf{y},i}$ and $M_{\mathbf{y}}$ become

$$\mathbb{P}[T_{\mathbf{y},i}=t|\Phi] = \zeta_{V_o}[1-\zeta_{V_o}]^{t-1},$$
$$\text{and } \mathbb{P}[M_{\mathbf{y}}=m|\Phi] = p_{\rm b}(\mathbf{y},\Phi)[1-p_{\rm b}(\mathbf{y},\Phi)]^{m-1},$$

for $1\le m,t$, respectively. Since $T_{\mathbf{y},i}$s are independent and identically distributed, we can apply the Wald's identity to (9) and obtain the mean of $X_{\mathbf{y},k}$ as

$$\mathbb{E}[X_{\mathbf{y},k}] = \mathbb{E}[T_{\mathbf{y},i}]\mathbb{E}[M_{\mathbf{y}}] = \zeta_{V_o}^{-1}p_{\rm b}(\mathbf{y},\Phi)^{-1}. \quad (11)$$

From (9), we have

$$X_{\mathbf{y},k}^2 = \sum\nolimits_{i=1}^{M_{\mathbf{y}}}T_{\mathbf{y},i}^2+\sum\nolimits_{i=1}^{M_{\mathbf{y}}}\sum\nolimits_{j=1,j\neq i}^{M_{\mathbf{y}}}T_{\mathbf{y},i}T_{\mathbf{y},j}.$$

Note that $T_{\mathbf{y},i}$ and $T_{\mathbf{y},j}$, for $i\neq j$, are independent because of each BS schedules its associated IoT devices uniformly at random in a given slot. Thus, $\mathbb{E}[X_{\mathbf{y},k}^2|M_{\mathbf{y}}=m]=$

$$m\mathbb{E}[T_{\mathbf{y},i}^2]+m(m-1)\mathbb{E}[T_{\mathbf{y},i}]^2=m1-\zeta_{V_o}\zeta_{V_o}^{-2}+m^2\zeta_{V_o}^{-2}.$$

Now, by averaging over the pmf of $M_{\mathbf{y}}$, we obtain

$$\mathbb{E}[X_{\mathbf{y},k}^2]=(1-\zeta_{V_o})\zeta_{V_o}^{-2}\mathbb{E}[M_{\mathbf{y}}]+\zeta_{V_o}^{-2}\mathbb{E}[M_{\mathbf{y}}^2],$$
$$=\frac{1-\zeta_{V_o}}{\zeta_{V_o}^2}\frac{1}{p_{\rm b}(\mathbf{y},\Phi)}+\frac{1}{\zeta_{V_o}^2}\frac{2-p_{\rm b}(\mathbf{y},\Phi)}{p_{\rm b}(\mathbf{y},\Phi)^2}. \quad (12)$$

Finally, by substituting (11) and (12) into (10), we obtain the conditional mean AoI as given in the following lemma.

Lemma 3. *The conditional mean AoI measured by the typical BS of the updates from the IoT device at $\mathbf{y}\in V_o$ is*

$$\Delta(\mathbf{y},\Phi)=\zeta_{V_o}^{-1}p_{\rm b}(\mathbf{y},\Phi)^{-1}. \quad (13)$$

Moments of conditional mean AoI: It is well-known that the correlation between $p_{\rm b}(\mathbf{y},\Phi)$ and K_{V_o} for given Φ is difficult to capture in the spatio-temporal analysis of cellular networks. Hence, similar to the existing stochastic geometry-based analyses (refer to [28] for an example), we rely on the assumption of independence of these two quantities. Thus, it is apparent from Lemma 3 that the n-th moment of $\Delta(\mathbf{y},\Phi)$ is equal to the product of $(-n)$-th moments $p_{\rm b}(\mathbf{y},\Phi)$ and ζ_{V_o} which can be directly obtained from Lemma 2 and the pmf of K_{V_o} given in (3), respectively. However, note that $\Delta_n=\mathbb{E}[\Delta(\mathbf{y},\Phi)]=\infty$ because $p_{\rm b}(\mathbf{y},\Phi)\to 0$ as $\|\mathbf{y}\|\to\infty$. To tackle this, we evaluate the moments of $\Delta(\mathbf{y},\Phi)$ under the condition of $R_{\rm b}<\mathcal{R}$ in the following theorem.

Theorem 1. *The n-th moment of the conditional mean AoI measured at the typical BS of the status updates generated from the IoT devices within distance $\mathcal{R}$ is $\Delta_n(\mathcal{R})=$*

$$\mathcal{Z}\mathbb{E}_{K_{V_o}}[K_{V_o}^n]\int_0^{\mathcal{R}}\exp\bigg(-\pi c_2\lambda_{\rm b}r^2-\pi\lambda_{\rm d}\mathcal{G}(r,-n)\bigg)r{\rm d}r, \quad (14)$$

where $\mathcal{Z}=\frac{2\pi c_2\lambda_{\rm b}}{1-\exp(-\pi\lambda_{\rm b}c_2\mathcal{R}^2)}$ and $\mathcal{G}(r,-n)$ is given in Lemma 2.

Proof. Using the assumption of independence of ζ_{V_o} and $p_{\rm b}(\mathbf{y},\Phi)$ and Lemma 3, the n-th moment of the conditional mean AoI can be obtained as

$$\Delta_n(\mathcal{R})=\mathbb{E}_{\mathbf{y},\Phi}[\Delta(\mathbf{y},\Phi)^n|R_{\rm b}\leq\mathcal{R}]$$
$$=\mathbb{E}_{K_{V_o}}[K_{V_o}^n]\mathbb{E}_{\mathbf{y},\Phi}[p_{\rm b}(\mathbf{y},\Phi)^{-n}|R_{\rm b}\leq\mathcal{R}].$$

Thus, we arrive at (14) by plugging the $(-n)$-th moment of $p_{\rm b}(\mathbf{y},\Phi)$ from Lemma (2) with the condition $R_{\rm b}\leq\mathcal{R}$. □

V. Numerical Analysis and Discussion

For the numerical analysis, we consider the system parameters as $\lambda_{\rm b}=10^{-4}$ BSs/m^2, $\lambda_{\rm d}=20\lambda_{\rm b}$ devices/m^2, $P_{\rm d}=30$ dBm, $P_{\rm b}=100P_{\rm d}$, $\alpha=4$, $q_{\rm d}=0.1$, and $R_{\rm d}=10$ m, unless mentioned otherwise. First, we verify the success probabilities derived in Section III through simulation results in Fig. 3. The curves correspond to the analytical results whereas the markers correspond to the simulation results. Fig. 3 (left) presents the success probability observed by the typical D2D link and Fig. 3 (right) presents the mean and variance of conditional success probability observed by the typical BS both match closely with the simulation results.

Fig. 4 (left) shows the mean AoI at the typical BS and the throughput of the D2D network. Note that the mean AoI is evaluated for the maximum IoT device-BS link distance $\mathcal{R}=120$ m where $\mathbb{P}[R_{\rm b}\leq\mathcal{R}]\approx 0.99$. The figure shows that both the throughput of D2D network and the mean AoI at the typical BS increase with the increase of density ratio ${\rm R}_\lambda=\frac{\lambda_{\rm d}}{\lambda_{\rm b}}$ which is expected. It is also apparent form the figure that $\Delta_1\to\mathbb{E}[K_{V_o}]\approx{\rm R}_\lambda$ for smaller $\beta_{\rm b}$ (i.e. when the success probability is almost one) and $\Delta_1\to\infty$ for larger values of $\beta_{\rm b}$ (i.e. when the success probability is almost zero). Let $\mathcal{T}_{\rm N}^*=\lambda_{\rm d}\mathcal{T}_{\rm L}^*$ and $\mathcal{T}_{\rm L}^*=\max_{\beta_{\rm d}}\mathcal{T}_{\rm L}$ be the optimal throughputs of the D2D network and typical D2D link, respectively, with respect to $\beta_{\rm d}$. Fig. 4 (middle) shows the behaviour of $\mathcal{T}_{\rm N}^*$ and $\mathcal{T}_{\rm L}^*$ with respect to the density ratio ${\rm R}_\lambda$. $\mathcal{T}_{\rm N}^*$ initially increases with ${\rm R}_\lambda$ (due to the increase in transmission of D2D links) and then start to decrease with further increase of ${\rm R}_\lambda$ (due to severe interference from D2D links).

Fig. 4 (right) depicts the mean AoI observed by the typical BS from the IoT devices within distance $\mathcal{R}$. The mean AoI increases with $\mathcal{R}$ because it includes more IoT devices with lower success probabilities. In addition, the mean AoI also increases with both $\beta_{\rm b}$ and ${\rm R}_\lambda$ since these two parameters negatively affect the success probability of the status update. *This conditional mean AoI is important from the perspective of devising a strategy for the collection of status updates such that the mean AoI remains below some threshold.* For instance, the figure indicates that updating the status from the IoT devices within distance 66 meters from the BSs (which are around 83% of total IoT devices) can bound the mean AoI below 100 slots when $\beta_{\rm b}=3$ dB and ${\rm R}_\lambda=20$. These numerical results provide useful insights for maximization of the D2D network throughput such that the mean AoI is below a limit.

VI. Conclusion

This paper considered a cellular-based IoT network where the IoT devices can communicate with each other over direct D2D links and also send status updates to BSs. We characterize the performance of these D2D and status update links in terms of throughput and average AoI, respectively. Using tools from stochastic geometry, we derived the success probability for the D2D link and the moments of the conditional success probability of the status updates link. Then, using these results, we obtained the achievable D2D network throughput and the moments of the conditional mean AoI. Our numerical results demonstrated that limiting the IoT device-BS link distance is necessary to keep the mean AoI below a threshold. They also revealed the impact of the ratio of densities of IoT devices and BSs on the network throughput and average AoI.

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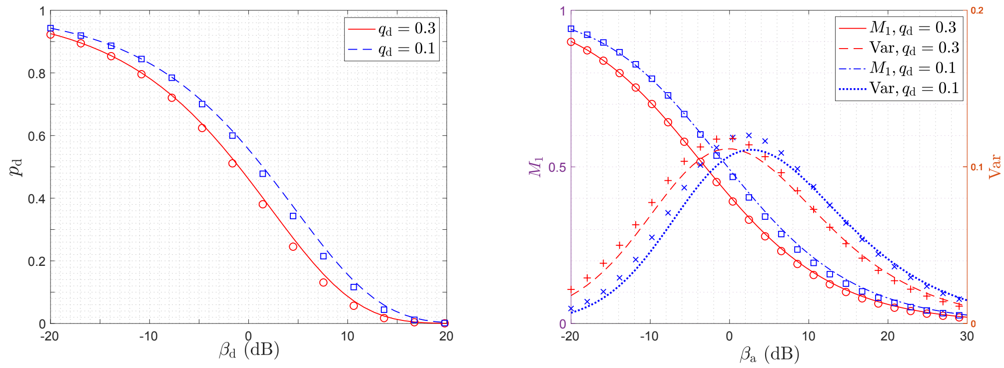

Figure 3. Left: success probability of regular message. Right: mean and variance of conditional success probability of status update. The curves correspond to the analytical results whereas the markers correspond to the simulation results.

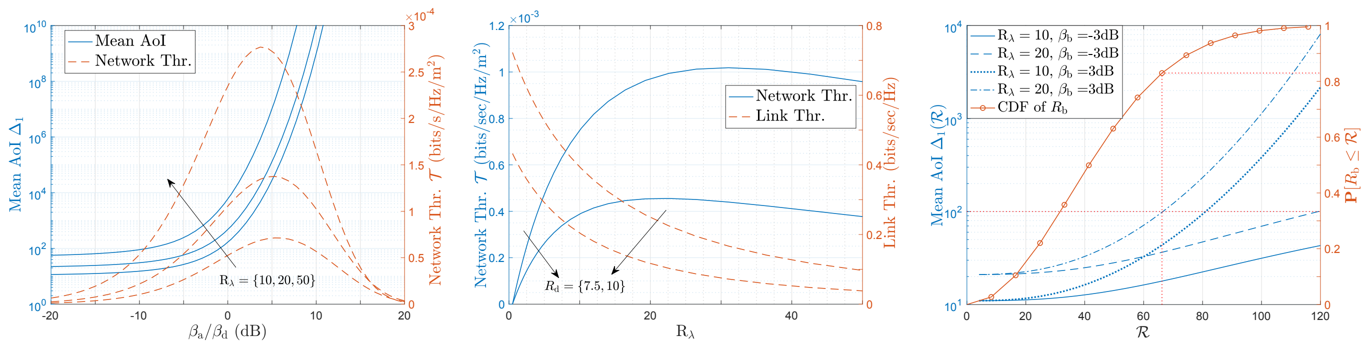

Figure 4. Left: mean AoI and D2D network throughput. Middle: optimal D2D network throughput for $q_{\rm d} = 0.5$. Right: mean AoI conditioned on the BS-IoT device link distance $R_{\rm b} < \mathcal{R}$ for $q_{\rm d} = 0.1$.

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