

Spectrum of tensor perturbations in warm inflation

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We compute the spectrum of tensor perturbations in warm inflation. We find that the spectrum, besides the standard component $\propto H^2/M_p^2$ associated to the amplification of the tensor vacuum fluctuations, acquires a component $\propto \ell_{\text{mfp}} T^5/M_p^4$, where ℓ_{mfp} and T are, respectively, the mean free path and the temperature of the thermal degrees of freedom. The new contribution is due to the direct production of gravitational waves by the thermal bath and can exceed the standard one in a viable region of parameter space. This contribution is dominated by thermal fluctuations at scales longer than ℓ_{mfp} .

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I. INTRODUCTION

In models of warm inflation [1,2], the inflaton interacts with a thermal bath of relativistic particles with a slowly evolving temperature T . In order to prevent the temperature from redshifting away, the thermal bath must be continuously replenished by some interaction with the inflaton—the form of interactions being model dependent. The spectrum of metric scalar perturbations in warm inflation has been studied in several works, see, e.g., [1–4], and its expression depends on the specific form of the interaction between the thermal bath and the inflaton. The tensor perturbations will see, as usual, their vacuum fluctuations amplified by the accelerated expansion, which will lead to a contribution to their power spectrum with amplitude $\mathcal{P}'_{\text{vac}} = \frac{2}{\pi^2} \frac{H^2}{M_p^2}$. The thermal bath will provide an additional source of tensors. In this work, we compute this contribution.

Since the interaction of gravitational waves with the thermal bath depends only on the properties of the latter, our results do not depend on the specifics of the inflaton sector. They do however depend on the strength of the interactions that maintain the thermal bath in equilibrium.

Besides the Hubble parameter H and the temperature T , a relevant scale for our system is given by the mean free path ℓ_{mfp} of the particles in the thermal bath. For thermal inflation to be at work, the hierarchy $T \gtrsim \ell_{\text{mfp}}^{-1} \gg H$ must be realized. The first inequality derives from the fact that one cannot define a mean free path shorter than the thermal wavelength. The second is equivalent to the requirement of thermal equilibrium in an expanding Universe. In Sec. III, we compute the contribution to the tensor spectrum from

modes at length scales much shorter than ℓ_{mfp} , whereas in Sec. IV, we compute the contributions from larger scales, that give the dominant effect.

II. THE SOURCED TENSOR SPECTRUM

We work in conformal time and consider only transverse-traceless perturbations $h_{ij}(\mathbf{x}, \tau)$ around a flat Friedmann-Robertson-Walker background $ds^2 = a(\tau)^2[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$. We approximate the inflating Universe with a de Sitter space, $a(\tau) = -1/(H\tau)$. Then, in the presence of a stress-energy tensor $T_{ab}(\mathbf{x}, \tau)$, that we assume to be generated by a bath of relativistic particles, the tensor fluctuations satisfy the equation

$$h''_{ij}(\mathbf{x}, \tau) + 2\frac{a'}{a}h'_{ij}(\mathbf{x}, \tau) - \Delta h_{ij}(\mathbf{x}, \tau) = \frac{2}{M_p^2}\Pi_{ij}{}^{ab}(\partial_{\mathbf{x}})T_{ab}(\mathbf{x}, \tau), \quad (1)$$

where $\Pi_{ij}{}^{ab}(\partial_{\mathbf{x}}) = \Pi_i^a(\partial_{\mathbf{x}})\Pi_j^b(\partial_{\mathbf{x}}) - \frac{1}{2}\Pi_{ij}(\partial_{\mathbf{x}})\Pi^{ab}(\partial_{\mathbf{x}})$ is the projector on the transverse-traceless modes, with $\Pi_{ij}(\partial_{\mathbf{x}}) = \delta_{ij} - \partial_i\partial_j/\Delta$, while a prime denotes a derivative with respect to the conformal time τ . The stress-energy tensor is defined in such a way that $T_{ab} \sim \partial_a\phi\partial_b\phi + \dots$ for a scalar field whose kinetic term is normalized as $\int d\tau d^3\mathbf{x} \frac{a^2}{2}\dot{\phi}^2$. A transformation to canonically normalized fields brings $T_{ab} \rightarrow \frac{1}{a^2}T_{ab}^{(c)}$, where the index superscript (c) refers to comoving quantities. Note that Eq. (1) does not assume thermalization of the gravitational waves. This possibility has been considered in [5], where it was shown that such a situation cannot be achieved consistently in warm inflation.

After taking the Fourier transform of Eq. (1) and solving it in terms of the Green's function $G_p(\tau, \tau')$, we obtain the correlator

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$$\begin{aligned} \langle h_{ij}(\mathbf{p}, \tau) h_{ij}(\mathbf{p}', \tau) \rangle_s &= \frac{4}{M_P^4} \int^\tau \frac{d\tau'}{a(\tau')^2} \int^\tau \frac{d\tau''}{a(\tau'')^2} \times G_p(\tau, \tau') G_{p'}(\tau, \tau'') \Pi_{ij}^{ab}(-i\mathbf{p}) \Pi_{ij}^{cd}(-i\mathbf{p}') \\ &\times \int \frac{d^3\mathbf{x} d^3\mathbf{x}'}{(2\pi)^3} e^{-i\mathbf{p}\mathbf{x} - i\mathbf{p}'\mathbf{x}'} \langle T_{ab}^{(c)}(\mathbf{x}, \tau') T_{cd}^{(c)}(\mathbf{x}', \tau'') \rangle, \end{aligned} \quad (2)$$

where $\langle \dots \rangle_s$ refers to the component of the correlator sourced by the thermal bath and where the propagator, in the approximation of exact de Sitter background, reads

$$G_p(\tau, \tau') = \frac{1}{p^3 \tau^2} [(1 + p^2 \tau \tau') \sin(p(\tau - \tau')) - (p(\tau - \tau')) \cos(p(\tau - \tau'))] \Theta(\tau - \tau'). \quad (3)$$

In what follows, we consider the tensor spectrum evaluated at the end of inflation, $\tau = -1/H$, at large scales $p \ll H$, so that we set $\tau = 0$ in the propagator.

III. CONTRIBUTION FROM SHORT WAVELENGTH MODES

Let us start by computing the contribution to the graviton two point function from the stress-energy correlators when both comoving distances and (conformal) time differences are much shorter than the comoving mean free path $\ell_{\text{mfp}}^{(c)}$. In this regime, we can neglect the effects of interactions and treat our theory as that of a free field.

For definiteness, we assume that our system is given by a conformally coupled, canonically normalized massless scalar field φ in thermal equilibrium at comoving temperature $T^{(c)}$. As a consequence, the stress-energy tensor correlator appearing in Eq. (2) takes the form

$$\begin{aligned} \langle T_{ab}^{(c)}(\mathbf{x}, \tau') T_{cd}^{(c)}(\mathbf{x}', \tau'') \rangle &= \partial_{y_1^a} \partial_{y_2^b} \partial_{y_3^c} \partial_{y_4^d} [\langle \varphi(\mathbf{y}_1, \tau') \varphi(\mathbf{y}_2, \tau') \varphi(\mathbf{y}_3, \tau'') \varphi(\mathbf{y}_4, \tau'') \rangle \\ &- \langle \varphi(\mathbf{y}_1, \tau') \varphi(\mathbf{y}_2, \tau') \rangle \langle \varphi(\mathbf{y}_3, \tau'') \varphi(\mathbf{y}_4, \tau'') \rangle] |_{\mathbf{y}_1=\mathbf{y}_2=\mathbf{x}, \mathbf{y}_3=\mathbf{y}_4=\mathbf{x}'}, \end{aligned} \quad (4)$$

where we ignored the part of the stress-energy tensor proportional to δ_{ab} that is projected out by $\Pi_{ij}^{ab}(\partial_{\mathbf{x}})$.

To compute $\langle \varphi(\mathbf{y}_1, \tau') \varphi(\mathbf{y}_2, \tau') \varphi(\mathbf{y}_3, \tau'') \varphi(\mathbf{y}_4, \tau'') \rangle$ in a thermal state, we Wick rotate to Euclidean spacetime with periodic imaginary (conformal) time, $\varphi(i\tau + 1/T^{(c)}) = \varphi(i\tau)$, and we use Wick's theorem to decompose the four-point correlator into products of thermal Green's functions. The thermal Green's function at comoving temperature $T^{(c)}$, in terms of the Euclidean conformal time $\tau_E = i\tau$, reads

$$\begin{aligned} G_T(x, \tau_E) &= -T^{(c)} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \frac{e^{2\pi i n T^{(c)} \tau_E + i\mathbf{k}\cdot\mathbf{x}}}{(2\pi n T^{(c)})^2 + \mathbf{k}^2} \\ &= -\frac{T^{(c)}}{4\pi x} \frac{\sinh(2\pi T^{(c)} x)}{\cosh(2\pi T^{(c)} x) - \cos(2\pi T^{(c)} \tau_E)}, \end{aligned} \quad (5)$$

that, rotating back to real conformal time, turns into [6]

$$G_T(x, \tau) = -\frac{T^{(c)}}{4\pi x} \frac{\sinh(2\pi T^{(c)} x)}{\cosh(2\pi T^{(c)} x) - \cosh(2\pi T^{(c)} \tau)}. \quad (6)$$

Note that in the limit $T^{(c)} \rightarrow 0$, we obtain the Minkowskian Green's function for a massless field,

$$G_0(x, \tau) = -\frac{1}{4\pi^2} \frac{1}{x^2 - \tau^2}. \quad (7)$$

To renormalize away the effects of the zero temperature fluctuations of φ , we work with the subtracted Green's function

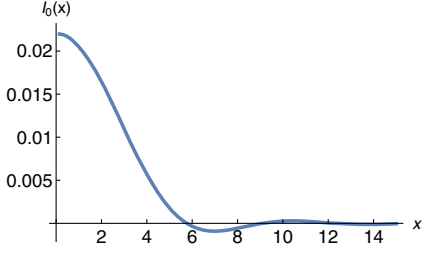
$$G_T^{\text{sub}}(x, \tau) = G_T(x, \tau) - G_0(x, \tau). \quad (8)$$

We are now in position to compute

$$\langle T_{ab}^{(c)}(\mathbf{x}, \tau) T_{cd}^{(c)}(\mathbf{0}, 0) \rangle = 2\hat{\mathbf{x}}_a \hat{\mathbf{x}}_b \hat{\mathbf{x}}_c \hat{\mathbf{x}}_d G_{T,xx}^{\text{sub}}(x, \tau' - \tau'')^2, \quad (9)$$

where we denote $G_{T,xx}^{\text{sub}}(x, \tau) = \partial_x^2 G_T^{\text{sub}}(x, \tau)$ and where a hat denotes a vector with unit length. We thus obtain

$$\langle h_{ij}(\mathbf{p}, 0) h^{ij}(\mathbf{p}', 0) \rangle_{s, \text{short}} = \frac{4}{M_P^4} \delta(\mathbf{p} + \mathbf{p}') \times \int \frac{d\tau'}{a(\tau')^2} \int \frac{d\tau''}{a(\tau'')^2} G_p(0, \tau') G_{p'}(0, \tau'') \mathcal{I}(p, \tau' - \tau''), \quad (10)$$

FIG. 1. The function $\mathcal{I}_0(x)$, defined in Eq. (12).

where we have defined

$$\mathcal{I}(p, \Delta\tau) \equiv 2\Pi_{ij}^{ab}(-i\mathbf{p})\Pi^{ijcd}(-i\mathbf{p}) \times \int d^3\mathbf{x} e^{-i\mathbf{p}\cdot\mathbf{x}} \hat{\mathbf{x}}_a \hat{\mathbf{x}}_b \hat{\mathbf{x}}_c \hat{\mathbf{x}}_d G_{T,xx}^{\text{sub}}(x, \Delta\tau)^2. \quad (11)$$

Since here we are considering only the short-distance modes, the upper limit of integration in $d\mathbf{x}$ in the integral above is given by $\approx \ell_{\text{mfp}}^{(c)}$, but, since $G_T^{\text{sub}}(x, \tau) \rightarrow 0$ for $2\pi T^{(c)}x \gtrsim 1$, we can approximate it by infinity assuming $2\pi T^{(c)}\ell_{\text{mfp}}^{(c)} \gg 1$.

Numerical evaluation then gives that for $p \ll 2\pi T^{(c)}$,

$$\mathcal{I}(p, \Delta\tau) \simeq \mathcal{I}_0(p|\Delta\tau|)T^{(c)5}, \quad (12)$$

where the function $\mathcal{I}_0(x)$ is plotted in Fig. 1. The modes with $p \gtrsim 2\pi T^{(c)}$ are suppressed and irrelevant.

The comoving temperature $T^{(c)}$ appearing in Eq. (12) is time dependent, as it is given by aT , where the physical temperature T is approximately constant during warm inflation. This raises the question of whether $T^{(c)}$ should be evaluated at time τ' or at time τ'' . The fact that we are considering short-distance modes helps us here. In fact, for those modes $|\tau' - \tau''| \lesssim \ell_{\text{mfp}}^{(c)} = -\tau'(\ell_{\text{mfp}}H)$ and since thermalization requires $(\ell_{\text{mfp}}H) \ll 1$, we have $|\tau' - \tau''| \ll |\tau'| \simeq |\tau''|$ in our integral. As a consequence, the short wavelength contribution to the graviton correlator will be confined to the region of integration with $\tau' \simeq \tau''$, and it makes no difference whether $T^{(c)}$ is evaluated at τ' or τ'' . To keep things symmetric, we assume $T^{(c)} = \sqrt{a(\tau')a(\tau'')}T$ inside the integral.

The condition $|\tau' - \tau''| \lesssim \ell_{\text{mfp}}^{(c)}$ also helps to simplify the next step. Since the propagators multiplied by the factor $T^{(c)5} = a(\tau')^{5/2}a(\tau'')^{5/2}T^5$ give suppressed contribution unless $|p\tau'| \approx |p\tau''| = \mathcal{O}(1)$, we obtain that $p|\tau' - \tau''| \lesssim |p\tau'|(\ell_{\text{mfp}}H) = \mathcal{O}(\ell_{\text{mfp}}H) \ll 1$, so that we can approximate

$$\mathcal{I}(p, \Delta\tau) \approx \begin{cases} \mathcal{I}_0(0)T^{(c)5} \simeq .02T^{(c)5}, & p|\Delta\tau| \lesssim (\ell_{\text{mfp}}H) \\ 0, & p|\Delta\tau| \gtrsim (\ell_{\text{mfp}}H). \end{cases} \quad (13)$$

We finally find the approximate result

$$\begin{aligned} & \int^\tau \frac{d\tau'}{a(\tau')^2} \int^\tau \frac{d\tau''}{a(\tau'')^2} G_p(\tau, \tau') G_{p'}(\tau, \tau'') \mathcal{I}(p, \tau' - \tau'') \\ & \approx \frac{(\ell_{\text{mfp}}H)}{p} \int^\tau \frac{d\tau'}{a(\tau')^4} G_p(\tau, \tau')^2 \times .02T^{(c)5} \\ & = \frac{5 \times 10^{-3}}{p^3} \ell_{\text{mfp}} T^5. \end{aligned} \quad (14)$$

Introducing the tensor power spectrum $\mathcal{P}^t$ through $\langle h_{ij}(\mathbf{p}, \tau) h_{ij}(\mathbf{p}', \tau) \rangle = \frac{2\pi^2}{p^3} \delta^{(3)}(\mathbf{p} + \mathbf{p}') \mathcal{P}^t(p)$, we finally obtain

$$\mathcal{P}_{s,\text{short}}^t(p) \approx 10^{-3} \frac{\ell_{\text{mfp}} T^5}{M_P^4}. \quad (15)$$

We now consider the contribution from hydrodynamic modes with wavelengths larger than the mean free path, and we find that they give the dominant contribution to the sourced correlator.

IV. CONTRIBUTION FROM HYDRODYNAMIC MODES

In the hydrodynamic regime (in which either distances or time differences are larger than the mean free path of the particles), we can apply a treatment analogous to that used in [7,8] for the case of a radiation-dominated Universe. We start from the relation [9]

$$\langle T_{ab}^{(c)}(\mathbf{x}, \tau) T_{cd}^{(c)}(\mathbf{x}', \tau') \rangle = 2T^{(c)} \left[\eta^{(c)} (\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) + \left(\zeta^{(c)} - \frac{2}{3}\eta^{(c)} \right) \delta_{ab}\delta_{cd} \right] \delta(\mathbf{x} - \mathbf{x}') \delta(\tau - \tau'), \quad (16)$$

where $\eta^{(c)}$ and $\zeta^{(c)}$ are, respectively, the comoving shear and the bulk viscosity. Inserting the expression above into Eq. (2), we obtain

$$\mathcal{P}_{s,\text{long}}^t(p) = \frac{24p^3}{\pi^2 M_P^4} \int \frac{d\tau'}{a(\tau')^4} G_p(0, \tau')^2 T^{(c)}(\tau') \eta^{(c)}(\tau'), \quad (17)$$

where we used $\Pi_{ij}^{ab}(-i\mathbf{p})\Pi_{ij}^{ab}(-i\mathbf{p}) = 3$.

Equation (17) is our main result. To proceed, we need to specify the expression of $\eta^{(c)}$ that depends on the details of the interactions within the thermal bath.

The shear viscosity can take values between two limits.

A lower bound on $\eta^{(c)}$ is conjectured [10] to be

$$\eta^{(c)} \geq \frac{s^{(c)}}{4\pi}, \quad (18)$$

where $s^{(c)} = \frac{2\pi^2}{45} g_{*,S} T^{(c)3}$ is the comoving entropy density of the thermal gas, with $g_{*,S}$ denoting the effective number of degrees of freedom in entropy. Applying the inequality (18), we obtain

$$\mathcal{P}'_{s,\text{long}}(p) \geq \frac{4}{15\pi M_P^4} g_{*,S} T^4 p^3 \int d\tau' G_p(0, \tau')^2, \quad (19)$$

where we have assumed that the physical temperature $T = T^{(c)}/a$ is approximately constant. Evaluation of the integral in $d\tau'$ gives

$$\mathcal{P}'_{s,\text{long}}(p) \geq \frac{2}{45M_P^4} g_{*,S} T^4 \simeq \frac{4}{3\pi^2} \frac{\rho_r}{M_P^4}, \quad (20)$$

where in the last step we have introduced the energy density in the radiation, $\rho_r = \frac{\pi^2}{30} g_* T^4$ assuming $g_* \simeq g_{*,S}$.

Since by assumption the radiation must be subdominant with respect to the inflaton energy, $\rho_r \ll 3H^2 M_P^2$, Eq. (20) shows that if the inequality (18) is saturated, $\mathcal{P}'_{s,\text{long}} \ll \mathcal{P}'_{\text{vac}} \equiv \frac{2}{\pi^2} \frac{H^2}{M_P^2}$.

An upper bound on $\mathcal{P}'_{s,\text{long}}$ is induced by an upper bound on $\eta^{(c)}$. The shear viscosity is approximately given by

$$\eta^{(c)} \approx \ell_{\text{mfp}}^{(c)} T^{(c)4}. \quad (21)$$

Imposing that the mean free path is much shorter than the horizon radius $\ell_{\text{mfp}}^{(c)} \ll (aH)^{-1}$, we obtain the upper bound

$$\mathcal{P}'_{s,\text{long}}(p) \ll \frac{p^3}{M_P^4} \int_0^\tau \frac{d\tau'}{a(\tau')^4} G_p(\tau, \tau')^2 \frac{T^{(c)}(\tau')^5}{a(\tau')H} \simeq \frac{T^5}{HM_P^4}, \quad (22)$$

that, for relatively large values of the temperature, can exceed $\mathcal{P}'_{\text{vac}}$ even in a regime in which the energy density in radiation is subdominant with respect to that in the background, $T \ll \sqrt{HM_P}$.

A. An example

To work out a specific example, let us consider a model where the thermal bath is given by a real scalar field φ with negligible mass and with self-interaction $V(\varphi) = \frac{\lambda}{4!} \varphi^4$.

The shear viscosity for this model, in the $\lambda \ll 1$ limit, was computed in [11], where it was found that $\eta^{(c)} \simeq 2860 T^{(c)3} / \lambda^2$. The mean free path is given by $[\ell_{\text{mfp}}^{(c)}]^{-1} = \sigma^{(c)} n^{(c)}$, where for a relativistic boson the comoving number density reads $n^{(c)} = \frac{\zeta(3)}{\pi^2} T^{(c)3}$, and the

cross section is $\sigma^{(c)} = \frac{\lambda^2}{32\pi^2 s_{\text{Man}}^{(c)}} \simeq \frac{\lambda^2}{128\pi^2 T^{(c)2}}$ [using the comoving Mandelstam invariant $s_{\text{Man}}^{(c)} \simeq (2T^{(c)})^2$]. Using these formulas, we obtain

$$\eta^{(c)} \simeq 2860 T^{(c)3} \times \frac{\zeta(3)}{128\pi^4} \ell_{\text{mfp}}^{(c)} T^{(c)} \simeq .2 \ell_{\text{mfp}}^{(c)} T^{(c)4}, \quad (23)$$

and going back to physical quantities, we finally obtain

$$\mathcal{P}'_{s,\text{long}}(p) \simeq .3 \frac{\ell_{\text{mfp}} T^5}{M_P^4}, \quad (24)$$

where thermalization requires the model-dependent quantity $\ell_{\text{mfp}} \ll 1/H$. Comparison of the amplitude of Eq. (24) with that of Eq. (15) shows that the hydrodynamic modes dominate the sourced component of the tensor spectrum.

If, to fix ideas, we set $\ell_{\text{mfp}} H \simeq .2$, we see that a tensor spectrum as large as $\sim 10^{-10}$ (that saturates the current observational bounds) can be obtained for temperatures $T \simeq 10^{13} (H/\text{GeV})^{1/5} \text{ GeV}$, where the condition that the radiation density is subdominant by a factor of at least 5 with respect to the background inflaton energy $\simeq 3H^2 M_P^2$ allows for a Hubble parameter during inflation as low as $\sim 2 \times 10^{12} \text{ GeV}$. For such a value of the Hubble parameter, one gets $\mathcal{P}'_{\text{vac}} \simeq 10^{-12}$. For this choice of parameters, therefore, the presence of the thermal bath enhances the tensor spectrum by about 2 orders of magnitude. This amplitude can be reduced by the effects of damping by anisotropic stress [12], whose effect is expected to be of the order of $T^4 / (H^2 M_P^2)$.

To sum up, we have found that the spectrum of gravitational waves generated during thermal inflation includes a component $\propto \ell_{\text{mfp}} T^5 / M_P^4$, sourced by long wavelength thermal modes, that can dominate over the vacuum component in a viable region of parameter space.

Our analysis has been agnostic regarding perturbations in the scalar sector that depend on the details of the interactions between the thermal bath and the inflaton. For this reason, in particular, we give no expression of the amplitude of the tensor-to-scalar ratio (note however that [13] discussed how mechanisms sourcing tensor modes will generally source scalar perturbations with higher efficiency). While, as we noted in the Introduction, the expression of the tensor spectrum does depend only on the details of the interaction among particles in the thermal bath, this quantity is also influenced by the interaction between those particles and the inflaton, which in turn affects the efficiency of the mechanism leading to warm inflation. In fact, cross sections (and therefore the shear viscosity) in the warm sector will get a contribution by processes where thermal particles are exchanging virtual quanta of the inflaton that are controlled by the same parameters that control the dissipation of inflaton energy into thermal degrees of freedom. Given the model

dependence of the dissipation mechanisms, an analysis of this process should be performed case by case.

A crucial prediction of standard inflation is a slightly red spectrum of tensor modes. One can see that this is not the case for warm inflation. For instance, for the simple example with quartic self-interaction described above, we get $\mathcal{P}_s' \propto T^4/(\lambda^2 M_P^4)$ that can lead to a blue tensor spectrum if the temperature T increases during inflation. This is not impossible and is even generally expected as dissipation (and therefore the temperature of the bath) is typically an increasing function of the inflaton velocity that

typically increases during inflation. In particular, this mechanism might lead to large amplitudes of tensor modes towards the end of inflation that might be detectable by gravitational interferometers, as discussed, for instance, in [14].

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